Energy-gap structure of a *t-J* bilayer

M. Biagini
Istituto Nazionale Fisica della Materia and Dipartimento di Fisica, Università di Modena, via Campi 213/A, I-41100 Modena, Italy
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The role of interlayer coupling in copper-oxide superconductors represents an open problem in high-T_c superconductivity. In this work, the energy gap and the density of states of a *t-J* bilayer are analyzed for different value of the doping δ within a mean-field approximation. It is shown that an interlayer single-electron hopping increases the ratio $R=2\Delta/KT_c$ and that the extent of such increase depends strongly on the doping δ. The density of states contains both BCS-like and logarithmic singularities and presents a multiple-peak structure, in qualitative agreement with recent tunneling experiments.

I. INTRODUCTION

All copper-oxide-based high-T_c superconductors have stratified structures, where superconducting layers alternate to metallic or insulating layers. Because of the short coherence length, the Cooper pairs are localized in the CuO_2 planes, which is a common feature of these systems. The weakly coupled layered structure of cuprates gives an origin to a strong anisotropy between the in-plane and the off-plane directions in their physical properties such as the normal-state resistivity,[1] the upper critical field,[2] the thermal conductivity,[3] etc.

A major problem in the theory of high-T_c superconductivity is to what extent the superconducting properties are bound to the two dimensionality of the CuO_2 planes, and how they are affected by the interaction between adjacent CuO_2 planes. In order to investigate the relationship between dimensionality and high-T_c superconductivity, many groups have studied the electrical transport properties of $YBa_2Cu_3O_7/PrBa_2Cu_3O_7$ (YBCO/PBCO) superlattices.[4] Several different models have been proposed in order to explain the T_c depression observed in the above systems, which are based upon Kosterlitz-Thouless transition and charge redistribution effects,[5–9] interlayer coupling,[10,11] proximity effect,[12–15] and hole filling.[16] The results obtained for YBCO/PBCO superlattices do not provide clear evidence of a relationship between interlayer coupling and critical temperature, since many other possible effects are involved, such as hole filling, localization effects, modification of the electronic structure of the superconducting planes, etc. On the other hand, none of the above effects have been found in recent first principles calculations of the electronic structure of YBCO/PBCO superlattices.[17]

The role of interlayer coupling in high-T_c superconductivity has been studied within the framework of BCS-like models by several authors.[18,19] On the other hand, many of the microscopic models that have been proposed to describe the properties of the high-T_c copper oxides are based on the two-dimensional Hubbard model or the *t-J* model.

The *t-J* model was proposed by Anderson,[20] as the simplest model containing the essential strong correlation physics of the oxide superconductors. Such a model attempts to describe both magnetism and superconductivity within the same framework and successfully accounts for many of the unconventional transport and collective mode properties observed in the metallic phases of cuprate superconductors.[21,22] Approximated solutions of the *t-J* model can be obtained by means of a mean-field decoupling of the Hamiltonian, and several mean-field phases have been suggested.[23–30]

In recent works,[31,32] I have shown that a single-electron hopping between two bidimensional *t-J* lattices can reduce the superconducting transition temperature to values that agree more with the experimental ones. Besides, I have shown that the above model can be applied to the interpretation of the T_c depression observed in iodine intercalated $Bi_2Sr_2CaCu_2O_x$.[33]

In the present paper, I analyze the effect of a single-electron interlayer coupling on the energy gap and the density of states of a *t-J* bilayer. The present model can be applied to materials like $Bi_2Sr_2CaCu_2O_{8+\delta}$ or $YBa_2Cu_3O_7$ that contains two CuO_2 planes within the unit cell. The knowledge of the density of states and energy gap structure can allow one to identify the mechanism responsible for the superconductivity in copper-oxide superconductors. Experimental information about these quantities can be obtained through tunneling spectroscopy.[34,35] Besides, very recently, information about the momentum dependence of the energy gap has been obtained through high-resolution angle-resolved photoemission spectroscopy.[36,37]

II. THEORY

The *t-J* model is defined by the Hamiltonian

$$H=-t\sum_{\langle i,j\rangle\sigma} c^{\dagger}_{i\sigma}c_{j\sigma}+J\sum_{\langle i,j\rangle}(\vec{S}_i\cdot\vec{S}_j-\tfrac{1}{4}n_in_j)-\mu_f\sum_i n_i, \qquad (1)$$

