

Phases of two coupled Luttinger liquids

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A model of two interacting one-dimensional fermion systems (“Luttinger liquids”) coupled by single-particle hopping is investigated. Bosonization allows a number of exact statements to be made. In particular, for forward scattering only, the model contains two massless boson sectors and an Ising-type critical sector. For general repulsive interactions, there is a spin excitation gap and either d -type pairing or orbital antiferromagnetic fluctuations dominate. It is argued that the same behavior is also found for strong interactions. A possible scenario for the crossover to a Fermi liquid in a many-chain system is discussed.

The properties of a strictly one-dimensional interacting fermion system are by now rather well understood.^{1,2} The typical phenomenology (called “Luttinger liquid”³) is characterized by a separation of the dynamics of spin and charge and by interaction-dependent power laws in many correlation functions, and is thus quite different from Fermi-liquid behavior familiar from higher-dimensional systems. On the other hand, the effects of coupling between parallel chains, present in any real *quasi*-one-dimensional system, are still a subject of debate.^{4–6} Considerable effort has been devoted to the understanding of the properties of many coupled chains,⁵ however, it is in many respects unclear how to connect these results to the strictly one-dimensional case. A possible bridge between the single- and many-chain cases are two (and possibly three, four, etc.) coupled chains. The two-chain case is also of relevance for experiments on $\text{Sr}_2\text{Cu}_4\text{O}_6$,⁷ $(\text{VO})_2\text{P}_2\text{O}_7$,⁸ and possibly the blue bronzes⁹ (in this last case three-dimensional phonons certainly play an important role).

The two-chain model has thus attracted considerable interest, both analytically^{10–13} and numerically.^{14–16} Nevertheless, there is little general information on the low-lying excitations or on the possible ground-state phases. In the present paper, I investigate this problem for a small interchain hopping integral and small intrachain interaction, but with their relative size left arbitrary. Using the standard bosonization procedure, a rather complete picture of the different possible phases and the excitation spectrum will emerge. It will further be shown that the low-energy properties found for weak interactions also exist in the strong-interaction limit, suggesting that weak and strong interaction are in the same phase of the coupled-chain model.

The model I consider is given by the Hamiltonian

$$H = H_1 + H_2 - t_\perp \int dx (\psi_{rs1}^\dagger \psi_{rs2} + \text{H.c.}). \quad (1)$$

Here $H_{1,2}$ are the (identical) Hamiltonians of the two chains,^{1,2} each characterized by a Fermi velocity v_F and forward and backward scattering interaction g_2 and g_1 , t_\perp is the interchain hopping amplitude, and ψ_{rsi} is the fermion field operator for right ($r = +$) or left ($r = -$) going particles of spin s on chain i . To start, I neglect the backward scattering g_1 . The following analysis is then initially identical to that of Ref. 10. The Hamiltonian is transformed by the fol-

lowing steps: (i) Introduce bonding and antibonding operators via $\psi_{rs0} = (\psi_{rs1} + \psi_{rs2})/\sqrt{2}$, $\psi_{rs\pi} = (\psi_{rs1} - \psi_{rs2})/\sqrt{2}$; (ii) introduce charge and spin boson fields $\phi_{\rho,\sigma;0,\pi}$ corresponding to the 0 and π fermions, following the standard procedure; (iii) form the linear combinations $\phi_{\nu\pm} = (\phi_{\nu 0} \pm \phi_{\nu\pi})/\sqrt{2}$ ($\nu = \rho, \sigma$). The noninteracting Hamiltonian (including t_\perp) then takes the form

$$H_0 = \frac{\pi v_F}{2} \sum_{\substack{\nu=\rho,\sigma \\ \alpha=\pm}} \int dx \left[\Pi_{\nu\alpha}^2 + \frac{1}{\pi^2} (\partial_x \phi_{\nu\alpha})^2 \right], \quad (2)$$

where $\Pi_{\nu\alpha}$ is the momentum density conjugate to $\phi_{\nu\alpha}$, and the interaction is

$$H_{\text{int},2} = \frac{1}{4} \int dx \sum_{\gamma=\pm} g_\gamma^{(2)} \left[\frac{1}{\pi^2} (\partial_x \phi_{\rho\gamma})^2 - \Pi_{\rho\gamma}^2 \right] + \frac{g_{00\pi\pi}^{(2)}}{2(\pi\alpha)^2} \int dx \cos 2\theta_{\rho-} (\cos 2\phi_{\sigma-} + \cos 2\theta_{\sigma-}). \quad (3)$$

