Ultrasound attenuation in gap-anisotropic systems

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Transverse ultrasound attenuation provides a weakly coupled probe of momentum current correlations in electronic systems. We develop a simple theory for the interpretation of transverse ultrasound attenuation coefficients in systems with nodal gap anisotropy. Applying this theory we show how ultrasound can delineate between extended-*s* and *d*-wave scenarios for the cuprate superconductors.

The development of gaps with nodal anisotropy is a recurrent property of highly correlated electron systems. Bulk probes, such as the NMR relaxation rate,[1] specific heat,[2] and penetration depth[3,4] indicate that gap nodes may be present in a wide variety of strongly correlated systems including heavy fermions, strong coupling, and cuprate superconductors[5–7] and the narrow gap Kondo insulators CeNiSn and CeRhSb.[8] However, with a few noted exceptions,[9,10] we have no direct information about the symmetry of the gap in these strongly correlated systems.

A versatile, but much under-utilized tool for probing electronic gap nodes is the use of transverse ultrasound attenuation. This method has been successfully used to locate the gap lines and point nodes in superconducting UPt_3.[11] Surprisingly, very little work has been done to enable the model-independent interpretation of transverse ultrasound measurements. In this paper we revisit this old problem, highlighting those aspects of ultrasound attenuation that are model independent and relevant to future experiments.

Ultrasound attenuation probes the relaxation of electronic momentum in a model-independent fashion. This information is encoded in the ''viscosity tensor,'' a high symmetry tensor with very few independent components. Here we develop a simple theory which links these components to the location of the gap nodes. We illustrate this theory in a vein of current interest, cuprate superconductivity, showing how ultrasound measurements can provide an unambiguous fingerprint of gap zeros lying on the diagonal of the Brillouin zone.[12]

In a metal, the phonon strain field $u_{ij}(x)$ couples linearly to the electron stress tensor $\sigma^{ij}(x)$

$$H_I = -\int d^3x \sigma^{ij}(x) u_{ij}(x). \tag{1}$$

This coupling is model independent. The stress tensor $\sigma^{ij}(x)$ describes the flow of electronic momentum: its divergence governs the rate of change of electronic momentum density $\nabla_j \sigma^{ij}(x) = -\dot{P}_i(x)$. When a sound wave propagates through a crystal, the dissipation rate is[13]

$$\dot{E} = -\int d^3x \sigma^{ij}(x) \dot{u}_{ij}. \tag{2}$$

In linear response $\sigma^{ij} = \eta^{ijkl} \dot{u}_{kl}$ where η^{ijkl} is the viscosity tensor. The sound attenuation coefficient is defined as the ratio of the time average energy dissipation to twice the energy flux in the wave,[14]

$$\alpha(\vec{q},\hat{u}) = \frac{q^2}{\rho c_s}\bar{\eta} \quad \text{where} \quad \bar{\eta} = \eta^{ijkl}\hat{u}_i\hat{q}_j\hat{q}_k\hat{u}_l, \tag{3}$$

ρ is the mass density and c_s is the speed of sound with wave vector $\vec{q}$ and polarization $\hat{u}$. From the coupling (1), it is straightforward to obtain the Kubo formula for the viscosity tensor:

$$\eta^{ijkl}(\vec{q}) = -\lim_{\omega\to 0}\frac{1}{i\omega}\langle \sigma^{ij}(\mathbf{q})\sigma^{kl}(-\mathbf{q})\rangle, \tag{4}$$

with $\mathbf{q} = (\vec{q}, \omega + i\delta)$.

In typical ultrasonic measurements, wavelengths λ are hundreds of microns and substantially exceed the electronic mean-free paths (l_e), so the attenuation is safely in the hydrodynamic limit, $l_e \ll \lambda$. In this case the momentum dependence of the viscosity tensor is irrelevant, permitting us to take the limit $\vec{q}\to 0$.[12]