where $\vec{S}_i=\frac{1}{2}c^{\dagger}_{iw}\hat{\vec{\sigma}}_{w\beta}c_{i\beta}$ and $n_i=\Sigma_\sigma c^{\dagger}_{i\sigma}c_{i\sigma}$. This Hamiltonian is under the constraint that no site is double occupied. Such a constraint can be satisfied by employing the slave-boson formalism,[38] in which the electron operator $c^{\dagger}_{i\sigma}$ is replaced by $c^{\dagger}_{i\sigma}=f^{\dagger}_{i\sigma}b_i$. The constraint of no double occupancy implies that $b^{\dagger}_ib_i+\Sigma_\sigma f^{\dagger}_{i\sigma}f_{i\sigma}=1$ at each site i; this request may be satisfied by adding to the Hamiltonian (1) a term $\Sigma_i\lambda_i(\Sigma_\sigma f^{\dagger}_{i\sigma}f_{i\sigma}+b^{\dagger}_ib_i-1)$, where λ_i is a Lagrangian multiplier. A mean-field theory for the Hamiltonian (1) can be

obtained by decoupling the four-fermion term $\vec{S}_i\cdot\vec{S}_j$. This leads to the introduction of the Hubbard-Stratonovich fields $\chi_{ij}=\langle\Sigma_\sigma f^\dagger_{i\sigma}f_{j\sigma}\rangle$ and $\Delta_{ij}=\langle f_{i\uparrow}f_{j\downarrow}-f_{i\downarrow}f_{j\uparrow}\rangle$. A mean-field theory is achieved by replacing the fields χ_{ij}, Δ_{ij}, b_i, and λ_i by their saddle-point values. The hopping term $t\Sigma_\sigma f^\dagger_{i\sigma}f_{j\sigma}b^\dagger_j b_i$ is replaced by $b_0^2 t\Sigma_\sigma f^\dagger_{i\sigma}f_{j\sigma}$, where b_0 is the saddle-point value of the boson field b_i. The mean-field Hamiltonian then becomes[29]

$$H^{\mathrm{MF}}=\frac{3J}{8}\sum_{\langle i,j\rangle}\left[|\chi_{ji}|^2+|\Delta_{ji}|^2-\chi^*_{ji}\sum_\sigma f^\dagger_{j\sigma}f_{i\sigma}-\mathrm{c.c.}-\Delta^*_{ji}(f_{j\uparrow}f_{i\downarrow}-f_{j\downarrow}f_{i\uparrow})-\mathrm{c.c.}\right]$$
$$-tb_0^2\sum_{\langle i,j\rangle}\left(\sum_\sigma f^\dagger_{j\sigma}f_{i\sigma}+\mathrm{c.c.}\right)-\mu_0\sum_{i\sigma}f^\dagger_{i\sigma}f_{i\sigma}+\mu_b\sum_i\left(\sum_\sigma f^\dagger_{i\sigma}f_{i\sigma}+b_0^2-1\right), \tag{2}$$

where $\mu_0=\mu_f-3J/4$. The decoupling in the particle-hole channel is supposed uniform; that is, $\chi_{ij}=\chi$ for all bonds $\langle i,j\rangle$. The decoupling in the particle-particle channel is instead chosen so that $\Delta_{ij}=+\Delta$ if $\mathbf{ij}\|\hat{x}$, $\Delta_{ij}=-\Delta$ if $\mathbf{ij}\|\hat{y}$: This choice corresponds to the d-wave phase.[24] The Hamiltonian (2) can be conveniently written in the Nambu formalism

$$H^{\mathrm{MF}}=\sum_{\vec{k}}\psi^\dagger_{\vec{k}}H_{\vec{k}}\psi_{\vec{k}}+\frac{3J}{4}N(\chi^2+\Delta^2)+\mu_b b_0^2N-\mu_0N, \tag{3}$$

where $\psi^\dagger_{\vec{k}}\equiv(f^\dagger_{\vec{k}\uparrow},f_{-\vec{k}\downarrow})$ and N is the number of lattice points. The Hamiltonian matrix $H_{\vec{k}}$ is given by

$$H_{\vec{k}}=\begin{pmatrix}A_{\vec{k}} & B_{\vec{k}}\\ B_{\vec{k}} & -A_{\vec{k}}\end{pmatrix}, \tag{4}$$

where $A_{\vec{k}}=-2(tb_0^2+3J\chi/8)[\cos(k_x)+\cos(k_y)]+\mu_b-\mu_0$ and $B_{\vec{k}}=-(3J\Delta/4)[\cos(k_x)-\cos(k_y)]$. I consider now the effect of a single-electron hopping between two bidimensional t-J lattices. Such a model can be applied to materials such as $Bi_2Sr_2CaCu_2O_{8+\delta}$ or $YBa_2Cu_3O_7$, which contains two CuO_2 planes in the unit cell, and is described by the Hamiltonian

$$H=\sum_{l=1,2}H_l^{\mathrm{MF}}+\sum_{\vec{k}}\sum_{\substack{l,l'=1,2\\ l\neq l'}}\alpha\psi^\dagger_{l,\vec{k}}\hat{\sigma}_z\psi_{l',\vec{k}}, \tag{5}$$

