Deviations from Fermi-liquid behavior in (2+1)-dimensional quantum electrodynamics and the normal phase of high-T_c superconductors

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We argue that the gauge-fermion interaction in multiflavor quantum electrodynamics in (2+1) dimensions is responsible for non-Fermi-liquid behavior in the infrared, in the sense of leading to the existence of a nontrivial (quasi)fixed point that lies between the trivial fixed point (at infinite momenta) and the region where dynamical symmetry breaking and mass generation occurs. This quasifixed-point structure implies slowly varying, rather than fixed, couplings in the intermediate regime of momenta, a situation which resembles that of (four-dimensional) "walking technicolor" models of particle physics. The inclusion of wave-function renormalization yields marginal $O(1/N)$ corrections to the "bulk" non-Fermi-liquid behavior caused by the gauge interaction in the limit of infinite flavor number. Such corrections lead to the appearance of modified critical exponents. In particular, at low temperatures there appear to be logarithmic scaling violations of the linear resistivity of the system of order $O(1/N)$. The connection with the anomalous normal-state properties of certain condensed-matter systems relevant for high-temperature superconductivity is briefly discussed. The relevance of the large (flavor) N expansion to the Fermi-liquid problem is emphasized. As a partial result of our analysis, we point out the absence of charge-density-wave instabilities from the effective low-energy theory, as a consequence of gauge invariance.

I. INTRODUCTION

One of the most striking phenomena associated with the high-temperature superconductors is their *abnormal* normal-state properties. In particular, these substances are known to exhibit deviations from the known Fermi-liquid behavior, which are remarkably stable with respect to variations in the relevant parameters.[1] Recently, Shankar[2] and Polchinski[3] have presented an intuitively appealing idea of using the renormalization-group (RG) approach, so powerful in particle and statistical physics, to systems of interacting electrons with a Fermi surface in order to understand, at least qualitatively, how deviations from Fermi-liquid behavior can appear *naturally* (as opposed to being fine-tuned). From this point of view Landau's Fermi liquid is nothing else but a system of free electrons, which has no relevant perturbations, in the RG sense, which can drive it away from its trivial infrared fixed point. In general, however, as we integrate out certain modes of our original theory, some interactions may become relevant in the RG sense; i.e., their effective coupling may grow as one lowers the momentum scale. Then two interesting possibilities arise.[3] (i) Fermion bound states are formed, symmetries are spontaneously broken, and the low-energy spectrum bears little resemblance to that of the original theory. In such a case one has to rewrite the effective theory in terms of the new degrees of freedom: For instance, in the superconducting case this is the Landau-Ginzburg effective action expressed in terms of the fermion condensate. (ii) Alternatively, the growth of the coupling is cut off by quantum effects at a certain low-energy scale, and in this way a *nontrivial* fixed-point structure emerges. The low-energy fluctuations still correspond to fields of the original theory despite their nontrivial interactions. This case leads to observable deviations from the Fermi-liquid behavior.

In the case of the high-T_c materials, the physically interesting question is whether one model theory can be found with a structure rich enough to describe *both* the non-Fermi-liquid behavior of the normal phase *and* the transition to (and phenomenology of) the superconducting phase. In this article we shall put forward a candidate model which, as we shall argue, seems to us to fulfill this role.

It is known that possibility (i) above can be caused by relevant interactions of superconducting (BCS) or charge-density-wave (CDW) type, both of which are accompanied by the formation of fermion condensates. Possibility (ii) has only rather recently begun to be seriously explored.[2–4] It has been known for a long time that the electromagnetic interaction of the vector potential can cause deviation from Fermi-liquid behavior[5] but its effects are suppressed by terms of $O[(v_F/c)^2]$, with v_F the Fermi velocity and c the light velocity. Its effects occur only at much lower energies than those relevant to the high-T_c materials. Nevertheless, the electromagnetic example is suggestive enough, perhaps, to motivate a search for other (nonelectromagnetic) gauge interactions in which the effective signal velocity would be of order v_F and which might be responsible for a nontrivial fixed-point behavior. It was precisely this sort of ("statistical") gauge-fermion interaction that was studied (in different forms) in Refs. 3 and 4, and which led to nontrivial fixed-point structure in the infrared.

Returning now to possibility (i), we recall that it has been shown[6] that a variant of QED in (2+1) dimensions (QED_3) leads to superconductivity, characterized, as appropriate to two space dimensions, by the absence of a local order parameter (Kosterlitz-Thouless mode). Thus the exciting possibility arises that a single fermion-gauge theory could describe both non-Fermi-liquid behavior in the normal phase and the transition to the superconducting phase.

Formulated in terms of N species of electromagnetically

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charged fermions, the model of Ref. 6 (to which we shall return in Sec. IV) consists of a CP^{N-1} σ model coupled to the fermions via the gauge field of the σ model representing magnetic spin-spin interactions. The main purpose of the present article is to present an (approximate) renormalization-group analysis of a simplified version of this model, namely, QED_3 itself, which indicates that QED_3 exhibits two quite different behaviors depending on the momentum scale. At very low momenta QED_3 enters a regime of dynamical mass generation (DMG), which in the full theory leads to superconductivity, but at "intermediate" momenta (see below) DMG does not occur and the dynamics is controlled by a nontrivial fixed point, leading to non-Fermi-liquid behavior. Thus we have the possibility of one theory encompassing both the normal and superconducting phases of the high-T_c cuprates.

We postpone until Sec. IV a fuller account of the realistic model we are advocating. Before that, in Secs. II and III, we shall consider for clarity the simpler case of QED_3, which as we shall see already exhibits the crucial dynamical features (however, as we shall see in Sec. IV, QED_3 describes only a part of the realistic model believed to simulate the physics of the high-T_c cuprates). From this we conclude that the essential dynamical ingredient in our model is simply that it is a U(1) *gauge theory in two space dimensions*.

At this point the reader might worry that applying renormalization-group techniques to a superrenormalizable theory like QED_3 is redundant, since the theory has no ultraviolet divergences. However, this is a mistaken view. In the modern approach to the RG and effective-field theories, one considers quite generally how a theory evolves as one integrates out degrees of freedom above a certain momentum scale, moving progressively down in scale. From this point of view an effective-field-theory description is equally applicable to nonrenormalizable, renormalizable, and superrenormalizable theories. However, there are some crucial features in the case of a superrenormalizable theory. First, the QED_3 coupling e introduces an intrinsic *intermediate* scale e^2 which has the dimension of mass, this being directly related to the superrenormalizability of the theory. The physical effect of this will be the existence of an intrinsic mass scale, and we can expect different physics in different regimes of momenta relative to this mass scale ($p \gg e^2$, $p \simeq e^2$, $p \ll e^2$).

The second distinctive feature of our RG analysis of QED_3, concerns the way in which we introduce a running coupling. Conventionally, such running couplings are dimensionless, and so, once again, the dimensionfulness of e^2 presents a distinctive feature. The way in which an effective dimensionless running coupling can be introduced into QED_3 has been shown by Kondo and Nakatani (KN),[7] building on work by Higashijima[8] for QCD_4. The crucial step is to consider the effect of wave-function renormalization in the Schwinger-Dyson (SD) equations, as controlled by a large-N approximation. In this case, one considers the theory at large N with $\alpha = e^2 N$ held fixed and the dimensionless coupling that runs is essentially $1/N$.

KN actually considered only the regime in which dynamical mass generation (chiral symmetry breaking) occurs, and of course here the gauge coupling is becoming strong and the use of a large-N expansion is unavoidable. What we shall do (in Sec. II) is to identify the "normal" (no dynamical mass generation) regime of the theory and extend the RG-type analysis of KN to this normal regime. We shall argue that there exists a nontrivial fixed point of the effective dimensionless coupling, which governs the dynamics for a range of *intermediate* momenta $p \simeq \alpha$, lying between the trivial fixed point at $p \gg \alpha$ and the region $p \ll \alpha$ of dynamical mass generation. Important to this analysis will be the introduction (following KN) of an infrared cutoff ϵ, which serves to delineate the different momentum regimes.

