Coulomb scattering lifetime of a two-dimensional electron gas

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Motivated by a recent tunneling experiment in a double quantum-well system, which reports an anomalously enhanced electronic scattering rate in a clean two-dimensional electron gas, we calculate the inelastic quasiparticle lifetime due to electron-electron interaction in a single loop dynamically screened Coulomb interaction within the random-phase approximation. We obtain excellent quantitative agreement with the inelastic scattering rates in the tunneling experiment without any adjustable parameter, finding that the reported large ($\geqslant$ a factor of 6) disagreement between theory and experiment arises from quantitative errors in the existing theoretical work and from the off-shell energy dependence of the electron self-energy.

A central quantity in the theory of interacting electron systems is the quasiparticle lifetime, which is the inverse of the scattering rate or the broadening of the quasiparticle state, and therefore, determines the width of the quasiparticle spectral function. The concept of inelastic lifetime is also important in electronic device operation, because it controls the electron energy dissipation rate. It is, therefore, of great significance that a recent *direct* measurement of inelastic broadening in a two-dimensional electron gas by Murphy *et al.*[1] reports a factor of six discrepancy between experimental results and the existing theory. In this paper, we develop a theory for inelastic Coulomb scattering lifetime in a degenerate two-dimensional electron system (2DES), finding essentially exact quantitative agreement with the tunneling results reported in Ref. 1. We also identify the reason for the factor of 6 disagreement reported in Ref. 1.

Over the past several decades two-dimensional electron systems have been extensively studied for both their fundamental and technological interest. The 2DES in high mobility GaAs/$Al_xGa_{1-x}As$ heterostructures has become an especially suitable system for studying electron-electron interaction effects, because of the reduced effect of impurity scattering arising from the modulation-doping technique. Many properties of the 2DES are strongly influenced by the presence of electron-electron interactions. One important property is the broadening of the electronic states by inelastic Coulomb scattering, which plays a major role in many physical processes, such as tunneling,[1] ballistic hot electron effects,[2] transport,[3] and localization.[4] The asymptotic properties of Coulomb scattering in a 2DES are well established from the existing theoretical work:[5–10] The electron inelastic lifetime τ_e in a pure 2DES becomes $\tau_e^{-1}(\xi)\propto\xi^2\ln\xi$ for $\varepsilon_F\gg\xi\gg k_BT$, and $\tau_e^{-1}(T)\propto T^2\ln T$ for $\varepsilon_F\gg k_BT\gg\xi$, where ξ is the quasiparticle energy with respect to the Fermi energy ε_F, k_B and T are the Boltzmann constant and temperature, respectively. Earlier experimental work on the inelastic lifetime of 2D electrons focused on the dephasing time,[11] while the recent experiment on tunneling[1] in a double quantum-well structure directly measures the inelastic broadening. One advantage of the tunneling experiment over the dephasing experiment in this context is that the subtlety associated with quantum interference effects can be avoided in a tunneling experiment, which directly obtains the inelastic broadening. In this sense, the lifetime measured from the tunneling experiment is an excellent candidate for a direct comparison with theoretical calculations. It is the aim of this work to calculate the inelastic quasiparticle lifetime, due to the Coulomb interaction in a clean 2DES and compare it with the results of the tunneling experiment.[1]

The scattering rate obtained from the tunneling experiment,[1] with the contribution from the residual impurity scattering excluded, is essentially due to electron-electron interaction. The effect of phonon scattering,[12] including both acoustic and LO phonons, are safely negligible[1] in the experimental temperature range. Unexpectedly (and as mentioned above), a very large quantitative disagreement between the tunneling experiment and the existing theoretical calculations was reported. For example, the measured Coulomb scattering rate[1] close to the Fermi surface at low temperatures is found to be more than six times larger than that of the quoted calculation of Giuliani and Quinn (GQ),[5] which has been extensively used to interpret experimental results.[3,11,13] This level of discrepancy is difficult to understand, since the essential approximation used in GQ's calculation is the random-phase approximation (RPA), and the corresponding three-dimensional RPA Coulomb scattering calculations are in excellent agreement with experiments.[14,15] The biggest inaccuracy in the RPA comes from treating the short range correlations poorly. These short range correlations should not be very important in the low temperature scattering rate of electrons close to the Fermi surface, where only long wavelength excitations are involved. This large discrepancy, if proved true, would cast serious doubt on the validity of the schemes of the existing theoretical work. This is particularly important in view of recent suggestions[16] that an interacting 2DES may not be a Fermi liquid and may have nonperturbative similar interaction effects akin to Luttinger liquids.[17] Another significant puzzle is that the Coulomb scattering rate measured at very low temperatures as a function of energy in the quantum interference experiment[11] seems to agree quite well with the GQ result. Since the calculation of GQ is the most widely used theoretical result in this subject, it is of considerable importance to investigate this discrepancy. For this purpose, we calculate the inelastic lifetime by obtaining imaginary part of the electron self-energy, using the RPA dynamically screened exchange inter-