where $\hat{\sigma}_z$ is the third Pauli matrix, α is the hopping constant, and $\psi_{l,\vec{k}}$ is the Nambu spinor corresponding to the lth t-J lattice. Equation (5) can be written in the form

$$H=\sum_{\vec{k}}\Psi^\dagger_{\vec{k}}\tilde{H}_{\vec{k}}\Psi_{\vec{k}}+\frac{3J}{2}N(\chi^2+\Delta^2)+2\mu_b b_0^2N-2\mu_0N, \tag{6}$$

where $\Psi^\dagger_{\vec{k}}\equiv(f^\dagger_{1,\vec{k}\uparrow},f_{1,-\vec{k}\downarrow},f^\dagger_{2,\vec{k}\uparrow},f_{2,-\vec{k}\downarrow})$ and the Hamiltonian matrix $\tilde{H}_{\vec{k}}$ is given by

$$\tilde{H}_{\vec{k}}=\begin{pmatrix}A_{\vec{k}} & B_{\vec{k}} & \alpha & 0\\ B_{\vec{k}} & -A_{\vec{k}} & 0 & -\alpha\\ \alpha & 0 & A_{\vec{k}} & B_{\vec{k}}\\ 0 & -\alpha & B_{\vec{k}} & -A_{\vec{k}}\end{pmatrix}. \tag{7}$$

The eigenvalues of the Hamiltonian (7) are given by

$$\pm E^-_{\vec{k}}=\pm\sqrt{A^2_{\vec{k}}+B^2_{\vec{k}}-2A_{\vec{k}}\alpha+\alpha^2},$$
$$\pm E^+_{\vec{k}}=\pm\sqrt{A^2_{\vec{k}}+B^2_{\vec{k}}+2A_{\vec{k}}\alpha+\alpha^2}.$$

The mean-field parameters are determined by the condition of minimum of the thermodynamic potential

$$\Omega=2N\left[\frac{3J}{4}(\chi^2+\Delta^2)+\mu_b b_0^2-\mu_0-T\ln(4)\right]$$
$$-2T\sum_{\vec{k}}\left[\ln\cosh\left(\frac{E^+_{\vec{k}}}{2T}\right)+\ln\cosh\left(\frac{E^-_{\vec{k}}}{2T}\right)\right]. \tag{8}$$

Besides, the chemical potential μ_0 is determined by the condition $\partial\Omega/\partial\mu_0=\delta-1$; the doping δ is the number of holes per plane so that the average number of electrons in each site is $1-\delta$. The parameter δ must be related to the number of holes per CuO_2 unit in the sample. Such a value can be obtained through different techniques: Hall coefficient measurements,[39] chemical methods,[40] or by assuming a given valence state for the ions in the coumponds.[41] Through a direct comparison of the self-consistent equations, one can readily obtain the values of $b_0^2=\delta$ and $\mu_b=4t\chi$.

The Green functions of the system can be calculated from the Gor'kov equations,[42] which can be written

$$(i\omega_n-A_{\mathbf{k}})G_{11}(\mathbf{k},\omega_n)+B_{\mathbf{k}}F^*_{11}(\mathbf{k},\omega_n)-\alpha G_{12}(\mathbf{k},\omega_n)=1,$$
$$(i\omega_n+A_{\mathbf{k}})F^*_{11}(\mathbf{k},\omega_n)+B^*_{\mathbf{k}}G^*_{11}(\mathbf{k},\omega_n)+\alpha F^*_{12}(\mathbf{k},\omega_n)=0,$$
$$(i\omega_n-A_{\mathbf{k}})G_{12}(\mathbf{k},\omega_n)+B_{\mathbf{k}}F^*_{12}(\mathbf{k},\omega_n)-\alpha G_{11}(\mathbf{k},\omega_n)=0,$$
$$(i\omega_n+A_{\mathbf{k}})F^*_{12}(\mathbf{k},\omega_n)+B^*_{\mathbf{k}}G^*_{12}(\mathbf{k},\omega_n)+\alpha F^*_{11}(\mathbf{k},\omega_n)=0,$$

where

$$G_{ij}(\mathbf{k},\omega_n)=\frac{1}{2}\int_{-1/KT}^{1/KT}d\tau e^{i\omega_n\tau}\langle T_\tau[f^\dagger_{i,\mathbf{k}\uparrow},f_{j,\mathbf{k}\uparrow}]\rangle, \tag{9}$$

$$F^*_{ij}(\mathbf{k},\omega_n)=\frac{1}{2}\int_{-1/KT}^{1/KT}d\tau e^{i\omega_n\tau}\langle T_\tau[f^\dagger_{j,\mathbf{k}\uparrow},f^\dagger_{i,\mathbf{k}\downarrow}]\rangle, \tag{10}$$

