Phase rigidity and $h/2e$ oscillations in a single-ring Aharonov-Bohm experiment

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We address the results presented recently by Yacoby *et al.* [Phys. Rev. Lett. **74**, 4047 (1995)] on the coherency and phase evolution in a resonant tunneling quantum dot embedded in an Aharonov-Bohm (AB) ring. The observed phase rigidity of the AB conductance oscillations is theoretically explained by invoking simple symmetry arguments applicable to two terminal measurements, and is corroborated experimentally in generic experiments. We show that under certain conditions $h/2e$ oscillations dominate completely the conductance of a single AB ring.

In a recent interference experiment [1] we devised a method to measure a system's phase evolution (which is not measured by usual conductance measurements). The chosen system was a *resonant tunneling* (RT) structure in a form of a *quantum dot* (QD) operating in the Coulomb blockade regime.[2] The QD was inserted in one arm of an Aharonov-Bohm (AB) ring,[3] and the *two-terminal* conductance of the modified AB ring was monitored as a function of a magnetic flux threaded through the center of the ring. The observed oscillatory conductance, periodic with the elementary flux quantum, measured when the QD was tuned to conduct resonantly, confirmed conclusively that the QD supports coherent transport. Moreover, the phase of the oscillating conductance, measured at different points on a single resonance peak and compared among different resonant peaks, revealed unexpected results, suggesting, at first sight, a complicated behavior that could have resulted due to deviation from the single-particle-based model. Here we offer a simple explanation for the seemingly bizarre behavior of the phase evolution in one resonant peak based on simple symmetry arguments, and support it with experimental verifications. We show that under certain controllable conditions the h/e component of the oscillation vanishes, and the $h/2e$ component is dominant. We conclude in general that a *direct* phase measurement of the transmission amplitude of any system via a two-terminal interference experiment is in principle not possible.

Our test system, a QD in the Coulomb blockade regime, is tuned to conduct by varying its electrostatic energy with an external metallic gate (called the *plunger* gate), so that the extra energy needed to add (or subtract) electrons to (from) the dot is only within the electrons' temperature. It is known then that the conductance at low temperatures $k_BT \ll \Gamma$ exhibits a Lorentzian line shape with respect to the electrostatic energy (a Breit-Wigner resonance with a natural line width $\Gamma \cong h/\tau_{\text{dwell}}$, with τ_{dwell} the dwell time in the QD). For higher temperatures $k_BT \gg \Gamma$, the conductance has the form of the derivative of the Fermi function with width $\approx 4k_BT$. The device of the phase-sensitive experiments consists of a modified AB ring: an AB ring and a QD inserted in its left arm. The QD is adjusted to couple weakly to the ring's arm by almost pinching off its two *point-contact-like* orifices. The ring is coupled to two-dimensional (2D) reservoirs, *drain (D)* and *source (S)*, via its own two point contacts and a small excitation voltage, V_{DS}, is applied across the ring. The two-terminal conductivity G_{DS} is then measured as a function of a magnetic flux Φ for different plunger gate voltages V_P, and a certain coupling strength of the QD to the ring (a certain QD resistance).

A simple *two-slit-like* model[4] based on summing two interfering paths (from left and right arms) would predict a one-to-one dependence between the phase of the oscillating conductance and the phase of the transmission amplitude of the QD. However, experiments revealed a totally different behavior: The conductance oscillations (with fundamental period h/e) belonging to different resonance peaks are all in phase while a rapid phase change of π, taking place on a scale of order of $k_BT/10$, occurs at some point along each resonance peak. A noninteracting 1D model for the QD in the two-slit configuration would predict the conductance oscillation to reverse its phase at subsequent peaks (ignoring spin)—not seen in the experiment. This phase evolution along one peak is expected to be smooth on the scale of $\approx 4k_BT$—also contradicting the experimental results.

