

Giant Josephson current through a single bound state in a superconducting tunnel junction

G. Wendin

Department of Applied Physics, Chalmers University of Technology and Göteborg University, S-41296 Göteborg, Sweden

V. S. Shumeiko

Department of Applied Physics, Chalmers University of Technology and Göteborg University, S-41296 Göteborg, Sweden and B. Verkin Institute for Low Temperature Physics and Engineering, 47 Lenin Avenue, 310164 Kharkov, Ukraine

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We study the microscopic structure of the Josephson current in a single-mode tunnel junction with a wide quasiclassical tunnel barrier. In such a junction each Andreev bound state carries a current of magnitude proportional to the *amplitude* of the normal electron transmission through the junction. Tremendous enhancement of the bound-state current is caused by the resonance coupling of superconducting bound states at both superconductor-insulator interfaces of the junction. The possibility of experimental observation of the single bound-state current is discussed.

The Josephson effect in a tunnel junction deals with coherent transmission of Cooper pairs through a tunnel barrier which separates superconducting electrodes. What is the mechanism of such a transmission? Conventional theory of the Josephson effect,^{1,2} based on the phenomenology of the transfer Hamiltonian model,³ does not provide adequate physical description of this process, e.g., similar to the quantum mechanical picture of tunneling of normal electrons. Instead, it treats tunneling as a perturbative transition, introducing a matrix element of coupling of electrons in different electrodes proportional to the amplitude of single electron tunneling.⁴

A more realistic description of Josephson tunneling, based on the Bogoliubov–de Gennes equation,⁵ was suggested by Furusaki and Tsukada.⁶ The crucial role in this picture is played by superconducting bound states, similar to Andreev bound states in *SNS* junctions.⁷ A bulk supercurrent, when approaching the tunnel interface, experiences transformation into current flowing through superconducting bound states which provide transmission of Cooper pairs through the barrier. The bound states are induced in the vicinity of the junction by the discontinuity of the superconducting phase and they appear as a consequence of the current.⁸ In a quantum junction the bound state spectrum consists of a single pair of levels per transverse mode, with symmetric position of the levels with respect to the chemical potential. The Josephson current is distributed among bound states in such a way that each bound state carries a current proportional to the normal electron transparency of the junction.

In this paper we show that the above picture of quantization of the Josephson coupling is valid only for an extremely narrow barrier and that the picture is qualitatively different for any realistic tunnel barrier with large width on an atomic scale. In the latter case, the structure of the bound-state spectrum is determined by the coupling of superconducting surface states situated at the two *SI* interfaces of the *SIS* junction. In a symmetric junction, the resonance coupling of these surface states provides tremendous enhancement of the current flowing through a single bound state, the magnitude being proportional to the *amplitude* rather than the probabil-

ity of normal electron tunneling. The currents are distributed among the bound states in such a way that they almost cancel each other in equilibrium, giving rise to a comparatively small residual current, including the contribution from the continuum. This current coincides with the conventional Josephson current given by Ambegaokar-Baratoff theory.² The large current of the single bound state can be revealed under nonequilibrium conditions when the bound level population is imbalanced by means of microwave pumping or tunnel injection.

We consider for the sake of clarity a single mode quantum constriction with a rectangular potential barrier of length *L* and height *V* (Fig. 1). The structure is described by the one-dimensional (1D) Bogoliubov-de Gennes equation:⁵

$$\left\{ \left[\frac{\hat{p}^2}{2m} - \tilde{\mu} + V\theta(L/2 - |x|) \right] \sigma_z + \hat{\Delta}(x) \right\} \Psi = E\Psi, \quad (1)$$

with the order parameter matrix given by

$$\hat{\Delta}(x) = \begin{pmatrix} 0 & \Delta e^{i \operatorname{sgn} x \phi/2} \\ \Delta e^{-i \operatorname{sgn} x \phi/2} & 0 \end{pmatrix} \theta(|x| - L/2); \quad (2)$$

