Generation of spin-polarized currents in Zeeman-split Tomonaga-Luttinger models

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In a magnetic field an interacting electron gas in one dimension may be described as a Tomonaga-Luttinger model comprising two components with different Fermi velocities due to the Zeeman splitting. This destroys the spin-charge separation, and even the quantities such as the density-density correlation involve the spins. Specifically, we have shown that the ratio of the up-spin and down-spin conductivities in a dirty system *diverges* at low temperatures like an inverse power of the temperature, resulting in a spin-polarized current. In finite, clean systems the conductance becomes different for up and down spins as another manifestation of the electron-electron interaction.

Recent studies of mesoscopic systems have brought to light many unusual features in their quantum transport properties.[1] This is heightened by recent advances in fabricating nanostructure quantum wires and quasi-one-dimensional (1D) crystal structures.

In 1D systems, the interactions between the electrons is so crucial due to the strong constraint in the phase space that the system becomes universally what is called the Tomonaga-Luttinger (TL) liquid as far as low-lying excitations are concerned no matter how small the interaction may be.[2] A most striking feature of this 1D model is the spin-charge separation. The transport properties[3] are also dominated by the spin-charge separation in the following sense. The low-temperature conductivity of the dirty TL liquid as studied by Luther and Peschel[4] exhibits a power law, $\sigma(T) \sim T^{2-K_\rho-K_\sigma}$. The power law comes from the degraded Fermi singularity in the TL liquid, while the critical exponents (which are functions of the interaction) enter as a sum of K_ρ for the charge phase of the system and K_σ for the spin phase (which is actually unity for spin-independent interactions). A recent experiment[7] for high-quality quantum wires seems to support this result. For clean systems Kane and Fisher[5] and Furusaki and Nagaosa[6] found that the conductance quantization in noninteracting mesoscopic systems becomes proportional to the exponent, $G=(e^2/\pi)K_\rho$ (where $\hbar=k_B=1$ is assumed hereafter).

Now we can raise an intriguing question: what happens if we degrade the spin-rotational [SU(2)] symmetry in some way? This is indeed realized by applying a magnetic field, which makes the Fermi velocities spin dependent due to the Zeeman splitting. In this paper, we show that the degraded spin-charge separation, which will make even the quantities such as the density-density correlation involve spins, may manifest itself in the transport, leading such a drastic effect as spin-polarized currents in a right condition.

The generation of spin-polarized currents itself has been of a long-standing interest for academic[8] as well as practical points of view, where typical applications include spin-polarized scanning tunneling microscopy[9] (STM) and the Mott detector.[10] Fasol and Sakaki[11] have suggested that in the spin-orbit split bands of GaAs quantum wires the curvature in the band dispersion (as opposed to the linearized dispersion in the Tomonaga-Luttinger model) will make the relaxation time due to the electron-electron interaction spin dependent and consequently make the outgoing current spin polarized. The mechanism proposed here is by contrast a purely electron-correlation effect, where the ratio $\sigma_\uparrow/\sigma_\downarrow$ diverges toward $T=0$.

We start from a clean, two-band Tomonaga-Luttinger model, which is similar to the one employed in Refs. 12 and 13. The Hamiltonian is given by

$$H_{\rm clean}=H_0+H_{\rm int}, \tag{1}$$

with the noninteracting Hamiltonian being

$$H_0=\sum_{k,s,i} v_{Fs}[(-)^{i+1}k-k_{Fs}]c^\dagger_{iks}c_{iks}. \tag{2}$$

Here $v_{F\uparrow}$ ($v_{F\downarrow}$) is the Fermi velocity of the up (down) spin subband in a magnetic field B, for which we denote an average $v_0\equiv(v_{F\uparrow}+v_{F\downarrow})/2$ and the difference $\Delta v\equiv(v_{F\uparrow}-v_{F\downarrow})/2$. Similarly, $k_{F\uparrow}$ ($k_{F\downarrow}$) is the Fermi momentum of the up (down) spin. $c^\dagger_{iks}$ creates a right-going ($i=1$) or left-going ($i=2$) electron with momentum k and spin s, and L is the system size.