The analysis of Sec. II is performed at zero temperature, and in Sec. III we shall try to connect this to finite-temperature calculations by interpreting the temperature as an effective infrared cutoff. We present an approximate computation, at finite temperature, of the electrical resistivity ρ of the fermionic system. We argue that it is the existence of the nontrivial RG fixed point which is responsible for the fact that the non-Fermi-liquid behavior (ρ approximately proportional to the temperature T) is observed over so large a temperature range. Wave-function renormalization effects, important at $O(1/N)$, lead to calculable logarithmic deviations from the linear-in-T behavior.

Before proceeding further, it is useful to compare and contrast our approach with two other recent explorations of gauge theories in (2+1) dimensions in a similar context, by Polchinski[3] and by Nayak and Wilczek.[4] Both works deal with fermions interacting with a statistical gauge field, the latter representing magnetic spin-spin interactions (as in our CP^{N-1} sector; see Sec. IV). In both, the fermions represent *spin* quasiparticle excitations (spinons), and they should therefore not be identified with the carriers of ordinary electric charge (holes or electrons). This is to be sharply contrasted with our own model of Sec. IV, in which the spin-charge separation is done differently, leading to the fermions in our model carrying both statistical and ordinary charge.

The model of Ref. 4 consists of a gauge-fermion interaction, in the presence of a modified four-fermion interaction of a long-range $1/k^x$ form, with k the momentum. An important role is also played by a P- and T-violating term in the form of a Chern-Simons interaction for the gauge field. The latter is responsible for enslaving gauge-field fluctuations to density fluctuations. In the case $x<1$ this results in a relevant gauge-fermion interaction. Nayak and Wilczek[4] have shown, by employing a systematic expansion in powers of $1-x$, the existence of a nontrivial infrared fixed point responsible for deviations from Fermi-liquid behavior. The importance of the Chern-Simons interaction lies in the fact that it allows, through the constraint implied by integrating out the temporal component of the statistical gauge field, a rewriting of the nonlocal $1/k^x$–four-Fermi interaction as a Maxwell-like term for the gauge field but with modified $1/k^x$ momentum behavior. The ordinary Maxwell term corresponds to $x=0$, while the Coulomb interaction corresponds to $x=1$. Up to its nonrelativistic form, which is a consequence of the nonrelativistic character of the fermion-gauge system with a Fermi surface, this situation is qualitatively similar to the dimensional reduction of the ordinary Maxwell term from four to three space-time dimensions.[6] Indeed, in that case, a three-dimensional Maxwell term for the electromagnetic field A_M, $M=1,2,3$, corresponding to the projection of a four-dimensional theory onto the spatial plane, results in a Coulomb-like form for the gauge field kinetic term

$$\int d^3x\, F_{MN}(A)\, \frac{1}{\sqrt{\nabla^2}}\, F^{MN}(A). \qquad (1)$$

This result is due to the fact that in three space-time dimensions the Green's functions for the dimensionally reduced Maxwell field are modified appropriately to yield the above "square-root-of-∇^2" behavior (1). It is natural, therefore, to imagine that a behavior $(\sqrt{\nabla^2})^{-1+\epsilon}$ may be attributed to quasiplanar geometries or to deviations from three space-time dimensions as in dimensional regularization $D=3+\epsilon$ with $\epsilon=1$ corresponding to the (Maxwell) four-dimensional kinetic term.

From this analogy one can understand that the parameter $1-x$ of Ref. 4 plays a role similar to that of the ϵ parameter of Wilson or of dimensional regularization. This is the advantage of the method of Ref. 4, in the sense of providing a controlled expansion in powers of $1-x$, which can lead to a nontrivial fixed point for the gauge-fermion interaction at weak coupling.

The above work makes explicit use of parity- (P-) and time-reversal- (T-) breaking effects of the ground state, which, however, is difficult to reconcile with experiment at present. To avoid this difficulty, Polchinski[3] examined the possibility of a nontrivial infrared fixed point in a P- and T-conserving situation in which the only nontrivial interaction in the effective Lagrangian of spinons is that with the statistical gauge field without any Chern-Simons term. This is formally the same as the essential fermion-gauge sector of our own model, but with the crucial physical difference—to repeat—that our fermions will (in Secs. III and IV) carry electric charge, whereas Polchinski's cannot. To have a controllable expansion Polchinski[3] employed a large-N analysis in the fermionic flavors by extending the SU(2) spin group to SU(N), $N\to\infty$. He presented a Schwinger-Dyson analysis for the propagators of the fermion and gauge fields, which he solved in a closed form to leading order in the $1/N$ expansion by invoking a tree-level *Ansatz* for the gauge-fermion vertex at large N at low energies. Renormalization, then, implies that the gauge-fermion interaction is promoted from irrelevant to *marginal*, thereby sowing the possibility of a nontrivial fixed point of this model in the infrared and, hence, its non-Fermi-liquid behavior. Because the kinetic term for the gauge field assumes the normal Maxwell form, the results of Polchinski can probably be classified as belonging to the $x=0$ universality class in the language of Nayak and Wilczek.[4] The criticism that one may make of Polchinski's approach is the fact that he neglects renormalization effects on the vertex, which can lead to a nonconsistent expansion in $1/N$. Such effects were crucial in the work of Nayak and Wilczek in order to get a controllable expansion in the fermion self-energy calculation at (resummed) one loop.

The important observation in Polchinski's work, which will be directly relevant for our purposes here, is that kinematics implies that the most important interactions among fermions are those which pertain to fermionic excitations whose momentum components tangent to the Fermi surface are parallel. This is the only way that the gauge-field momentum transfer can still be relatively large as compared to the distance of the fermion momenta from the Fermi surface, as required by special kinematic conditions.[3] There are two cases where such conditions are met in condensed-matter physics. The first pertains to nested Fermi surfaces, at which the points with momenta k_0 and $-k_0$ have parallel tangents. This is the situation relevant to BCS or CDW interactions. The other situation, which is the bulk of Polchinski's work and will be of interest to us as well, is the case where the fermions are close to a single point on the Fermi surface. This means that the most important fermion interactions are those which are local on the Fermi surface, and hence qualitatively this situation can be extended to relativistic (Dirac) fermions as well, since the dispersion relations become effectively linear.[6]

