Nonlinear electron-wave directional coupler

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We propose a class of quantum interference device that is an electronic counterpart of the nonlinear optical directional coupler. Nonlinear coupled wave equations including Coulomb charging effects in coupled electron waveguides predict coupling lengths depending on the amplitude of the input electron waves, and also predict that for the initial condition, the electron wave function is equally distributed to two electron waveguides; a small initial fluctuation of the wave function causes the localization of the electron wave in one of the two waveguides by symmetry-breaking instability. These operating characteristics would be useful for constructing ultrafast self-switching devices and logic gates.

Many kinds of quantum interference devices have been proposed up to now.[1] These devices differ in two major respects from ordinary electronic devices. They employ quantum effects and do not rely on drift and diffusion of carriers, but rather ballistic, non-phase-destroying transport. The reason for the interest in these devices is their potential for ultrafast signal processing in compact structures. Especially, there are many theoretical and experimental reports on electron-wave directional couplers in recent years.[2–12]

In this paper, we propose an electrical counterpart of a nonlinear optical directional coupler.[13–18] By introducing the Coulomb charging (electrostatic) potential into the coupled equations for the electron-wave directional coupler,[5] we obtain new differential coupled nonlinear equations. From the numerical analysis of the nonlinear coupled equations, a self-switching (or self-discriminating) phenomenon for different amplitudes of electronic signals is predicted. This operating characteristic would be useful for constructing exceedingly fast electronic switching and electronic logic (AND, XOR) gates.

A nonlinear electron-wave directional coupler is schematically shown in Fig. 1. The device consists of two closely spaced, parallel electron waveguides with extremely small capacitances. For small input signals, the device behaves as a linear directional coupler. Because of evanescent coupling, the signals introduced into waveguide *a* transfer completely to waveguide *b* in one coupling length L_0. For large input signals, on the other hand, the electronic charge carried by the signals induces non-negligible changes of the electrostatic potential in the waveguides and detunes the coupler. Coupling is inhibited for input intensities above a critical intensity as described below.

Zaslavsky *et al.*[19] and Sollner[20] have pointed out that the electrostatic potential induced by the input charge, which is a main concern in this paper, plays an important role in the advent of the intrinsic bistability in the resonant tunneling diode. Jensen and Buot[21] have also observed dynamical bistability in the resonant tunneling diode manifested by an unstable buildup of electrons in the well when the bias is swept in the positive direction, and an unstable depletion of electrons when the bias is swept in the reverse direction: dynamical charge accumulation in the well induces the time-dependent change of the electrostatic potential in the wells.

Yang and Xu[5] have derived the general formulation of guided waves in two coupled waveguides *a* and *b* from the Schrödinger equation. They write the electron wave $\psi(z)$ propagating in the *z* direction as a linear combination of the eigenfunctions of the individual waveguides ψ_a and ψ_b, i.e., $\psi(z)=a(z)\psi_a+b(z)\psi_b$, and obtain two coupled differential equations for $a(z)$ and $b(z)$ [see Eqs. (36) and (37) or Eqs. (46) and (47) in Ref. 5]. We intend to introduce the charging potential (electrostatic potential) into these coupled equations. When some of the electrons are localized in the waveguide, an electrostatic potential $\phi(Q)$ can arise between the waveguide and the surrounding materials (Q is the charge on the waveguide per unit length). We adopt the simple approximation of expressing ϕ in terms of an effective capacitance per unit length C between the waveguide and surroundings, $\phi(Q)=Q/C$. For the coupled waveguides, the electrostatic potential of each waveguide is given by $\phi_j=Q_j(z)/C_j$ $(j=a,\ b)$, where $Q_j(z)$ is the amount of the distributed electron charge per unit length in the waveguide *j*, i.e., $Q_j(z)=P_j(z)Q_0$, and C_j is the effective capacitance per unit length of the waveguide *j*. $P_j(z)$ is the existing

