Effects of weak disorder on two coupled Hubbard chains

E. Orignac and T. Giamarchi
*Laboratoire de Physique des Solides, Université Paris-Sud, Bâtiment 510, 91405 Orsay, France**
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We consider the effect of weak nonmagnetic disorder on two chains of interacting fermions (with and without spins) coupled by interchain hopping. For the spinless case, interchain hopping increases localization for repulsive interactions but *stabilizes* the *s*-wave superconducting phase for attractive interactions. For the case with spin, the *d*-wave phase arising from purely repulsive interactions in the clean system is destroyed by an infinitesimal disorder, while for attractive interactions, the *s*-wave superconductivity is more resistant to disorder than in the one-chain case. In each case we compute the localization length and the conductivity. [S0163-1829(96)52616-0]

One-dimensional electronic systems are known to be the simplest realizations of non-Fermi liquids, and to have generic properties known as Luttinger liquids.[1] Despite the good understanding of purely one-dimensional systems, the effects of interchain hopping, allowing us to move from one to higher (two or three) dimensions, are much less known. Whether non-Fermi-liquid properties can be retained even in the presence of finite hopping or not is still a controversial issue.[2] As a theoretical understanding of quasi-one-dimensional strongly correlated electronic systems (especially the crossover from Luttinger to Fermi liquid) is crucial for the physics of organic conductors, there has been in recent years, a growing interest in systems of coupled interacting electron chains, and mostly in systems of two chains. They present the advantage of allowing a careful study of the effects of hopping by being tractable by powerful analytical[3–9] and numerical techniques.[10] In addition, there exists good experimental realizations of coupled spin chains such as $Sr_{n-1}Cu_{n+1}O_{2n}$ (Ref. 11) and $VO_2P_2O_7$ (Ref. 12) compounds that will give coupled Hubbard chains upon doping. Although the complete phase diagram of such systems is still under study, a generic property of a two-coupled-chains system is the appearance of a *d*-wave-like superconducting phase for repulsive interactions.

In this work we study the effects of nonmagnetic disorder on two-chain systems, both for the case of spinless electrons and for electrons with spins. Such a study is relevant for various problems: first, in real two-chain systems, disorder will be present, and it is therefore essential to know the stability of the phases found in the pure system, as for a one-chain system an arbitrarily weak disorder destroys superconductivity except for exceedingly attractive interactions.[13] Second, on a more theoretical level, the two-chain problem is the simplest one for studying the effects of interchain hopping onto the Anderson localization in the presence of interactions, giving some clues into the unsatisfactorily understood physics of such a transition in more than one dimension. It also allows for the study of the effects of interactions on persistent currents. Finally, classical systems of coupled planes, studied in the context of vortices in type-II superconductors,[14] can be mapped to coupled quantum chains so that the results and methodology developed here are relevant for them. We show here that for the spinless model the superconducting phase for attractive interactions is *stable* towards weak disorder, the opposite of what happens for a one-chain system. For the model with spins and weak enough attractive interactions an arbitrarily weak disorder destroys the superconductivity, as in the one-chain case. Nevertheless, the threshold in attraction strength to induce superconductivity is much smaller for disordered two-chain systems and can be reached for a pure Hubbard attraction, at variance to the one-chain case.[15] On the other hand, the *d*-wave–type superconductivity found for repulsive interactions is completely unstable with respect to arbitrarily weak disorder. In two spinless chains, attractive interactions *reduce* localization compared to the case of a single chain, whereas repulsive ones *enhance* localization. For the case with spin, two chains are always less localized than their one-chain counterpart. For each case we also compute physical quantities such as localization length and conductivity.

Let us consider first two chains of spinless fermions coupled by an interchain hopping $t_\perp$. Such a model can also be mapped to two spin chains coupled by an exchange X-Y term, in the presence of a magnetic field. For simplicity we will just consider here a nearest-neighbor interaction V since longer-range interactions do not change the main physical results. Details will be given elsewhere.[16] The disorder is a random on-site potential $\epsilon_{i,p}$ uncorrelated from site to site and from chain to chain. The Hamiltonian then reads

$$H=-t\sum_{i,p} c^\dagger_{i,p}c_{i+1,p}+\text{H.c.}+V\sum_i n_{i,p}n_{i+1,p}$$
$$+t_\perp\sum_i c^\dagger_{i,1}c_{i,-1}+\text{H.c.}+\sum_{i,p}\epsilon_{i,p}n_{i,p}, \qquad (1)$$

