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Symmetry breaking of the admittance of a classical two-dimensional electron system in a magnetic field

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The symmetry properties under magnetic-field reversal are reported for the elements of the admittance matrix of a set of probes that couple capacitively to a nondegenerate two-dimensional electron system, which in itself is not connected to an electron reservoir. Strong asymmetries are observed in multiterminal measurement configurations (three probes or more), whereas two-terminal measurements are symmetric. The asymmetries are explained in terms of classical edge magnetoplasmons and satisfy the generalized Onsager-Casimir symmetry requirements. [S0163-1829(96)50320-6]

Measurement techniques involving one or more capacitively coupled probes have been extensively used to investigate the properties of two-dimensional electron systems (2DES's). They have been used for studies of the energy level structure of the 2DES-hosting semiconductor structures,[1] for the electronic density of states (DOS) determination of a 2DES in a magnetic field,[1,2] and for convenient contactless measurements of the 2DES (magneto) conductivity.[3] In spite of this, an incomplete basic understanding of capacitance techniques still leads to ambiguities in the interpretation of experimental data in terms of DOS or conductivity effects.[4,5] A new interpretation, implying that the capacitance is determined simply geometrically by the *conducting* part of the area of the 2DES, was proposed recently.[5] In the modern concept of edge states,[6] the conducting area of a 2DES varies from the total area to a series of small strips near the edges, depending on whether the Fermi level in the bulk is near the center or in between Landau levels.[7]

Another issue of fundamental importance concerns the symmetry properties of the capacitance tensor of a 2DES in a magnetic field.[8] In a multilead (number of leads >2) measurement configuration an almost complete *asymmetry* was observed in the capacitances between a gate and a dc contact under magnetic-field reversal. In order to explain the physical origin of this asymmetry a model was presented based on current flow along edge states.[8] No interpretation in classical terms was given, but it was clearly stated in Ref. 8 that the asymmetry should be caused by a classical effect, as it was observed at temperatures above 100 K. The observations were consistent with the fundamental Onsager-Casimir symmetry requirements,[9] as were worked out for capacitances in Ref. 10. Partial density of states concepts as used in Ref. 8 are general and can be applied to either quantum or classical systems, just like the Onsager-Casimir symmetry requirements.

Capacitance methods form the only method for measuring the (low-frequency) transport properties of the 2DES formed by surface state electrons (SSE's) on liquid helium.[11] Therefore, the method is highly developed here, even in the case of high magnetic fields.[12] In many respects, the SSE system is far more simple than the 2DES's in semiconductors. Because the system is nondegenerate, quantum edge states are absent. In this paper, we report the magnetic-field symmetry relations for the *admittance* of the capacitively coupled SSE system. It is shown that these are qualitatively very similar to those for the pure capacitances of the degenerate system.[8] For the present case, however, they can be directly understood in terms of the classical phenomenon of edge magnetoplasmons (EMP's). This is a direct result of the Hall effect and is believed to be the low-frequency classical analog of the quantum edge state, although the direct relation is not clear. Because of the absence of any direct electrical contact, the present 2DES will be described by a canonical ensemble, as opposed to the degenerate systems where the chemical potential usually is fixed by direct electrical contacts, forcing one to use a grand-canonical ensemble. In addition to the standard geometries with the contacts at the edges,[8] here also the Corbino geometry has been investigated where edges are not important.

The system under study consists of an assembly of N conductors, which all capacitively couple to the electrically floating 2DES.[13] An oscillating potential v_k applied to terminal k will cause an oscillating current i_l to flow through terminal l $(k,l=1,\dots,N)$. Analogous to Ref. 10, an admittance matrix is defined as $Y_{kl}=i_l/v_k$. Y_{kl} is a complex quantity and its phase may have any value due to phase shifts caused by series resistance effects in the 2DES. This is a more general situation than in Ref. 8, where only pure capacitances (phase shift $\pi/2$) were considered. Only in the low-frequency limit does the admittance become purely capacitive. The field dependence of Y_{kl} is determined by the conductivity σ of the 2DES. In the presence of a magnetic field, σ is an antisymmetric tensor because of the Hall effect. Generalizing the symmetry relations found for pure resistances[9] and pure capacitances,[8] it is expected that $Y_{kl}(B)=Y_{lk}(-B)$.

