Depinning transition in Mott-Anderson insulators

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We have studied the I-V curves of amorphous insulators at very low temperature which exhibit a crossover from variable range hopping to a low-temperature activation law. Around a critical voltage V_c, the current increases by several orders of magnitude. This is interpreted as resulting from an interplay between Coulomb interactions and disorder: below V_c charges are collectively "pinned," while above V_c free charges exist. We find a critical scaling of the I-V curves near V_c, at least when hysteretic effects can be neglected.

In the last decades, taking into account both strong disorder and Coulomb interactions in the description of electronic transport turned out to be both a necessity and a tough challenge. This is particularly true in disordered insulators, where Coulomb interaction is poorly screened. In systems with a small number of electrons such as lightly doped semiconductors, disorder is thought to dominate over charging effects, leading to Mott's famous variable range hopping (VRH) law for the resistance at low temperature T: $R \propto \exp(T_0/T)^{\gamma}$, where $\gamma = 1/(d+1)$. Electron-hole interactions were later included by Efros and Shklovskii (ES),[1] who argued that, at sufficiently low T, γ should take the value $\frac{1}{2}$.

Very recently, this interplay between disorder and interactions was addressed within a rather different framework, in the context of quantum dots where the interaction is dominant, and leads to Coulomb blockage effects. Middleton and Wingreen[2] have shown that an array of such quantum dots should, in the presence of local disorder, undergo a depinning transition as the external voltage V is increased. For small voltages, the current I is *zero* (at $T=0$), and increases from zero above a critical value V_c of the voltage, with a nontrivial exponent: $I \propto (V-V_c)^{\beta}$. This prediction was rapidly corroborated by a series of experiments on model quantum dot systems.[3] The central point of the present paper is to show that this depinning transition could also be relevant in certain real samples—in our case sputtered films of Y_xSi_{1-x}. In such samples, some kind of small-scale granularity is unavoidable, in particular near the metal-insulator transition (MIT), leading to the presence of more metallic droplets within a more insulating matrix. If the charging energy of these droplets becomes large, Coulomb blockade effects, such as Middleton-Wingreen's collective pinning, should be expected. For example, Coulomb blockade oscillations were observed in mesoscopic In_2O_{3-x} samples, confirming the importance of charge effects near the MIT.[4] Another consequence of a strong on-site Coulomb repulsion[5] is the activated behavior ($\gamma=1$) of the resistance, even if some disorder is present—at least in an intermediate range of temperatures. This is indeed the case in our Y_xSi_{1-x} samples in the immediate vicinity of the MIT ($x \simeq 0.2$), for T in the range 20–500 mK, where VRH holds at high temperatures and becomes simply activated for lower temperatures. (Such a behavior was also reported in Ref. 6.) It is thus our belief that, close to the MIT, the *on-site* repulsive electron-electron interaction dominates over disorder: more insulating samples of the same series exhibit simple VRH or ES behavior.[7] This is expected if disorder supersedes interactions, which indeed occurs when the localization length ξ becomes small.[8] Close to the MIT, the I-V characteristics are rather unusual. A critical voltage V_c appears around which the current increases by several orders of magnitude, which we interpret along the lines of Ref. 2 as resulting from the interplay between Coulomb interactions and disorder. By scaling considerations inspired from Ref. 2, we account for the critical behavior around V_c. The existence of a critical depinning transition is common to many physical systems, ranging from charge-density waves to domain walls in magnets.[9,10]

Our basic physical assumption is to model our samples as an assembly of metallic droplets of typical size a, separated by insulating regions of typical size b, where for consistencey $(a,b) \gtrsim \xi$. (The distinction between a metal and an insulator indeed only makes sense beyond ξ.) Our model is thus a Hubbard-like lattice where the metallic droplets play the role of coarse-grained sites with on-site repulsion U given by the charging energy e^2/C, where C is the nearest-neighbor capacitance of the order of $\epsilon\xi$.[11] As shown in Coulomb blockade experiments, a charging energy has to be introduced if many electrons stay on a droplet connected to reservoirs by tunnel barriers with a conductance less than e^2/h. The energy $U \simeq e^2/\epsilon\xi$ is such a typical energy, and is assumed to be the dominant energy scale: we thus argue that the system as a whole behaves much as a Mott insulator (although U is much smaller than usual) with a gap of order U and an activated conductance $\simeq \exp -U/T$ at low T. In our case, however, random sizes and locations of grains implies that energy levels close to the Fermi energy are randomly distributed, leading to pinning effects, which is the problem considered in Ref. 2. As mentioned above, one finds a $T=0$ phase transition induced by the bias voltage V applied to the system. For a d-dimensional array with N capacitances in series, and for $V < V_C \propto N(e/2C)$, the current I does not flow (insulating phase). V_c corresponds to an exact compensation by the voltage source of Coulomb interactions along the best conducting path through the sample, along which the sum of charging energies encountered is minimal. For $V > V_c$, I should behave as $(V-V_c)^{\beta}$, since the number of conducting

