Theory of Andreev reflection in a junction with a strongly disordered semiconductor

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We study the conduction of a normal-metal–semiconductor–superconductor junction, where the semiconductor is strongly disordered. The differential conductance dI/dV of this structure is predicted to have a sharp peak at $V=0$. Unlike the case of a weakly disordered system, this feature persists even in the absence of an additional (Schottky) barrier on the boundary. The zero-bias conductance of such a junction is smaller only by a numerical factor than the conductance in the normal state G_N. Implications for experiments on gated heterostructures with superconducting leads are discussed.

Since the seminal work of Andreev[1] on the theory of electron transport through an ideal interface between a normal (N) metal and a superconductor (S), significant efforts were undertaken to understand the transport in real NS junctions. It was shown[2] that a barrier at the N-S interface reduces strongly the conductance of the boundary between a clean normal metal and a superconductor. Later, experiments[3] with semiconductor (Sm) -superconductor junctions revealed in the differential conductance $G_{NS}(V)$ a broad maximum at zero bias. This feature was explained[4] as an interference effect due to the scattering off the Schottky barrier and of the imperfections in the semiconductor. Recent technological advances have resulted in fabrication of low-resistance contacts between a two-dimensional electron gas (2DEG) and a superconducting lead.[5] Because of the absence of a Schottky barrier at the interface, the subgap conductance is determined by the propagation of electron pairs through the 2DEG itself rather than by two-electron tunneling at the interface, and there is no peak[6] in the differential conductance at zero bias.

The advantage of a gated heterostructure lies in the controllable level of carrier density in the 2DEG. Depending on the density, the 2DEG may behave as a good conductor or as an insulator with an adjustable localization length. Whereas the former case has been extensively studied both theoretically and experimentally,[6] the latter case has received no attention as of yet. In this paper we study two-electron transport through a disordered insulator. We will show that the zero-bias conductance of the N-Sm-S junction differs from the conductance of the same structure in the normal state by a numerical factor only. This is similar to the properties exhibited in the metallic regime. However, the differential conductance $G_{NS}(V)$ drops abruptly with increasing voltage, in contrast to the behavior in the metallic regime in the absence of the Schottky barrier. The development of this feature of the I-V characteristic under the progressive depletion constitutes the signature of the crossover between the metallic and insulating regimes.

Deep in the insulating regime, the conductance of the normal (N-Sm-N) structure is dominated by tunneling via those configurations of localized states in the semiconductor layer that facilitate resonant transmission of electrons.[7] An example of such a configuration is a state with energy close to the Fermi level and location symmetric with respect to the leads. The transmission coefficient for an electron tunneling through a resonant configuration is close to unity, and the conductance is proportional to the probability w of finding such a configuration. This probability scales exponentially with the length of the Sm region, L.

The zero-bias conductance of the N-Sm-S junction, $G_{NS}(0)$, is determined by the tunneling of pairs of electrons at the Fermi level.[1] Clearly, these tunneling processes are also facilitated by the same resonant configurations that control single-electron transport. Thus, the conductance $G_{NS}(0)$ is also proportional to w and, therefore, it has the same exponential dependence upon L as the does normal conductance. If a finite bias eV is applied to the junction, the energies ϵ_1, ϵ_2 of the two electrons in the pair are different: $\epsilon_1 - \epsilon_2 = 2eV$. If eV exceeds the width of the resonant level with respect to tunneling, this level cannot provide a large tunneling coefficient for both electrons. It results in a sharp drop of the conductance with voltage.