Like the elasticity tensor, the symmetry properties of the viscosity tensor significantly reduce the number of its independent components.[15] This tensor is symmetric, not only in the first and second pair of indices, but also under the interchange of these pairs:

$$\eta^{iklm} = \eta^{kilm} = \eta^{ikml} = \eta^{lmik}. \tag{5}$$

Symmetry under the crystal point group further reduces the number of independent components. For example, inversion symmetry eliminates those components with an odd number of identical suffixes, e.g., $\eta^{xxxy} = 0$, $\eta^{xyyy} = 0$. In a square or cubic environment, 90° rotation symmetry restricts the viscosity tensor to the form

$$\eta^{ijkl} = A(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}) + \tilde{C}\delta^{ij}\delta^{kj} + B\delta^{ij}\delta^{jk}\delta^{kl} \tag{6}$$

with no sum on indices implied, so for a cubic or square lattice the ultrasound attenuation is proportional to

$$\bar{\eta} = A + B\sum_{i=1,d}\hat{u}_i^2\hat{q}_i^2 + C(\hat{u}\cdot\hat{q})^2, \tag{7}$$

where $C = \tilde{C} + A$ and d is the dimension. We shall restrict our attention to transverse ultrasound attenuation, for which the

last term vanishes. In other words, in a cubic crystalline environment, there are only two independent transverse ultrasound attenuation coefficients. In a two-dimensional hexagonal system, 60° rotation symmetry means that B must also vanish. In a three-dimensional hexagonal system, polynomial terms involving $\hat{u}_z$ and $\hat{q}_z$ must be added to the above expression, giving

$$\bar{\eta}=A+B(\hat{u}_z^2+\hat{q}_z^2)+C\hat{u}_z^2\hat{q}_z^2, \tag{8}$$

where we have dropped terms which vanish for transverse ultrasound. These strong symmetry constraints mean that only a few different propagation directions are required to measure the full electronic viscosity tensor.

We now turn to the relationship between the quasiparticle gap structure and the electron viscosity tensor. The electron contribution to ultrasonic attenuation only becomes substantial at low temperatures in a regime where a quasiparticle description of the excitations is valid. For a fluid of quasiparticles with dispersion $E_{\vec{k}}$, the quasiparticle group velocity is $\vec{v}_{\vec{k}}=\nabla_{\vec{k}}E_{\vec{k}}$. The traceless stress tensor pertinent to transverse ultrasonic measurements is then

$$\sigma^{ij}=\sum_{k\sigma}\ \psi^{\dagger}_{\vec{k}\sigma}\sigma^{ij}_{\vec{k}}\psi_{\vec{k}\sigma}, \tag{9}$$

where $\psi^{\dagger}_{\vec{k}\sigma}$ creates a quasiparticle of momentum $\vec{k}$ and

$$\sigma^{ij}_{\vec{k}}=\frac{1}{2}(k^i v^j_{\vec{k}}+k^j v^i_{\vec{k}})-\frac{1}{d}\delta^{ij}\vec{k}\cdot\vec{v}_{\vec{k}} \tag{10}$$

is the momentum flux of a single quasiparticle. The quasiparticle contribution to the viscosity tensor is then

$$\eta^{ijkl}=\sum_{\vec{k}}\ \left(-\frac{\partial f}{\partial E_k}\right)\tau_{\vec{k}}\sigma^{ij}_{\vec{k}}\sigma^{kl}_{\vec{k}}, \tag{11}$$

where f is the Fermi distribution function and $\tau_{\vec{k}}$ is the relaxation time of the quasiparticle. From this relation, we see that a simple expression for the viscosity for transverse ultrasound is

$$\bar{\eta}=\frac{1}{4}\sum_{\vec{k}}\ \left(-\frac{\partial f}{\partial E_k}\right)\tau_{\vec{k}}([\vec{v}_{\vec{k}}\cdot\hat{u}][\vec{k}\cdot\hat{q}]+[\vec{k}\cdot\hat{u}][\vec{v}_{\vec{k}}\cdot\hat{q}])^2. \tag{12}$$

We shall consider the situation where the temperature is low enough for the quasiparticles to be entirely concentrated within the gap nodes of the excitation spectrum. To simplify our discussion, we shall assume that the gaps are small enough in comparison with the Fermi energy, that to a good approximation $\vec{v}_{\vec{p}}\approx v_F\hat{p}_F(\partial E_{\vec{p}}/\partial\epsilon_{\vec{p}})$, where $\epsilon_{\vec{p}}$ and v_F are respectively the "bare" energy and Fermi velocity of the excitations prior to gap formation. Consider the case where the gap nodes are simple points in momentum space, located at positions $\vec{p}_o(i)$. The attenuation from a given node will depend on the orientation of the sound wave vector and polarization. If neither of these vectors is perpendicular to $\vec{p}_o(i)$ then the quasiparticles at the node can couple to the sound wave. In this configuration, the node is *"activated"* [Fig. 1(a)]. If, however, either the wave vector direction $\hat{q}$ or the polarization $\hat{u}$ are perpendicular to the node, it is *"inactive"* and quasiparticles at the bottom of the node will not couple to the ultrasound [Fig. 1(b)]. In this configuration, the attenuation produced by the node is strongly suppressed. For most orientations of the ultrasound, the nodes are "activated," and their contribution to the ultrasound attenuation may be written as