Since the actual measurements were done effectively in a two-terminal configuration, the applicable Onsager relation, based on time-reversal symmetry, for the transmission amplitude of a single-channel wire in the presence of a magnetic field is $t_{\text{DS}}(B) = t_{\text{SD}}(-B)$. With conservation of current $|t_{\text{DS}}(B)|^2 = |t_{\text{SD}}(B)|^2$, we find[5]

$$G_{\text{DS}}(B) = G_{\text{DS}}(-B), \tag{1}$$

that is, a symmetric conductance with magnetic field around $B=0$. This basic relation immediately revokes the notion, based on the two-slit model, that in a two-terminal interference experiment a smooth change in the phase accumulated in one path leads to a similar smooth change in the phase of the oscillating conductance of the whole system. On the contrary, the phase of the oscillating conductance, with a flux period h/e, has to be *rigid* or change abruptly by π as the accumulated phase in one arm is being varied. Note that some of the pioneering works with AB rings, such as these by Benoit *et al.*,[6] Washburn *et al.*,[6] and de Vegvar *et al.*[7] presented asymmetric conductance with respect to magnetic field—however, this asymmetry probably resulted solely from a *four-terminal-like* coherent measurement related to a modification of the AB rings by their coherent leads.

A detailed derivation of the (symmetric) conductance of an AB ring as a function of a threading magnetic flux was already derived by Gefen, Imry, and Azbel.[8,9] In these works a single-channel AB ring was assumed to contain a single arbitrary scatterer in each of its arms (labeled 1 and 2) characterized by transmission and reflection amplitudes from the left t_1, r_1 and t_2, r_2, and from the right t_1', r_1' and t_2', r_2'. The incoming (S) and outgoing (D) leads were considered to be part of such a system, and were coupled to the ring via a scattering matrix describing the relation between the electronic flux in the leads and fluxes in the ring's arms. The two-terminal conductance of such a system was calculated to be

$$G_{\mathrm{DS}}=\frac{2e^2}{h}\left|\frac{2(Ae^{i\varphi}+Be^{-i\varphi})}{De^{2i\varphi}+Ee^{-2i\varphi}+C}\right|^2, \quad (2)$$

where $\varphi=\pi\Phi/\Phi_0$ with $\Phi_0=h/e$, and the constants, defined in Ref. 8, are functions of the transmission and reflection amplitudes of the individual scatterers.

We now replace one of the scatterers in the ring with a QD, and replace the other scatterer with a fixed phase. The QD is being simulated with a 1D-RT structure composed of two barriers (denoted a and b), with transmission from the left t_a and t_b and reflection from the left r_a and r_b, with primed coefficients related to flux impinging from the right. The plunger gate voltage is simulated by a phase accumulated between the barriers, θ, with QD transmission and reflection coefficients

$$t_{\mathrm{QD}}=\frac{t_a t_b e^{i\theta}}{1-r_b r_a' e^{2i\theta}}, \quad (3a)$$

$$r_{\mathrm{QD}}=r_a+\frac{t_a t_b' r_b e^{2i\theta}}{1-r_b r_a' e^{2i\theta}}. \quad (3b)$$

We also introduce arbitrary phases between the QD and S and D junctions. Figure 1(a) shows the conductance of the bare QD and that of the modified AB ring for two sequential resonances as a function of θ at zero flux. Similar to the conductance of the bare QD which exhibits Lorentzian-like resonances with width Γ, the conductance of the modified AB ring also has highly peaked resonances (width depending on the choice of the different phases) with peaks slightly shifted relative to the peaks of the QD. When a magnetic field threads the modified AB ring, its conductance, for a certain phase θ of the QD, changes in a periodic manner with a flux period h/e. The amplitude of the first and second harmonics of the conductance oscillations (via a Fourier analysis) is plotted for different θ in Fig. 1(b). As the resonant peak is approached from the left, the amplitude of the oscillation grows and so does the h/e component. Near the peak maximum the h/e component drops dramatically, and the $h/2e$ component grows, peaking at a phase θ where the h/e harmonic vanishes. Upon further increase in θ the h/e component reverses sign and grows again, while the $h/2e$ component gradually vanishes. This explains the rigidity of the phase of the oscillator conductance observed in the experiment.[10,11] Note the dominance of $h/2e$ oscillations in the single AB ring—known before to exist only in an array of rings or in a cylinder.[3]