Following Ref. 8, we model transport through the depleted region as resonant tunneling via isolated localized states (impurities). We will show that for a wide range of lengths L, it suffices to consider the single-impurity configurations only. In order to calculate the conductance, we first calculate the contribution to the conductance due to tunneling through a single impurity $g_{NS}(eV)$ and then sum these partial conductances over all the impurities. Each localized state is characterized by its energy ϵ_j and by the level widths $\Gamma_{l(r)}^{(j)}$ due to the decay into the left (right) lead, see Fig. 1(a). The amplitude of electron transmission through the barrier via the resonant state, $t_{(j)}(\epsilon)$, and the amplitude of reflection, $r_{(j)}(\epsilon)$, at energy ϵ are given by the single-channel Breit-Wigner formula:

$$r_{(j)}(\epsilon) = \frac{\epsilon - \epsilon_j + i(\Gamma_l^{(j)} - \Gamma_r^{(j)})}{\epsilon - \epsilon_j + i(\Gamma_l^{(j)} + \Gamma_r^{(j)})},$$

$$t_{(j)}(\epsilon) = \frac{-2i\sqrt{\Gamma_l^{(j)}\Gamma_r^{(j)}}}{\epsilon - \epsilon_j + i(\Gamma_l^{(j)} + \Gamma_r^{(j)})}. \tag{1}$$

The tunneling widths $\Gamma_{l(r)}^{(j)}$ depend exponentially on the distance of the impurity from the middle of the barrier, x_j [see also Fig. 1(a)]:

$$\Gamma_l^{(j)} = E_0 e^{-(L+2x_j)/a_0}, \quad \Gamma_r^{(j)} = E_0 e^{-(L-2x_j)/a_0}, \tag{2}$$

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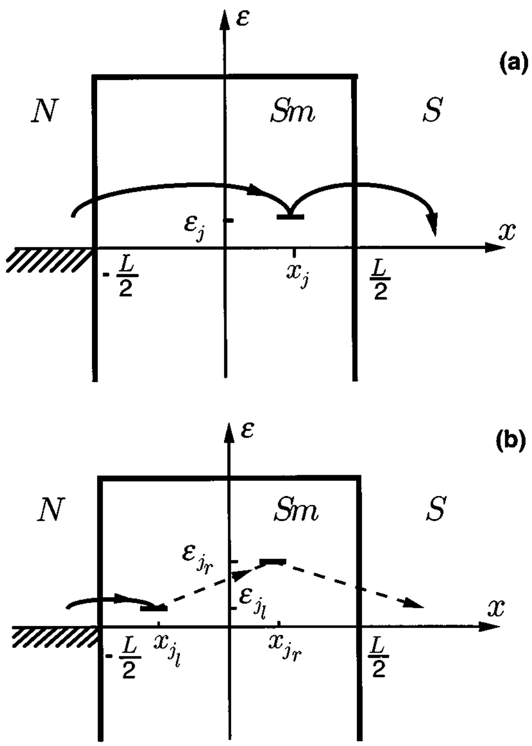

FIG. 1. Schematic picture of the *N-Sm-S* junction with (a) single localized state and (b) two-impurity chains.

where a_0 is the localization radius of the impurity state, and the energy E_0 can be estimated as $E_0 \simeq \hbar^2/ma_0^2$, with m being the electron mass in the *Sm* layer. The Andreev reflection probability can be expressed[6] in terms of the one-electron amplitudes $t_{(j)}(\epsilon)$ and $r_{(j)}(\epsilon)$. The corresponding contribution $g_{NS}(eV;x_j,\epsilon_j)$ of localized state j to the conductance is

$$g_{NS}(eV;x_j,\epsilon_j) = \frac{2e^2}{\pi\hbar}\left|\frac{t_{(j)}(eV)t^*_{(j)}(-eV)}{1+r_{(j)}(eV)r^*_{(j)}(-eV)}\right|^2. \quad (3)$$

(We will restrict our discussion to the most interesting regime of the bias being small compared to the superconducting gap.)

Now we sum up the contributions to the conductance from different impurities. Assuming that the density of the localized states ρ is independent of energy, we obtain

$$G^{(1)}_{NS}(eV) = \rho W\int_{-\infty}^{\infty} d\epsilon_j \int_{-L/2}^{L/2} dx_j g_{NS}(eV;x_j,\epsilon_j). \quad (4)$$

Here W is the width of the barrier.