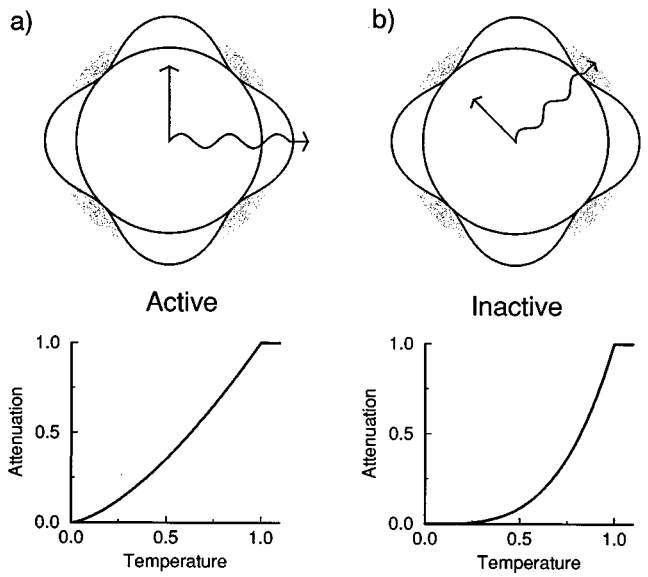

FIG. 1. (a) The nodes are "activated"; (b) $\hat{q}$ or $\hat{u}$ are perpendicular to the nodes, the nodes are "inactive."

$$\bar{\eta}_A=(v_F p_F)^2\sum_i\ [\hat{u}\cdot\hat{p}_o(i)]^2[\hat{q}\cdot\hat{p}_o(i)]^2\mathscr{A}_i(T), \tag{13}$$

where

$$\mathscr{A}_i(T)=\overline{[2N_i(E)\Gamma_i(E)]^{-1}}=\int_{|\vec{p}-\vec{p}_o(i)|<\Lambda}\frac{d^dp}{(2\pi)^d}\left(-\frac{\partial f}{\partial E_{\vec{p}}}\right)\left(\frac{\partial E_{\vec{p}}}{\partial\epsilon_{\vec{p}}}\right)^2\tau_{\vec{p}} \tag{14}$$

is the thermal average of the inverse product of twice the relaxation rate $\Gamma_i(E)$ and the quasiparticle density of states $N_i(E)$ in the gap node.

Suppose instead the gap node is "inactive," with the polarization vector $\hat{u}$ at right angles to $\vec{p}_o(i)$, then the contribution to the ultrasonic attenuation contains the additional factor $\cos^2(\theta)$, where $\cos\theta=(\hat{k}\cdot\hat{u})$. In this case

$$\bar{\eta}_I(i)=(v_F p_F)^2[\hat{q}\cdot\hat{p}_o(i)]^2\mathscr{B}_i(T), \tag{15}$$

where

$$\mathscr{B}_i(T)=\overline{\cos^2\theta_{\vec{p}}[2N_i(E)\Gamma_i(E)]^{-1}}=\int_{|\vec{p}-\vec{p}_o(i)|<\Lambda}\frac{d^dp}{(2\pi)^d}\left(-\frac{\partial f}{\partial E_{\vec{p}}}\right)\left(\frac{\partial E_{\vec{p}}}{\partial\epsilon_{\vec{p}}}\right)^2\tau_{\vec{p}}\cos^2\theta^2_{\vec{p}}. \tag{16}$$

For a point node where the size of the node grows linearly with energy, $\mathscr{B}_i(T)\sim T^2\mathscr{A}_i(T)$. A similar result holds for a line node which lies in a plane. It thus follows that if a