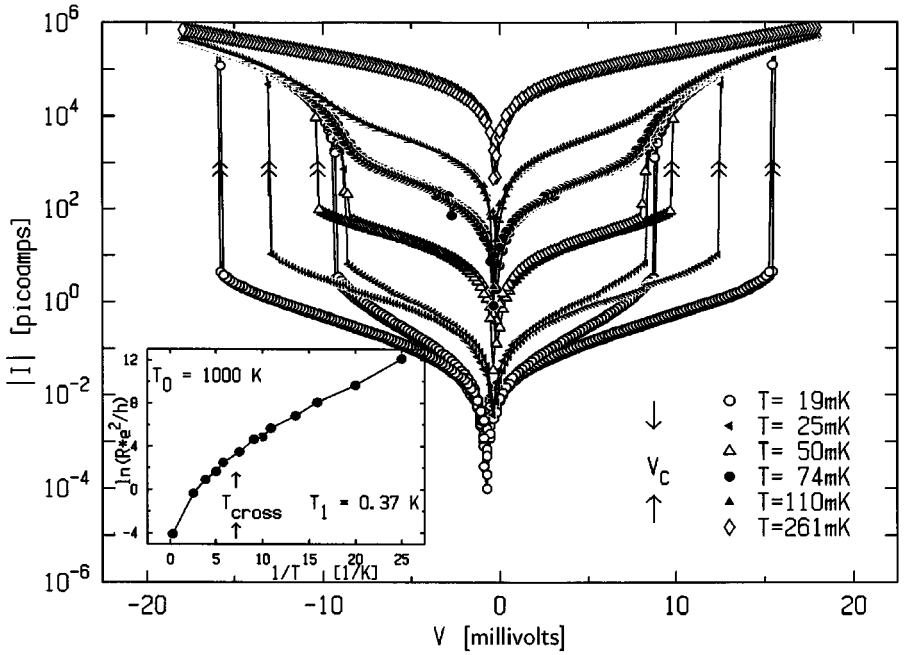

FIG. 1. $I(V)$ in the semilog scale for $S3d$ (120 μm long) at $H=0$ T and various T. Around $V_c \simeq 9.0$ mV, note the sudden decrease of I by a huge factor at low T. This behavior is hysteretic: when V rises (see the arrows), the discontinuity occurs at a voltage that is strongly T dependent. Inset: $R_{V=0} \times e^2/h$ in the log scale as a function of T^{-1}. Above $T_{\rm cross}$ Mott's law holds: $R \propto \exp(T_0/T)^{1/4}$. Below $T_{\rm cross}$, the divergence of R is *stronger* since it is *simply activated*: $R \propto \exp(T_1/T)$.

paths increases with V. The critical exponent β depends on the dimension d and was estimated in Ref. 2 using results on optimal paths and the KPZ equation.[12] For $T>0$, this phase transition disappears, and leads to a crossover which we postulate to be of the form (see Ref. 9)

$$\frac{I}{I_0} = \left(\frac{V}{V_c} - 1\right)^{\beta} F\left[\left(\frac{V}{V_c} - 1\right)\left(\frac{T_1}{T}\right)^{\phi}\right] \quad (V \simeq V_c), \qquad (1)$$

where ϕ is the crossover exponent, and F a crossover function, which we determine experimentally. As shown below, this picture allows us to describe satisfactorily the I-V characteristics of all our samples, at least for temperatures that are not too low. At very low temperature, however, the transition becomes first order and hysteretic, as expected in the presence of some (even very small) heating. This hysteretic behavior is actually also observed in model quantum dots systems.[3]