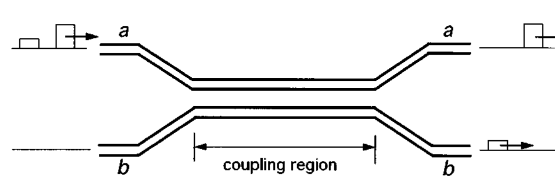

FIG. 1. Schematic of a nonlinear electron-wave directional coupler.

probability of the electron wave in the waveguide j at the distance z, i.e., $P_a(z)=a(z)a(z)^*$, $P_b(z)=b(z)b(z)^*$, and Q_0 is the total amount of the electron charge launched into the coupler. Therefore, the energy difference (phase mismatching) can arise between the waveguides a and b:

$$\Delta\phi=\phi_b-\phi_a=(Q/C)[|b|^2-|a|^2]$$
$$=(Q/C)[1-2|a|^2]. \quad (1)$$

Here we assumed a symmetric coupled waveguide structure, i.e., $C_a=C_b=C$. Introducing the detuning (or phase mismatching) term $\Delta\phi$ into the couple equations [Eqs. (36) and (37) or Eqs. (46) and (47) in Ref. 5], we obtain generalized coupled equations including the effects of the charging potential:

$$\frac{da(z)}{dz}=i\Omega(1-2|a(z)|^2)a-i\kappa b(z), \quad (2a)$$

$$\frac{db(z)}{dz}=-i\Omega(1-2|a(z)|^2)b-i\kappa a(z), \quad (2b)$$

where *a(z)* and *b(z)* are probability amplitudes, $\psi(z)=a(z)\psi_a+b(z)\psi_b$, $\Omega=Q/2C$, and κ the wave-coupling coefficient per unit length between waveguide a to waveguide b. The Eqs. (2) have the same form as the coupled Eqs. (5) in Ref. 22 if we replace the differential of time t with the differential of space z. Neglecting the Coulomb charging effects, the Eqs. (2) become the linear coupled equations, which have a simple oscillatory solution with respect to the propagation distance z, i.e., $P_a(z)=\cos^2\kappa z$ and $P_b(z)=\sin^2\kappa z$.

We numerically solved Eqs. (2) for the initial condition of $a(0)=1$ and $b(0)=0$. Figure 2 shows the $P_a(z)$ (solid line) and $P_b(z)$ (dashed line) as a function of κz for a wide range of the charging potential normalized by κ, i.e., Ω/κ. For $\Omega/\kappa=0.5$ [Fig. 2(a)], the solution shows the simple sinusoidal oscillation, because the nonlinear term due to charging energy does not affect strongly the result in Eqs. (2). As the value of Ω/κ approaches 2, the period of the oscillation is remarkably increased and the evolution becomes nonsinusoidal. For $\Omega/\kappa=1.96$, the oscillation period becomes nearly two times longer than that for $\Omega/\kappa=0.5$ and its shape is significantly deformed from the sinusoidal oscillation. Further increasing the charging potential beyond $\Omega/\kappa=2.0$, the oscillation amplitude becomes abruptly less than 50%. For $\Omega/\kappa=2.04$, the oscillation amplitude becomes about 40% and the oscillation period is nearly equal to that for $\Omega/\kappa=0.5$. From the detailed calculations with various values of Ω/κ, we can obtain the resultant Fig. 3 which shows the maximum oscillation amplitude $P_{\max}$ and the distance of the oscillation (transfer length) L_c normalized by that for the $\Omega/\kappa=0(L_0)$ against the normalized charging potential Ω/κ.

We can see from Figs 2 and 3 that there are two distinct types of solutions. For low amplitudes of the input electron wave satisfying $\Omega/\kappa<2$, the device acts as a linear directional coupler and the complete coupling ($P_{\max}=1.0$) between the waveguides can be achieved. At large amplitudes of the input electron wave satisfying $\Omega/\kappa>2$, the device acts as a phase mismatched coupler and only a part of the input electron wave (at most 50%) is transferred from waveguide a to waveguide b.