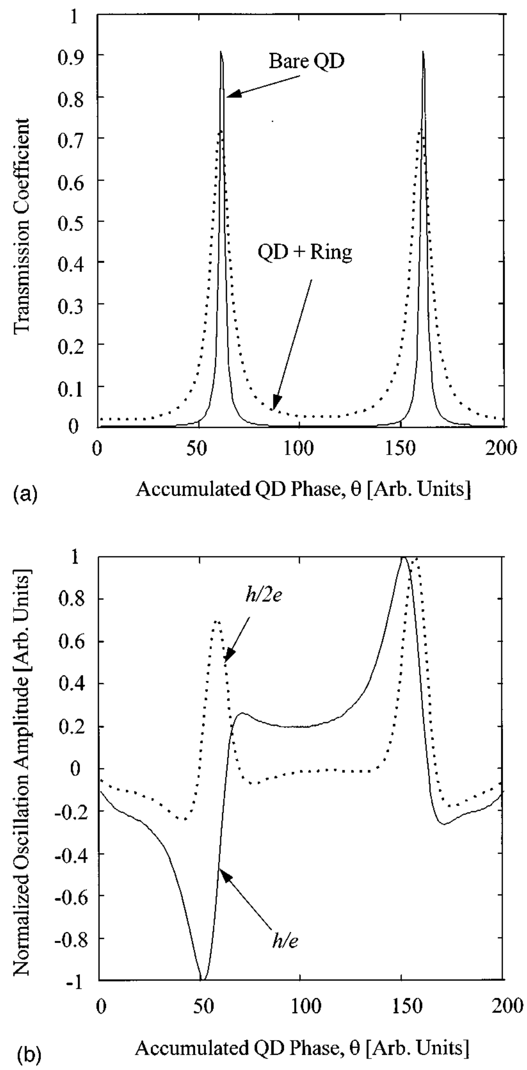

FIG. 1. A calculated response of a modified AB ring (ring plus QD). (a) The transmission of a bare 1D resonance tunneling QD and that of the modified AB ring. (b) The amplitude of the AB oscillation harmonics. Note the large $h/2e$ signal when the h/e signal vanishes.

To test the validity of our explanation we experimentally realized an *artificial impurity* with a controlled phase in one of the ring's arms [the inset of Fig. 2(b)]. Our device consists of a patterned two-dimensional electron gas (2DEG) formed in a $GaAs/Al_xGa_{1-x}As$ heterostructure. An added metallic air bridge changes the potential of a small gate deposited inside one arm, thus changing the height of a local potential barrier. The two-terminal resistance oscillations of this system, measured at 100 mK, and their Fourier transforms, are shown in Fig. 2. Again, one can clearly see that the h/e component has a rigid phase (0 or π) as a function of the voltage applied to the metallic gate. The $h/2e$ component can clearly be seen to peak when the h/e component vanishes. This demonstrates that under certain phase conditions in a single AB ring only the $h/2e$ oscillations exists—suggesting that only paths surrounding once the full ring's circumference survive.

Another demonstration of this phase rigidity can be demonstrated by changing the relative phase between the arms of the ring via changing the energy of the injected electrons.