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action, which is the same level of approximation as the work of GQ. Our calculation, with all the input parameters taken from the real samples, i.e., with no adjustable parameters, shows very good quantitative agreement with the tunneling experiment. We further find that the originally reported large discrepancy is due to some quantitative errors in the previous theoretical work and the negligence of the energy dependence of the Coulomb scattering rate. The Coulomb scattering broadening studied in this work is caused almost entirely by quasiparticle excitations. Plasmon excitation can contribute only at much higher electron energies.[5]

We first present our calculation of the electron self-energy for a pure and ideal 2DES. The corrections from finite well thickness, vertex correction, diffusive effects, and phonon scattering, all of which are included in our numerical work, will be briefly discussed at the end. The finite temperature electron self-energy in the RPA is

$$\Sigma(k,ip_n)=-\frac{1}{\nu}\sum_{\mathbf{q}}\frac{1}{\beta}\sum_{i\omega_k}V_{\rm sc}(\mathbf{q},i\omega_k)\mathscr{G}^0(\mathbf{k}+\mathbf{q},i\omega_k+ip_n),$$

where $\beta^{-1}=k_BT$, ν is the area of the 2D system, and $V_{\rm sc}=v(q)[\epsilon(q,i\omega)]^{-1}$ and $\mathscr{G}^0$ are, respectively, the screened Coulomb interaction and the electronic Green's function. After a standard procedure of analytical continuation,[15,18] the imaginary part of the self-energy is obtained as

$$\mathrm{Im}\Sigma(k,\omega)=\frac{1}{\nu}\sum_{\mathbf{q}}v(q)\mathrm{Im}\frac{1}{\epsilon(q,\xi_{\mathbf{q}+\mathbf{k}}-\omega+i0^+)}\times[n_F(\xi_{\mathbf{q}+\mathbf{k}})+n_B(\xi_{\mathbf{q}+\mathbf{k}}-\omega)],\qquad(1)$$

where $\xi_k=\hbar^2k^2/2m^*-\varepsilon_F$, is the electron energy relative to the Fermi energy, $n_{F(B)}$ is the fermion (boson) distribution function $n_{F(B)}(x)=[e^{\beta x}\pm1]^{-1}$, $v(q)=2\pi e^2/\epsilon_s q$ is the Coulomb potential. The RPA dielectric function is

$$\epsilon(q,\omega)=1-v(q)\chi_c^0(q,\omega),\qquad(2)$$

where

$$\chi_c^0(q,\omega)=\frac{(\omega+i\gamma)\chi_0(q,\omega+i\gamma)}{w+i\gamma\chi_0(q,\omega+i\gamma)/\chi_0(q,i0^+)},\qquad(3)$$

with γ related to the mobility broadening by $\gamma=e/(m^*\mu)$. The above particle-conserving polarizability[19] includes the essential effect of disorder scattering: The motion of electrons becomes diffusive rather than ballistic at large time and length scales. This expression allows a simple quantitative treatment of the diffusive effect arising from the finite value of mobility, due to impurity and phonon scattering. For the experimental high mobility samples, the low temperature mobility is high, $\mu\geqslant10^6$ cm^2/V s, making the effect of disorder and phonon scattering practically negligible.[1,12] The use of χ_c^0 does not change the results within the numerical accuracy, however, it helps to improve the numerical integrations by suppressing the singularities associated with plasmon excitation. The noninteracting density-density response χ^0 in the above expression is

$$\chi_0(q,\omega+i\gamma)=\frac{2}{\nu}\sum_{\mathbf{p}}\frac{n_F(\xi_{\mathbf{q}+\mathbf{p}})-n_F(\xi_p)}{\omega+\xi_{\mathbf{q}+\mathbf{p}}-\xi_p+i\gamma}.\qquad(4)$$