The calculation of the conductance is thus reduced to the integration in Eq. (4), with the help of formulas (1)–(3). The result of this calculation is presented in Fig. 2 and the limiting cases are discussed later. In the low-bias limit, $eV \ll \Gamma_1$, we find

$$G^{(1)}_{NS} = \frac{e^2}{\hbar}(\rho a_0 W\Gamma_1)\left[\frac{\Gamma(3/4)^2}{\sqrt{\pi}} + \frac{\Gamma(1/4)^2}{96\sqrt{\pi}}\left(\frac{eV}{\Gamma_1}\right)^2\right], \quad (5)$$

where $\Gamma_1 \equiv E_0 e^{-L/a_0}$ is the level width of a state localized at $x_j=0$, and $\Gamma(x)$ is the Γ function. It is instructive to express this result in terms of the normal state conductance of the same junction:

$$G^{(1)}_N = \frac{e^2\rho W}{\pi\hbar}\int_{-\infty}^{\infty} d\epsilon_j \int_{-L/2}^{L/2} dx_j g_N(eV;x_j,\epsilon_j),$$

$$g_N(eV;x_j,\epsilon_j) = \frac{e^2}{\pi\hbar}|t_{(j)}|^2. \quad (6)$$

A simple calculation based on (1) and (6) gives[8,9]

$$G^{(1)}_N = \frac{e^2}{\hbar}(\pi\rho a_0 W\Gamma_1). \quad (7)$$

Comparing Eqs. (5) and (7) we obtain

$$G^{(1)}_{NS} = G^{(1)}_N\left[0.27 + 0.049\left(\frac{eV}{\Gamma_1}\right)^2\right]. \quad (8)$$

Results (5) and (7) can be easily understood. The contribution of the single-site resonant states to both G_N and G_{NS} is determined by the number of the states with the energy near the Fermi level within the strip of width Γ_1 and positioned within the strip of width a_0 around the middle of the barrier, so that $\Gamma_l \simeq \Gamma_r$. Therefore, the factor $\rho a_0 W\Gamma_1$ is just the number of such states. Result (8) at $V=0$ was obtained independently in Ref. 10.

Let us estimate the domain of parameters within which the mechanism of tunneling through rare single resonant states dominates over direct tunneling through the potential barrier created by the depleted *Sm* layer. The contribution of the latter mechanism can be estimated as $G_N^{\mathrm{dir}} \sim (e^2/\hbar)(k_F W)e^{-2L/a_0}$, where k_F is the Fermi wave vector in the N lead. Thus direct tunneling is irrelevant for not too short barriers:

$$L \gg a_0 \ln\left(\frac{ma_0k_F}{\hbar^2\rho}\right). \quad (9)$$

This condition is easily met in the experimental situation.[7,11]

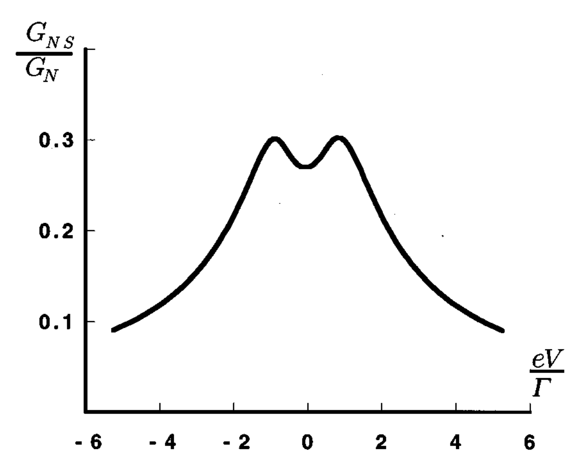

FIG. 2. Voltage dependence of the differential *N-Sm-S* conductance contributed by single-impurity configurations, $G^{(1)}_{NS}(V)$.