Another important point, which was recently pointed out by Shankar[2] in connection with the RG approach to interacting fermions, is the use of an effective large-N expansion in cases where the effective momentum cutoff Λ is much smaller than the size of the Fermi surface k_F, $\Lambda/k_F\to 0$. Such a situation is encountered in a RG study of (deviations from) Fermi-liquid theories, the Landau Fermi-liquid theory being defined as a trivial infrared fixed point in a RG sense. To understand the connection of a large-N expansion with infrared behavior of excitations, one should recall the work of Ref. 9 where the RG approach to the theory of the Fermi surface has been studied in a mathematically rigorous way. The basic observation of Ref. 9 is that, unlike the case of relativistic field theories, in systems with an extended Fermi surface, the fermionic excitation fields exhibiting the correct scaling are not the original excitations, ψ_x (x a configuration space variable), but rather *quasiparticle* excitations defined as

$$\psi_x=\int_{|\mathbf{\Omega}|=1} d\mathbf{\Omega}\; e^{ik_F\mathbf{\Omega}\cdot\mathbf{x}}\psi_{x,\Omega}=\int_{|\mathbf{\Omega}|=1} d\mathbf{\Omega}\; e^{i(k_F\mathbf{\Omega}-\mathbf{K})\cdot\mathbf{x}}\tilde{\psi}_{\mathbf{K},\mathbf{\Omega}}, \qquad (2)$$

where for the sake of simplicity we assumed that the Fermi surface is spherical with radius k_F, $\mathbf{\Omega}$ is a set of angular variables defining the orientation of the momentum vector of the excitation at a point on the Fermi surface, and the tilde denotes ordinary Fourier transform in a momentum space $\mathbf{K}$. These quasiparticle fields have propagators with the correct scaling,[9] which allows ordinary RG techniques, familiar from relativistic field theories, to be applied, such as the appearance of renormalized coupling constants, scaling fields, etc. Indeed, it is not hard to understand why this is so. For this purpose it is sufficient to observe that for large k_F the exponent of the exponential in (2) is nothing other than the *linearization*, $\mathbf{k}\equiv\mathbf{K}-k_F\mathbf{\Omega}$, about a point on the Fermi surface, which makes these quasiparticle excitations identifiable with ordinary field variables of the low-energy limit of these condensed-matter systems. The latter is a well-defined field theory.[6] The crucial point in this interpretation is that now the field variables will depend on "internal degrees of freedom" $\mathbf{\Omega}$, which denote angular orientation of the momentum vectors on the Fermi surface. In two spatial dimensions, which is the case of interest, $\mathbf{\Omega}$ is just the polar angle θ. Following Ref. 2, we discretize this angular space into small cells of extent $f(\Lambda/k_F)\ll 1$, e.g., $f=\Lambda/k_F$:

$$\int \frac{d^2k}{4\pi^2}\equiv\int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi}\int_{f(\Lambda/k_F)}^{f(\Lambda/k_F)} k_F\,\frac{d\theta}{2\pi}, \qquad (3)$$

where k denotes a linearizing momentum about a point on the Fermi surface. Doing so, we observe[2] that when looking at interaction terms involving fermionic particle-antiparticle pairs $\bar{\psi}\psi$ the leading interactions are among those fermion-antifermion pairs for which the creation and anihilation operators lie within the same angular cell. This is for purely kinematic reasons in the infrared regime $\Lambda \ll k_F$, similar to those mentioned previously,[3] which implied that the most important fermion interactions on the Fermi surface must be among excitations which have their tangents to the Fermi surface parallel. It is, then, straightforward to see that interaction terms involving either gauge excitations or just fermions resemble those in large-N relativistic field theories, given that the only Λ dependence appears through proportionality factors $f(\Lambda/k_F) \ll 1$ in front of the interactions, in the infrared. One, then, identifies $1/N$ with $f(\Lambda/k_F) \ll 1$, and the only difference from ordinary particle-physics large-N expansions is the dependence of this effective N on the cutoff Λ: that is to say, $1/N$ runs.

As we shall show in the next section, however, large-N expansions in three-dimensional QED can exhibit such a scale dependence. Wave-function renormalization leads to a renormalized ''running'' $1/N$. Instead of finding a nontrivial infrared fixed point, we shall demonstrate the existence of an (intermediate) regime of momenta, where the effective running of the gauge coupling, which is essentially $1/N$ times a spontaneously appearing scale, is slowed down considerably, so that one encounters a quasifixed-point situation. As we shall argue, this quasifixed-point structure is sufficient to cause (marginal) deviations from the Fermi-liquid picture. In view of the above, this makes such theories plausible candidates for a correct qualitative description of deviations from Landau Fermi-liquid theory. This has obvious relevance to the normal-phase properties of (realistic) condensed-matter systems,[6] advocated in Sec. IV, which are believed to simulate the physics of the high-T_c cuprates.

II. QED$_3$: SUPERRENORMALIZABILITY, RUNNING COUPLINGS, AND NONTRIVIAL (QUASI)FIXED-POINT STRUCTURE

A. Wave-function renormalization and running flavor number

Three-dimensional quantum electrodynamics (QED$_3$) has recently received a great deal of attention[10–18] not only as a result of its potential application to the study of planar high-temperature superconductivity,[6] mentioned in the Introduction, but also because of its use as a prototype for studies of chiral symmetry breaking in higher-dimensional (non-Abelian) gauge theories.[19]

However, despite the theory's apparent simplicity, the situation is not at all clear at present. A great deal of controversy has arisen in connection with the role of wave-function renormalization. In the early papers[10] the wave-function renormalization $A(p)$ was argued to be 1 in Landau gauge to leading order in $1/N$, where N is the number of fermion flavors, and thus was ignored. More detailed studies, however, showed[14] that the precise form, within the resummed $1/N$ graphs, of $A(p)$ is

$$A(p)=\left(\frac{p}{\alpha}\right)^{8/3N\pi^2}, \tag{4}$$

where $\alpha = e^2 N$ is the dimensionful coupling constant of QED$_3$, which is kept fixed as $N\to\infty$. It is clear from (4) that, although at energies $p \simeq \alpha$ the wave function is of order 1 however at low momenta $p \ll \alpha$ relevant for dynamical generation of mass, the wave-function renormalization yields logarithmic scaling violations which could affect[14] the existence of a critical number of flavors, N_c, below which, as argued in Ref. 10, dynamical mass generation occurs. However, this result was not free of ambiguities either, given that the inclusion of wave-function renormalization necessitates the introduction of a nontrivial vertex function. The exact expression for the latter is not tractable, even to order $O(1/N)$, and one has to assume various *Ansätze*[14] that can be questioned. The situation became clearer after the work of Ref. 7, which showed that the introduction of an infrared cutoff affects the results severely, depending on the various *Ansätze* used for the vertex function. In particular, as the authors of Ref. 7 showed, there are extra logarithmic scaling violations in the expression for N_c, depending on the form of the vertex function assumed, which render the limit where the infrared cutoff is removed not well defined.

For our present purposes, however, we are not so much interested in whether the inclusion of wave-function renormalization leads to a critical N_c or not, as in the more general point that, as noted by Kondo and Nakatani (KN),[7] following Higashijima,[8] the vacuum polarization contribution to A produces effectively a running coupling, even in the case of the superrenormalizable theory of QED$_3$. KN's analysis was restricted to the regime of dynamical mass generation, and our main purpose in this section is to extend that to the ''normal'' regime where mass is not dynamically generated. We emphasize now, however, that if A is set equal to unity at the outset, the power of the running coupling concept to unify both regimes is completely lost.

We therefore continue with a brief review of the analysis of Ref. 7. Their vertex *Ansatz* was assumed to be

$$\Gamma_\mu(q,p)=\gamma_\mu A(p)^n \equiv \gamma_\mu G(p^2), \tag{5}$$

where p denotes the momentum of the photon. The Pennington-Webb[14] *Ansatz* corresponds to $n=1$, where chiral symmetry breaking occurs for arbitrarily large N.[20] It is this case that was argued to be consistent with the Ward identities that follow from gauge invariance.[14] In this paper we shall concentrate on the generalized *Ansatz*, with $n\neq 1$, and in particular we shall discuss its finite-temperature behavior. We keep the exponent n arbitrary[7] and discuss qualitatively the implications of the vertex *Ansatz* for various ranges of the parameter n. As we shall argue below, this is crucial for the low-energy renormalization-group structure of the model.