From Eqs. (1)–(4), $\mathrm{Im}\Sigma(k,\omega)$ can be computed. It is helpful to make clear the relationship between the lifetime obtained from this self-energy and the lifetimes calculated from the Fermi's Golden rule before we move on to discuss the numerical result. The lifetimes of electrons and holes from the Golden rule are

$$\tau_e^{-1}(k)=\frac{2\pi}{\hbar}\frac{1}{\nu^2}\sum_{\mathbf{pq}\sigma'}n_{F\sigma'}(\xi_{\mathbf{p}})[1-n_{F\sigma'}(\xi_{\mathbf{p}-\mathbf{q}})]\times[1-n_{F\sigma}(\xi_{\mathbf{k}+\mathbf{q}})]|V_{\mathbf{kq}}|^2\delta(\xi_{\mathbf{k}+\mathbf{q}}+\xi_{\mathbf{p}-\mathbf{q}}-\xi_k-\xi_p),$$

$$\tau_h^{-1}(k)=\frac{2\pi}{\hbar}\frac{1}{\nu^2}\sum_{\mathbf{pq}\sigma'}n_{F\sigma'}(\xi_{\mathbf{p}})[1-n_{F\sigma'}(\xi_{\mathbf{p}-\mathbf{q}})]\times n_{F\sigma}(\xi_{\mathbf{k}+\mathbf{q}})|V_{\mathbf{kq}}|^2\delta(\xi_{\mathbf{k}+\mathbf{q}}-\xi_{\mathbf{p}-\mathbf{q}}+\xi_k-\xi_p),$$

$$n_F(\xi_k)\frac{1}{\tau_e(k)}=[1-n_F(\xi_k)]\frac{1}{\tau_h(k)},\qquad(5)$$

with $V_{\mathbf{kq}}=v(q)/\epsilon(q,\xi_k-\xi_{\mathbf{k}+\mathbf{q}})$. The last equation above is the equilibrium condition. Defining the broadening $\Gamma(k,\xi_k)=-2\ \mathrm{Im}\Sigma(k,\xi_k)/\hbar$, it is straightforward to show[18]

$$\Gamma(k,\xi_k)=\frac{1}{\tau_e(k)}+\frac{1}{\tau_h(k)}.\qquad(6)$$

It is, therefore, clear that the lifetime $\Gamma^{-1}(k,\xi_k)=[-2\ \mathrm{Im}\Sigma(k,\xi_k)/\hbar]^{-1}$ is the relaxation time of the electron momentum occupation number $n_{\mathbf{k}\sigma}$. In general, it differs from either the electron lifetime or the hole lifetime. In particular, Γ^{-1} and τ_e differ by a factor of 2 at the Fermi surface. It is readily recognized that the lifetime obtained from the measured spectral function in a tunneling experiment is Γ^{-1}, not τ_e. This,[5] we believe, is one source of error (by a factor of 2) in interpreting the experimental results of Ref. 1.

In Fig. 1, we show, respectively, the numerical results of Γ as functions of temperature T, energy ξ, and electron density N_s. Several of the familiar features are easy to see from the figure: $\Gamma(k_F,0)\propto T^2\ln T$ for small T, $\Gamma(k,\xi)\propto\xi^2\ln\xi$ for small ξ at $T=0$, and $\Gamma(k_F,0)\propto1/N_s$ at small T. It is interesting to compare the numerical results in Fig. 1 to the analytical expressions obtained from the low T and small ξ asymptotic expansions of $\mathrm{Im}\Sigma$ in Eq. (1):

$$\Gamma(k,\xi_k)=\frac{2}{\tau_e(T)}=-\frac{\pi\varepsilon_F}{4\hbar}\left(\frac{k_BT}{\varepsilon_F}\right)^2\ln\frac{k_BT}{\varepsilon_F}\quad\text{for}\ \ \varepsilon_F\gg k_BT\gg\xi_k,\qquad(7)$$

$$\Gamma(k,\xi_k)=-\frac{\varepsilon_F}{4\pi\hbar}\left(\frac{\xi_k}{\varepsilon_F}\right)^2\ln\frac{\xi_k}{\varepsilon_F}\quad\text{for}\quad\varepsilon_F\gg\xi_k\gg k_BT.\qquad(8)$$

The above asymptotic expressions are consistent with our full numerical results (the inserts of Fig. 1). It should be noted that the prefactor in Eq. (7) is different from that of the work by GQ.[5] The cause of this difference, we believe, is that the corresponding expansion of GQ is incorrect by a missing factor of $(\pi/2)^2$. This can partially account (by providing a factor of 2.5) for the fact that many studies have reported a Coulomb scattering rate significantly larger than