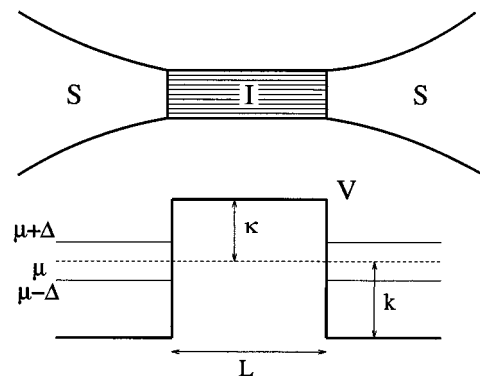


FIG. 1. Single-transverse-mode adiabatic junction modeled as a 1-dim *SIS* junction with a wide barrier of length *L* and height *V*.

$\tilde{\mu} = \mu - E_{\perp}$ is the chemical potential shifted by the energy of the transverse mode, $\hbar = 1$.

Consider first an isolated *SI* surface, assuming in Eq. (1) $L = \infty$. Constructing the ansatz for $|E| < \Delta$ as a superposition of eigenfunctions, with complex wave vectors $k_{\pm} = \sqrt{2m(\tilde{\mu} \pm i\zeta)}$ ($\zeta = \sqrt{\Delta^2 - E^2}$) in the superconducting region and with decay rates $\kappa_{\pm} = \sqrt{2m[V - (\tilde{\mu} \pm E)]}$ in the insulating region, we find the dispersion equation, keeping the difference between electron and hole wave vectors to first order,

$$\frac{\zeta}{E} = -\frac{\delta\kappa k}{\kappa^2 + k^2}. \quad (3)$$

The surface level in Eq. (3) crucially depends on the finite difference $\delta\kappa = \kappa_+ - \kappa_-$ of the decay rates of electron and hole wave functions inside the insulator, while dephasing inside the superconductor is not important and gives small corrections omitted in Eq. (3). The level lies close to the gap edge within the superconducting gap; assuming a typical relation among the energies of the problem: $\Delta \ll V - \tilde{\mu} \leq \tilde{\mu}$, one has $\Delta - E \approx (\Delta^3/2V^2)(k/\kappa)^2$. The wave function of the surface state decays into superconductor on the characteristic length scale $l \sim \xi_0\Delta/\sqrt{\Delta^2 - E^2}$ (ξ_0 is the coherence length) and into insulator on the characteristic length scale $l \sim 1/\kappa$.

Proceeding to calculation of the coupling of surface states at finite $L < \infty$ in Eq. (1), we construct wave functions by means of a transfer matrix formalism,⁹ which yields the following dispersion relation:

$$E^2 = \Delta^2 \cos^2\left(\frac{|\beta| \pm \alpha}{2}\right) \quad (|\beta| \pm \alpha > 0), \quad (4)$$

$$\cos\alpha = \tilde{R} + \tilde{D} \cos\phi, \quad \sin\beta = \tilde{D} \operatorname{Im}(a_+ a_*^*), \quad (5)$$

where $\tilde{D} = \sqrt{D_+ D_-}$, $\tilde{R} = \sqrt{R_+ R_-}$, and the transmission $D_{\pm} = |a_{\pm}|^{-2}$ and reflection $R_{\pm} = 1 - D_{\pm}$ coefficients of normal electrons with energy $\pm E$ are given in terms of the inverse transmission amplitudes

$$a_{\pm} = \cosh\kappa_{\pm} L - \frac{i}{2} \left(\frac{\kappa_{\pm}}{k} - \frac{k}{\kappa_{\pm}} \right) \sinh\kappa_{\pm} L. \quad (6)$$

Equation (4) is valid for all values of the barrier length L and barrier heights V .¹⁰ A numerical solution of this equation is presented in Fig. 2, which also shows the corresponding wave functions. To get an explicit expression for the level energy we consider barriers which are long on the scale of quasiparticle decay κ^{-1} but short on the scale of quasiparticle dephasing $|\delta\kappa|^{-1}$: $\kappa^{-1} \ll L \ll |\delta\kappa|^{-1}$ [the last condition is equivalent to $(L/\xi_0)(k/\kappa) \ll 1$]. In this case, the junction transparency \tilde{D} , the dephasing angle β , and the coupling factor α are all small: $\tilde{D} \ll 1$, $D_+ - D_- \approx -2\tilde{D}\delta\kappa L$,

$$\beta \approx -\frac{2k\delta\kappa}{k^2 + \kappa^2}, \quad |\beta| \ll 1, \quad (7)$$

$$\alpha \approx 2\sqrt{\tilde{D}} \sqrt{\sin^2(\phi/2) + (\delta\kappa L/2)^2} \ll 1. \quad (8)$$