Using the *Ansatz* (5), Kondo and Nakatani[7] proceeded to analyze the Schwinger-Dyson (SD) equations, in the regime of dynamical mass generation, in terms of a running coupling as follows. Their (approximate) SD equation for $A(p)$ is (in the Landau gauge)

$$A(p)=1-\frac{g_0}{3}\int_\epsilon^\alpha dk\,\frac{kA(k)G(k^2)}{k^2A^2(k)+B(k^2)} \times\left\{\left(\frac{k}{p}\right)^3\theta(p-k)+\theta(k-p)\right\}, \tag{6}$$

where $g_0 = 8/\pi^2 N$, N is the number of fermion flavors, and ϵ is an infrared cutoff. In the low-momentum region relevant for dynamical mass generation, $p \ll \alpha$ and the first term in the right-hand side of (6), cubic in k/p, may be ignored. Then, taking into account that $G(k^2) = A(k)^n$ and using the bifurcation method in which one ignores the gap function $B(k)$ in the denominators of the SD equations, one obtains easily

$$A(t) = 1 - \frac{g_0}{3} \int_t^0 ds\ A^{n-1}(s), \tag{7}$$

which has the solution

$$A(t) = \left(1 + \frac{2-n}{3} g_0 t \right)^{1/(2-n)}, \quad t \equiv \ln(p/\alpha). \tag{8}$$

Substituting to the SD equation for the gap, one then obtains a running coupling[7] in the low-momentum region,

$$g^L = \frac{g_0}{1 + \frac{2-n}{3} g_0 t}, \tag{9}$$

which, we note, is actually independent of ϵ. The existence of the dimension less parameter g^L in QED_3 may be associated with the ratio of the gauge coupling e^2/α, given that in the large-N analysis the natural dimensionful scale α has been introduced. Thus a renormalized running N^{-1} might be thought of expressing ‘‘charge’’ scaling in this superrenormalizable theory. In particular, (9) implies that the β function corresponding to g^L is of ‘‘marginal’’ form,

$$\beta^L \equiv -\frac{dg^L}{dt} = \frac{2-n}{3} (g^L)^2. \tag{10}$$

Thus, depending of the sign of $2-n$, one might have *marginally* relevant or irrelevant couplings $g^L \propto e^2/\alpha$. The first derivative of the β function with respect to the coupling g^L is

$$\frac{d}{dg^L} (\beta^L) = 2 \frac{2-n}{3} g^L, \tag{11}$$

and since $g^L > 0$ by construction, its sign depends on the sign of $n-2$. For $n<2$ (the marginally relevant case), the gauge interaction decreases rapidly as one moves away from low momenta, and the theory is ‘‘asymptotically free.’’[7] If $n>2$ (marginally irrelevant), on the other hand, then $g^L(t)$ tends to zero in the low-momentum region, while for $n=2$ the coupling is exactly marginal and one recovers the results of Refs. 10 and 15 about the existence of a critical flavor number. Gauge invariance, in the sense of the Ward-Takahashi identity, seems to imply[14,15] $n \leq 2$, and this is the range we shall explore in this article.

Our problem now is to extend (9) beyond the region $p \ll \alpha$. Consider first the true ultraviolet region $p \to \infty$. Assuming for the moment that (9) were correct for $p \gg \alpha$, one finds a zero of the β function at the point $t \to \infty$, the trivial fixed point $g^* = 0$, which is an ultraviolet fixed point. However, (9) or (10) is not reliable for the range of momenta, $p \gg \alpha$. Both formulas have been derived in the regime of momenta relevant to the dynamical mass generation, $p \ll \alpha$.

This being so, do we have an alternative argument for a trivial ultraviolet fixed point? The answer is affirmative. To this end we use the results of Ref. 21 employing a quenched fermion approximation in large-N QED. The result of such an investigation is that once fermion loops are ignored, and hence only tree-level graphs (ladder) are taken into account, the wave-function renormalization is rigorously proved to be trivial in the Landau gauge:

$$A(p)^{\text{quenched}} = 1. \tag{12}$$

This result is a consequence of special mathematical relations of resummed ladder graphs in Schwinger-Dyson equations. Now, in our case, one observes that in the high-energy regime $p \to \infty$ the $(1/N)$-resummed gauge-boson polarization tensor vanishes as $\Pi(p \to \infty) \simeq \alpha/8p \to 0$. Thus the situation is similar to the quenched approximation, which implies the absence of any wave-function renormalization (12) and, therefore, the vanishing (triviality) of the effective (running) coupling constant g in the ultraviolet regime of momenta. This is in qualitative agreement with the naive estimate made above, based on the formulas (9) and (10).

The situation is, therefore, as follows. The coupling grows from the trivial fixed point (ultraviolet regime), where there is no mass generation to stronger values as the momenta become lower. According to the naive formula (10), this coupling grows indefinitely for low momenta and the perturbation expansion breaks down. But—to repeat—(9) was derived for the regime $p \ll \alpha$, and the question now arises whether nothing new happens from this regime all the way up to $p \to \infty$, or whether there is interesting structure at intermediate scales. In particular, we might envisage a ‘‘quasifixed-point’’ situation, in which g remains more or less stationary around the value $g(0)$ for a wide range of t below $t=0$, before commencing to grow rapidly at very low momenta.

B. Nontrivial (quasi)fixed-point structure at intermediate momenta

The answer to the above question turns out to reside, essentially, in the infrared cutoff ϵ [which, as we noted above, actually disappeared from (9)]. The coupling of (9) is ‘‘asymptotically free’’ (i.e., grows rapidly in the far infrared) for $n<2$, *provided* that the ratio α/ϵ is large enough, and in this case dynamical mass generation (DMG) occurs. To get to the region where DMG does not occur, we must consider smaller values of α/ϵ, tending ultimately to unity. This is the region that will yield the effective nontrivial fixed-point structure. In this case, $p \simeq \alpha$ and hence the only allowed region for the momentum k in (6) is $k \leq p$, which now eliminates the *second* term in (6). Solving then (6) in this approximation (and taking $B=0$ since DMG does not occur), with the vertex (5), one obtains

$$A(p) = 1 - \frac{g_0}{3} \int_\epsilon^p \frac{dk}{k} \left(\frac{k}{p}\right)^3 A^{n-1}(k)$$
$$= 1 - \frac{g_0}{3} \int_{t_0 - t}^0 ds\ e^{3s} A^{n-1}(s), \tag{13}$$

which can be easily solved with the result

$$A(t)=\left(\text{const}+\frac{2-n}{9}\,g_0e^{3t_0-3t}\right)^{1/(2-n)}, \qquad (14)$$

where the "const" is a positive one and can be found from the value of the wave-function renormalization at $t=\ln(\epsilon/\alpha)\equiv t_0$, namely, $A(t_0)=1$. From (13) this yields the value

$$\text{const}=1-\frac{2-n}{9}\,g_0.$$

Substituting (14) back to the gap equation, one obtains a running coupling constant in this new intermediate regime,

$$g^I\equiv\frac{g_0e^{3t}}{\left(1-\frac{2-n}{9}\,g_0\right)e^{3t}+\frac{2-n}{9}\,g_0e^{3t_0}}$$

$$=\frac{g_0}{1-\frac{2-n}{9}\,g_0+\frac{2-n}{9}\,g_0\left(\frac{\epsilon}{p}\right)^3}. \qquad (15)$$

We note that just as the "lower scale" ϵ disappeared from (9), so the "intermediate scale" α is absent from (15).

Let us study the fixed-point structure of this renormalization-group flow. To this end, consider the β function obtained from (15):

$$\beta^I=-\frac{dg^I}{dt}=-3g^I+\frac{3}{g_0}\left(1-\frac{2-n}{9}\,g_0\right)(g^I)^2. \qquad (16)$$

Taking into account that $g_0=8/\pi^2N$, we observe that the vanishing of β^I occurs not only at $g^I=0$, but also at the nontrivial point,

$$g^I_*=\frac{8}{\pi^2N}\left(1-\frac{2-n}{9}\,\frac{8}{\pi^2N}\right)^{-1}, \qquad (17)$$

which indicates the existence of a fixed point lying at a distance of $O(1/N)$, for $N\to\infty$, from the trivial one.