In the interaction, $H_{\rm int}$, the charge and spin are no longer decoupled for $\Delta v\neq 0$, in sharp contrast to the usual Tomonaga-Luttinger model. If we introduce the usual phase field $\theta_+(x)$ [$\phi_+(x)$] and the dual field $\theta_-(x)$ [$\phi_-(x)$] that correspond to the charge (spin) degree of freedom, the spin-charge separated part is cast into the usual phase Hamiltonian with v_0 playing the role of the Fermi velocity, while additionally θ-ϕ coupled terms appear as

$$\begin{aligned}H_{\rm clean}=&\frac{v_\rho}{4\pi}\int dx\left\{\frac{1}{K_\rho}[\partial_x\theta_+(x)]^2+K_\rho[\partial_x\theta_-(x)]^2\right\}\\&+\frac{v_\sigma}{4\pi}\int dx\left\{\frac{1}{K_\sigma}[\partial_x\phi_+(x)]^2+K_\sigma[\partial_x\phi_-(x)]^2\right\}\\&+\frac{\Delta v}{2\pi}\int dx\{[\partial_x\theta_+(x)][\partial_x\phi_+(x)]+[\partial_x\theta_-(x)]\\&\times[\partial_x\phi_-(x)]\},\end{aligned} \tag{3}$$

where K_ρ (K_σ) is the critical exponent of the charge (spin) phase. In the following we assume that the coupling constants between electrons have no significant magnetic-field dependences, so that we set $K_\sigma=1$, $v_\sigma=v_0$, $K_\rho=1/\sqrt{1+4g}$, and $v_\rho=v_0/K_\rho=v_0\sqrt{1+4g}$, where g ($\sim U/2\pi v_0$ for the Hubbard model) is the dimensionless, forward-scattering coupling constant. Here we have neglected the backward-scattering and umklapp-scattering processes, since they have large momentum transfers.

We can diagonalize $H_{\rm clean}$, as is done for the electron-hole system in a two-channel Tomonaga-Luttinger study of the excitonic phase by Nagaosa and Ogawa,[14] via a linear transformation to two new phases,

$$\begin{pmatrix} \theta_+(x) \\ \phi_+(x) \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\frac{1}{y}\sin\alpha \\ y\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \tilde{\theta}_+(x) \\ \tilde{\phi}_+(x) \end{pmatrix}, \qquad (4)$$

where α is the "rotation angle in the spin-charge space" ($\propto \Delta v$ for small Δv) with $\tan 2\alpha = 2(\Delta v/v_0)\sqrt{2(K_\rho^{-2}+1)}/(K_\rho^{-2}-1)$ and $y^2=\frac{1}{2}(K_\rho^{-2}+1)$. The diagonalized phases have gapless, linear dispersions, in which the new velocities are given by

$$\tilde{v}^{\,2}_{\rho,\sigma} = \Delta v^2 + \tfrac{1}{2} v_0^2 [K_\rho^{-2}+1 \pm (K_\rho^{-2}-1)\sqrt{1+\tan^2 2\alpha}], \qquad (5)$$

where the $+$ $(-)$ sign corresponds to $\tilde{v}_\rho$ ($\tilde{v}_\sigma$).

Now we can turn to the calculation of the conductivity in a dirty system. We then add to the Hamiltonian the impurity-scattering part,

$$H_{\rm imp} = \sum_s \sum_l \int dx N_s(x) u(x-x_l), \qquad (6)$$

where $u(x-x_l)$ is the impurity potential situated at x_l and $N_s(x)$ is the density operator of spin s electrons, whose phase representation is $N_s = (1/2\pi)\,\partial_x(\theta_+ + s\phi) + (1/\pi\Lambda)\cos[2k_{Fs}x + \theta_+ + s\phi]$ with Λ being a short-range cutoff. The conductivity σ_s of spin s subband is given by $\sigma_s = n_e e^2 \tau_s / 2m_s^*$, where τ_s is the relaxation time for spin s, n_e the density of electrons, and $m_s^* \propto v_s^{-1}$ the effective mass of the spin s subband. In 1D we have $n_e = 2k_F/\pi$, but we ignore the trivial magnetic-field dependence of $k_{F\uparrow}$ and $k_{F\downarrow}$ to single out the effect of $v_{F\uparrow}/v_{F\downarrow} \neq 1$.

