

Percolation transition of the vortex lattice and c -axis resistivity in high-temperature superconductors

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We use the three-dimensional Josephson-junction array system as a model for studying the temperature dependence of the c -axis resistivity of high-temperature superconductors, in the presence of an external magnetic field H applied in the c direction. We show that the temperature at which the dissipation becomes different from zero corresponds to a percolation transition of the vortex lattice. In addition, the qualitative features of the resistivity versus temperature curves close to the transition are obtained starting from the geometrical configurations of the vortices. The results apply to the cases $H \neq 0$ and $H = 0$.

Strong thermal fluctuations and anisotropy make the physics of the vortex lattice in high- T_c materials much more rich and complicated than predictions of mean-field theories.¹ This shows up, in particular, in the complicated structure of the field-temperature (H - T) phase diagram of the high T_c 's. It seems to be clear that there is a line in the H - T phase diagram that separates a low-temperature phase (known as the vortex glass phase²) where vortex lines are frozen in space, and a high-temperature phase in which the vortex lines move through the material due to thermal activation. The passage to the normal state when increasing temperature is likely to be a crossover, instead of a well-defined transition. We call the curve that separates the low- and high-temperature phases the irreversibility line (IL). The V - I characteristics when an external current is applied perpendicularly to the magnetic field is different above and below the IL. Below the IL the V - I curves are well fitted by $V \sim \exp[-I_c/I]^\mu$ with μ and I_c being two parameters,^{2,3} and the resistivity ρ of the system—which is defined as $\rho = \lim_{I \rightarrow 0} (V/I)$ —is strictly zero. Above IL the behavior is ohmic, i.e., $V \sim I$.⁴

When the current is applied parallel to the field, the mean force exerted on the vortices is zero. However there are local forces—due to misalignment of the local magnetic field—that may give rise to dissipation. The most important mechanism for dissipation in this configuration at intermediate temperatures is the thermal activation of vortex loops, which gives a voltage $V_c \sim \exp[-I_c/I]$,¹ implying zero resistivity. In this work we show that when temperature is increased there is a phase transition at a temperature T_p that reflects a thermodynamic property of the vortex system and is signaled by the occurrence of a nonzero resistivity. In fact, the I - V characteristic for Y-Ba-Cu-O, when current and magnetic field are parallel to the c -axis, show the following behavior:^{5,6} for small currents and high temperature the response is ohmic, the range of currents that gives a linear response is reduced as the temperature decreases, and at a well-defined temperature T_p the linear behavior disappears. Moreover, the I - V curves can be scaled on two universal curves, corresponding to $T > T_p$ and $T < T_p$ respectively.⁷ This behavior—similar to what occurs when the current is applied in the ab -plane—supports the idea of a thermodynamic transition that we iden-

tify with a percolation transition of the vortex system. Experimentally, it is observed⁵ that the dissipation in the c axis appears at different temperatures than in the ab plane. This implies that the “irreversibility line” for a current parallel to the field is different than that corresponding to the ab plane.

Here we explore the following idea. At zero temperature the vortices are straight lines and the net force on each of them when a small current in the c axis is present is zero. At low temperature, vortex lines start to wander and vortex loops are created due to thermal activation. However, if the temperature is not too high, vortex loops and vortex lines are still isolated from each other and dissipation in the linear regime is zero—except for surface effects (see below). When increasing the temperature, vortex lines and thermally generated vortex loops start to touch each other and for temperatures greater than a critical value T_p , there will be a vortex path crossing the sample along the ab plane. The net force exerted by the current on this path is different from zero, and a finite dissipation will be observed. In this way we qualitatively see that the existence of paths perpendicular to the current in the sample—i.e., the transversal percolation of the vortex lattice—is crucial for the dissipation in the c axis.⁸

The model used to test this idea is the three-dimensional (3D) Josephson-junction array on a discrete lattice, that has been described in detail elsewhere.^{9,10} The dynamics of the model is contained in the evolution of the phases $\varphi^i(t)$, which are defined on the nodes of a cubic lattice and represent the phase of the order parameter. Between nearest-neighbor nodes there are Josephson junctions characterized by a critical current I_0 and a normal resistance R_0 . The equations describing the model are

$$j^{ii'} = I_0 \sin(\varphi^i - \varphi^{i'} - A^{ii'}) + \frac{1}{R_0} \frac{\partial(\varphi^i - \varphi^{i'})}{\partial t} + \eta^{ii'}(t), \quad (1)$$

$$\sum_{\{i'\}} j^{ii'} = j_{\text{ext}}^i. \quad (2)$$

Equation (1) gives the current $j^{ii'}$ between nearest-neighbor nodes i and i' with phases φ^i and $\varphi^{i'}$. Here $A^{ii'}$ is the vector potential of the external magnetic field, and $\eta^{ii'}(t)$ is an uncorrelated Gaussian noise which incorporates

the effect of temperature. Equation (2) assures the current conservation on each node, and j_{ext}^i is the external current applied at node i .