Let us now describe our experimental results. $S3d$ and $S2d$ films differ mainly by their thickness h: For $S3d$, $h \simeq 7$ μm while for $S2d$ $h \simeq 0.04$ μm$\simeq a,b$ (see below). Both $S3d$ and $S2d$ were made by sputtering from a single $Y_{0.2}Si_{0.8}$ target onto a substrate cooled at liquid-nitrogen temperature in order to minimize atomic segregation.[13] All reported electrical measurements are two-probe dc results using a voltage source and detection of current with a Keithley 617 electrometer. Contacts of 2500 μm^2 are made by ultrasonic soldering of aluminum wires directly on the surface of the sample. To ensure that contact resistances are negligible at low T, we first realized four-probe measurements showing

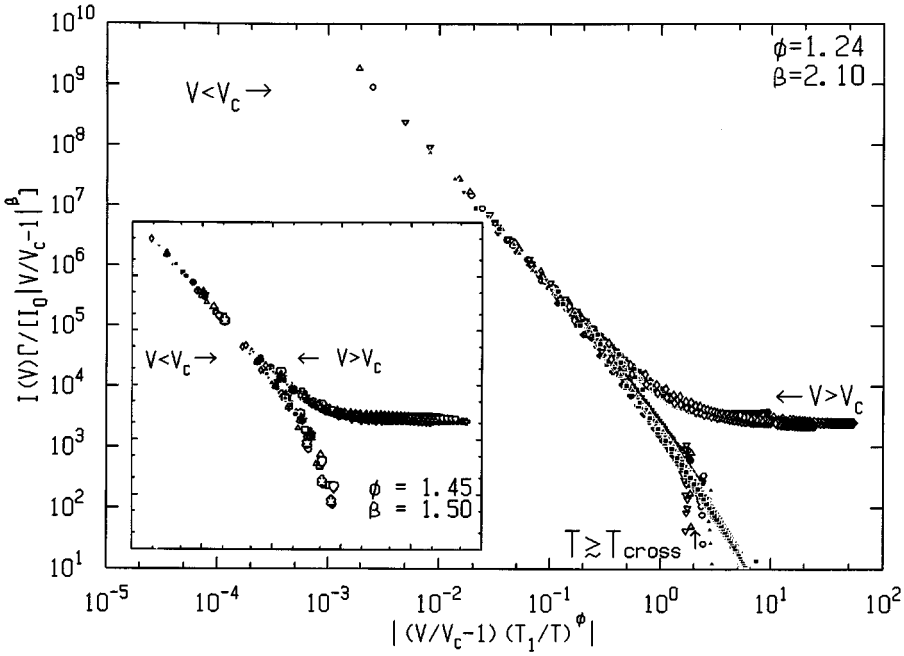

FIG. 2. Rescaling for $S3d$ using Eq. (1). 2594 I-V points rescaled from $T \simeq 20$ mK to $T=300$ mK and $H=0$ to $H \simeq 6$ T. β, ϕ, and F do not depend on H. T_1 is extracted from Fig. 1. At $H=0$, $T_1=0.37$ K, and $V_c=8.7\pm0.2$ mV. At $H=6$ T, $T_1=0.42$ K, and $V_c=20.0\pm0.5$ mV. Γ is the simplest function allowing a linear regime: $\Gamma(V<V_c)=1+V_c/V$, and $\Gamma(V>V_c)=2$. I_0 is a typical current slightly depending on H. For $V<V_c$ scaling fails either when VRH dominates the measurement (i.e., $T \geqslant T_{\rm cross} \simeq 0.15$ K and V is small) or in the low phase current when discontinuity at V_c is large: these latter points were removed from the plot. Inset: same kind of rescaling for the $2d$ sample (same ranges of H and T, and arb. units for the log-log plot). T_1 is extracted from $R(V=0)$ experiments. At $H=0$, $T_1=0.57\pm0.05$ K, and $V_c \simeq 2.5\pm0.05$ mV. At $H=6$ T, $T_1=0.8\pm0.05$ K, and $V_c \simeq 6.5\pm0.15$ mV.

that the voltage drop at the contacts is negligible. Furthermore, increasing the surface of contacts did not change the measured two-probe resistance. Then we varied the distance L between the contacts in two-probe experiments which affected the resistance by a factor *independent of T*. Varying L allowed us to test ten different samples.[14] They all behaved similarly, and for simplicity we focused on two samples of the *same* length $L \simeq 120$ μm, recorded on $S3d$ and $S2d$ films.