We can calculate τ_s following Götze and Wölfle in the Mori formalism for the conductivity[15,16] in the second order in $H_{\rm imp}$ as

$$\frac{1}{\tau_s} \approx 4\pi v_{Fs} n_i u^2(2k_F) \sum_q \lim_{\omega\to 0} \frac{{\rm Im}\Pi_s(2k_{Fs}+q,\omega)}{\omega}, \qquad (7)$$

where n_i is the density of impurities and $u(q)$ is the Fourier transform of $u(x)$. Here Π_s is the density-density correlation function for spin s, which is related to the density operator $\rho_s(x)$ as

$$\lim_{\omega\to 0} \sum_q \frac{{\rm Im}\Pi_s(2k_{Fs}+q)}{\omega} = \frac{1}{2T} \sum_{s'} \int_{-\infty}^{\infty} dt \langle \rho_s(0,t) \rho_{s'}(0,0) \rangle. \qquad (8)$$

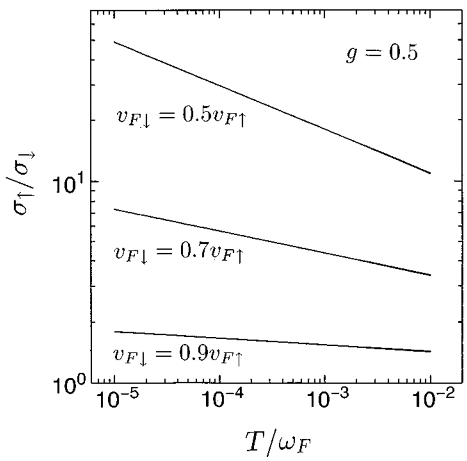

FIG. 1. The result for the temperature dependence of the ratio $\sigma_\uparrow/\sigma_\downarrow$ with a fixed interaction $g=0.5$, and with several ratios $v_{F\uparrow} v_{F\downarrow}$.

In the summation over the spin s' we can readily show that the cross term $\langle \rho_\uparrow(0,t)\rho_\downarrow(0,0)\rangle$ vanishes. Then the conductivity becomes a sum of the two spin components, each of which has a simple power-law temperature dependence as in the usual Luttinger theory,

$$\sigma_s(T) = \sigma_0 \left(\frac{v_{Fs}}{v_0}\right)^2 \left(\frac{T}{\omega_F}\right)^{2-K_s}, \qquad (9)$$

where $\sigma_0 \equiv \sigma(T=\omega_F)$ (whose dependence on k_{Fs} is again ignored here) and $\omega_F \sim \epsilon_F$ is the high-energy cutoff.

Here the spin-dependent exponent K_s is given by

$$K_s = (\cos\alpha + sy\sin\alpha)^2 \tilde{K}_\rho + \left(\cos\alpha - \frac{s}{y}\sin\alpha\right)^2 \tilde{K}_\sigma, \qquad (10)$$

which involves both the critical exponent, $\tilde{K}_\rho$, for the phase $\tilde{\theta}$ and the critical exponent, $\tilde{K}_\sigma$, for $\tilde{\phi}$ given by

$$\tilde{K}^2_{\rho(\sigma)} = y^{\mp 2}[K_\rho^{-2}+3 \pm (K_\rho^{-2}-1)\sqrt{1+\tan^2 2\alpha}][3K_\rho^{-2}+1 \pm (K_\rho^{-2}-1)\sqrt{1+\tan^2 2\alpha}]^{-1}, \qquad (11)$$

where the upper (lower) sign corresponds to $\tilde{K}_\rho$ ($\tilde{K}_\sigma$).