The sample cell is a cylindrical, parallel plate capacitor of height 3 mm and radius 7.5 mm. The sample space is filled

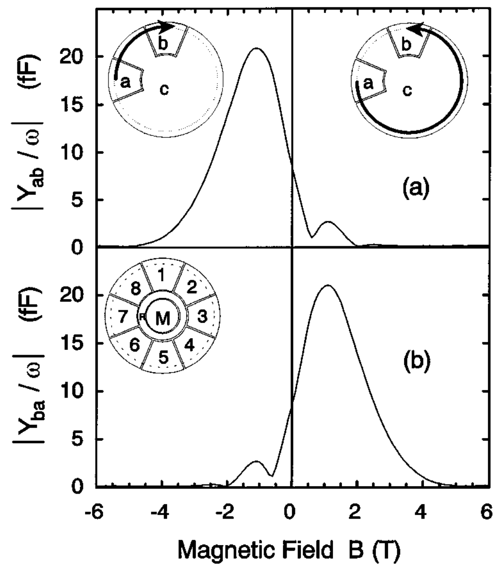

FIG. 1. Absolute magnitude of Y (divided by angular frequency) for a three-terminal measurement; $T = 1.9$ K, areal density $n = 1.78 \times 10^{11}$ m^{-2}, frequency $f = 30$ kHz. The inset of (b) shows the electrode layout of the bottom plate. The insets in (a) show the a, b, and c terminals. The arrows indicate the direction of propagation of the EMP for both field directions. For (b), the voltage and current terminals are interchanged.

up to a height of 1 mm with superfluid ^{4}He. The surface is charged by pulse heating a tungsten filament. Suitable dc potentials applied to the top plate and wall electrodes provide the holding fields. The radius of the 2DES is typically 6.6 mm. The bottom plate is divided into ten electrodes as shown in the inset of Fig. 1. The mobility μ of the 2DES at temperatures $T < 2$ K exceeds 2 m^2 V^{-1} s^{-1}. For magnetic fields $B > 0.5$ T the system is in the classical strong magnetic-field regime ($\mu B > 1$). Different configurations can be realized by electrically combining different electrodes into N groups for a measurement with N terminals. The terminals in a three-terminal configuration are denoted by a, b, and c, where a and b are either voltage or current terminals, and c is at fixed potential (ground). The Y_{kl}'s are determined by phase-sensitive current measurements at frequencies of the order of 10 kHz. It is sufficient for the present discussion to consider only the modulus of Y.

Figure 1(a) shows the strong asymmetry of Y_{ab} with respect to the direction of the magnetic field in a three-terminal measurement. Both excitation and detection terminals are single electrodes at the perimeter of the system. The third terminal is made up of the remaining electrodes of the bottom plate. In zero field Y_{ab} is almost entirely capacitive since the mobility is sufficiently high to cause only a very small phase shift. In this case the value of Y_{ab} agrees well with what is to be expected from perfect capacitive coupling and the geometry. As a magnetic field is applied, Y_{ab} increases for negative fields and has a maximum around -1 T. At more negative fields the signal decays and has almost vanished at about -6 T. For positive magnetic fields Y_{ab} first decreases with B and then falls off to zero in a slightly oscillatory fashion, resulting in an asymmetric B dependence of Y_{ab}. The phase of Y continuously varies and may attain any value.[12,14]

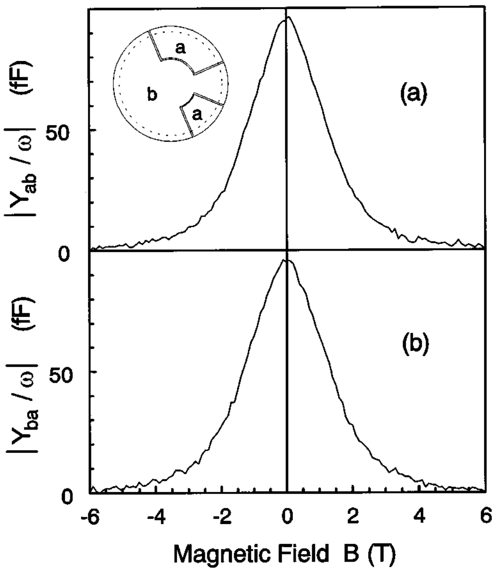

FIG. 2. Absolute magnitude of Y (divided by angular frequency) for a two-terminal measurement for the same parameters as for Fig. 1. The a and b terminals for (a) are as indicated in the inset to (a). For (b), the voltage and current terminals are interchanged.