Here α is a short distance cutoff, $\partial_x \theta_{\beta\gamma} = \pi \Pi_{\beta\gamma}$, $g_\gamma^{(2)} = g_{0000}^{(2)} + \gamma g_{0\pi\pi 0}^{(2)}$, and I use the notations of Ref. 11: $g_{abcd}^{(2)}$ is the coupling constant for an interaction scattering two particles from states (a, b) into (d, c) . Initially, all the g 's in Eq. (3) equal g_2 , but renormalization will give rise to differences. At energy scales higher than t_\perp an additional process of type $g_{0\pi 0\pi}^{(2)}$ also exists and is responsible for the fact that g_2 is not renormalized in the purely one-dimensional problem $t_\perp = 0$ (this process also only involves the ρ_- and σ_- fields). At energies below t_\perp the $g_{0\pi 0\pi}^{(2)}$ process becomes, however, forbidden due to energy and momentum conservation, and Eq. (3) is then indeed the full forward scattering Hamiltonian.

One now can notice that the ρ_+ and σ_+ parts of the Hamiltonian remain bilinear, and the corresponding fields are thus massless. On the other hand, there are nontrivial interaction effects for the coupled ρ_- and σ_- fields: one finds coupled Kosterlitz-Thouless-type renormalization-group equations for $g_{00\pi\pi}^{(2)}$ and $g_{-}^{(2)}$.^{10,17} For the initial conditions appropriate here, these equations always scale to strong cou-

pling, and the standard interpretation¹⁰ then is that there is a gap $\Delta_0 \approx t_\perp \exp(-\pi^2 v_F / |g_2|)$ for both the ρ_- and σ_- degrees of freedom.

That things are actually a bit more subtle can be seen noting that the σ_- part of the Hamiltonian is the continuum transfer matrix of a two-dimensional classical XY model with twofold anisotropy field $\cos 2\phi_{\sigma_-}$ [the XY spins then are $(S = \cos\phi_{\sigma_-}, \sin\phi_{\sigma_-})$].^{18,19} This model has Ising-type symmetry, with order parameter $\sin\phi_{\sigma_-}$, and the symmetry of the Hamiltonian under the duality transformation $\phi_{\sigma_-} \leftrightarrow \theta_{\sigma_-}$ implies that the model is critical. The duality symmetry is related to the fact that the left- and right-going fermions are independently invariant under spin rotation, i.e., there is a chiral $SU(2) \times SU(2)$ symmetry in the fermionic model.

What are the physical properties of the pure forward scattering model? First, there are massless modes in the ρ_+ and σ_+ channels, giving a total specific heat $C(T) = (\pi T/3)(1/u_{\rho_+} + 1/u_{\sigma_+} + 1/2v_F)$, where the total charge and spin velocities are given by $u_{\rho_+}^2 = v_F^2 - (g_2/\pi)^2$ and $u_{\sigma_+} = v_F$, and the factor 1/2 in the last term comes from the Ising critical behavior [with central charge $c = 1/2$ (Ref. 20)]. The compressibility is determined by the ρ_+ modes only and given by $\kappa^{-1} = \pi \rho_0^2 u_{\rho_+} / 4K$, where ρ_0 is the equilibrium particle density and $K^2 = (\pi v_F - g_2) / (\pi v_F + g_2)$. Similarly, the (Drude) weight of the zero-frequency peak in the conductivity is $\sigma_0 = 4u_{\rho_+} K$. As in the one-chain case,⁶ these relations can in particular be used to determine the coefficient K which determines power laws of different correlation functions.