The linear conductance $G_{NS}^{(1)}(0)$ differs only by a numerical factor from $G_N^{(1)}$ because the optimal configurations contributing to both quantitites are the same. It changes drastically when the the bias increases, $eV \gg \Gamma_1$. In this regime we obtain, from Eq. (4),

$$G_{NS}^{(1)}(V)=G_N^{(1)}\frac{\Gamma_1}{eV}. \tag{10}$$

The main contribution to $G_{NS}^{(1)}(V)$ comes not from the impurities located near the middle of the barrier but rather from ones shifted closer to the S lead.

Result (10) can be understood using the following arguments. Consider the impurity lying at the Fermi level, $\epsilon_j=0$, that is completely decoupled from the normal lead, $\Gamma_l=0$. The tunneling of an electron pair between the superconductor and the impurity mixes the states $|0\rangle$ and $|2\rangle$, corresponding to zero and two electrons occupying the impurity. As the result, the fourfold degeneracy of the impurity level is partially lifted: two singly occupied states still have energy $\epsilon_j=0$, but the other two states, $|0\rangle-|2\rangle$ and $|0\rangle+|2\rangle$, are split by $2\Gamma_r$ symmetrically with respect to the Fermi level. The even wave function is the ground state of the impurity, and the smallest excitation energy (to the singly occupied state) is Γ_r. Let us now turn on a small coupling between the impurity and the normal lead, $\Gamma_l \ll \Gamma_r$. This coupling enables the electron to tunnel from the Fermi level in the N lead to the impurity level. This tunneling is resonant if the potential drop on the contact eV is close to the energy of excitation, $|eV-\Gamma_r|\lesssim\Gamma_l$. A similar consideration is valid for all the impurities with energies $|\epsilon_j|\lesssim\Gamma_r$. Now, we find the impurities that give the maximal contribution to the conductance. The transmission coefficient is close to unity for the impurities with $\Gamma_r=eV$, i.e., at $eV>\Gamma_1$ the optimal impurities are shifted towards the superconducting lead. The energies of these impurities may lie in the strip of width Γ_r about the Fermi level. The coordinate x_j of the impurity may deviate from its optimal value by no more than $\pm a_0\Gamma_l/\Gamma_r$, as follows from the condition $|eV-\Gamma_r|\lesssim\Gamma_l$, and Eq. (2). This consideration shows that the number of relevant impurities is of the order of $\rho a_0\Gamma_l\Gamma_r/eV$, which with the help of Eqs. (2) and (7) immediately yields Eq. (10).

As the barrier length is increased, the dominant mechanism of electron transport shifts from the single-state configurations to the configurations containing "chains" of two localized states.[8] These configurations are more complex than the ones considered before, nevertheless Eq. (3) is still valid. The only difference is that the one electron reflection and transmission amplitudes now depend upon the energies $\epsilon_{j_l},\epsilon_{j_r}$ and positions $\mathbf{r}_{j_l}$, $\mathbf{r}_{j_r}$ of the two impurities, see Fig. 1(b). Each impurity is characterized by its coupling to the *nearest* lead, see Fig. 1(b),

$$\Gamma^{(j_l)}=E_0e^{-(L+2x_{j_l})/a_0}, \quad \Gamma^{(j_r)}=E_0e^{-(L-2x_{j_r})/a_0}. \tag{11}$$

The coupling between the two impurities is given by

$$h_{j_lj_r}=E_1\left(\frac{a_0}{|\mathbf{r}_{j_l}-\mathbf{r}_{j_r}|}\right)^{1/2}e^{-|\mathbf{r}_{j_l}-\mathbf{r}_{j_r}|/a_0}, \tag{12}$$

where the energy E_1 is of the order of E_0.