For what momenta is this fixed point reached? Accepting (15) at face value, the answer would be that it is reached for $p\to\infty$. But of course (15) is not valid for $p\gg\alpha$, being appropriate for $\epsilon<p<\alpha$, where the ratio ϵ/α is smaller than unity, though not so very small that p can enter the region of DMG. Referring then to the right-hand side of the second equality in (15), we see that when $p\simeq\alpha$ the quantity g^I will be very close to g^I_*, differing from it by terms of order

$$\left(\frac{\epsilon}{\alpha}\right)^3\frac{1}{N^2},$$

which is negligible. Indeed, as p moves down to $p\simeq\epsilon$, g^I arrives at g_0, which is still within $(1/N^2)$ of g^I_*. Thus the crucial point is that there is, on the basis of this admittedly approximate analysis, a significant momentum region over which the coupling g^I varies very slowly, and we are in a "quasifixed-point" situation. In a sense, this slow variation of g^I in the range $\epsilon<p<\alpha$ (for not too small ϵ) provides a reconciliation between the normalizations adopted in the two different approximations (9) and (15), namely, between $g^L(p=\alpha)=g_0$ and $g^I(p=\epsilon)=g_0$.

The new fixed point occurs at weak coupling for large N. This is consistent with the interpretation that such a fixed point should characterize a regime of the theory, as determined by the ratio α/ϵ, where dynamical mass generation does not occur.

In summary, then, our analysis suggests a significant modification of the picture presented by Kondo and Nakatani.[7] Whereas those authors only considered $\epsilon\ll\alpha$, which is the regime of "asymptotic freedom" and DMG, we have explored also the region of smaller values of α/ϵ and have concluded that here quantum corrections create a quasifixed point with weak coupling. *Both* regions of α/ϵ will be important in our application of these results to the cuprates, as we discuss in Sec. III, where we shall try to relate the ϵ of this QED_3 with the temperature T of QED_3 at finite temperature.

At this stage, it is worth pointing out the similarity of the above-demonstrated "slow running" of the effective gauge coupling g at intermediate scales with (four-dimensional) particle physics models of "walking technicolor" type.[22] Such models pertain to gauge theories with asymptotic freedom and involve regions of momentum scale at which effective running couplings move very slowly with the scale, exactly as happens in our (asymptotically free) QED_3 case. [A similarity of QED_3 with walking technicolor had also been pointed out previously,[23] but from a different point of view. In Ref. 23, a formal analogy of QED_3 with walking technicolor models was noted, based on the role of fermion loops in softening the logarithmic confining gauge potential to a Coulombic $1/r$ type, in the infrared regime of momenta. This $1/r$ behavior of the potential, and its relevance to dynamical chiral symmetry breaking, is common in both theories. The formal analogy between QED_3 and walking technicolor theories is achieved[23] by replacing the coupling g^2 of the four-dimensional theory by $1/N$ of QED_3. However, N of Ref. 23 does not vary with the energy scale, since wave-function renormalization effects have not been discussed in their case. This is the crucial difference in our case, where there is more precise analogy with walking technicolor theories, due to the slowing down of the variation of the "effective" N (15) with the (intermediate-)energy scale.] This slow running of the coupling results in such theories in a significant enhancement of the size of the fermion condensate. In our case, such condensates are responsible for an opening of a superconducting gap, and therefore one could associate the slow running of the coupling at intermediate scales with the suppression of the coherence length of the superconductor (inverse of the fermion condensate) in the phase where dynamical mass generation occurs. Such a suppression, as compared to the phonon (BCS) type of superconductivity, which is an experimentally observed and quite distinctive feature of the high-T_c cuprates,[24] appears then, in the context of the above gauge theory model,[6] as a natural consequence of the nontrivial quasifixed-point renormalization-group structure. Note that in Ref. 6 the enhancement of the superconducting-gap–to–critical-temperature ratio, as compared to the standard BCS case, had been attributed to the superrenormalizability of the theory and the T independence of quantum corrections, features which are both associated with the above quasifixed-point (slow running) situation as discussed above. It is understood, of course, that before we arrive at definite conclusions about the actual size of the coherence length in the model, we should be able to perform exact calculations by resumming the higher orders in $1/N$ to see whether these features persist. At present this is impossible analytically, but one could hope for (nonperturbative) lattice simulations of the above systems.[6,25]

C. Effect of wave-function renormalization on the effective fermion-fermion interactions and nontrivial (quasi)fixed points

Despite the important physical differences between the models, it is worth comparing the above results with the model of Ref. 4, where a nontrivial infrared fixed point in the running of the effective gauge-fermion coupling was associated with the presence of a modified fermion-fermion interaction, of long-range $1/p^x$ type, with p the momentum. As mentioned in the Introduction, the model made explicit use of a P- and T-violating Chern-Simons interaction for the statistical gauge field. The nonrelativistic nature of the system of Ref. 4 was not important. What was important was the deviation from pure Coulombic behavior $x=1$, which itself leads only to marginal deviations from Fermi-liquid behavior. In our case, as we shall argue below, the role of x is played by $1-O(1/N)$. The deviation from the Coulombic interactions among fermions is caused by the nontriviality of $1/N$, and the (marginal) Coulomb interaction would be recovered in the infinite fermion flavor limit.

To make formal contact with the results of Ref. 4, it is essential to compute the (zero-temperature) effective potential among our fermions, with wave-function renormalization included. This is straightforward and we present the result below. The zero-temperature static potential among fermions is given by the $\mu=0$, $\nu=0$ form of the gauge-boson propagator. In Ref. 6 the effects of wave-function renormalization were ignored, which is an accurate result only in the $N\rightarrow\infty$ limit, where the "mean-field" theory is recovered. This is the Landau Fermi-liquid fixed point. The $1/N$ corrections yield a nontrivial wave-function renormalization effect. Resumming the $1/N$ corrections, in an improved renormalization-group framework, and using the *Ansatz* (5) for the effective vertex, we can compute the effective static potential in a straightforward manner with the result

$$V(p)\propto\frac{\alpha}{8}\,p^{16n/3\pi^2N}p^{-1}, \tag{18}$$

which makes contact with the effective potential of Ref. 4 if one identifies $x=1-16n/x^2 3N<1$.

D. Comments and comparison with other works

Before closing this section it would be useful to compare our results with the works of Refs. 26 and 27, concerning existence, as well as gauge invariance, of a critical number of flavors. As we have mentioned above, our work does not deal directly with this issue, which pertains to the infrared momentum regime, but rather with the effects of the wave-function renormalization at intermediate momenta, in the presence of an infrared cutoff, which, as we shall argue below, could be interpreted as expressing finite-temperature effects. In the presence of an infrared cutoff, a critical number of flavors has been shown to exist, albeit depending on it.[7,16] The issue of gauge invariance of the result is still unresolved. The complexity of the situation can be understood probably better if we draw an analogy of the (finite) infrared cutoff with the temperature scale. In such a case, there are known[17] unresolved ambiguities appearing in the low-momentum regime of the theory, due to nonanalyticities of the effective action.