The last term in Eq. (8) is essential only in the close vicinity of $\phi = 0$: $|\phi| \sim |\delta\kappa|L$. Since the energy dispersion of β can be neglected, one finds from Eq. (4)

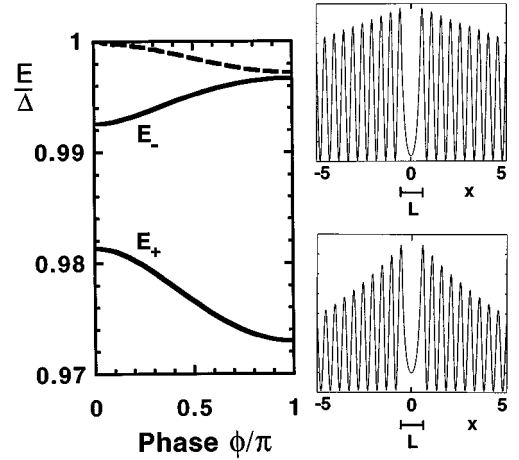


FIG. 2. Andreev level energies and wave functions in a single-mode tunnel junction. Left part: Andreev level energies $E(\phi) > 0$ as functions of phase difference ϕ across the junction. Full lines: Long *SIS* junction ($L = 1$, $V = 50$); Dashed line: Short *SIS* junction ($L = 0.2$, $V = 270$) having the same transparency D as the long junction but showing only a single Andreev level, E_+ (Ref. 12). Right part: Andreev wave functions $|\psi|^2 = |u|^2 + |v|^2$ for the long *SIS* junction at $\phi = \pi$; each of the u and v components are symmetric or antisymmetric around $x = 0$, making E_{\pm} nearly “bonding” or “antibonding” states. In this figure the choice of parameters is beyond the approximate Eqs. (8),(9) to achieve suitable resolution.

$$E_{\pm}^2 = \Delta^2 \left[1 - \left(\frac{|\beta|}{2} \pm \sqrt{D} \left| \sin\frac{\phi}{2} \right| \right)^2 \right], \quad (9)$$

where $|\beta|/2 \pm \sqrt{D} |\sin(\phi/2)| > 0$, $D \approx \tilde{D}$, $|\phi| \gg |\delta\kappa|L$.

The solution presented in Eqs. (4),(9) generally consists of two pairs of bands $E(\phi)$: $\pm E_+(\phi)$ and $\pm E_-(\phi)$, lying inside the superconducting gap symmetrically with respect to the chemical potential. The dephasing angle β determines the position of the preexisting surface states at the single *SI* interface [cf. Eq. (3)] and the amplitude \sqrt{D} determines the level splitting due to coupling via tunneling. All branches of the bound-state spectrum are fully developed if $|\beta|/2 > \sqrt{D}$ or $\kappa L > \ln|\kappa/\delta\kappa|$.¹²

The dispersion of the bands $E(\phi)$ determines the current flowing through the bound states: $I = 2e(dE/d\phi)$.¹³ Taking the derivative of Eq. (4), one gets under conditions (7),(8) the current of the single level in the form

$$I(E_{\pm}) = -\operatorname{sgn}E \frac{e\Delta D}{2} \sin\phi \times \left(1 \pm \frac{|\beta|}{2\sqrt{D}} \frac{1}{\sqrt{\sin^2(\phi/2) + (\delta\kappa L/2)^2}} \right). \quad (10)$$