where $p=-1,1$ is the chain index and i is the site index. It is convenient to rewrite the Hamiltonian in a boson representation.[1] We therefore linearize the fermions dispersion relation around k_F, introduce right (R) and left movers (L) for each chain, and take the continuum limit $c_{n,r,p}\to\sqrt{\alpha}\psi_{r,p}(n\alpha)$ with $r=L,R$, $p=\pm 1$ the chain index, and α the lattice spacing. We use the bonding $\psi_0=(\psi_1+\psi_{-1})/\sqrt{2}$ and antibonding $\psi_\pi=(\psi_1-\psi-1)/\sqrt{2}$ bands base and introduce the densities $\rho_{r,0,\pi}(x)$

$=:\psi^{\dagger}_{r,0,\pi}(x)\psi_{r,0,\pi}(x):$. We then define the canonically conjugate fields $\phi_{\rho,\|}$ and $\Pi_{\rho,\|}$ via $\partial_x\phi_{\rho,\|}=-(\pi/\sqrt{2})(\rho_{L,0}+\rho_{R,0}\pm\rho_{L,\pi}\pm\rho_{R,\pi})$ and $\Pi_{\rho,\|}=1/\sqrt{2}(\rho_{R,0}\pm\rho_{R,\pi}-\rho_{L,0}\mp\rho_{L,\pi})$ and the field $\theta_{\rho,\|}(x)=\int_{-\infty}^{x}\Pi_{\rho,\|}(x')dx'$. In terms of these fields the Hamiltonian becomes, for the pure case ($\epsilon_{i,p}=0$),

$$H=H_\rho+H_\|,\quad H_\rho=\int\frac{dx}{2\pi}\left[u_\rho K_\rho(\pi\Pi_\rho)^2+\frac{u_\rho}{K_\rho}(\partial_x\phi_\rho)^2\right],$$

$$H_\|=\int\frac{dx}{2\pi}\left[u_\| K_\|(\pi\Pi_\|)^2+\frac{u_\|}{K_\|}(\partial_x\phi_\|)^2\right]+\int dx t_\perp\frac{\sqrt{2}}{\pi}\partial_x\phi_\|+\int dx\left[\frac{2g_\perp}{(2\pi\alpha)^2}\cos(\sqrt{8}\phi_\|)+\frac{2g_f}{(2\pi\alpha)^2}\cos(\sqrt{8}\theta_\|)\right]. \tag{2}$$

The expressions of K,u,g in terms of the original parameters of the Hamiltonian can easily be obtained.[9,16] For the pure t-V model one has $K_\rho<1$ ($K_\rho>1$) and $g_f<0$ ($g_f>0$) for repulsive (attractive) interactions and $K_\|=1$ for all t,V. By adding interchain interactions, one has access to $K_\rho>1$ and $g_f<0$ or $K_\rho<1$ and $g_f>0$. The complete phase diagram in the pure case has been obtained in Ref. 9 by a mapping on a problem of one chain of fermions with spin and spin-anisotropic interactions in a magnetic field.[17] The $t_\perp$ term suppresses $\cos(\sqrt{8}\phi_\|)$ so that $\theta_\|$ develops a gap and acquires a nonzero expectation value determined by minimizing the ground-state energy. The operators with divergent associated susceptibilities are then

$$O_{\mathrm{CDW}^\pi}=\psi^{\dagger}_{R,0}(x)\psi_{L,\pi}(x)+\psi^{\dagger}_{R,\pi}\psi_{L,0}(x)\sim e^{i\sqrt{2}\phi_\rho}\cos(\sqrt{2}\theta_\|),\quad O_{\mathrm{SC}^s}=\psi_{L,0}(x)\psi_{R,\pi}+\psi_{L,\pi}\psi_{R,0}\sim e^{i\sqrt{2}\theta_\rho}\sin(\sqrt{2}\theta_\|),$$

$$O_{\mathrm{OAF}}=i[\psi^{\dagger}_{R,0}(x)\psi_{L,\pi}(x)-\psi^{\dagger}_{R,\pi}(x)\psi_{L,0}(x)]\sim e^{i\sqrt{2}\phi_\rho}\sin(\sqrt{2}\theta_\|),\quad O_{\mathrm{SC}^d}=\psi_{L,0}\psi_{R,\pi}-\psi_{L,\pi}\psi_{R,0}\sim e^{i\sqrt{2}\theta_\rho}\cos(\sqrt{2}\theta_\|).$$