Below we would like first to compare the results of Ref. 7 to those of Refs. 26 and 27. In Ref. 26, it has been argued, on the basis of a power-counting analysis, which did not make any use of the Ward-Takahashi identities, that there is no renormalization of N to any order in $1/N$, in the infrared regime of the model. The arguments were based on the softened Coulombic form of the gauge-boson propagator in the infrared, as a result of fermion vacuum polarization: $D_{\mu\nu}\simeq(1/q)[g_{\mu\nu}-(1-\xi)q_\mu q_\nu/q^2]$, in an arbitrary ξ gauge, for small momentum transfers $q\ll\alpha$. It is worth noticing that such arguments appear to apply equally well to Abelian as well as non-Abelian theories, since in the latter case non-Abelian three or four-gluon interactions could not contribute to the potential scaling-violating interactions. This analysis has been performed without implementing an infrared cutoff, due to the infrared finiteness of the (zero-temperature) theory. In the work of Ref. 7, which is applied to the infrared regime, an infrared cutoff is introduced, which changes the scaling properties of the gauge-boson propagator. In this case, the scale-invariant situation seems to occur only for the value $n=2$ in the vertex *Ansatz*, which notably does not satisfy the Ward-Takahashi identities.[14] As we have seen, gauge invariance requires $n=1$, and in that case there exists a running N, at infrared momentum scales, as well as a finite critical flavor number, which, however, is infrared cutoff dependent and diverges in the limit where the cutoff is removed.

We can also compare this result with that of Ref. 27, which claims to have proven the gauge invariance of the critical number of flavors in QED_3. There, a nonlocal gauge fixing was used; this mixes orders in $1/N$ expansion, in the sense that the gap function in the SD equations contains now graphs of $O(1/N^2)$, while the wave-function renormalization still remains of $O(1/N)$. In contrast, the analysis of Ref. 7 remains consistently at leading order in $1/N$ and in the Landau gauge. The meaning of the nonlocal gauge fixing is not clear if one stays consistently within an order-by-order $1/N$ expansion. Nor does gauge invariance make complete sense in the presence of an infrared cutoff.

Thus the key to a possible explanation of the discrepancy between the works of Refs. 26 and 27 and Ref. 7 seems to be hidden in the higher orders in the large-N expansion, as well the presence of the infrared cutoff. Notice that a naive removal of the infrared cutoff might lead to ambiguities, as becomes clear from the work of Ref. 17, for finite-temperature field theories, provided that one makes[16] the (physically sensible) identification and/or analogy of the infrared cutoff with the temperature scale, at least within a condensed-matter effective-theory framework.

Now we come to our case. As we shall argue, our results can offer a way out of the above-mentioned discrepancy. For us, the momentum regime of interest is not the infrared one, where dynamical mass generation occurs, but the intermedi-

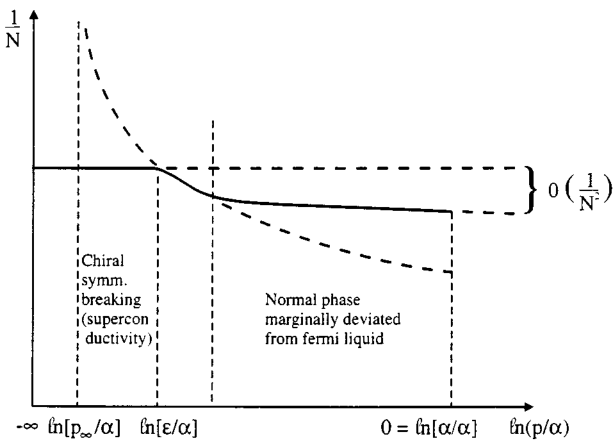

FIG. 1. Running flavor number in QED_3. The coupling is asymptotically free upon the Pennington-Webb choice for the vertex function (5), corresponding to $n=1$, as dictated by gauge invariance. The increase of the coupling is cut off at the infrared, as a result of the Coulombic form of the gauge-boson propagator due to fermion vacuum polarization. Above a certain infrared scale ϵ, the coupling starts running slowly, a situation resembling that of walking technicolor. This kind of behavior is argued to be responsible for (marginal) deviations from the Fermi-liquid picture in a condensed-matter framework.

ate scale. In this regime, the power-counting arguments of Ref. 26 do not apply, since the gauge-boson propagator does not have a simple Coulombic behavior. Thus the wave-function renormalization effects, which appear to exist in our, admittedly rough, truncation of the SD equations, might not be incompatible with the results of Ref. 26, pertaining to the existence of a critical flavor number. From our point of view, this would mean that, although there is a (slow) running of an effective N and thus scale invariance is marginally broken, however, the running of the coupling is even more suppressed in the infrared, where strong quantum effects cut off the increase of the (asymptotically free) coupling. The infrared cutoff then, appears as the (spontaneous?) scale, above which a slow running of the (asymptotically free) coupling becomes appreciable. In a condensed-matter-inspired framework, such a spontaneously appearing scale makes perfect sense, if one associates the infrared cutoff with the temperature scale.[16] For momenta sightly above the infrared cutoff, then, the situation of KN (Ref. 7) seems to be valid. This regime may be viewed as the boundary regime for which dynamical mass generation still can happen. Below the infrared scale, which is a regime that makes perfect sence in a infrared-finite theory such as QED_3, dynamical mass generation certainly occurs, and the arguments of Ref. 26 apply, leading to an effective cutoff of the increase of the coupling constant. In this regime, the gauge-boson propagator assumes a softened Coulombic $1/r$ form, which has been argued to be important for a (superconducting) pairing attraction among fermions (holes) in the model of Ref. 6. Such a situation, which is depicted in Fig. 1, was envisaged in Ref. 8 for the case of chiral symmetry breaking in four-dimensional QCD, which in this way was deassociated from the confining properties of the theory.

In the work of KN Ref. 7 and ours, all these issues could be confirmed only if a more complete analysis of the SD equations, including higher-order $1/N$ corrections, is performed. Whether resummation to all orders in $1/N$ washes out completely the wave-function renormalization effects at intermediate momenta, leading to an *exactly marginal* (scale-invariant) situation or keeps this effect at a RG marginal level remains an unresolved issue at present. On the basis of the above discussion, one would expect that marginal deviations from scale-invariant behavior at intermediate momenta, such as the ones studied in the present work, survive higher-order analyses, but they also lead to a critical number of flavors, since the latter is an entity pertaining to the infrared regime of the theory. Moreover, for us, who are interested in performing the analysis in a condensed-matter rather than particle-theory framework, there is the issue of the ambiguous infrared limit of the theory at finite temperatures, which is by no means a trivial matter.[17] It seems to us that all these important questions can only be answered if proper lattice simulations of the pertinent systems are performed. At present, the existing computer facilities might not be sufficient for such an analysis.

However, as we shall argue below, the slow running of the coupling constant of the model at intermediate-momentum scales, if true, is a desirable effect from a condensed-matter point of view, where both infrared and ultraviolet cutoffs should be kept. The wave-function renormalization effects, discussed above, prove sufficient in leading to a (marginal) deviation of the theory from the Fermi-liquid fixed point. At finite temperatures, this effect can have observable consequences and might be responsible for the experimentally observed abnormal normal-state properties of the high-T_c cuprates, the physics of which the above gauge theories are believed to simulate. We stress once again that such effects would be absent in an exactly marginal situation, like the one suggested in Ref. 26.

III. LINEAR BEHAVIOR OF THE RESISTIVITY IN QED_3 WITH THE TEMPERATURE SCALE

In this section we want to connect the above picture of the behavior of QED_3 at zero temperature to that of the same theory at finite temperature, T. In the absence, again, of anything like an exact solution in the $T\neq 0$ case, approximations (quite possibly severe ones) will have to be made. However, the physical aim is clear: We want to connect the experimental observation that the electrical resistivity in the normal phase of the high-T_c superconductors varies linearly with T over a wide range in T from low temperatures up to a scale of 600 K, to the existence of the nontrivial quasifixed-point structure of QED_3 found in the previous section. Qualitatively, the way we shall make the connection is to interpret the temperature in finite-T QED_3 as (related to) an effective infrared cutoff. This will follow from the form of the gauge-boson propagator for $T>0$, to which we now turn.