At room temperature both $S3d$ and $S2d$ exhibit a conductivity below (but close to) Mott's critical conductivity.[15] At zero bias $V=0$, when $0.15\ \mathrm{K} < T \leqslant 4.2\ \mathrm{K}$, $S3d$ exhibits standard VRH behavior ($\gamma=\frac{1}{4}$) with $T_0 \simeq 1000$ K. However, below $T_x \simeq 0.15$ K, γ shifts to 1:[16] $R \propto \exp(T_1/T)$, with $T_1 \simeq 0.37$ K (see the inset of Fig. 1). This suggests that some charging effects are present. In $S2d$, this simply activated regime extends up to 0.5–1.0 K, and T_1 is stronger ($T_1 \simeq 0.57$ K).

In addition to these departures from VRH at low voltage, nonlinearities qualitatively differ from the one predicted in VRH,[17] where the logarithm of the conductance is predicted to increase linearly with the applied voltage. The most striking example appears in Fig. 1 for $S3d$: around a critical voltage V_c, the current I varies by a large factor θ at low T (at $T=20$ mK, $\theta \simeq 1000$). For $S2d$, I-V nonlinearities are qualitatively the same. Note that this behavior is hysteretic at low T: when I is imposed, the V-I curve is S shaped;[18] when V is imposed (Fig. 1), the position of the discontinuity when rising V takes place at a voltage $V_{\mathrm{hyst}} > V_c$, which is strongly T dependent. When decreasing V, the discontinuity occurs at a critical voltage V_c, nearly T independent. This jump disappears for higher temperatures.

The simplest assumption is that heating becomes important at low temperatures, and opens up the low current part of the S, leaving the high current branch practically unaffected. We can roughly account for the shape of the S using a conventional heating model, whereby the injected power P induces an effective electronic temperature T_{el} larger than the sample temperature T_{ph}, and given by $P=\Gamma\Omega(T_{\mathrm{el}}^5 - T_{\mathrm{ph}}^5)$,[19] where Ω is the sample volume, and Γ characterizes phenomenologically the strength of the electron-phonon coupling. Since the conductance is activated, a small increase of T_{el} can induce a very large decrease of the resistance, leading to an overall decrease of observed V. The fit of the S-shaped part of the data leads to a parameter $\Gamma \simeq 510^{-4}$ nW $\mu\mathrm{m}^{-3}\,\mathrm{K}^{-5}$, 3–4 orders of magnitude smaller than in typical metals. Note, however, that heating alone cannot explain the shape of the high current branch (where the small temperature increase becomes irrelevant), nor does it account for the coexistance of two low current branches (see Fig. 1).

In order to rescale points according to Eq. (1), we thus focus on the high current branch. As shown in Fig. 2, for both $S2d$ and $S3d$, all the points of the high current branch, corresponding to different temperatures and magnetic fields, can be accommodated by the scaling assumption Eq. (1). In particular, *the same exponents and function F hold for different magnetic fields*, as expected by universality (magnetic-field effects will be reported in a separate publication). This rescaling allows us to estimate $\beta\phi = 2.60 \pm 0.05$, with $\beta = 2.10 \pm 0.20$ for $S3d$, and $\beta\phi = 2.25 \pm 0.05$, with $\beta = 1.5 \pm 0.15$ for $S2d$. This compares satisfactorily with the predictions of Refs. 2 and 12 for β: $\beta_{3d} = 2.20$–2.25 and $\beta_{2d} = \frac{5}{3}$, whereas a classical mean-field analysis leads to $\beta_{\mathrm{MF}} = 2$.[20] Extending the arguments of Ref. 2 to nonzero T suggests $\beta\phi_{3d} \simeq 2.95$ and $\beta\phi_{2d} = \frac{5}{2}$. Deviations from theoretical values might be due to finite-size effects, as discussed in Ref. 2. Since the model studied in Ref. 2 deals with grains capacitively coupled, the above agreement comforts our basic assumption of strong inhomogeneities or droplets in our samples.