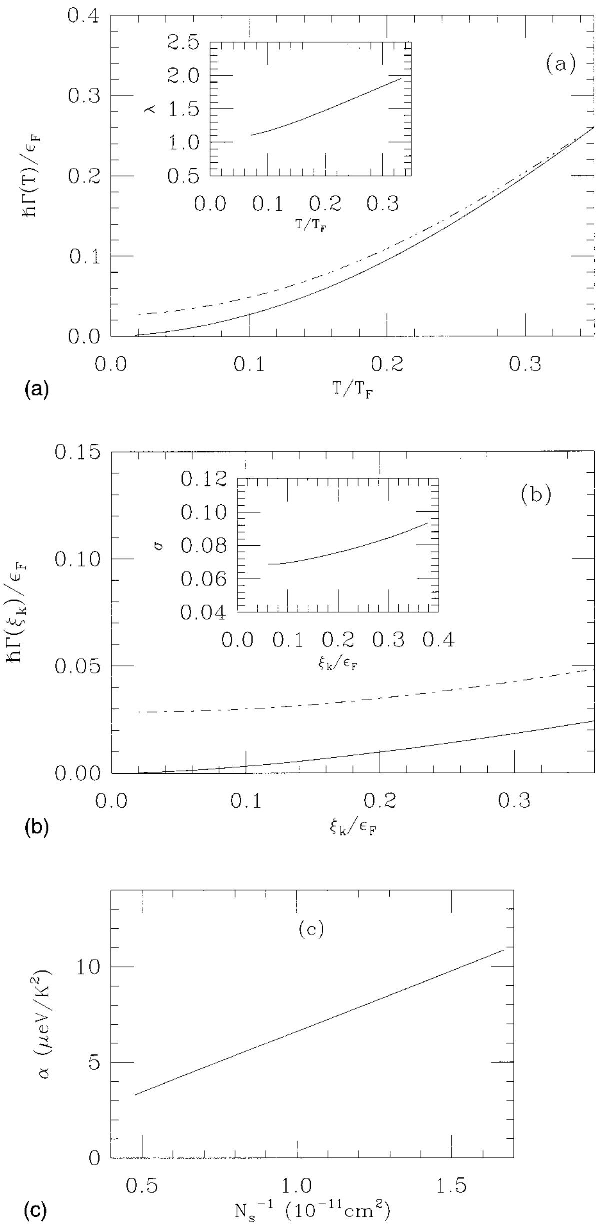

FIG. 1. (a) The Coulomb scattering rate $\Gamma(k,\omega)$, as a function of temperature T/T_F, where $T_F=\varepsilon_F/k_B$. The solid line is for $\Gamma(k_F,0)$. The dashed line is for $\Gamma(k,\xi_k)$, with $\xi_k=0.4\varepsilon_F$. The inset shows $\lambda=\hbar\Gamma(k_F,0)\varepsilon_F/[(k_BT)^2\ln(T_F/T)]$, as a function of T/T_F. The density of the 2DES is $N_s=1.6\times10^{11}\text{cm}^{-2}$. (b) The Coulomb scattering rate $\Gamma(k,\omega)$, as a function of electron energy ξ_k/ε_F. The solid line is for $\Gamma(k,\xi_k)$, at $T=0$. The dashed line is for $\Gamma(k,\xi_k)$, with $T=0.1T_F$. The inset shows $\sigma=\hbar\Gamma(k,\xi_k)\varepsilon_F/[\xi_k^2\ln(\varepsilon_F/\xi_k)]$, as a function of ξ_k/ε_F at $T=0$. The density of the 2DES is $N_s=1.15\times10^{11}$ cm^{-2}. (c) $\Gamma(k_F,0)/T^2$, as a function of the density N_s, at $T=3$ K.

the GQ prediction. Note that the asymptotic expression in energy, Eq. (8), is the same as that in GQ, explaining the puzzle of why the measurement of energy dependent scattering rate agrees with the GQ result.[11]

Next, we directly compare our calculation with the recent tunneling experiment[1] in a double quantum-well system. For the case of equal electron densities on each layer, the tunneling current as a function of the external bias potential V is sharply peaked at $V=0$. The resonance width V_{HWHM}, the bias potential at half maximum, is a measure of the quasiparticle lifetime. Under the condition (satisfied in Ref. 1) that the Fermi energy is much larger than all the other energy scales involved, the resonance width $\hbar\Gamma_{\text{eff}}=V_{\text{HWHM}}$ is[1]