A. Gauge-boson propagator at finite $T>0$

The gauge-boson propagator $\Delta_{\mu\nu}$ is given by the expression

$$\Delta^{-1}_{\mu\nu}(p_0,P,\beta)=\Delta^{(0)-1}_{\mu\nu}(p_0,P,\beta)+\Pi_{\mu\nu}(P,p_0,\beta), \qquad (19)$$

where the vacuum polarization is given by

$$\Pi_{\mu\nu}=\Pi_T(P,p_0,\beta)P_{\mu\nu}+\Pi_L(P,p_0,\beta)Q_{\mu\nu}. \qquad (20)$$

The transverse $P_{\mu\nu}$ and longitudinal $Q_{\mu\nu}$ tensors are given, respectively, by

$$P_{\mu\nu}=-\delta_{\mu i}\left(\delta_{ij}-\frac{P_iP_j}{P^2}\right)\delta_{j\nu},$$

$$Q_{\mu\nu}=-\left(g_{\mu 0}-\frac{p_\mu p_0}{p^2}\right)\frac{p^2}{P^2}\left(g_{0\nu}-\frac{p_0 p_\nu}{p^2}\right),$$

$$Q_{\mu\nu}+P_{\mu\nu}=g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}. \qquad (21)$$

The zero-temperature polarization tensor of the gauge boson is $\Pi(p,\beta\rightarrow\infty)=\alpha p/8$. Thus, for low-energies, relevant for the definition of resistivity, the p^{-2} behavior of the gauge-boson propagator is softened to p^{-1}. For finite temperatures, on the other hand, this behavior is softened even more. In the instantaneous approximation, one finds[12] a "longitudinal" gauge-boson mass term proportional to

$$\Pi_{00}(P\rightarrow 0,p_3=0,\beta)=\frac{2\alpha\ln 2}{\pi\beta}\equiv 2\omega_\pi^2, \qquad (22)$$

where P is the magnitude of the spatial momentum. Thus we see that, in this approximation, the temperature has introduced an effective infrared cutoff $\sim\sqrt{\alpha/\beta}$. Interpreting this as the ϵ of the previous section, we find that the role of the all-important ratio α/ϵ is played by $\sqrt{\beta\alpha}$. The "intermediate"-momentum region is then $\sqrt{\beta\alpha}\gtrsim 1$, while the DMG region $\sqrt{\beta\alpha}\gg 1$ (or $T\ll\alpha$).

In the instantaneous approximation the transverse gauge bosons remain massless.[12] However, beyond the instantaneous approximation,[18] one obtains temperature-dependent corrections also to the transverse parts. The low-momentum behavior of these polarization tensors is not smooth,[18,17] and in particular one has the following ambiguities, depending on the order of the various limits:

$$\Pi_L(P\rightarrow 0,p_3=0,\beta)\rightarrow 2\omega_\pi^2,$$

$$\Pi_L(P=0,p_3\rightarrow 0,\beta)\rightarrow \omega_\pi^2,$$

$$\Pi_T(P\rightarrow 0,p_3=0,\beta)\rightarrow 0,$$

$$\Pi_T(P=0,p_3\rightarrow 0,\beta)\rightarrow \omega_\pi^2, \qquad (23)$$

where, in Euclidean formalism, p_0 is replaced by ip_3. For our purposes, however, an approximate form given in Ref. 18 will be sufficient:

$$\Pi_L\simeq\Pi_T\simeq\left(\frac{\alpha p^2}{64}+4\omega_\pi^4\right)^{1/2}, \qquad (24)$$

where $p^2=p_3^2+P^2$. In this approximation the gauge-boson propagator reads

$$\Delta_{\mu\nu}(p)=\frac{g_{\mu\nu}-\dfrac{p_\mu p_\nu}{p^2}}{p^2+\Pi(P,p_3,\beta)}, \qquad (25)$$

where Π is given by (24). In the limit $p\rightarrow 0$, which is relevant for the definition of resistivity (see below), one may then replace Π by $2\omega_\pi^2$, with the same qualitative association $\epsilon\sim\sqrt{\alpha/\beta}$ as before. The net effect of retardation on the gauge-boson propagator, in the large-N approximation, is summarized by the form (25).

B. Wave-function renormalization and vertex function at finite $T>0$

In view of the importance of wave-function renormalization in the $T=0$ case, as stressed in Sec. II, it is clear that we must include it also at $T\neq 0$. We shall find (see below) that its effect is to provide logarithmic (in T) corrections to the linear T dependence of the resistivity which is characteristic[28,29] of the gauge interactions.

Wave-function renormalization effects in QED_3 at $T>0$ were studied in Ref. 16, using the Pennington-Webb vertex *Ansatz* [$n=1$ in the notation of (5)] and making the instantaneous approximation, at least initially. The approximate SD equation for $A(P,\beta)$ then becomes (noting that the A of Ref. 16 is our $A-1$)

$$A(P,\beta)\simeq 1+\frac{\alpha^2}{16N\pi^2}\int_0^\alpha dK\, I(P,K,\beta)$$

$$\times\frac{\tanh\dfrac{\beta}{2}\sqrt{K^2+\mathcal{M}(K,\beta)^2}}{\sqrt{K^2+\mathcal{M}(K,\beta)^2}}, \qquad (26)$$

where $\mathcal{M}$ is the modified mass function B/A and

$$I(P,K,\beta)=\frac{K}{\alpha}\int_0^{2\pi}d\phi\,\frac{(P^2-K^2)^2-Q^4}{P^2Q^2[Q^2+\Pi_{00}(Q,\beta)^2]}, \quad (27)$$

with $Q=|P-K|$.

However, it was found[16] that the use of (26) led to a plainly unphysical result; viz, $A>1$. The trouble was traced to the use of the instantaneous approximation, which turns out to make a dramatic impact on A, essentially because of the effective reduction in the dimensionality of the integration in (26) from three to two dimensions.

An exact treatment is very difficult, but it was argued in Ref. 16 that a plausible improvement to (26), taking noninstantaneous terms into account in an approximate way, is obtained by replacing Π_{00} by a Q-independent constant Δ^2 which is of order α^2 and at the same time setting the factor (K/α) in (27) equal to unity. Certainly, the numerical results then obtained, in the region of dynamical mass generation, seemed physically sensible: In particular, as $T\rightarrow 0$, they were in good qualitative agreement with previous zero-temperature results and A was less than unity. In this case, the kernel I is replaced by the temperature-independent quantity

$$I_\Delta=-\frac{2\pi}{P^2}\left\{1-\frac{|P^2-K^2|}{\Delta^2}\right.$$

$$\left.+\frac{[P^2-K^2+\Delta^2][P^2-K^2-\Delta^2]}{\Delta^2\sqrt{[(P-K)^2+\Delta^2][(P+K)^2+\Delta^2]}}\right\}. \qquad (28)$$

Although originally discussed, in Ref. 16, within a context of dynamical mass generation, the above approximate formula for A can just as well be considered in the regime $\mathcal{M}=0$. It is for this regime that we now estimate the resistivity, introducing the effects of A.

C. Resistivity of QED$_3$ in the normal phase

Our aim in this subsection is to exhibit non-Fermi-liquid behavior of the resistivity and associate it with the quasifixed-point structure at intermediate scales revealed in the previous section via the qualitative connection $\alpha/\epsilon \sim \sqrt{\beta\alpha}$. The resistivity of the model is found by first coupling the system to an external electromagnetic field A and then computing the response of the effective action of the system, obtained after integrating out the (statistical) gauge-boson and fermion quanta, to a change in A.