After one finds the partial conductance g_{NS} of a single two-impurity "chain," the net contribution $G_{NS}^{(2)}(eV)$ of these chains to the total conductance can be calculated in a manner similar to that used previously for the single-impurity configurations:

$$G_{NS}^{(2)}(eV)=\rho^2W\int\int\int_{-\infty}^{\infty}d\epsilon_{j_r}d\epsilon_{j_l}dy_{j_l}\int_{-L/2}^{L/2}dx_{j_r}\int_{-L/2}^{x_{j_r}}dx_{j_l} \times g_{NS}(eV;\epsilon_{j_r},\epsilon_{j_l},x_{j_l},x_{j_r},y_{j_l}-y_{j_r}). \tag{13}$$

The formulas for the reflection and transmission amplitudes entering g_{NS} for an arbitrary impurity pair are quite cumbersome. Fortunately, the dominant part of the average (13) comes from the impurity pairs with sufficiently large energy difference, $|\epsilon_{j_l}-\epsilon_{j_r}|\gg h_{j_lj_r}$. This means that one of the two impurities serves as a resonant level for the incoming electron, while the second impurity provides a virtual state that modifies the escape rate from the resonant level into the lead. Therefore, for the relevant "chains" the reflection and transmission amplitudes are given by Eq. (1) with the level widths renormalized by the tunneling through a virtual state. Depending on which component of the pair is in resonance, we find

$$\epsilon_j\rightarrow\epsilon_{j_l}, \quad \Gamma_l^{(j)}\rightarrow\Gamma^{(j_l)}, \quad \Gamma_r^{(j)}\rightarrow\Gamma^{(j_r)}\left(\frac{h_{j_lj_r}}{\epsilon_{j_l}-\epsilon_{j_r}}\right)^2 \tag{14a}$$

if the left impurity is in resonance, or

$$\epsilon_j\rightarrow\epsilon_{j_r}, \quad \Gamma_l^{(j)}\rightarrow\Gamma^{(j_l)}\left(\frac{h_{j_lj_r}}{\epsilon_{j_l}-\epsilon_{j_r}}\right)^2, \quad \Gamma_r^{(j)}\rightarrow\Gamma^{(j_r)} \tag{14b}$$

if the right impurity is the resonant one. The corresponding contributions to g_{NS} are obtained from Eqs. (3) and (1) by the substitutions (14a) and (14b), respectively.

Similar to the case of single-impurity configurations, it is convenient to express the result in terms of the contribution of two-impurity chains to the normal conductance. This contribution $G_N^{(2)}$, is given by[8]

$$G_N^{(2)}(eV)=\rho^2W\int\int\int_{-\infty}^{\infty}d\epsilon_{j_r}d\epsilon_{j_l}dy_{j_l}\int_{-L/2}^{L/2}dx_{j_r}\int_{-L/2}^{x_{j_r}}dx_{j_l} \times g_N(eV;\epsilon_{j_r},\epsilon_{j_l},x_{j_l},x_{j_r},y_{j_l}-y_{j_r}), \tag{15}$$

where the partial conductances g_N are obtained from Eqs. (6) and (1) by the substitution of (14a) and (14b). Simple integration in Eq. (15) yields

$$G_N^{(2)}=3\sqrt{\frac{\pi}{2}}\rho L^2E_1G_N^{(1)}, \tag{16}$$

where $G_N^{(1)}$ is the single-impurity result, Eq. (7). Two-impurity chains dominate the conductance[8,9] if the disordered region is long enough:

$$L\gtrsim a_0\left(\frac{m}{\hbar^2\rho}\right)^{1/2}. \tag{17}$$

(We used here the estimate $E_1 \sim E_0 \sim \hbar^2/ma_0^2$.) Deep in the insulating regime, where the separate impurity states overlap weakly, the parameter $\hbar^2\rho/m$ is small, and condition (17) is more restrictive than (9).

If the length of the barrier increases further, more complex configurations may contribute. The effects associated with these configurations have not been observed experimentally for the normal conductance and we will not consider their contributiton to G_{NS}.