These operators describe out-of-phase charge-density waves (CDW's), an orbital antiferromagnetic (OAF) phase, and chain symmetric (SC) ''s'' and antisymmetric ''d'' type superconductivity. For $g_f<0$ we have $\langle\theta_\|\rangle=0$ giving an SC^d phase for $K_\rho>1$ and the CDW^π for $K_\rho<1$, whereas for $g_f>0$ $\langle\theta_\|\rangle=\pi/\sqrt{8}$ giving the SC^s phase for $K_\rho>1$ and the OAF phase for $K_\rho<1$.[18]

Now, we consider the effect of the disorder. Taking the continuum limit for the on-site random potential, keeping only the $2k_F$ terms in the bosonized expressions (as forward scattering does not induce localization[19]), and finally going to bonding and antibonding bands, one finds that the coupling to disorder is represented by two terms:

$$H_s=\int\frac{dx}{\pi\alpha}\xi_s(x)e^{i\sqrt{2}\phi_\rho}\cos(\sqrt{2}\phi_\|)+\mathrm{H.c.}, \tag{3}$$

$$H_a=\int\frac{dx}{\pi\alpha}\xi_a(x)e^{i\sqrt{2}\phi_\rho}\cos(\sqrt{2}\theta_\|)+\mathrm{H.c.} \tag{4}$$

where $\xi_{s,a}$ are two uncorrelated Gaussian random potentials such that $\overline{\xi_n(x)\xi^*_{n'}(x')}=D_n\delta_{n,n'}\delta(x-x')$ with $n,n'=s,a$. In the original lattice problem, the role of $\xi_{s,a}$ would be played, respectively, by $\epsilon_{n,1}\pm\epsilon_{n,-1}$. We consider in the following a disorder weak enough ($D_n\ll t_\perp$) not to destroy the gaps opened by the interchain coupling in the pure system. The other limit where $t_\perp$ and D_n are small compared to the other parameters but of arbitrary relative magnitude is only important in the vicinity of the noninteracting point. It can be studied by similar methods and will be discussed elsewhere.[16] In the weak disorder limit, $\phi_\|$ has huge quantum fluctuations, and consequently D_s is always less relevant than D_a. We can therefore focus on the latter and forget about the former.

First, we consider $g_f<0$ (i.e., $V>0$ for the t-V model). In that case, we can replace $\cos(\sqrt{2}\theta_\|)$ by its (nonzero) mean value. Then the coupling to disorder (4) reduces to $C\int dx\xi_a(x)e^{i\sqrt{2}\phi_\rho(x)}+\mathrm{H.c.}$ The RG equations for the disorder are derived, following[13]

$$\frac{dD_a}{dl}=D_a(3-K_\rho), \tag{5}$$

where $l=\ln(\alpha)$ is the standard logarithmic scale associated with cutoff renormalization. (5) implies a localization-delocalization transition[13] at $K_\rho=3$. As a consequence, the d-wave superconducting phase is unstable in the presence of disorder except for huge attractive interactions. For a simple t-V model for which $K_\rho<1$, the CDW ground state is also unstable to disorder. The localized phase is a pinned-charge-density-wave phase, with a localization length given by $L_{2\mathrm{ch}}=(1/D)^{1/(3-K_\rho)}$. $L_{2\mathrm{ch}}$ is smaller than the localization length of a one-dimensional spinless system $L_{1\mathrm{ch}}=(1/D)^{1/(3-2K_\rho)}$. For repulsive interactions the effects of the interchain hopping is therefore to make the two-chain system more localized. The conductivity above the pinning temperature $u/L_{2\mathrm{ch}}$ can be obtained by methods similar to Ref. 13 and varies as $\sigma(T)\sim T^{2-K_\rho}$.

On the other hand, if one considers $g_f>0$, i.e., attractive interactions for a t-V model, then $\langle\theta_\|\rangle=\pi/\sqrt{8}$ and in a first approximation $\langle\cos(\sqrt{2}\theta_\|)\rangle=0$ so that there is apparently no coupling at all to the disorder. Obviously, this approximation is too crude and we must integrate the fluctuations of $\theta_\|$ around its mean value. This gives the following effective action for ϕ_ρ:

$$S_\rho=\int dx d\tau\left[\frac{(\nabla\phi_\rho)^2}{2\pi K_\rho}+[\xi(x)e^{i\sqrt{8}\phi_\rho(x,\tau)}+\mathrm{H.c.}]\right] \tag{6}$$