Naturally, the present model does not have broken symmetry ground states, but as in the one-chain case there are divergent susceptibilities of different types, indicating incipient instabilities. I first consider $g_2 > 0$. To obtain the long-wavelength (low-energy) asymptotics of correlation functions one has to analyze the consequences of the nonlinear term in Eq. (3) which scales to strong coupling ($g_{00\pi\pi}^{(2)} \rightarrow \infty$). A semiclassical treatment is appropriate, and then the energy is minimized by $\theta_{\rho_-} = 0$ (there are different degenerate solutions which all lead to identical physical results). Following standard arguments¹ long-range order of the θ_{ρ_-} field implies exponentially decaying ϕ_{ρ_-} correlations. On the other hand, from the Ising analogy for the σ_- sector correlations of the order parameter $\sin\phi_{\sigma_-}$ and its dual $\sin\theta_{\sigma_-}$ then decay as $r^{-1/4}$ whereas correlation of the non-ordering $\cos\phi_{\sigma_-}$ and $\cos\theta_{\sigma_-}$ fields decay exponentially. These points have not been appreciated in previous work on this model. Consider now, for example, charge density oscillations which are out of phase between the two chains, described by the operator $O_{CDW\pi} \approx e^{i\phi_{\rho_+} + \cos\phi_{\sigma_+} + \sin\theta_{\sigma_-}}$.²¹ From the massless modes the CDW $_{\pi}$ correlations then decay as $r^{-(3+2K)/4}$, giving rise to a susceptibility diverging as $T^{(2K-5)/4}$. The analogous spin (SDW $_{\pi}$) correlations obey the same power law, whereas in-phase correlations decay exponentially.

Similar considerations apply to BCS-type instabilities. It turns out that long-range correlations exist for the pairing operator

$$O_{SCd} = \sum_s s (\psi_{-, -s, 0} \psi_{+, s, 0} - \psi_{-, -s, \pi} \psi_{+, s, \pi}) \quad (4)$$

and its triplet analog. It seems appropriate to call this form “ d wave” because pairing amplitudes of the “transverse” modes 0 and π intervene with opposite sign. The bosonic form of this operator is $O_{SCd} \approx e^{i\theta_{\rho_+} + \sin\phi_{\sigma_+} \sin\theta_{\sigma_-}}$. The corresponding susceptibilities diverge like $T^{(2/K-5)/4}$. Because for $g_2 > 0$ one has $K < 1$ this divergence is subdominant compared to the CDW $_{\pi}$ and SDW $_{\pi}$ ones. It may seem surprising that the exponents do not tend to zero as $g_2 \rightarrow 0$, however, one should notice that the power laws are valid for $T < \Delta_0$, and because $\Delta \rightarrow 0$ for $g_2 \rightarrow 0$ there is a nontrivial crossover in the noninteracting limit. In all other pairing correlations, “ s -wave” superconductivity in particular a plus instead of the minus sign in Eq. (4), the leading divergent terms cancel and one therefore has exponential decay of correlation functions and finite susceptibilities as $T \rightarrow 0$.

For negative g_2 the picture changes quite drastically, because now scaling goes to $g_{00\pi\pi}^{(2)} \rightarrow -\infty$, and consequently the Ising order parameter is $\cos\phi_{\sigma_-}$. Now $K > 1$, and the dominant divergent susceptibility is then easily found to be standard s -wave superconductivity, with exponent $(2/K - 5)/4$. The subdominant divergence occurs for orbital antiferromagnetic operators²² of the form $\psi_{+, s\pi}^\dagger \psi_{-, s0} - \psi_{+, s0}^\dagger \psi_{-, s\pi}$ and its triplet analog (the spin nematic).

Consider now the backscattering interaction g_1 . I will only treat the repulsive case $g_1 > 0$. In a purely one-dimensional system this then scales to zero as $g_1(E) = g_1 / [1 + g_1 / (\pi v_F) \ln(v_F/E\alpha)]$ when the running cut-off E goes to zero. In the coupled chain problem, the one-dimensional scaling breaks down for $E \approx t_\perp$. For small t_\perp the effective $g_1^* = g_1(t_\perp)$ will then indeed be a perturbation. Simultaneously, g_2 is renormalized to $g_2^* = g_2 - g_1/2 + g_1^*/2$. The backscattering Hamiltonian takes the form

$$\begin{aligned} H_{\text{int},1} = & \frac{g_1^*}{2(\pi\alpha)^2} \int dx \{ \cos 2\phi_{\sigma_+} (\cos 2\theta_{\rho_-} + \cos 2\phi_{\sigma_-} \\ & - \cos 2\theta_{\sigma_-}) - \cos 2\theta_{\rho_-} \cos 2\theta_{\sigma_-} \} \\ & - \frac{g_1^*}{4} \int dx \left[\frac{1}{\pi^2} \{ (\partial_x \phi_{\rho_+})^2 + (\partial_x \phi_{\sigma_+})^2 \} \right. \\ & \left. - \Pi_{\rho_+}^2 - \Pi_{\sigma_+}^2 \right]. \end{aligned} \quad (5)$$