The model allows for the existence of vortices, which consist in singularities of the phases $\varphi(t)$ around a given closed path. Self-inductance and disorder effects are not considered and the system is taken to be isotropic for simplicity—i.e., I_0 and R_0 are taken to be constant throughout the lattice.

We numerically integrate Eqs. (1) and (2) in time. Voltages at different points of the sample are calculated as the temporal mean value of the time derivative of the phases φ . The resistivity of the sample in a given direction is calculated by injecting a small external current (typically around $\sim 1/20$ of the critical current of the junctions) by one of the faces of the sample and withdrawing it from the opposite face. The small value of the external current is chosen in order to be in the linear regime, in which the voltage drop is proportional to the applied current.

The boundary conditions (BC) are taken to be open in the ab plane. However, if open BC in the c axis are used, there will be a finite force on an isolated vortex at a finite temperature if the top and bottom ends of the vortex are not aligned. The dissipation—which is nonzero even in the linear regime—caused by this net force turns out to be independent of the thickness of the sample,¹⁰ and in this sense, is only a surface effect. In order to eliminate this spurious surface effect it is crucial to use BC for the c direction that assure that each vortex line leaving the sample at a given point of the bottom plane reenters at the same point of the top plane. Strict periodic BC on the phases φ have this property, however, we would obtain that the voltage difference between top and bottom planes is identically zero. We use, instead, open BC for the mean value of the phases in the top ($\bar{\varphi}_T$) and bottom ($\bar{\varphi}_B$) planes, and periodic BC for all the phase differences $\varphi_T^i - \varphi_T^j$ and $\varphi_B^i - \varphi_B^j$. This guarantees the periodicity of the vortex configurations and permits the calculation of the c -axis resistivity.

We have to define a criterium for percolation: in our model there is a typical length which is the lattice parameter a . Distances smaller than a cannot be resolved. Flux conservation implies that every flux line going into a unit cell of our lattice also goes out of the cell. When two vortices go into the same elemental cell we cannot tell which one of the two outgoing vortices corresponds to each one of the ingoing vortices. We interpret this situation as the meeting of two vortex lines. In a real material this corresponds to two vortex lines being at a distance lower than the core size of the vortex. At high enough temperatures the vortex structure may percolate perpendicularly to the applied field: starting from one side of the sample we can follow a vortex line and arrive at the opposite side of the sample. Due to the finite size of the systems used and the dynamical evolution, percolation is not expected to occur every time, but only at a given fraction of the total time, which depends on temperature. We evaluate the probability that there exists a vortex line crossing the system from one side to the opposite as a function of temperature. Because a sharp percolation transition can only be seen in the thermodynamic limit, we do scaling with the size of the system.

In Fig. 1(a) we show the resistivity of a cubic

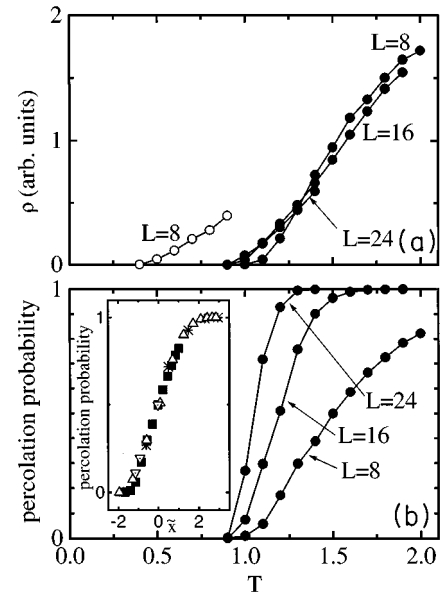


FIG. 1. (a) Resistivity along the c axis (solid circles) and along the ab plane (open circles), and (b) probability of percolation across the sample for a cubic lattice of size L vs temperature (in units of the Josephson energy of an individual junction). Inset: percolation probability vs scaled temperature \tilde{x} (see the text for definition). Different symbols correspond to different sizes of the sample.