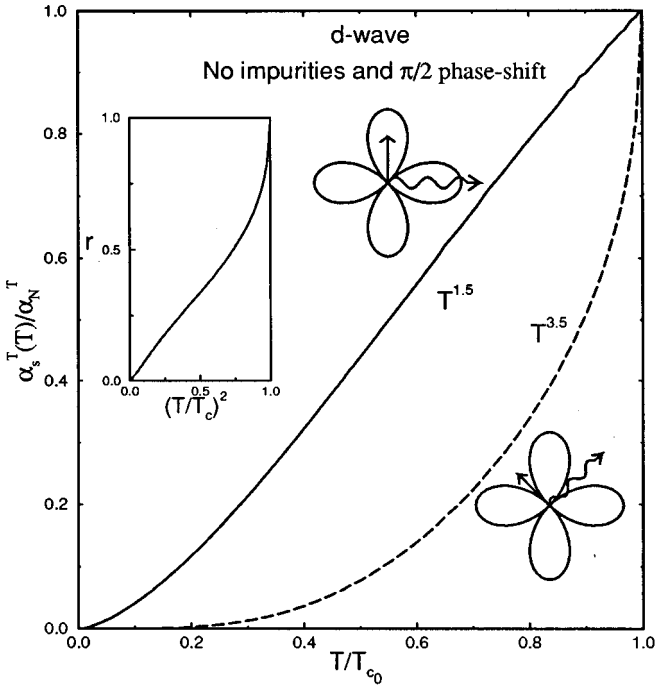

FIG. 2. Ultrasound attenuation for a d-wave state at two angles: $\phi=0$ (solid line) and $\phi=\pi/4$ (dashed line). In the inset the ratio $r=\alpha_s(\pi/4)/\alpha_s(0)$ as a function of $(T/T_c)^2$.

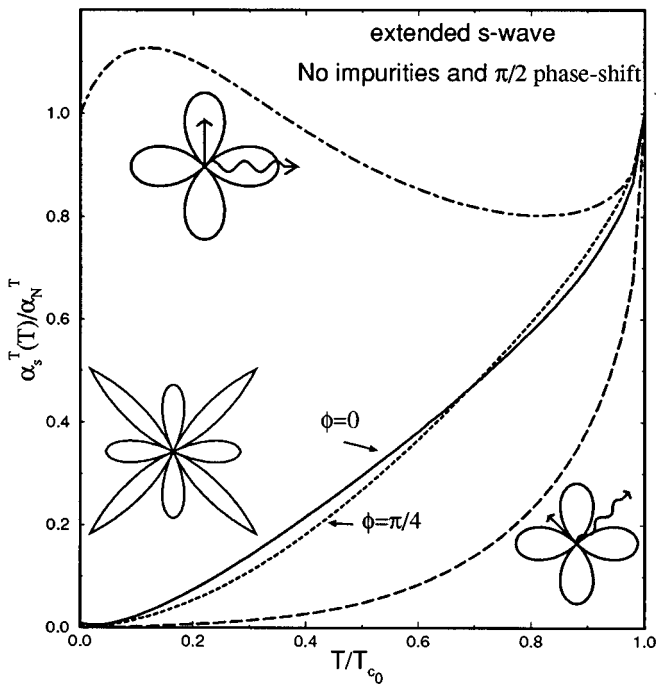

FIG. 3. Ultrasound attenuation for two extended s-wave states: $\Delta_s(\theta_p)=\Delta_0|\cos 2\theta_p|$ at angles $\phi=0$ (dot-dashed line) and $\phi=\pi/4$ (dashed line), and $\Delta_s(\theta_p)=\Delta_0[|\cos 2\theta_p|-(2/\pi)]$ at $\phi=0$ (solid line) and $\phi=\pi/4$ (dotted line).

configuration can be found where all nodes are simultaneously inactive, then the ultrasonic attenuation will exhibit a relaxation rate a factor of T^2 smaller. Such a situation will occur for point nodes situated at 90° to each other. It will also occur for line nodes lying in a plane, or lying in many planes that intersect at 90°. If the nodes do not lie in such 90° configurations, then ultrasound attenuation will not show an anisotropic dependence of the power laws.

To illustrate this discussion, we now make a more detailed application to a two-dimensional model relevant to the cuprate superconductors. A number of recent experiments have provided strong evidence for an anisotropic gap with nodes at the Fermi surface.[4,16] Superconducting interference experiments sensitive to the phase of the gap function[17] support an order parameter with d-wave symmetry with nodes lying at 45° to the a and b axis. Ultrasound measurements provide a complimentary approach which is sensitive to the precise location of the nodes. In particular, it offers the potential to distinguish between a $d_{x^2-y^2}$ state and an anisotropic s-wave state with nodes located either side of the 45° position, and there are no ambiguities associated with the interpretation of results for bilayer compounds.