with $\overline{\xi(x)\xi^*(x')}=D\delta(x-x')$ and $D\sim D_a^2$. The renormalization of the disorder is again given by an equation similar to (5), but with a coefficient $(3-4K_\rho)/2$ in front of D_a. The disorder is now relevant only for $K_\rho<3/4$, leading to three different phases for $g_f>0$: a random orbital antiferromagnet

for $K_\rho<3/4$ (with a localization length $L_{2\mathrm{ch}}=(1/D)^{2/(3-4K_\rho)}$), an ordered orbital antiferromagnet for $3/4<K_\rho<1$ and an s-wave superconducting phase for $K_\rho>1$. For the t-V model, $K_\rho>1$, and the ''s''-wave superconducting phase is therefore *stable* with respect to weak disorder, at variance to the single-chain problem. For the latter the delocalization only occured for extremely attractive interactions, i.e., $K_\rho>3/2$. For the two-chain problem the localization-delocalization transition arises in the immediate vicinity of the noninteracting point. In contrast to the case of repulsive interactions, interchain hopping now strongly reduces the localization effects. The determination of the critical properties at the boundary between the repulsive (localized) regime and the attractive (superconducting) one requires that we treat the case where the gaps induced by the hopping and the disorder have arbitrary relative strength.[16] The conductivity now behaves as $\sigma(T)\sim T^{2-4K_\rho}$, and diverges as $T\to 0$ since the ground state is superconducting. In addition, since the disorder is less relevant for attractive interactions than for repulsive ones, one can also expect the charge stiffness[20] and persistent currents for a disordered finite-length two-chain system to be larger for the attractive case than for the repulsive one, similarly to the one-chain[21,15] system, but with much more dramatic effects.

Let us consider now the problem with spins. Here again, we will make the simplifying assumption of a local Hubbard interaction. More general interactions can be treated by the same method, giving rise to a richer phase diagram.[6] The Hamiltonian is now

$$H=-t\sum_{i,\sigma,p} c^\dagger_{i+1,\sigma,p}c_{i,\sigma,p}+\mathrm{H.c.}-t_\perp\sum_{i,\sigma,p} c^\dagger_{i,\sigma,p}c_{i,\sigma,-p}$$
$$+U\sum_{i,p} n_{i,\uparrow,p}n_{i,\downarrow,p}+\sum_{i,\sigma,p}\epsilon_{i,p}n_{i,\sigma,p}\,. \qquad (7)$$

We proceed as for the spinless case, introduce boson fields for each spin degree of freedom, and make the symmetric (charge) $\phi_\rho=\phi_\uparrow+\phi_\downarrow$ and antisymmetric (spin) $\phi_\sigma=\phi_\uparrow-\phi_\downarrow$ linear combinations. The bosonized Hamiltonian, which contains four bosonic fields instead of two for the spinless case, is quite lengthy and will not be reproduced here for reasons of space. It can be found in Ref. 6, and we use in the following the notations of this paper. All physical quantities depend on a parameter $K_{\rho+}$ of the symmetric charge mode, analogous to the K_ρ of the spinless problem. For the purely repulsive case $U>0$, only one of the four bosonic fields ($\phi_{\rho+}$) is gapless.[6] The mean values of the three other fields are determined by minimizing the energy of the ground state, giving $\langle\theta_{\rho-}\rangle=0$, $\langle\phi_{\sigma+}\rangle=\pi/2$, $\langle\phi_{\sigma-}\rangle=\pi/2$, leading to a d-wave superconductive phase.[6] For the attractive case $U<0$, $\phi_{\rho+}$ is again massless in the pure case. But now we have $\langle\theta_{\rho-}\rangle=0$, $\langle\phi_{\sigma+}\rangle=0$, $\langle\phi_{\sigma-}\rangle=0$. Here, the most divergent fluctuations are associated with the operator $O_{\mathrm{SC}^s}\sim e^{\imath\phi_{\rho+}}\cos(\phi_{\sigma+})\cos(\phi_{\sigma-})$, which is the order parameter for s-wave superconductivity.

The coupling to disorder arises again via two terms:

$$H_a=\int \xi_a(x)O_{\mathrm{CDW}^\pi}(x)+\xi_a^*(x)O^\dagger_{\mathrm{CDW}^\pi}(x)dx, \qquad (8)$$

$$H_s=\int \xi_s(x)O_{\mathrm{CDW}^0}(x)+\xi_s^*(x)O^\dagger_{\mathrm{CDW}^0}(x)dx, \qquad (9)$$

where $\overline{\xi_n(x)\xi_{n'}(x')^*}=D_n\delta_{n,n'}\delta(x-x')(n,n'=a,s)$, the ξ_n being random Gaussian distributed potentials. The operator O_{CDW^0} represents the in-phase charge-density wave, and O_{CDW^π} the out-of-phase one.