Figure 2(a) shows the data for a two-terminal measurement, where the geometry of the electrode arrangement has no (simple) symmetry axis. Y_{ab}, however, displays perfect magnetic field symmetry, as expected for a two-terminal measurement.[8]

For the measurement in Fig. 1(b) [and 2(b)], excitation and detection terminals are interchanged with respect to Fig. 1(a) [and 2(a)]. The admittance element Y_{ba} exhibits the same behavior as Y_{ab} but with a reverse field dependence, i.e., we observe $Y_{ab}(B) = Y_{ba}(-B)$ within experimental error, which corresponds to the *Onsager-Casimir reciprocity relation*.[9]

The asymmetry in $Y_{ab}(B)$ for the three-terminal case can be understood in terms of EMP's, which are reviewed in Ref. 15. These modes have been investigated in detail for the SSE under the present circumstances.[12,14] For the present discussion, EMP's can be viewed as voltage waves localized near the edge of the sample and propagating along the edge in a direction determined by the magnetic-field direction. Referring to the data in Fig. 1(a), in zero magnetic field, terminal a excites a voltage wave in the 2DES, which spreads out isotropically from the edge near a to the bulk of the 2DES. When a magnetic field is applied, the Lorentz force will direct this wave along the edge of the system, towards terminal b at negative fields and in the opposite direction for positive fields, which eventually results in the edge mode.[12,14] The currents induced in the electrodes are proportional to the voltage in the 2DES. Therefore at b, for negative fields, the signal first increases, whereas for positive fields, it decreases [see insets to Fig. 1(a)]. Near the maximum at -1 T the edge mode, or EMP, still has a width of the order of the sample radius. The edge mode now also induces a signal at b for positive field after traveling over an angle of $3\pi/2$ along the

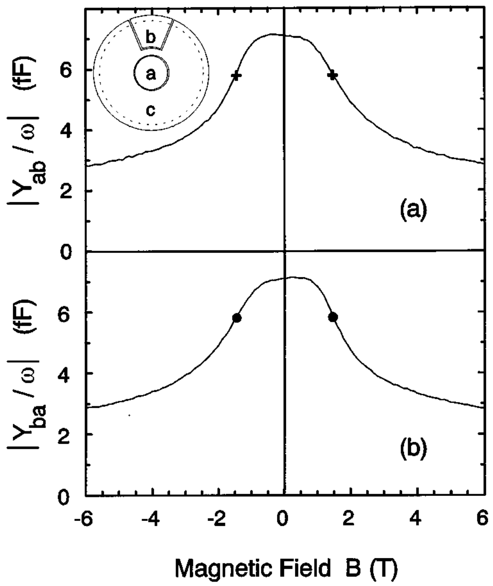

FIG. 3. Absolute magnitude of Y (divided by angular frequency) in a three-terminal Corbino-type measurement, T = 1.9 K, areal density $n=1.78\times10^{11}$ m^{-2}, frequency $f=10$ kHz. The a, b, and c terminals for (a) are as indicated in the inset. For (b), the voltage and current terminals are interchanged. The symbols correspond to the field value for which the calculations of Fig. 4 are done.

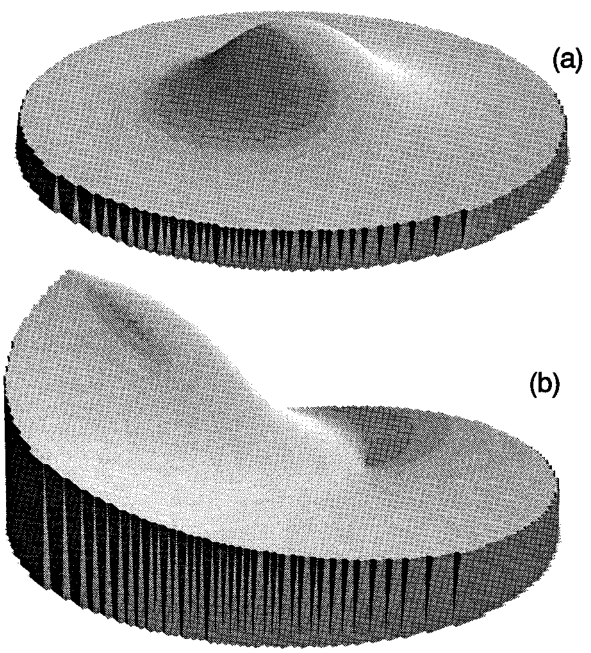

FIG. 4. Absolute magnitude of the potential in the 2DES at $B=1.5$ T for the parameters of Fig. 3; (a) excitation on central electrode and (b) excitation on one of the outer electrodes.