First, the $\theta_{\rho_-} - \theta_{\sigma_-}$ interaction now breaks the self-duality of the ϕ_{σ_-} fields. As $\cos 2\theta_{\rho_-}$ already has a nonzero expectation value from the g_2 interaction, one now also finds a gap in the σ_- modes, of order $\Delta_\sigma = (g_1^*/g_2^*)\Delta_0$. In the Ising model language, this corresponds to a deviation from criticality, long-range order, and exponentially decaying $\sin\theta_{\sigma_-}$ correlations. Secondly, the forward scattering interaction also leads to a nonzero expectation value of $\cos 2\theta_{\rho_-} + \cos 2\phi_{\sigma_-} - \cos 2\theta_{\sigma_-}$ which by spin rotation invariance has to be positive. The leading order effect of the first term in Eq. (5) then is to open a gap also in the σ_+ degrees of freedom, given, up to numerical factors, by Δ_σ . In the presence of the backscattering interaction *there thus is a gap in all the magnetic excitations*.

In correlation functions, to leading order one now replaces ϕ_{σ_+} by its classical value $\pi/2$. One then finds for $g_2 > 0$ a decay of the SCd correlations as $r^{-1/2K}$, giving rise

to a divergence of the corresponding susceptibility as $T^{1/2K-2}$, where now $K^2 = (\pi v_F - g_2 + g_1/2) / (\pi v_F + g_2 - g_1/2)$. On the other hand, the CDW_π and SDW_π operators contain the Ising disorder field, and therefore these correlations decay exponentially. A divergent density response exists for correlations of the form $\langle [O_{CDW\pi}(r)]^2 [O_{CDW\pi}(0)]^2 \rangle \propto \cos[2(k_{F0} + k_{F\pi})r] r^{-2K}$,¹³ because here the operator $\sin^2 \theta_{\sigma-} \approx 1/2$ appears. Perturbative and symmetry arguments show that the same contribution also exists in the density correlations: $\langle n(r)n(0) \rangle \propto \cos[2(k_{F0} + k_{F\pi})r] r^{-2K}$, in analogy with the $4k_F$ oscillations of a single chain. However, at least for weak interactions ($K \rightarrow 1$) the corresponding susceptibility is much weaker than the SCd pairing, i.e., *the two-chain model has predominant pairing fluctuations even for purely repulsive interactions*.^{11,23} In the regime of negative g_2 the leading divergent susceptibility is of orbital antiferromagnetic type, with exponent $K/2 - 2$. The precise boundary between the two regimes can be determined from the scaling equations of Ref. 17 and is given by $g_1 = 2g_2$. This again is an Ising-type critical line. The triplet susceptibilities (spin density wave or triplet superconductivity) are suppressed by the spin gap. The spin gap gives rise to ‘‘anomalous flux quantization,’’²⁴ and there is also a gap for single-particle excitations.

The power laws discussed above apply in the temperature region below Δ_0 . In the intermediate region $\Delta_0 < T < t_\perp$ the $g_{00\pi\pi}^{(2)}$ term in Eq. (3) has little effect, and one then can obtain the temperature dependence of different correlation functions from a purely bilinear Hamiltonian. For example, for CDW_0 susceptibilities one finds a power law $T^{(K-1)/2}$, whereas in the one-dimensional region $T > t_\perp$ one has a behavior as T^{K-1} . The important point here is that in the intermediate region the interaction-dependent exponent is smaller than in the high-temperature region, i.e., below t_\perp the system behaves more closely like a Fermi liquid than at high temperatures.

I now consider the strongly interacting case. For sufficiently strong intrachain interactions, i.e., small parameter K_ρ of the individual chains, single-particle hopping is renormalized to zero, however, simultaneously particle-hole tunneling processes appear.^{5,6} Introducing $\phi_{\nu\pm} = (\phi_{\nu1} \pm \phi_{\nu2})/\sqrt{2}$, where $\phi_{\nu1,2}$ are the boson fields of the individual chains, for the purely forward scattering case, these terms take the form $J \cos 2\phi_{\rho-} (\cos 2\phi_{\sigma-} + \cos 2\theta_{\sigma-})$. One again has a duality symmetry, $\phi_{\sigma-} \leftrightarrow \theta_{\sigma-}$, and the same types of power-law correlations as in the weak-coupling case appear. Introducing now intrachain backscattering, the duality symmetry is broken and, again as in the weak-coupling case, only SCd correlations (exponent $1/2K_\rho$) and $4k_F$ charge correlations (exponent $2K_\rho$) remain. The types of possibly divergent response functions and the scaling relations between different exponents are thus identical for weak and strong interaction. This strongly suggests that *this type of behavior actually holds for arbitrary interaction strength*.²⁵ Note that the density correlations decay more slowly than the pairing correlations only for $K_\rho < 1/2$. This typically corresponds to rather strong repulsion: for example, in the one-dimensional Hubbard model one reaches $K_\rho = 1/2$ only for infinite repulsion.⁶ Another interesting strong-coupling model is the ‘‘ t - J ladder.’’¹⁵ Here in the limit of strong in-