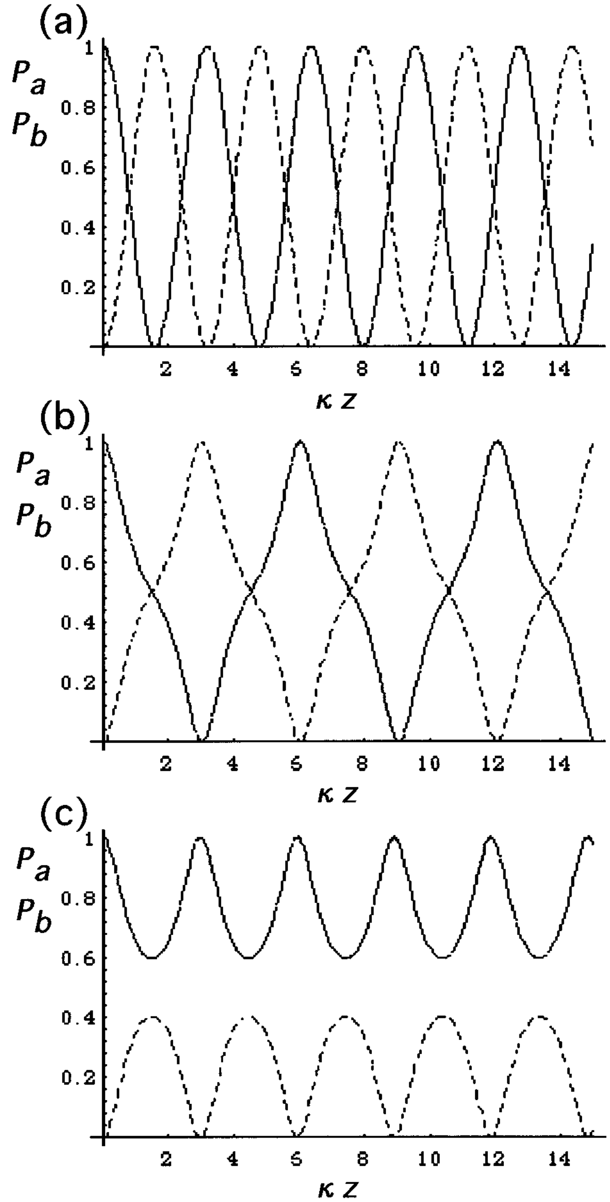

FIG. 2. Distributed functions $P_a(z)$ (solid line) and $P_b(z)$ (dashed line) as a function of the normalized propagation distance κz for the initial condition of $a(0)=1$ and $b(0)=0$: (a) $\Omega/\kappa=0.5$, (b) $\Omega/\kappa=1.96$, and (c) $\Omega/\kappa=2.04$.

This behavior is qualitatively explained in the following way. Here we assume that the input electron wave is launched into waveguide a. For small input signals ($\Omega/\kappa<2$), the phase mismatching is still so small that a relatively large coupling exists between waveguides a and b. As the electron wave gradually couples from waveguide a to waveguide b, the charging potential of waveguide a becomes smaller while that of waveguide b becomes larger. Then the input electron wave is equally distributed to the waveguides a and b, resulting in the equal charging potential in both waveguides, and hence the phase mismatching between waveguides becomes zero. As more of the electron wave couples into waveguide b, i.e., $P_a(z)<P_b(z)$, the phase mismatching reverses and

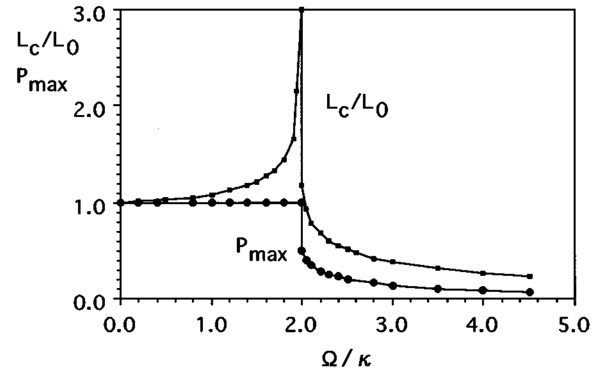

FIG. 3. Maximum oscillation amplitude $P_{\max}$ and normalized transfer length L_c/L_0 against the normalized charging potential Ω/κ.

then the electron wave transfers completely into channel b like a $\Delta\beta$ reversal coupler.[23] For large input signals ($\Omega/\kappa>2$), the electron wave is not completely transferred to waveguide b. This is because the large charging potential (or phase mismatching) leads to poor coupling between waveguides a and b, and the coupling of the input electron wave to waveguide b never exceeds 50%. Therefore, the phase mismatching is not reversed and only a part (less than 50%) of the input electron wave (signal) couples to waveguide b.