($L_{ab} \times L_{ab} \times L_c$, $L_{ab} = L_c \equiv L$) sample for an external field of 0.2 (in units of quantum fluxes per plaquette) as a function of temperature (which is measured in units of the Josephson energy of the junctions) for three different sizes of the system: $L=8, 16$, and 24 . For comparison, the resistivity when the current is applied perpendicularly to the field is also shown for the case of $L=8$. It is clearly seen that the onset temperature for the dissipation in the c -axis T_p is higher than that corresponding to the ab plane.¹¹ Figure 1(b) shows the probability that the vortex lines have percolated through the sample along the ab plane. We see a percolation transition around T_p that becomes narrower the greater the size of the system. This indicates that there exists a sharp percolation transition in the thermodynamic limit. As an additional check, in the inset of Fig. 1(b) the data of Fig. 1(b) are plotted versus a rescaled variable \tilde{x} : $\tilde{x} = L_{ab}^\alpha \{1/2 - [1 - \exp(-\Delta/T)]^{L_c}\}$, where $\alpha=0.7$, and $\Delta=3.75$ are numerically found parameters. This scaling comes up by using a simple model for the percolation.¹² It strongly suggests that a (percolative) thermodynamical phase transition is occurring in the system.

By comparing Figs. 1(a) and 1(b) it can be seen that the temperature T_p where c -axis resistivity starts to be different from zero is the same temperature at which the percolation probability becomes finite. This indicates—as qualitatively discussed above—that the percolation transition is a necessary condition for the existence of dissipation in the c direction. Moreover, T_p decreases with the thickness of the sample. The form of this dependence is obtained by putting $\tilde{x}=0$, and solving for T as a function of L_c . We thus obtain for large L_c the result $T_p \sim 1/\ln(L_c)$.

In addition, we would like to have a more quantitative estimation of the resistivity, based on the geometrical configurations of the vortex system. This can be accomplished in

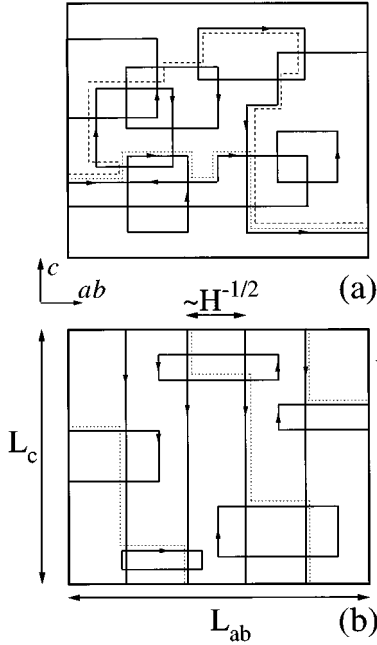


FIG. 2. Schematic two-dimensional representation of percolation paths (broken lines) of vortex lines (solid lines) across the sample for the cases $H=0$ (a) and $H>0$ (b). Note in (a) the two different percolation paths with the same end points, and in (b) the percolation through the externally generated vortex lines.

the following way. Consider a sample of size L_c (L_{ab}) in the c (ab) direction. The resistivity ρ of the sample in the c direction is proportional to the number of paths n that cross the sample in the ab plane per unit of area, times the velocity v this path acquires under the external force, divided by the external current density j : $\rho \sim nv/j$. The velocity v is given—using a viscous fluid argument—by the external force F divided by a total viscosity η , which is equal to a specific viscosity coefficient η_0 times the total length of the vortex path, which we will call l , i.e., $v = F/\eta_0 l$. The force F is given in term of the external current and the size of the system: $F \sim jL_{ab}$. We obtain $\rho \sim nL_{ab}/\eta_0 l$. The coefficient η_0 depends on temperature; however, on small ranges near the percolation threshold we will take it as a constant. The determination of n and l is a difficult task, because the percolation paths across the sample are not uniquely defined due to the crossing of vortex lines [see Fig. 2(a)]. We will use the following estimation: we assume that $n \times l \times L_{ab} \times L_c$ is the volume S of the percolation cluster in a sample of volume $L_{ab} \times L_{ab} \times L_c$. The value of S can be easily evaluated from the numerical simulation. We obtain $\rho \sim S/\eta_0 l^2 L_c$. The need to estimate the value of l remains. This length l depends both on temperature and the size of the system. As we said, a direct numerical determination of l is difficult due to indeterminacies at the crossing points of the vortex lines. We will use the most crude estimation (see Fig. 2): when the magnetic field H is close to zero—i.e., $H < H_{\text{cross}}$, where H_{cross} is a crossover field which is defined below—we take $l \sim L_{ab}$. However, for $H > H_{\text{cross}}$, percolation proceeds via the external field generated vortices and the length of a percolation path is much larger, and can be estimated to be