and $\omega_n=(2n+1)\pi KT$. One then finds

$$G_{11}^{*}(\mathbf{k},\omega_n)=\frac{(\omega_n+A_{\mathbf{k}})(\omega_n^2-A_{\mathbf{k}}^2-B_{\mathbf{k}}^2)-\alpha^2(\omega_n-A_{\mathbf{k}})}{[\omega_n^2-(E_{\mathbf{k}}^{-})^2][\omega_n^2-(E_{\mathbf{k}}^{+})^2]},$$

$$G_{12}^{*}(\mathbf{k},\omega_n)=\frac{-\alpha B_{\mathbf{k}}^2+\alpha(\omega_n+A_{\mathbf{k}})^2-\alpha^3}{[\omega_n^2-(E_{\mathbf{k}}^{-})^2][\omega_n^2-(E_{\mathbf{k}}^{+})^2]},$$

$$F_{11}^{*}(\mathbf{k},\omega_n)=-\frac{B_{\mathbf{k}}^{*}(\omega_n^2-A_{\mathbf{k}}^2-B_{\mathbf{k}}^2)-\alpha^2 B_{\mathbf{k}}^{*}}{[\omega_n^2-(E_{\mathbf{k}}^{-})^2][\omega_n^2-(E_{\mathbf{k}}^{+})^2]},$$

$$F_{12}^{*}(\mathbf{k},\omega_n)=\frac{-2\alpha B_{\mathbf{k}}^{*}B_{\mathbf{k}}}{[\omega_n^2-(E_{\mathbf{k}}^{-})^2][\omega_n^2-(E_{\mathbf{k}}^{+})^2]}.$$

The density of states (DOS) in the superconducting state can be calculated from the Green functions

$$N(\omega)=\frac{1}{\pi}\sum_{i,\mathbf{k}}\ \mathrm{Im}G_{ii}(\mathbf{k},\gamma)\big|_{\gamma\to-i\omega+\epsilon}, \tag{11}$$

where $\epsilon=0^{+}$. The density of states can be experimentally investigated through tunneling spectroscopy: At $T=0^{0}$ the conductance of a tunnel junction is directly proportional to the quasiparticle density of states, while at finite temperature the tunneling characteristics consist of a convolution of the DOS curve with the Fermi distribution factor

$$\frac{dI(V)}{dV}\propto\int N(\omega-eV)\frac{\beta e^{\beta\omega}}{(e^{\beta\omega}+1)^2}d\omega, \tag{12}$$

where $\beta=1/KT$ and V is the bias voltage.

I consider now the effect of a Heisenberg exchange interaction between spins $\vec{S}_{i,1}$ and $\vec{S}_{i,2}$ in the different layers. This leads to an additional term in the Hamiltonian (1) of the form $J_{\perp}\Sigma_i\vec{S}_{i,1}\cdot\vec{S}_{i,2}$, where $J_{\perp}$ is the interlayer Heisenberg coupling constant. A realistic value of $J_{\perp}$ is $\approx 0.085J$, which has been found experimentally from a spin-wave analysis of the $YBa_2Cu_3O_6$ compound.[43] The four-fermion term $\vec{S}_{i,1}\cdot\vec{S}_{i,2}$ can be decoupled within the present mean-field theory, through the introduction of two additional Hubbard-Stratonovich fields $\phi_i=\langle\Sigma_\sigma f_{1,i\sigma}^{\dagger}f_{2,i\sigma}\rangle$ and $\xi_i=\langle f_{1,i\uparrow}f_{2,i\downarrow}-f_{1,i\downarrow}f_{2,i\uparrow}\rangle$, which are then replaced by their saddle-point values. The mean-field Hamiltonian then becomes

$$\tilde{H}=H+\frac{3J_{\perp}}{8}\sum_i\left[\phi_i^2+\xi_i^2-\phi_i^{*}\sum_\sigma f_{2,i\sigma}^{\dagger}f_{1,i\sigma}-\text{c.c.}-\xi_i^{*}(f_{2,i\uparrow}f_{1,i\downarrow}-f_{2,i\downarrow}f_{1,i\uparrow})-\text{c.c.}\right], \tag{13}$$

where H is the Hamiltonian given in Eq. (5). The decoupling is supposed uniform in both channels, so that $\phi_j=\phi$ and $\xi_j=i\xi$ for all sites j (the choice of a real value for the mean-field ξ_j does not modify the final results). Observe that the term proportional to ϕ can be conveniently absorbed in the effective value of the coupling constant α (actually such a parameter has been also calculated independently, and it has been found that it gives a negligible contribution to α). The energy eigenvalues of the Hamiltonian (13) are