We can see from Fig. 2 that the small input signal with $\Omega/\kappa=0.5$ introduced into waveguide a transfers completely to waveguide b at $\kappa z\sim1.6$ and oscillates back into waveguide a at twice the coupling length ($\kappa z\sim3.2$). The large input signal with $\Omega/\kappa=1.96$; on the other hand, complete transfer to waveguide b is achieved at the same distance ($\kappa z\sim3.2$). This indicates that the input electron waves with different amplitudes ($\Omega/\kappa=0.5$ and 1.96) are self-discriminated at the output ports of the device. Furthermore, we can see, comparing Figs. 2(b) and 2(c), that the input signal can be switched by changing the small amount of the input amplitude from $\Omega/\kappa=1.96$ to 2.04 at $\kappa z\sim3.2$.

To make quantitative statements, we consider a device with GaAs-Al$_x$Ga$_{1-x}$As heterostructure waveguides. The capacitance between the waveguide and surroundings is, by neglecting the neighboring waveguide, approximately given by $C=\varepsilon\varepsilon_0 I_c(d_1+d_2)/s$.[24,8] Here, I_c is the length of the directional coupler, d_1 and d_2 are the waveguide transverse dimensions, s is the average position of the electrons in the waveguide, and ε (ε_0) the dielectric constant of the waveguide (vacuum). If we assume $I_c=1$ μm, $d_1=d_2=10$ nm, and $s=5$ nm (the average position of the electrons in the waveguide is at the center of the waveguide), we get a capacitance of $C=0.4$ fF. We further assume the coupling energy between the waveguides to be $\kappa=1$ meV. Since the critical value of the charge is given by $Q=4\kappa C$ ($\Omega/\kappa=2.0$), we obtain $Q=1.6\times10^{-18}$ C. This amount of the charge corresponds to the charge of 10 electrons. It seems that this value is not far from realistic conditions, because this corresponds to the carrier density of 10^{11} cm^{-2}.

The operating characteristics described above would be useful for constructing self-switching devices (or self-discriminators) for different amplitudes of the electron waves and also logic (AND/XOR) gates. These devices are the real electronic counterpart (*all electrical*) of the *all optical* nonlinear directional coupler.

For the initial condition that the electron wave function is equally distributed to the two electron waveguides, a small initial fluctuation in the distribution of the electron wave function causes the symmetry-breaking instabilities of the occupation probabilities of the electrons in the waveguides, leading to the localization of electrons in one of the electron waveguides. The results of a few numerical simulations for the initial condition of $a(0)=1/\sqrt{2}+0.001$ [0.001 is introduced as the initial fluctuation as an example, but the numerical results with another small fluctuation (0.0001–0.01) hardly change the qualitative results] and $b(0)=[1-|a(0)|^2]^{1/2}$ are presented in Fig. 4. For $\Omega/\kappa=0$ [Fig. 4(a)] $P_a(z)$ and $P_b(z)$ show simple oscillations with a small