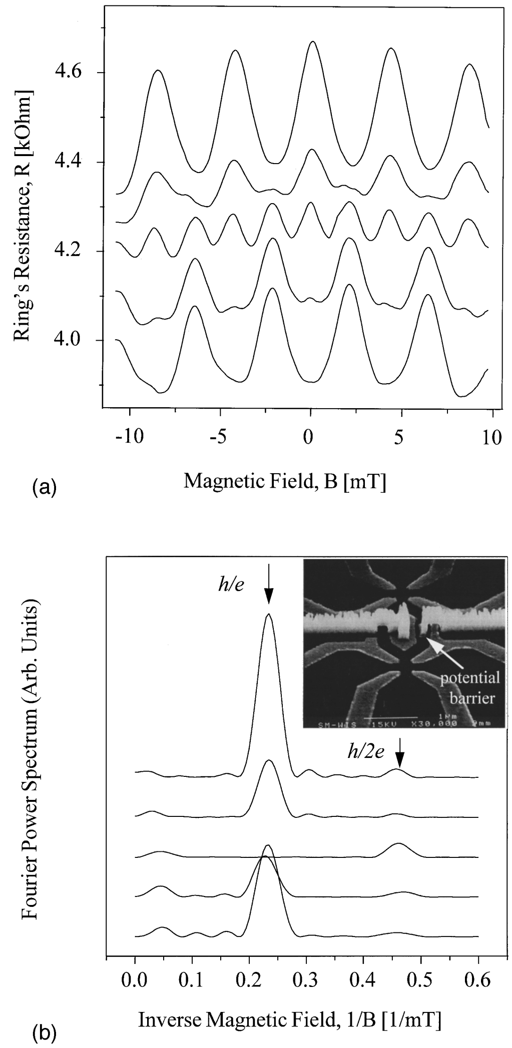

FIG. 2. The response of a modified AB ring (ring plus a potential barrier) as seen in the micrograph in the inset. (a) The AB oscillations for gradually increasing potential barrier heights. (b) The corresponding Fourier transforms of the oscillations. The lower graph in (a) is correct, while the others are shifted for clarity. The inset in (b) is a SEM micrograph of the fabricated device (gray areas are metallic gates and rough metal is the air bridge).

Carriers were thus injected via a dc voltage applied across the ring with an added small ac component, allowing us to sample only electrons in a narrow band (the ac voltage or

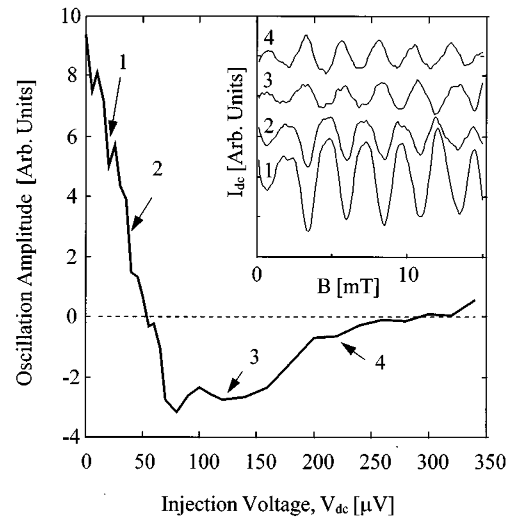

FIG. 3. The response of an AB ring for different injection energies. The inset shows the AB oscillations with amplitude (and polarity) drawn in the figure.

k_BT). The inset in Fig. 3 gives a few representative oscillations at different injection energies. The polarity is abruptly reversed around 50–60 μV, as depicted in the plot of the amplitude of the h/e component as a function of the injection energy (Fig. 3). note that the overall amplitude is reduced with injection energy as a result of increasing dephasing mediated by electron-electron interactions.[11] This experiment also demonstrates the phase rigidity we are alluding to.

We demonstrated the phase rigidity of a two-terminal measurement and explained it by invoking basic time-reversal symmetry. This rigidity *masks* the detailed phase evolution in interference experiments and leads, under certain phase accumulation conditions, to the appearance of oscillations with period of $h/2e$, and the vanishing of the h/e component, in a single AB ring.

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