$$\pm E_{\vec{k}}^{-}=\pm\sqrt{A_{\vec{k}}^2+B_{\vec{k}}^2+C_{\vec{k}}^2-2A_{\vec{k}}\alpha+\alpha^2},$$

$$\pm E_{\vec{k}}^{+}=\pm\sqrt{A_{\vec{k}}^2+B_{\vec{k}}^2+C_{\vec{k}}^2+2A_{\vec{k}}\alpha+\alpha^2},$$

where $A_{\vec{k}}=-2(tb_0^2+3J\chi/8)[\cos(k_x)+\cos(k_y)]+\mu_b-\mu_0$, $B_{\vec{k}}=-(3J\Delta/4)[\cos(k_x)-\cos(k_y)]$, and $C_{\vec{k}}=-(3J_{\perp}\xi/8)$.

The mean-field parameters can then be found through the condition of the minimum of the thermodynamic potential

$$\Omega=2N\left[\frac{3J}{4}(\chi^2+\Delta^2)+\frac{3J_{\perp}}{16}\xi^2+\mu_b b_0^2-\mu_0-T\ln(4)\right]-2T\sum_{\vec{k}}\left[\ln\cosh\left(\frac{E_{\vec{k}}^{+}}{2T}\right)+\ln\cosh\left(\frac{E_{\vec{k}}^{-}}{2T}\right)\right]. \tag{14}$$

From the solution of the self-consistent equations it has been found that the stable phase corresponds to $\xi=0$, for $J_{\perp}\leqslant 0.3J$. This implies that for realistic values of the interlayer Heisenberg coupling constant, the Heisenberg interlayer term has no effect on the superconducting properties of the system.

III. RESULTS AND DISCUSSION

Copper-oxide high-T_c superconductors present an unusually large value of the ratio $R=2\Delta_{\max}/KT_c$, where $\Delta_{\max}=3J\Delta/2$ corresponds to the maximum value of the superconducting gap $|B_{\mathbf{k}}|$ at zero temperature. Two recent tunneling experiments,[44,45] have given, respectively, a value $R=6.2$ and $R=7.4$ in $Bi_2Sr_2CaCu_2O_{8+\delta}$ while average over pre-1992 data suggest a value $R\approx 5$.[46] These results are sensibly larger than the BCS prediction of $R=3.5$. The ratio $R(T)=2\Delta_{\max}(T)/KT_c$ has been calculated for the t-J bilayer and is represented as a function of the temperature T in Figs. 1 and 2, for two different values of the doping δ. The dotted, dashed, and solid lines correspond, respectively, to an effective coupling constant $\alpha=0$, $\alpha=0.4J$, and $\alpha=0.6J$. From Figs. 1 and 2 one can see that an interlayer single-electron hopping increases the ratio $R(T)$ at any value of the doping δ and of the coupling constant α. The extent of such variation depends strongly on both δ and α. This can be seen also in Fig. 3 where $R(T=0)$ is represented as a function of α: The solid, dashed, and dotted curves refer, respectively, to $\delta=0.1$, $\delta=0.15$, and $\delta=0.2$. For $\delta=0.2$, $R(0)$ increases almost linearly with α up to a maximum variation of $\approx 5\%$

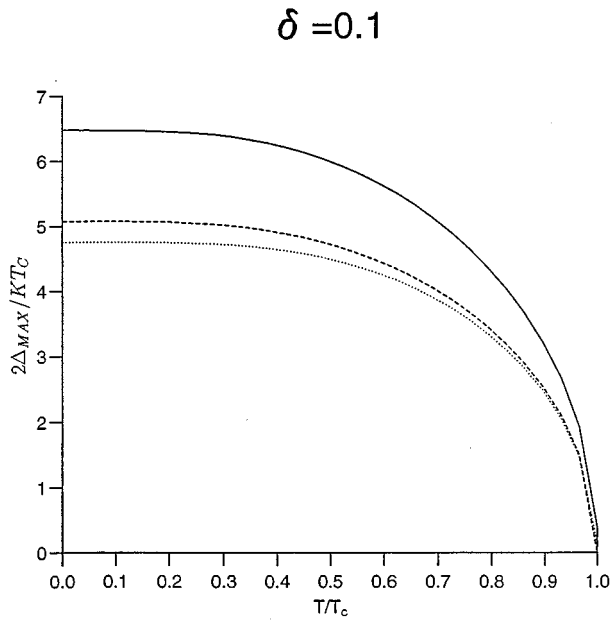

FIG. 1. The ratio $R(T)=2\Delta_{\max}(T)/KT_c$ is plotted as a function of the temperature at a doping $\delta=0.1$. $\Delta_{\max}$ is defined as the maximum value of the superconducting gap at $T=0$. The dotted, dashed, and solid lines correspond, respectively, to an effective coupling constant $\alpha=0$, $\alpha=0.4J$, and $\alpha=0.6J$.

at $\alpha=0.6J$. On the other hand, for $\delta=0.1$, the value of $R(0)$ is practically constant up to $\alpha=0.2J$, and increases sensibly at larger values of the hopping term up to variation of $\approx 30\%$ at $\alpha=0.6J$.