The above equations (10)–(12) are the key result of this paper: the electron-electron interaction does indeed make the conductivity dependent on the spin, where the power-law dependence in T is retained so that the spin dependence becomes more enhanced at lower temperatures. The ratio $\sigma_\uparrow/\sigma_\downarrow \propto T^{-(K_\uparrow - K_\downarrow)}$ diverges toward $T\to 0$, which implies a spin-polarized current there. Note that the divergence is an effect of the electron correlation.

We display in Fig. 1 the temperature dependence of $\sigma_\uparrow/\sigma_\downarrow$ numerically calculated for various values of $v_{F\uparrow}/v_{F\downarrow}$ for a fixed electron-electron interaction g. Figure 2 shows the dependence of $\sigma_\uparrow$ ($\sigma_\downarrow$) on the ratio $v_{F\uparrow}/v_{F\downarrow}$ at a fixed temperature with a fixed g.

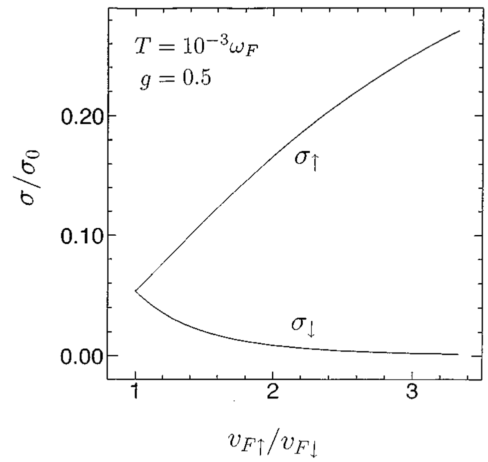

FIG. 2. The result for the dependence on $v_{F\uparrow}/v_{F\downarrow}$ (keeping $v_{F\uparrow}+v_{F\downarrow}$=const) of the conductivity $\sigma_\uparrow/\sigma_0$ and $\sigma_\downarrow/\sigma_0$ with a fixed temperature $T=10^{-3}\omega_F \sim 100$ mK and with a fixed interaction $g=0.5$.

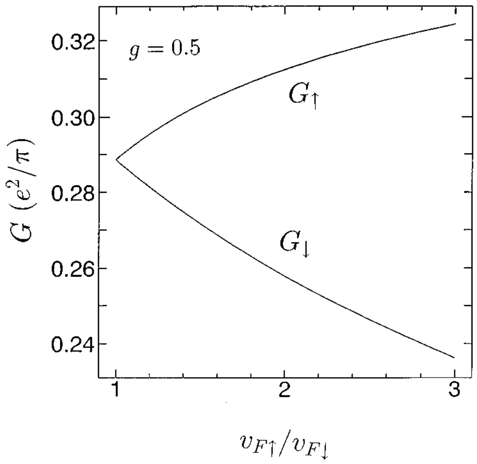

FIG. 3. The result for the dependence on $v_{F\uparrow}/v_{F\downarrow}$ (keeping $v_{F\downarrow}+v_{F\uparrow}$=const) of the conductance $G_\uparrow$ and $G_\downarrow$ normalized by e^2/π with a fixed interaction $g=0.5$.

The result shows that the more conductive channel is the spin having a larger v_F, since $K_\uparrow > K_\downarrow$ for $\Delta v \propto (v_{F\uparrow} - v_{F\downarrow}) > 0$. Physically, the effective electron-electron interaction is smaller (larger) in the lighter (heavier) spin subband, since it is the ratio of the electron-electron coupling constant to the kinetic energy ($\propto v_F$) that matters. Thus the result is roughly consistent with the observation in a single TL liquid that the electron-electron repulsive interaction suppresses the conductivity.[4]