Using the symmetry arguments advanced above, if the wave vector $\vec{q}$ forms an angle ϕ with the x axis, the transverse ultrasound attenuation $\alpha^T(\phi)$, will have the form

$$\alpha^T(\phi)=[\alpha^T(\pi/4)-\alpha^T(0)]\sin^2 2\phi+\alpha^T(0). \tag{17}$$

Suppose the quasiparticle excitation spectrum has the BCS form $E_{\vec{p}}=\sqrt{\epsilon_{\vec{p}}^2+\Delta_{\vec{p}}^2}$. Using the results obtained above, this leads to

$$\frac{\alpha(\vec{q},\hat{u})}{\alpha_N(\vec{q},\hat{u})}=\frac{1}{\langle\Pi^2(\vec{q},\hat{u})\rangle}\int d\omega\left(-\frac{\partial f}{\partial\omega}\right)\frac{\tau(\omega)}{\tau_N}\times\int_0^{2\pi}\frac{d\theta_p}{2\pi}\,\mathrm{Re}\left(\frac{\sqrt{\omega^2-|\Delta_p|^2}}{\omega}\right)\Pi^2(\vec{q},\hat{u}), \tag{18}$$

where $\Pi(\vec{q},\hat{u})=(\hat{p}\cdot\hat{u})(\hat{p}\cdot\hat{q})$ and α_N and τ_N are the normal attenuation and relaxation times.

To represent the d-wave order parameter we have chosen the function

$$\Delta_d(\theta_p)=\Delta_d\cos 2\theta_p\,. \tag{19}$$

The anisotropic s wave is represented by

$$\Delta_s(\theta_p)=\Delta_s\Phi_s(\theta_p)+\langle\Delta\rangle, \tag{20}$$

where $\Phi_s(\theta_p)=|\cos(2\theta_p)|-(2/\pi)$. We have treated impurity scattering in the standard self-consistent T-matrix approximation.[18]

For a pure d-wave state (Fig. 2) the transverse ultrasound attenuation is proportional to $T^{1.5}$ when the angle is zero (the wave vector and polarization of the sound wave are parallel to the symmetry axis of the crystal) and it is proportional to $T^{3.5}$ when the angle is $\pi/4$. When there is a finite density of impurities a flat region appears in both attenuation coefficients (the impurities induced a finite density of quasiparticle states at zero temperature), but at higher temperatures the power laws are unchanged. Then, in accord with our earlier arguments,

$$\frac{\alpha^T(\pi/4)}{\alpha^T(0)} \propto \frac{T^{3.5}}{T^{1.5}} = T^2. \tag{21}$$

Even considering nonresonant scattering (taking the cotangent of the scattering phase shift $c=1$) this ratio shows a quadratic behavior.

The results for a pure s-wave state are very different (Fig. 3). We have chosen two different anisotropic s waves, $\langle\Delta\rangle = \frac{2}{\pi}\Delta_s$ and $\langle\Delta\rangle = 0$. Unlike d-wave pairing, here the anomalous scattering off impurities is finite and resonant scattering does not ever develop.[19] Consequently, the attenuation for an s-wave gap with nodes at 45° is even more anisotropic, it is finite when the angle is zero, and approaches zero when $\phi = \pi/4$. On the other hand, when the nodes of the gap function depart from $\pi/4$ the ultrasound attenuation anisotropy disappears, in fact at temperatures close enough to T_c, the attenuation at $\phi = \pi/4$ actually becomes bigger than at $\phi = 0$. *In neither case* is the ratio $\alpha^T(\pi/4)/\alpha^T(0)$ proportional to T^2.

The generality of our approach lends itself naturally to many other strongly correlated systems with point nodes. One particularly interesting case is the narrow gap Kondo insulators, CeNiSn and CeRhSn, where Miyake *et al.*[20] have suggested point nodes on the c axis as an explanation of the unusual NMR relaxation rate and the anisotropy of the conductivity. There are several other strongly correlated superconductors that deserve examination, such as V_3Si and UPd_2Al_3 which exhibit T^3 NMR relaxation rates characteristic of gap lines. The heavy fermion superconductor UBe_{13} is noteworthy here; early ultrasound measurements,[21] found no anisotropy, despite the clear suggestion of line nodes from NMR measurements.[22] This result suggests the presence of domains with different orientations, and it would be interesting to repeat the measurements using field cooling to obtain an aligned single domain superconductor.

This paper has emphasized the model-independent aspects of ultrasound attenuation as a probe of nodal gap structure. This attenuation is independent of the mechanism driving superconductivity and sensitive only to the intrinsic symmetries of quasiparticle excitation spectrum. Its tensorial character permits the measurement of several independent components at the same time. The simple methods developed provide an economic way to extract vital information about the gap anisotropy in a general class of gap-anisotropic systems, and they appear to provide a discriminating tool for elucidation of gap structure in the cuprates.

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