Tunneling experiments have been very important to verify the BCS theory in the conventional superconductors. The tunneling characteristics obtained with high-T_c superconductors show a strong broadening of the superconducting-gap structure, with a finite contribution at zero voltage. This behavior is definitely different from the one expected from an ideal BCS density of states. A correct interpretation of the experimental tunneling characteristics requires an adequate model for the copper-oxide superconductors.

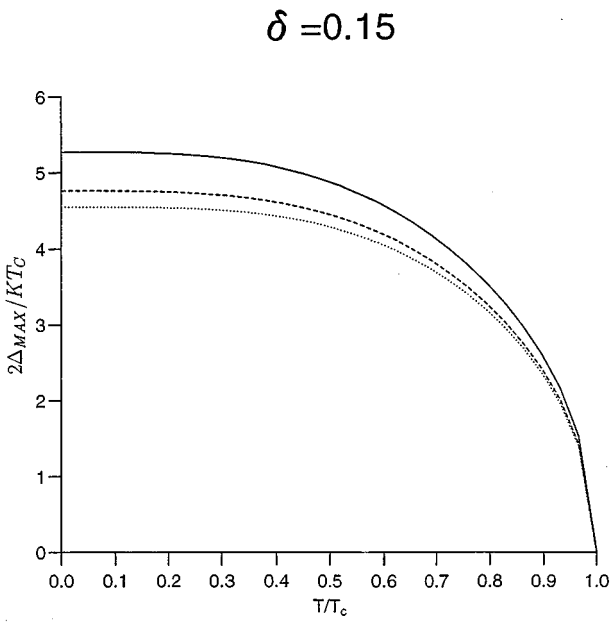

FIG. 2. As in Fig. 1 with a doping $\delta=0.15$.

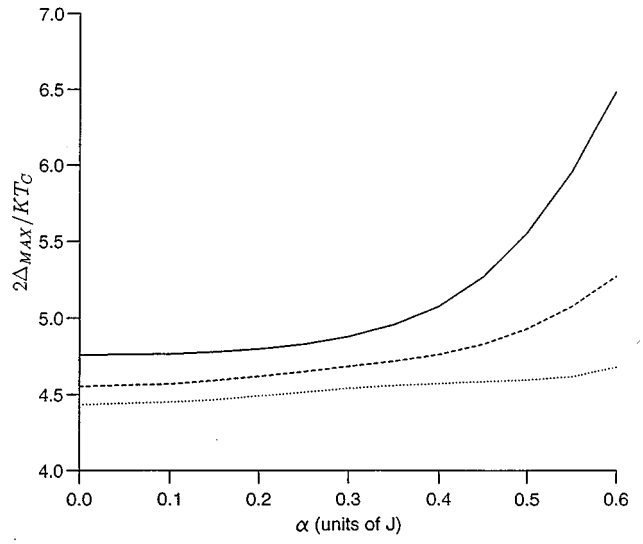

FIG. 3. The ratio $R=2\Delta_{\max}/KT_c$ is plotted as a function of the interlayer coupling constant α. $\Delta_{\max}$ is defined as the maximum value of the superconducting gap at $T=0$. The solid, dashed, and dotted curves refer, respectively, to $\delta=0.1$, $\delta=0.15$, and $\delta=0.2$.

The density of states and the tunneling characteristics of a t-J monolayer and bilayer are represented in Figs. 4–7. The