terchain exchange a mapping onto an effective single-chain hard core boson model can be made, leading again to the same low-energy properties as in the weak-coupling limit.²⁶ Recent numerical results^{14,16} confirm this point.

The exponents $K-1$ and $(K-1)/2$ valid for the single- and double-chain problems suggest that for N chains coupled by near-neighbor interchain hopping one might have an anomalous exponent $(K-1)/N$ at $T < t_\perp$. To see how such a behavior can possibly arise, in analogy to the two-chain case one can go to momentum space in the transverse direction. The noninteracting bosonized Hamiltonian then is

$$H_0 = \frac{\pi v_F}{2} \sum_{\nu=\rho,\sigma} \int_{k_\perp} dx \left[\Pi_{\nu k_\perp}^2 + \frac{1}{\pi^2} (\partial_x \phi_{\nu k_\perp})^2 \right]. \quad (6)$$

Following standard arguments²⁷ I now only consider forward-scattering interactions which for states at the Fermi energy are consistent with both energy and momentum conservation. The analog of the first term in Eq. (3) then is

$$H_{\text{int},2} = \frac{g_2}{2} \int dx \left[\frac{1}{\pi^2} (\partial_x \phi_{\rho 0})^2 - \Pi_{\rho 0}^2 \right], \quad (7)$$

where $\phi_{\rho x}$ is the Fourier transform of $\phi_{\rho k_\perp}$ with respect to k_\perp . The important point here is that *only the mode at $x=0$ is affected by the interactions*. A standard calculation then leads to a decay of CDW correlations as $r^{-2-(K-1)/N}$, giving rise to a susceptibility behaving as $T^{(K-1)/N}$. Similarly, the single-particle Green function decays as $r^{-1-\delta}$, with $\delta = (K+1/K-2)/4N$, leading to a singularity of the momentum distribution function as $|k-k_F|^\delta$.²⁸ In the limit of a large number of coupled chains the anomalous exponents now vanish, and in particular one recovers a Fermi-liquid-like momentum distribution function in this description. More realistically, v_F is k_\perp dependent. The model then can still be solved, even though details are more complicated. The leading $1/N$ corrections to Fermi-liquid theory still hold, and in addition the $x=0$ mode becomes the zero sound mode of a Fermi liquid, well separated from a particle-hole quasicontinuum.

Clearly, a number of interactions have been neglected in this argument. First, there are Cooper-type $[(k, -k) \rightarrow (k', -k')]$ and possibly nesting interactions, the prototype of which is given by the $g_{00\pi\pi}^{(2)}$ term in Eq. (3). By analogy with that case I expect these interactions to give rise to a gap of order Δ_0 , and to ordered ground states for $N \rightarrow \infty$. Thus the power laws of the preceding paragraphs are valid in the temperature region $\Delta_0 < T < t_\perp$. Moreover, there are interactions that involve at least one state not exactly at the Fermi energy. Though these interactions cannot directly affect the low-energy physics, they in general will lead to renormalizations of g_2 . The above arguments remain valid if these renormalizations are nonsingular. To which extent this is correct is currently under investigation.

In conclusion, I have investigated the phase diagram and excitation spectrum of two Luttinger liquids coupled by single-particle hopping, and proposed a possible extension to many coupled chains. The conclusions are valid for small hopping amplitude, but the same types of divergent responses (d -type superconductivity and $4k_F$ charge density in the case of repulsion) occur for both weak and strong inter-

actions, suggesting that this type of behavior is to be found for rather general interactions. The fact that for strong interaction interchain hopping renormalizes to zero^{6,5,11} only affects properties at intermediate energy scales (above the spin gap). Contrary to the case of a single chain, the pure forward-scattering model is found to be a singular line in the phase diagram, with Ising-type criticality.

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