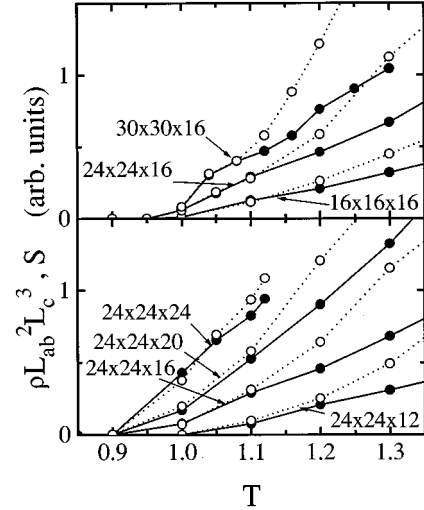


FIG. 3. Numerically calculated resistivity times $L_{ab}^2 L_c^3$ (solid symbols) and volume of the percolation cluster S (open symbols) vs temperature for different sizes of the sample ($L_{ab} \times L_{ab} \times L_c$), and $H=0.2$.

$l \sim L_c L_{ab} / H^{-1/2}$. In this way we obtain the following scaling for the resistivity near the percolation threshold:

$$\rho \sim S / L_{ab}^2 L_c \quad \text{for } H < H_{\text{cross}}, \quad (3)$$

$$\rho \sim S / L_{ab}^2 L_c^3 \quad \text{for } H > H_{\text{cross}}. \quad (4)$$

This scaling is expected to be valid only close to the percolation threshold.

The crossover field H_{cross} is estimated as $H_{\text{cross}} \sim 1/L_c^2$, and corresponds to the zero-temperature lattice parameter of the vortex structure being equal to the thickness of the sample. (The value of this field is about 20 G for a 1 μm thick sample.) For the value $H=0.2$ used in Fig. 1 we find the case of $H \gg H_{\text{cross}}$ for all the values of L_z considered.

In order to check the previous estimations, in Fig. 3(a) we compare the values of $\rho L_{ab}^2 L_c^3$ and S versus temperature when L_{ab} is varied between 16 and 30, for $H=0.2$. In Fig. 3(b) $\rho L_{ab}^2 L_c^3$ and S versus temperature are compared when L_c is varied between 12 and 24, for the same field $H=0.2$. The only free parameter of the fitting is a global factor, which is the same in Figs. 3(a) and 3(b). The agreement between the numerically calculated values and the estimated ones close to the threshold is fairly good if we take into account all the approximations made in order to obtain Eqs. (3) and (4). A more precise estimation of the resistivity using only the geometrical configurations of vortex lines seems to be difficult because of the following facts: the percolation paths across the sample are not uniquely defined [see Fig. 2(a)], and the real movement of vortex lines under the external force will depend on the cutting energy. The viscosity η_0 is not a constant, but a function of temperature. In addition, the supposition of a phenomenological viscous motion of vortex lines may not be accurate at low temperatures, when vortices creep.

The existence of two resistive transitions (in the c axis and the ab plane) has been experimentally observed in Y-Ba-Cu-O.⁶ The values of the two characteristic tempera-

tures depend on the pinning, vortex elasticity, and magnetic field. In Y-Ba-Cu-O, as the thickness of the sample increases the two temperatures become closer to each other. In our simulations we find that the temperature at which the percolation transition occurs decreases as $\sim 1/\ln(L_c)$,¹² as it can be deduced from the scaling in inset of Fig. 1(b).

The thermal excitations in the form of vortex lines crossing the sample along the ab plane destroy the phase coherence along the c axis. For $T > T_p$ the coherence length ξ_c is of the order of the mean distance between percolation paths, i.e., $\xi_c \sim L_c/n^{1/2}$. We conclude that the mechanism that leads to the 2D-3D transition in high- T_c materials with moderate anisotropy is the percolation of vortex line perpendicular to the external field.

In summary, for a model high-temperature superconductor we have shown by using qualitative arguments and numerical simulations, that the onset of the resistivity in the c di-

rection is related to a percolation transition of vortex lines in the ab plane. The results hold for $H \neq 0$ and $H = 0$. A qualitative estimation of the resistivity near the threshold, and its finite-size scaling has been given. For the sizes of the isotropic systems used, the percolation transition occurs at a higher temperature than the resistive transition in the ab plane, and corresponds to a thermodynamic transition that should be characterized by critical exponents different from those obtained for the vortex glass transition when current is applied parallel to the ab plane. We expect these results to be valid also for anisotropic systems, at least in the case of moderate anisotropy, as in Y-Ba-Cu-O.

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¹¹In real samples the ab -plane resistivity vanishes at a finite temperature due to the pinning generated by the presence of disorder. In our simulations pinning comes from the discrete structure of the Josephson-junction-array lattice. See Ref. 10 where this point is discussed in more detail.

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