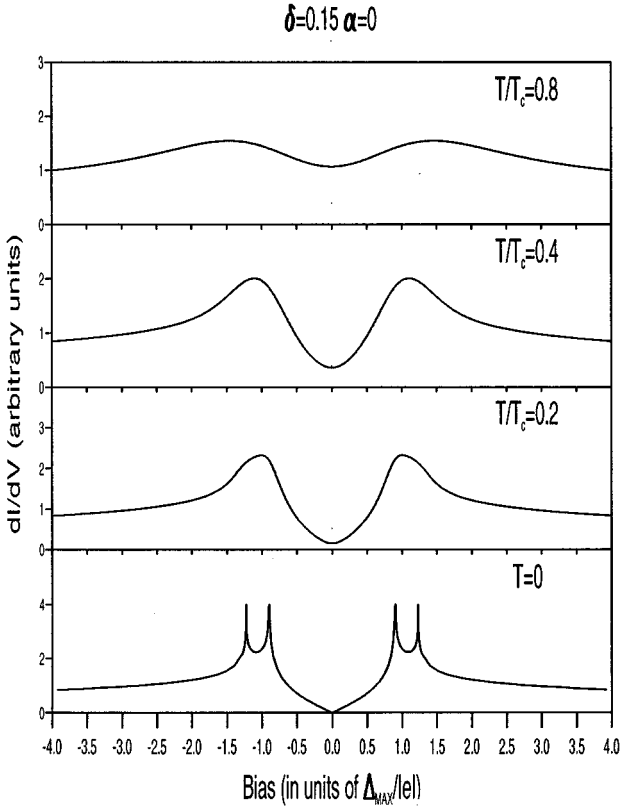

FIG. 4. Density of states at zero temperature (lower panel) and tunneling characteristics at finite temperature (upper panels) of a t-J monolayer at a doping $\delta=0.15$.

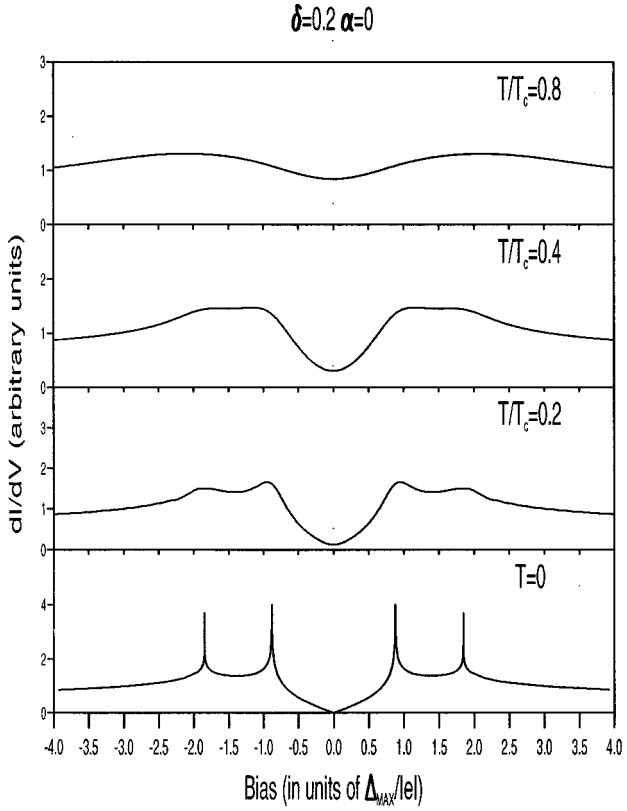

FIG. 5. Density of states at zero temperature (lower panel) and tunneling characteristics at finite temperature (upper panels) of a t-J monolayer at a doping $\delta=0.2$.

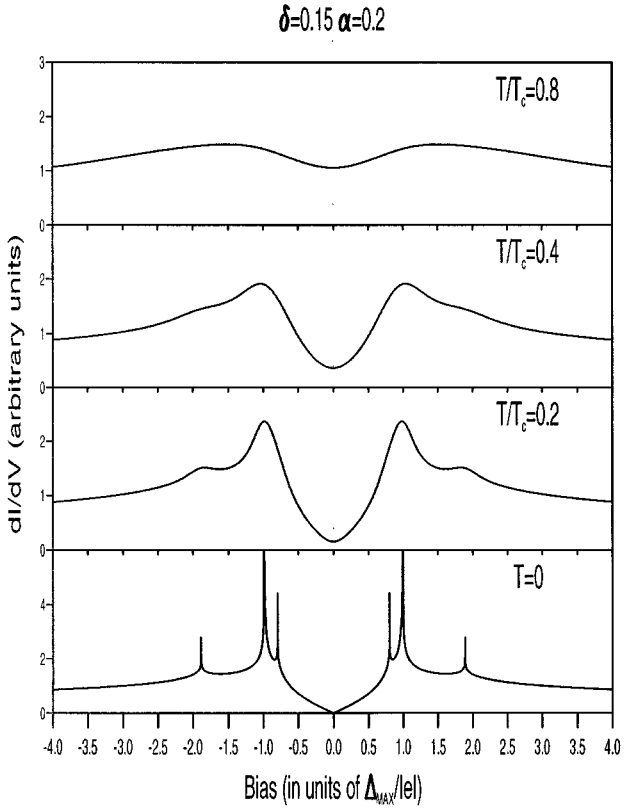

FIG. 6. Density of states at zero temperature (lower panel) and tunneling characteristics at finite temperature (upper panels) of a t-J bilayer at a doping $\delta=0.15$.