From the above theoretical discussions we can estimate the value of both V_c and T_1 which, as we shall see, turn out to be quite realistic. Since T_1 is the energy needed to create and ionize an electron-hole pair locally, we obtain $T_1(3d) \simeq (e^2/4\pi C) + \Delta$, and $T_1(2d) \simeq (e^2/2\pi C) + \Delta$, where $C = \epsilon_0 \epsilon_r a$ and Δ is the mean energy spacing on a droplet of size a. From experiment, we obtain $T_1(3d) = 0.37$ K and $T_1(2d) = 0.57$ K, which allows us to obtain $e^2/4\pi C \simeq \Delta \simeq 0.2$ K. Note that this is consistent with our assumption that Coulomb effects are important. This latter statement was not obvious *a priori*. Indeed, in disordered insulators, the relative dielectric constant ϵ_r is very large, since[21] it goes as $\epsilon_r \simeq (\xi/\lambda_{\mathrm{TF}})^2$, where $\lambda_{\mathrm{TF}} \simeq 0.2$ nm is the Thomas-Fermi screening length. In our systems, an estimate of $g(E_F)$ and the measurement of $T_0 \simeq 21/g(E_F)\xi^3$ leads[13] to $\xi \simeq 10$ nm and thus to $\epsilon_r \simeq 2500$. This huge value decreases Coulomb interactions, but not enough to prevent these interactions to dominate transport at low T.

Secondly, as explained in Ref. 2, $V_c \simeq (L/\Lambda)[E_{\mathrm{charg}}(\Lambda)/e]$, where Λ is the scale above which an electron and its hole are independent and $E_{\mathrm{charg}}(\Lambda)$ is the energy required to add a charge in a domain Λ^d. In the $3d$ case, the Coulomb interaction goes as r^{-1}, and thus Λ_{3d} is small: $\Lambda_{3d} \simeq a+b$. Conversely, in the $2d$ case $\Lambda_{2d} \simeq \sqrt{\epsilon_r}(a+b) \simeq 50(a+b)$. Indeed, due to the huge value of ϵ_r, the electric field is confined within the film up to Λ, enhancing the electron-hole interaction. Beyond Λ_{2d}, $3d$ electrostatics is valid, and charges are independent. Expressing $E_{\mathrm{charg}}(\Lambda)$ leads to $V_c(3d) \simeq (L/\Lambda_{3d})(e/8\pi C)$. For the $2d$ case, we obtain $V_c \simeq (L/\Lambda_{2d})(e/4\pi C)\ln[\Lambda_{2d}/(a+b)]$. From experiment we measure $V_c(2d) \simeq 2.5$ mV and $V_c(3d) \simeq 8.7$ mV.

Using the previous estimates of Coulomb terms, we find that *both values* of V_c lead to: $a+b \simeq 100$ nm $\simeq 10\xi$. Moreover, the large-V conductance $\mathcal{G}_{V\to\infty}$ allows us to extract b. Indeed, since $S2d$ is basically square shaped, one obtains $\mathcal{G}_{V\to\infty} \simeq 5.0\times10^{-6}\ \Omega^{-1} \simeq (e^2/h)\exp(-b/\xi)$ for $S2d$ (a similar argument holds for $S3d$). Thus $b \simeq 2-3\xi$, allowing us to obtain $a \simeq 6-8\xi$. Note that, since $\Delta \simeq T_0\xi^3/21a^3$ and $a \simeq 7\xi$ as above, we recover $\Delta \simeq 0.15$ K, and $e^2/2\pi\epsilon_0\epsilon_r a \simeq 0.2$ K, consistently with the respective values of 0.17 and 0.4 K extracted independently from T_1 measurement.

To summarize, we argued and demonstrated experimentally that the idea of a depinning transition in an electric field, introduced in the context of arrays of quantum dots, is also relevant to describe real $2d$ and $3d$ insulating samples. These samples exhibit a crossover from VRH to a *low-temperature* activation law near the MIT. This crossover is interpretated in a model of Coulomb blockade. This is confirmed by a scaling of I-V curves, which shows that an electric field indeed induces a second-order depinning transition, with exponents rather close to their theoretical values. At very low temperatures, however, the situation is complicated

by heating (and possibly other) effects which induce a hysteretic behavior. Note that the V-I characteristics of disordered superconducting films or charge-density waves (CDW's) should display similar features. In contrast with artificial structures, metal-metalloid alloys are produced easily, allowing experiments on arrays containing a large number of capacitively coupled grains in *any d*.

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