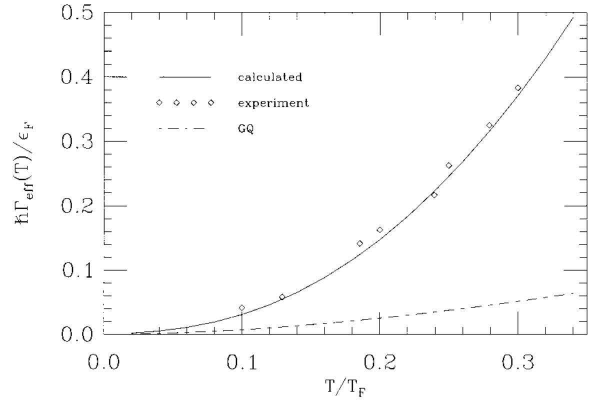

FIG. 2. The Coulomb scattering determined tunneling resonance width Γ_{eff}, as a function of temperature T. The solid line is the present calculation with density $N_s=1.6\times10^{11}$ cm^{-2}, well-thickness $b=200$ Å, and mobility $\mu=10^6\text{cm}^2/\text{V s}$. The diamonds and the dashed line are, respectively, the experimental data and the theoretical result of GQ quoted from Ref. 1.

$$\Gamma_{\text{eff}}=\frac{1}{2}\Gamma(k_F,0)+\frac{1}{2}\Gamma(k_F,V_{\text{HWHM}}). \quad (9)$$

It is important to note that the finite bias potential V_{HWHM} introduces an *off-shell* energy dependence into the scattering rate. This kind of energy dependence is a direct consequence of simultaneous momentum and energy conservation[1,20] in the tunneling process. Taking the value of V_{HWHM} along with the values of all other sample parameters from Ref. 1, we numerically calculate Γ_{eff} as a function of temperature T.[21] In order to make a realistic comparison, we include the effects of finite well thickness, vertex correction, and a finite value of mobility. The method to include these effects is discussed below. The calculated Γ_{eff} is shown as the solid line in Fig. 2 together with the experimental data and the theoretical result of GQ (dashed line) quoted from Ref. 1. One can see that the agreement of the present calculation with the experiment is excellent. The discrepancy between the GQ result and the experiment on the other hand is very large. This large discrepancy is due partly to the error (a factor of $\pi^2/2\sim5$) in GQ's work, which was discussed following Eqs. (6) and (7), and partly to the negligence of the off-shell energy dependence of the scattering rate, which contributes a 30–40 % quantitative effect. The excellent agreement between the present calculation and the experiment suggests that the commonly adapted Fermi liquid RPA-like many-body treatments for the Coulomb scattering rate are well valid in GaAs-based 2DES. This also shows that a clean interacting 2DES is, in fact, a Fermi liquid[17] similar to a three-dimensional system.

Finally, we briefly discuss how the corrections from finite well thickness, vertex correction, phonon and impurity scattering are incorporated into our calculation. The finite well thickness, which tends to weaken the interaction at short distances, can be represented by replacing $v(q)$ by $v(q)F(q)$. The form factor may be chosen as $F(q) = (2/qb)[1+1/qb(e^{-qb}-1)]$ with b as the well thickness.[22] The influence of vertex correction, which tends to decrease screening through a local field correction, may be estimated by the replacement of $\chi_c^0(q,\omega)$ by $\chi_c^0(q,\omega)[1-G(q)]$, with the Hubbard local field approximation[23] $G(q)=0.5q/(q^2+k_F^2)^{1/2}$. The effect of LO-phonon-mediated electron-electron interaction can be included[12] by adding the factor $(1-\epsilon_\infty/\epsilon_0)/(\epsilon_\infty/\epsilon_0-\omega^2/\omega_{\rm LO}^2)$ to the dielectric function $\epsilon(q,\omega)$, with the parameters for GaAs materials as $\epsilon_0=12.9$, $\epsilon_\infty=10.9$, and $\omega_{\rm LO}=36.8$ meV. For low energy excitations, the LO phonons act as a small source of static screening. The effect of a finite mobility, due to impurity and acoustic phonon scattering, which is small under the present conditions, can be taken into account by putting into χ_c^0 the appropriate value of $\gamma=e/m^*\mu$. For computational reasons, all the data presented here are calculated with $\mu=10^6$ cm^2/V s.

In summary, we have calculated the inelastic scattering rate, due to electron-electron interaction for a two-dimensional electron gas. Our work is motivated by the large disagreement between the recent tunneling experiment[1] and the existing theoretical calculations. Using experimental sample parameters, we have obtained excellent quantitative agreement with the tunneling experiment. Our work suggests that a clean interacting 2DES is a Fermi liquid and that RPA-based perturbative many-body calculations are of quantitative validity.

Note added in proof. After submission of our work we learned of a closely related work by T. Jungwirth and A. H. MacDonald [Phys. Rev. B **53**, 7403 (1996)], which gives essentially the same results at this work.

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