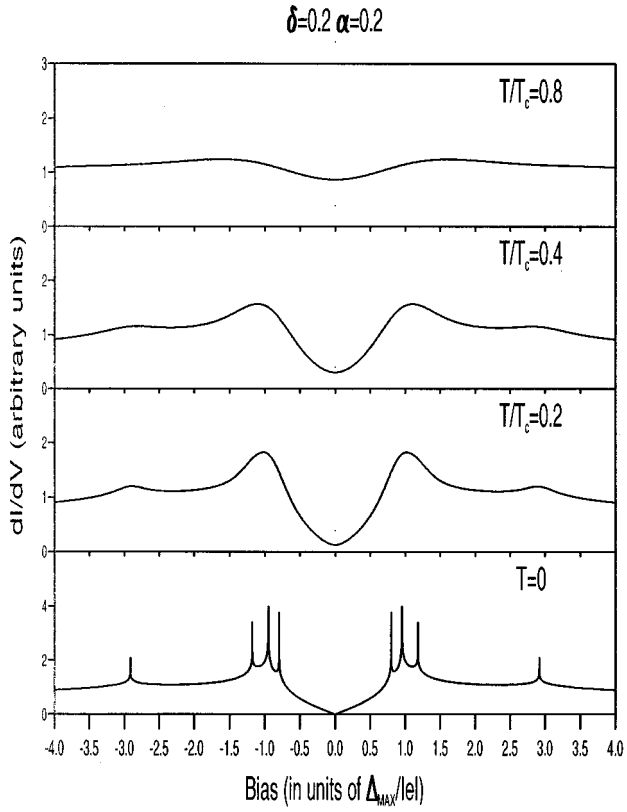

FIG. 7. Density of states at zero temperature (lower panel) and tunneling characteristics at finite temperature (upper panels) of a t-J bilayer at a doping $\delta=0.2$.

curves at zero temperature correspond to the density of states, calculated according to Eq. (11), while the curves at finite temperature represent the tunneling characteristics, obtained through Eq. (12). The densities of states present both logarithmic and BCS-like $(\omega^2-\bar{\omega}^2)^{-1/2}$ singularities. This result is in contrast with the one recently obtained by Liu and Klemm[18] for a BCS-like bilayer model, where only logarithmic singularities were found. The BCS-like singularities occur at $\omega=\sqrt{(4t\chi\pm\alpha)^2+(3J\Delta/2)^2}$. The logarithmic singularities may eventually occur at $\omega=\left|(3J\Delta/4)[4t\chi\pm\alpha \pm 4(t\delta+3J\chi/8)]\right|/\sqrt{(3J\Delta/4)^2+4(t\delta+3J\chi/8)^2}$.

In Figs. 4 and 5 the tunneling characteristics of a t-J monolayer are represented for, respectively, $\delta=0.15$ and $\delta=0.2$. The density of states at zero temperature presents two peaks, which position depends strongly on the value of the doping δ. None of these peaks is located at the BCS value $\omega=\Delta_{\max}$. When the interlayer coupling is introduced, the density of states presents a more complex structure, as can be seen in Figs. 6 and 7: Both the number and the position of the peaks change when the doping δ is varied.

At finite temperature, because of the thermal broadening, the tunneling characteristics present a much more smooth structure, containing one or two peaks. The main peak is in general located near the energy $\omega=\Delta_{\max}$. Nevertheless, the current assumption that the position of the main peak corresponds to the value $\Delta_{\max}$ can lead to small errors in the interpretation of experimental tunneling characteristics. The energy separation between the main and the minor peak increases when the doping δ or the coupling constant α is

increased. Recent tunneling data[45,47,48] have shown the presence of a minor peak or a dip beyond the superconducting gap in $Bi_2Sr_2CaCu_2O_{8+\delta}$. Such features are qualitatively well reproduced by the tunneling characteristics represented in Figs. 5–7. The minor peaks disappear when the temperature is raised, in agreement with experiments.[48]

In summary, the energy-gap structure of a t-J bilayer has been studied within a mean-field approximation. It has been shown that an interlayer single-electron hopping increases the ratio $R=2\Delta_{\max}/KT_c$. The extent of such increase depends strongly on the doping δ. The density of states presents a multiple-peak structure, in qualitative agreement with recent experimental results for $Bi_2Sr_2CaCu_2O_{8+\delta}$.[45,47,48] The densities of states contain both BCS-like and logarithmic singularities, in contrast with the results obtained for a BCS-like bilayer model.[18] The current assumption that the position of the main peak in the experimental tunneling characteristics corresponds to the value $\Delta_{\max}$ is approximately satisfied in the present model.

The effect of a Heisenberg exchange interaction between spins $\vec{S}_{i,1}$ and $\vec{S}_{i,2}$ in the different layers has been considered. It has been found that for realistic values of the interlayer Heisenberg coupling constant $J_\perp$, the Heisenberg interlayer term has no effect on the superconducting properties of the system.

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