The first term in Eq. (10) is consistent with calculations within the tunnel model, coinciding with the Ambegaokar-Baratoff current of a single-mode junction at $T = 0$.² The second term dominates at small transparency $\sqrt{D} \ll \beta/2$. It possesses an anomalous square-root dependence on the junction transparency and anomalous [close to $\cos(\phi/2)$] dependence on the phase difference (Fig. 3). This current corresponds to the transition of Cooper pair between electrodes with the probability proportional to the amplitude of the normal elec-

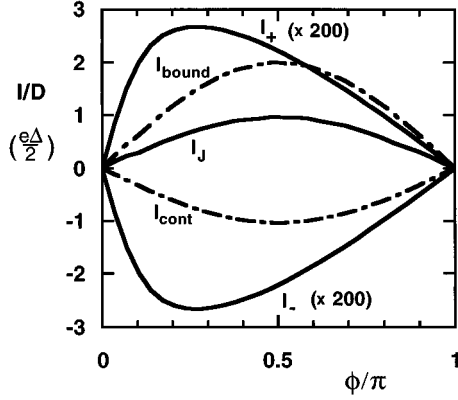


FIG. 3. dc Josephson currents in a single-mode tunnel junction at $T=0$ (long SIS junction: $D \approx 2.4 \times 10^{-9}$, $\beta \approx 3 \times 10^{-2}$). $I_+ = I(-E_+)$ and $I_- = I(-E_-)$ are giant Josephson currents associated with individual Andreev levels. $I_{\text{bound}} = I_+ + I_-$ is the small residual current from compensating giant bound-state currents. I_{cont} is the continuum contribution to the current [Eq. (12)]. $I_J = I_{\text{bound}} + I_{\text{cont}}$ is the (total) Josephson current of the junction.

tron transition. From a physical point of view, this enhancement of the tunnel current results from resonance coupling of the surface states with the equal energies situated at SI interfaces of the symmetric junction, Eq. (3). This resonance coupling causes large dispersion of the bound state bands, in analogy with the energy level splitting in the Schrödinger symmetric double well potential.¹⁴ We stress that the resonant enhancement concerns only the tunneling of Cooper pairs, while the normal electron tunneling is nonresonant. In asymmetric junctions resonance coupling may be destroyed if the relative shift of preexisting surface levels exceeds the level splitting due to the tunnel interaction, $\beta\sqrt{D}\Delta$. A restriction on the difference of the order parameters in the left and right electrodes, $|\Delta_R - \Delta_L| \leq \beta\sqrt{D}\Delta$, results from Eq. (9) if the term $[\beta^2 D \sin^2(\phi/2) + (\Delta_R - \Delta_L)^2/\Delta^2]^{1/2}$ is allowed to replace the cross term $\sqrt{D}|\beta \sin(\phi/2)|$.

The bound levels in symmetric low-transmission tunnel junctions carry a current which tremendously exceeds the known critical Josephson current. However, this current is not manifested in equilibrium. Indeed, assuming Fermi distribution $n_F(E_{\pm})$ for the occupation numbers of the bound levels, one finds with the assumed accuracy the current of all bound states

$$I_{\text{bound}} = \sum_{\text{sgn}E, \pm} I(E_{\pm}) n_F(E_{\pm}) = e\Delta D \tanh(\Delta/2T) \sin\phi. \quad (11)$$

This current is twice bigger than the magnitude of the Josephson current calculated for a single-mode junction within the tunnel model² and beyond it.¹⁵ However, in order to get the total current one has to take into account the contribution from the continuum

$$I_{\text{cont}} = \frac{e}{\pi} \int dE \{ [|t^N(\phi)|^2 - |t^A(\phi)|^2] - (\phi \rightarrow -\phi) \} n_F(E). \quad (12)$$

The normal transmission amplitude t^N in Eq. (12) has the form:

$$t^N = \tilde{D} \frac{E[(E-\xi)a_+ e^{i\phi/2} - (E+\xi)a_- e^{-i\phi/2}]}{\Delta^2(\cos\alpha + \cos\beta) - 2E^2 \cos\beta + 2i\xi E \sin\beta}, \quad (13)$$

where $\xi = \sqrt{E^2 - \Delta^2}$. The Andreev transmission amplitude t^A is symmetric in ϕ and drops out of Eq. (12), which together with Eqs. (7), (8), and (13) yields

$$I_{\text{cont}} = -\frac{e\Delta}{2} D \tanh(\Delta/2T) \sin\phi = -\frac{1}{2} I_{\text{bound}}. \quad (14)$$