edge, causing the small maximum near +1 T. The decrease at large fields (positive or negative) has two causes. First, the mode narrows (after having reached the maximum voltage amplitude), decreasing the overlap with the current terminal and therefore the signal. Second, there is a field-dependent damping due to magnetoresistance.[16] The EMP's play a role similar to the quantum edge channels in Ref. 8, in the sense that they also confine the current to the edge and give it a definite direction. The asymmetry in Ref. 8 is caused by a second contact at the 2DES, which drains off the current carried by the edge channel, and a signal is only observed when the current in the edge channel passes the measuring terminal. In the present work, the current along the edge damps out by itself, due to the low μB value. At lower temperatures (high μB), EMP's manifest themselves as strong traveling wave resonances when their wavelength matches the sample perimeter.[17] This leads to a far more symmetric pattern. Very recently, asymmetries as sharp as those of Ref. 8 were reported in Ref. 18 for resonating EMP's in a degenerate system. This was established by draining off the EMP current at an additional contact to the 2DES. It shows that the difference in sharpness of the asymmetry in the present case as compared to Ref. 8 is a matter of electron scattering rate only and is not of fundamental importance. The weak oscillatory structure at positive field in Fig. 1(a) is a result of interference effects of the damped wave going around the sample. The data in Fig. 1(a) have a remarkable analogy to results reported in Ref. 19, where asymmetries and oscillations similar to those in Fig. 1 were observed in the transmission matrix elements of a mesoscopic conductor. In that case,[19] the effects are caused by individual electron trajectories under the influence of the Lorentz force.

Another consequence of the reciprocity relations is demonstrated by Corbino-type measurements, where an inner disk and an outer ring are used, respectively, as voltage and current terminals. In this symmetric arrangement EMP's are not excited, and the signal depends on the diagonal conductivity component σ_{xx} only.[12] Even if the current is measured on a part of the outer ring, for example, on only one of the outer electrodes, the only effect (neglecting the voltage drop across the current amplifier) is a reduction of the signal, in this case by a factor of 8. Figure 3(a) shows the matrix element Y_{ab} for a three-terminal arrangement where a is the central electrode and b is one of the outer electrodes. The circularly symmetric ac potential distribution at a certain magnetic field in the 2DES for an excitation on terminal a is shown in Fig. 4(a), as obtained by numerical simulation of the system.[20] Figure 3(b) shows the case when voltage and current terminals are interchanged. The corresponding asymmetric potential distribution shown in Fig. 4(b) shows the existence of a damped EMP wave. In spite of these completely different potential distributions, the reciprocity relation $Y_{ab}(B)=Y_{ba}(-B)$ is still satisfied, as follows from Fig. 3. This forces us to conclude that it is not possible to deduce the reciprocity relation for the elements of the admittance matrix from the symmetry properties of the induced potential distribution in the 2DES. This is different from the three-terminal measurement of Fig. 1, where the reciprocity relation can be directly deduced from the potential distribution. The magnetic-field symmetry of the admittance matrix elements Y_{ab} and Y_{ba} is a result of the azimuthal symmetry of a Corbino-type measurement. Thus not only azimuthal symmetry in the excitation terminal but also in the detection terminal can lead to symmetric behavior under magnetic-field reversal. In the case of Y_{ab}, EMP's cannot be excited because of the axial symmetry of the excitation probe, and in the case of Y_{ba}, EMP's are excited but their different behavior under $B \rightarrow -B$ is not detectable because of the axial symmetry of the detection terminal. The slight magnetic-field asymmetry in the experimental data is due to a small misalignment of the capacitor plates with respect to the ^{4}He surface.

In summary, asymmetries in the elements of the admittance matrix of a capacitively coupled, nondegenerate 2DES as a function of magnetic field were observed in a three-terminal measurement. This asymmetry can be understood in terms of the propagation of EMP's that originate from Lorentz forces. Reversing the magnetic field and exchanging current and voltage terminals give results for the elements of the admittance matrix that are in complete agreement with the *Onsager-Casimir reciprocity relations*. The validity of these relations can be demonstrated very clearly in Corbino-type measurements. These also show that EMP's no longer break the magnetic-field symmetry of the elements of the admittance matrix regardless of whether they are excited or not.

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