Assuming again that the disorder is weak enough not to destroy the gaps, the O operators have the simple form for repulsive interactions,

$$O_{\mathrm{CDW}^0}\sim e^{\imath\phi_{\rho+}}\sin(\phi_{\rho-}), \qquad (10)$$

$$O_{\mathrm{CDW}^\pi}\sim e^{\imath\phi_{\rho+}}\sin(\theta_{\sigma-}). \qquad (11)$$

These two operators have exponentially decaying correlation functions and no direct coupling with disorder would exist if one just took into account the mean values of the fields $\phi_{\rho,-}$ and $\theta_{\sigma,-}$. As in the spinless case one should integrate over fluctuations to get the effective coupling,

$$S^{\mathrm{disorder}}_{\rho+}=\int \xi_{\mathrm{eff}}(x)e^{\imath 2\phi_{\rho+}(x,\tau)}dx\,d\tau+\mathrm{H.c.} \qquad (12)$$

One can also view (10) as the coupling of the fermions with the $k_{F0}\pm k_{F\pi}$ Fourier component of the disordered potential. The problem has in fact been reduced to a problem of spinless fermions. The localization-delocalization would occur at $K_{\rho+}=3/2$ (Ref. 22) but purely repulsive interaction implies $K<1$. The d-wave phase is therefore unstable to abitrarily weak disorder. The symmetric (9) and the antisymmetric (8) part of the disorder contribute equally to destroy the d-wave superconductivity, in contrast to the spinless case where the antisymmetric part was the most relevant. The localization length in that phase is $L_{2\mathrm{ch}}\sim(1/D)^{2/(3-2K_{\rho+})}$, and therefore longer than the corresponding one, $L_{1\mathrm{ch}}\sim(1/D)^{1/(2-K_{\rho+})}$, of the one chain with spin. The two-chain problem is less localized than the corresponding one-chain one even for repulsive interactions, in contrast to the spinless case. This is in qualitative agreement with what one expects in the absence of interactions where the localization length is proportional to the number of channels in the system.

For the attractive case, the O operators take a different simplified form, due to the different gaps in the system:

$$O_{\mathrm{CDW}^0}\sim e^{\imath\phi_{\rho+}}\cos(\phi_{\rho-}), \qquad (13)$$

$$O_{\mathrm{CDW}^\pi}\sim e^{\imath\phi_{\rho+}}\sin(\theta_{\sigma-})\sin(\phi_{\sigma+}). \qquad (14)$$

By substituting in (8) and (9) and integrating over fluctuations we end with an effective coupling of the form (12). This time, $K_{\rho+}>1$, so that we can attain the localization-delocalization transition at $K=3/2$. This transition arises for much weaker attraction than in the one-dimensional case[13] where $K_\rho=3$. For the two-chain problem the critical value of K can be attained for a Hubbard model[23,15] whereas the one-chain Hubbard model is always localized even for very negative U.[15] In addition, the localization length is increased: $L_{2\mathrm{ch}}=(1/D)^{2/(3-2K_{\rho+})}$, whereas in the one-chain case $L_{1\mathrm{ch}}=(1/D)^{1/(3-K_\rho)}$. Opposite to what happens for the one-dimensional case where the attractive localization length was smaller than the repulsive one,[24,13] here the two lengths are

the same, up to prefactors. Therefore, the enhancement of charge stiffness by repulsive interactions found in the one-chain case[15] should be absent or strongly reduced for two chains. This issue would need a more detailed study. The conductivity behaves as $\sigma(T) \sim T^{2-2K_{\rho+}}$.

Clearly, these effects are due to the existence of a spin gap and to the freezing of interchain charge excitations.[6] As a consequence, it would be worth studying the localization effects in a three-chain model (where there should be no spin gap) to see if the delocalizing effect of attractive interaction does persist or if we fall back on the one-chain case.

Note added: Recently, we learned about the work of Kawakami and Fujimoto.[25] These authors considered the related, albeit different, problem of disordered coupled Hubbard chains with a ferromagnetic Hund's exchange and no hopping. They also found reduction of the localization effects in this system.

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*Laboratoire associé au CNRS.

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