The calculation presented above, as well as the results of direct numerical calculations in Fig. 3, uncover a remarkable fact, namely that the Josephson current in a symmetric junction with an extended tunnel barrier results from cancellation of large resonant currents flowing through individual bound states. This situation resembles the situation in a normal junction: while individual scattering states of electrons carry a finite current, the total current through the junction is equal to zero in equilibrium due to cancellation of currents of the modes incident from the right and from the left. To reveal the net current of a single mode, one has to create a current imbalance, connecting the junction to reservoirs with different chemical potentials. In a similar way, creation of current imbalance in the Josephson junction by means of nonequilibrium population of the bound states is able to reveal the current of a single bound state. For example, it is possible to equalize the level populations within one of the bound level pairs (e.g., $\pm E_-$) by means of resonant electromagnetic pumping, as suggested in Ref. 11. In principle, one might then suppress the current of this level pair and thereby reveal the current of the second pair of levels ($\pm E_+$), which will show up in a Josephson current enhanced by the factor of $|\beta|/2\sqrt{D}$, flowing in the same direction as the equilibrium current. Suppression of the current of the other level pair $\pm E_+$, by a proper choice of the frequency of the pumping field, will show up in an enhanced current flowing in the opposite direction. Another possibility is to inject excess quasiparticles into one of the bound levels by means of tunnel coupling to an additional normal electrode, i.e., to use a three-terminal device similar to the one suggested by van Wees *et al.*,¹⁶ in order to reveal the currents of individual Andreev states in a SNS junction (see also Ref. 17).

A crucial condition for observing the current of a single bound state in experiment is sufficient resolution of the bound level structure, with respect to both junction asymmetry and level broadening Γ : $\beta\sqrt{D}\Delta \geq |\Delta_R - \Delta_L|, \Gamma$. Combination of the resolution condition with the condition of a long junction yields: $\beta^2\Delta \geq |\Delta_R - \Delta_L|, \Gamma$. An estimate of the rate of quasiparticle recombination due to electron-phonon scattering in the superconducting banks yields $\Gamma/\Delta \sim 10^{-2}$ at the critical temperature, decreasing exponentially at low temperature.¹⁸ The factor β^2 depends on the height of the tunnel barrier and varies between $(\Delta/\mu)^2 < \beta^2 < \Delta/\mu$ in junctions with high ($V \sim \mu$) and low ($V - \mu \sim \Delta$) tunnel barriers. In line with these estimates, one might expect to observe the effect—giant Josephson current, Eq. (10)—in junctions with very low tunnel barriers.

A more realistic approach, however, would be to increase the energy level splitting, increasing the β factor by making *SNI* wells at the interfaces. Precisely such a *SNINS* structure is realized in a gate controlled S-2DEG-S device¹⁹ where the tunnel barrier is produced in the quantum 2DEG constriction by the gate potential. The advantage of such a device is the potential possibility to control the asymmetry of the junction by varying the relative size of normal regions. For a symmetric *SNINS* structure we find the same result for the bound-state spectrum as in Eq. (4), but with a factor $\beta = 4d/\xi_0$ ($d \ll \xi_0$, where d is the width of normal region) corresponding to a preexisting surface state in each *SNI* well.²⁰ This can easily provide a sufficiently large β factor, say $\beta \sim 0.1$. The asymmetry due to different effective length of the normal regions does not destroy the effect provided $|d_R - d_L| \ll \xi_0 \sqrt{D}$.

In conclusion, we have studied a microscopic mechanism of Josephson current transport in single-mode tunnel junctions with wide quasiclassical tunnel barriers. We found that each Andreev bound state in such junctions carries a giant current proportional to the amplitude of normal electron transmission through the junction. Experimental observation of the current of a single bound state is possible under non-equilibrium conditions.

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