Now we present our results for the two-impurity contribution to *N-Sm-S* conductance obtained by performing the integration in Eq. (13). At zero bias, we found

$$G_{NS}^{(2)}(0) = \frac{\Gamma(3/4)^2}{\sqrt{\pi}} G_N^{(2)} \approx 0.27 G_N^{(2)}. \tag{18}$$

It is important to emphasize that the relationships between the contributions to the *N-Sm-S* and *N-Sm-N* conductances are identical for one- and two-impurity configurations, see Eq. (8). This is because within the optimal two-impurity chains only one level is in resonance, and the other level is responsible only for the tunneling width. The same argument also applies for the one-dimensional chains consisting of a larger number of impurities. Therefore, we believe that the relation $G_{NS}(0) = 0.27 G_N$ is a universal property of disordered junctions for which the conductivity occurs via the tunneling through quasi-one-dimensional chains containing an arbitrary number of impurities.

For a finite bias, however, the relationship beween $G_{NS}^{(2)}$ and $G_N^{(2)}$ is quite different from Eq. (10): the differential conductance $G_N^{(2)}$ drops only logarithmically with the increase of V. The strongest variation of the conductance,

$$G_{NS}^{(2)}(eV) = G_{NS}^{(2)}(0) \frac{4a_0^2}{3L^2} \ln^2\left(\frac{\Gamma_2}{eV}\right), \tag{19}$$

occurs in the region $\Gamma_1 \lesssim eV \lesssim \Gamma_2$. Here $\Gamma_2 \equiv E_0 \exp(-L/2a_0) = \Gamma_1 \exp(L/2a_0)$ is the characteristic width of a resonant level formed by a two-impurity chain.

The reason for the logarithmic dependence (19) is the following. As we already discussed, the transmission coefficient for two-electron tunneling through the localized level is close to unity, if two conditions are met: (i) the tunneling rates Γ_l, Γ_r are approximately the same, and (ii) the difference between the energies of the two tunneling electrons is less than the level width, $eV < \Gamma_{l,r}$. For the single-impurity configuration, the first condition can be met only for the impurities located in the vicinity of the middle of the barrier, therefore, the sharp drop occurs at $eV \sim \Gamma_1$. However, for the two-impurity configuration, the first condition can be satisfied for various configurations $\{x_{j_l}, x_{j_r}\}$. The largest level width compatible with the condition (i), $\Gamma_2 = \Gamma_1 e^{L/2a_0}$, corresponds to the largest separation $X = L/2$ between the two impurities. On the other hand, condition (ii) restricts the separation from below, $X \gtrsim a_0 \ln(eV/\Gamma_1)$, which eventually leads to Eq. (19).

Comparison of Eq. (10) with Eq. (19) shows that even if at low biases conductance is dominated by single-impurity channels, a crossover to the two-impurity chain configurations may occur at larger eV. This crossover from a sharp function (10) to a much slower logarithmic dependence (19) takes place at voltage

$$eV^* \simeq \frac{e^{-\lambda}}{\lambda^2}\Gamma_2, \quad \lambda \equiv \frac{L}{2a_0} - \ln\left(\frac{m}{\hbar^2\rho}\right).$$

At this bias, the optimal configurations changes from single impurity to two-impurity chains. To observe the crossover, the junction parameters should satisfy the condition

$$1 \ll \lambda \ll \left(\frac{m}{\hbar^2\rho}\right)^{1/2}.$$

In conclusion, we studied the conductance of the *N-Sm-S* junction where *Sm* is a strongly disordered semiconductor. The electron transport is due to resonant tunneling through the levels localized in *Sm*. We find that at zero-bias *N-Sm-S* conductance is proportional to the conductance of the same junction in the normal state, $G_{NS}(0) = 0.27 G_N$. At larger biases, the conductance $G_{NS}(V)$ drops drastically. This drop represents the signature of Andreev reflection in a junction with a strongly disordered semiconductor.

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