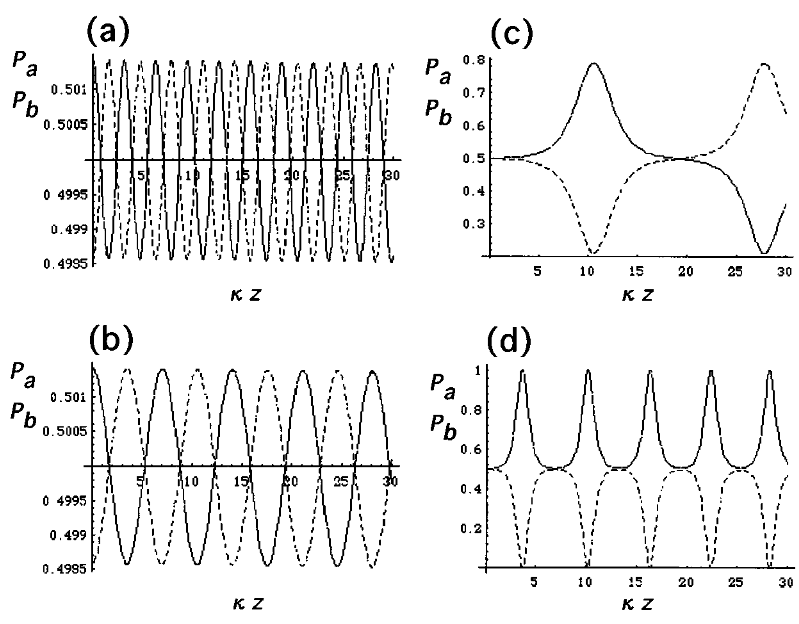

FIG. 4. Distributed functions $P_a(z)$ (solid line) and $P_b(z)$ (dashed line) as a function of the normalized propagation distance κz for the initial condition of $a(0)=1\sqrt{2}+0.001$ and $b(0)=[1-|a(z)|^2]^{1/2}$: (a) $\Omega/\kappa=0$, (b) $\Omega/\kappa=0.8$, (c) $\Omega/\kappa=1.1$, and (d) $\Omega/\kappa=2.0$.

amplitude of $|a(0)|^2-|b(0)|^2=0.0014$, because the nonlinear coupled equation becomes a linear coupled equation under this condition. For $\Omega/\kappa=0.8$ [Fig. 4(b)], the oscillation period in distance increases and the oscillation amplitude gradually varies against κz. Figures 4(c) and 4(d) show the symmetry-breaking instabilities induced by the small initial fluctuation in the distribution of the wave function. For $\Omega/\kappa=1.1$ [Fig. 4(c)], the electrons localize one of the waveguides and then oscillate back and forth between the waveguides, while for $\Omega/\kappa=2.0$ the quasiperiodic oscillation with modulation amplitudes less than 50% follows the symmetry-breaking instabilities. This operating characteristic would also be useful for constructing switching devices and logic gates. For example, two signals with an equal amplitude launched at the input ports of the waveguides *a* and *b* can be localized in one of the two waveguides at the output ports by adding a very small control signal to one of the two input signals.

It should be noticed that Eqs. (2) have the same form as those for the nonlinear optical directional coupler, which was first proposed by Jensen.[13] In the optical coupler, the nonlinear term arises from the nonlinear interaction of guided modes with themselves through the nonlinear refractive index n_2. Similarly, in the electron-wave coupler, the nonlinear term arises from the charging (electrostatic) potential induced by the guided electron waves (signals) themselves.

This device is capable of exceedingly fast switching times. Because the electron waves within the device interact in a spatially and temporally local fashion (self-switching phenomenon), the switching time of this device is not limited by *RC* time and transit time but limited only by the tunneling (or transfer) time. This is because the electrostatic potential follows the coherent electron oscillation; in other words, the Coulomb charging potential builds up immediately after the changing of the charge distribution of the electron waveguides.

To conclude, we have proposed a class of quantum interference device that is an electronic counterpart of the nonlinear optical directional coupler. By introducing the Coulomb charging (electrostatic) potential into the linear coupled wave equations, we obtained *the nonlinear coupled wave equations*, which predict coupling lengths depending on the amplitudes of the input electron waves (electric signals). This operating characteristic would be useful for constructing ultrafast self-switching (or self-discriminating) devices and logic gates. Furthermore, it was shown that for the initial condition that the two waveguides are equally excited, a small initial fluctuation in the input signals causes the symmetry-breaking instabilities of the guiding electron waves and leads to the localization of the electrons in one of the two electron waveguides. This would also be useful to control the operation of a nonlinear electron-wave directional coupler.

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