We can further give an intuitive interpretation of the present result, if we mimic the single TL model of spin-1/2 electrons as a double-chain system of spinless electrons. The present situation is then regarded as a generalization of our previous model,[16] where we have considered two equivalent chains having intrachain and interchain interactions in the absence of interchain tunneling. When the two "chains" are made inequivalent by the differentiated v_F, this modifies both the "intrachain" (parallel-spin) dimensionless coupling constant, g/v_F, and "interchain" (antiparallel-spin) g/v_F. Note that if the g parameters derive from an SU(2) symmetric interaction (such as the Hubbard U), the g's for intrachain and interchain interactions are the same. It is then a highly nontrivial question what conductivities will come out. The present result indicates that the chain that has a smaller g/v_F does indeed remain more conductive, so that the effect of parallel-spin interaction eventually prevails. This is consistent with the double-chain result that the intrachain repulsion suppresses the conductivity, while the effect of interchain interactions, which incidentally enhances the conductivity, is only of the second order.

Next we consider the *conductance* of finite, clean systems in magnetic fields. The conductance G_s of spin s subband is calculated from the current-current correlation function as[5]

$$G_s = \lim_{\omega\to 0} \sum_{s'} \frac{1}{\omega L} \int d\tau \int dx e^{i\omega\tau} \langle T_\tau J_s(\tau) J_{s'}(0) \rangle, \quad (12)$$

where $J_s = e\partial_\tau(\theta + s\phi)/2$ is the spin s current, and τ the imaginary time. Then we end up with

$$G_s = \frac{e^2}{2\pi}\left[\left(\cos^2\alpha + \frac{ys}{2}\sin 2\alpha\right)\tilde{K}_\rho + \left(\frac{1}{y^2}\sin^2\alpha - \frac{s}{2y}\sin 2\alpha\right)\tilde{K}_\sigma\right]. \quad (13)$$

Thus the conductance too depends on the spin in contrast to the ordinary case with $G_\uparrow = G_\downarrow = (e^2/2\pi)K_\rho$. We may emphasize that the Fermi velocity of the system does not appear in conductances as shown in the Landauer formula[17] (while in the conductivity the Fermi velocities do appear in the power of the temperature), so the spin-dependent conductance here is an effect of the electron-electron interaction in a more manifest manner. Figure 3 numerically depicts the way in which $G_\uparrow$ ($G_\downarrow$) increase (decrease) with the ratio $v_{F\uparrow}/v_{F\downarrow}$ with a fixed g.

Finally let us make a comment on the Anderson localization. In order to discuss this effect at very low temperatures, we must treat the impurity scattering beyond the simple perturbation to consider the renormalization effect due to the impurities. In fact, Giamarchi and Schulz have shown in the absence of magnetic fields that the temperature at which the localization sets in shifts downward for larger K_ρ.[19] We can actually show that such a renormalization for the present system results in a flow diagram divided into three regions.[18] In region I with $K_\uparrow, K_\downarrow < 3$, the impurity scatterings for both spins are monotonically enhanced at low temperatures. In region II with $K_\uparrow < 3, K_\downarrow > 3$, the impurity scattering of the heavier spin subband is suppressed (except at lower temperatures), while that of the lighter spin is enhanced. Then we expect large spin polarization of the current. At lower temperatures the impurity scattering of the heavier subband too will be eventually enhanced. In region III with $K_s > 3$ both spins will remain delocalized for $T \to 0$.

We believe these many-body effects can be experimentally measured in quantum wires by taking appropriate fillings of the up- and down-spin subbands in a given magnetic field. Unfortunately, in the case of the usual electron-doped

GaAs quantum wires the small g factor (~ -0.4 in the bulk) will require sufficiently low electron fillings for the Zeeman splitting effect to appear. However, if we can prepare, e.g., InSb quantum wires (whose g factor is as large as ~ -50 in the bulk[20]), the strength of the Zeeman splitting in a typical magnetic field of 1 T amounts to $g\mu_B H \sim$ 3.0 meV. In such cases a significant deviation of $v_\uparrow/v_\downarrow$ from unity may be expected.

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