In the case at hand, in the model of Ref. 6 (τ_3-QED) the effective action of the electromagnetic field, after integrating out hole and statistical gauge fields, assumes the form

$$S_{\rm eff}=\int A^{\mu}(p)\Delta_{\mu\nu}A^{\nu}(-p), \quad \Delta_{\mu\nu}=\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2+\Pi} \tag{29}$$

in a resummed $1/N$ framework, with Π the one-loop polarization tensor due to fermions. [Because of the τ_3 structure, as a result of the bipartite lattice structure,[6] there are no cross terms between the statistical and the electromagnetic gauge fields to lowest nontrivial order of a derivative expansion in the effective action. This implies that in this model the resistivity is determined by the polarization tensor of the hole (fermion) loop. On the other hand, in models where only a single sublattice is used,[29,30] such cross terms arise, which are responsible, after the statistical gauge-field integration, for the appearance of a conductivity tensor proportional to $\Pi_F\Pi_B/(\Pi_B+\Pi_F)$, with $\Pi_{B,F}$ denoting (respectively) polarization tensors for the boson fields of the CP^1 model and for the fermions (holes) in a resummed $1/N$ framework. In such a case, the conductivity is determined by the lowest conductivity among the subsystems.[30] In condensed-matter systems of this type, relevant for the physics of the normal state of the high-T_c cuprates, it is the bosonic contribution that determines the total electrical resistivity.[29]] The functional variation of the effective action with respect to A yields the electric current j. From (29) this is proportional to the electric field $E(\omega)=\omega A$, in, say, the $A_0=0$ gauge, with ω the energy. In the normal phase of the electron system, the proportionality tensor, evaluated at zero spatial momentum, is $\sigma_f\times\omega$, with σ_f the conductivity.[30] From (29), then, we have

$$\sigma_f=\left.\frac{1}{p^2+\Pi}\right|_{\underline{P}=0}, \tag{30}$$

where $\underline{P}$ denotes spatial components of the momentum.

If the effective action were real, then the temperature (T) dependence of the resistivity of the model would be given by the T dependence of the finite-temperature vacuum polarization of the gauge boson. Thus, following the approximation (24) for the polarization tensor in the resummed-$1/N$ framework,[18] we would have immediately obtained a linear T dependence for the resistivity. Such a temperature dependence would actually be valid for a wide range of temperatures above the critical temperature of dynamical mass generation,[6] due to specific features[18] of (24).

However, things are not so simple. As shown by Landau,[31] the analytic structure of the vacuum polarization graphs entering the effective action (29) is such that there are imaginary parts in a real-time formalism.[32] These imaginary parts are associated with dissipation caused by physical processes involving (on-shell) processes of the type fermion→fermion+gauge boson. It turns out that these constitute the major contributions to the (microscopic) resistivity.[33,28,29] In this picture, the latter is determined by virtue of the Green-Kubo formula[34] in the theory of linear response, and it turns out to be inversely proportional to the imaginary part of the two-point function of the "electric" current $j^{\psi}_{\mu}=\bar{\psi}\gamma_\mu\psi$, evaluated at zero spatial momentum. In our case, in the leading $1/N$-resummed framework, the two-point function of the electric current is given by the graph of Fig. 1. Adopting the *Ansatz* (5) for the vertex function, the result for the current-current correlator is

$$\langle J_\mu(p)J_\nu(-p)\rangle \propto [A(p)]^n\Delta_{\mu\nu}(p)[A(p)]^n. \tag{31}$$

To compute the imaginary parts of (31) would require a real-time formalism, taking into account the processes of Landau damping,[17] which are not an easy matter to compute in a resummed-$1/N$ approximation, especial in the limit of zero-momentum transfer, relevant for the definition of resistivity. Indeed, as shown in Ref. 17 and mentioned briefly above, there is a nonanalytic structure of the imaginary parts of the one-loop polarization tensors appearing in the quantum corrections of the gauge-boson propagator. Such nonanalyticities result in a nonlocal effective action. This nonlocality persists upon coupling the system to an *external* electromagnetic field A. Since the resistivity of the system is defined as the response of the system to a variation of A, then the Landau processes, which constitute the major contribution to the (microscopic) resistivity, complicate the situation enormously. At present, only a numerical treatment of these nonanalyticities is possible.[17,18]

We can circumvent this difficulty and use only the real parts of the gauge-boson polarization tensor and the approximate expression (24) to estimate the temperature dependence of the resistivity by making use of the fact that in "realistic" many-body systems,[6,28,29] believed to be relevant for a description of the physics of the cuprates, there is the phenomenon of spin-charge separation of the relevant excitations, discussed briefly in Sec. IV. According to this picture, the statistical current (responsible for spin transport) is opposite to the hole current (electric charge transport),

$$j_\psi+j_z=0, \quad j^{\psi}_{\mu}=\bar{\psi}\gamma_\mu\psi, \quad j^{z}_{\mu}=2z^*\partial_\mu z, \tag{32}$$

and this constraint is implemented by the statistical gauge field a_μ that plays the role of a Lagrange multiplier.[29] The gauge field, on the other hand, is identified for physical (on-shell) processes, with the current j_z (of the CP^1 model), and thus, on account of (32), the electric charge is transported in such systems with a velocity which equals the propagation velocity v_F of the statistical gauge fields a_μ. [Again, the model of Ref. 6 is different from those of Refs. 28 and 29 in that the (independent) statistical gauge field a_μ is related (through its equations of motion) to the sum of the currents

$j_\psi + j_z$. To apply our arguments in this model one has to assume that for the electric resistivity the boson part plays no role, which is justified by the formula (30) above. This allows one to consider only static configurations for the z fields, thereby justifying the assumption that the electric charge in the model propagates with the a_0 gauge-boson velocity.] In nontrivial vacua, such as the one pertaining to our system, the velocity v_F receives quantum corrections[35] from vacuum polarization effects. In a thermal vacuum such corrections are temperature (T) dependent.

If we represent the (observable) average of the electric current as j_ψ=charge$\times v_F$, and use Ohm's law to relate it with an (T-independent) externally applied electric field E, $j_\psi = \sigma . E$, then one observes that in this picture the main T dependence of the resistivity σ^{-1} comes from v_F, as a result of (thermal) vacuum polarization effects.[35] [Of course, it is understood that the above argument is only heuristic and a proper (microscopic) computation of the resistivity, using real-time Green function calculus, combined with kinetic transport theory, appears necessary in order to arrive at rigorous results.[28,29] However, the heuristic picture above captures the particular characteristics of the gauge interactions, responsible for yielding a linear T dependence, as we show below, and for our purposes it will be sufficient.]

To compute $v_F(T)$ we shall use its definition in the case of an (on-shell) relativistic massless particle[35] (in this case the gauge boson),

$$v_F = \frac{\partial E}{\partial Q}, \quad E^2 \equiv q_0^2 = Q^2 + \Pi(Q,\beta). \tag{33}$$

Only the real parts of the gauge-boson polarization tensor are relevant for the computation of (33).[35] Using (24), it is then straightforward to evaluate (33) in the limit of vanishing momentum transfer, appropriate for the definition of resistivity. The result is

$$v_F \propto \frac{Q}{T^{3/2}}, \quad Q \to \epsilon. \tag{34}$$

Using the association of the momentum infrared cutoff $Q \simeq \epsilon$ with $\sqrt{\alpha/\beta} \propto \sqrt{T}$, one gets from (34) a linear T dependence for v_F^{-1} and, thus, for the resistivity ρ. Such a linear T dependence is a characteristic feature of the gauge interactions and, as we shall discuss below, is valid for a wide range of T.

Above, we have ignored wave-function renormalization effects. We now proceed to include them explicitly and demonstrate the existence of (logarithmic) deviations from this linear T behavior. This part of the analysis does not require an explicit computation of the imaginary part of the correlator (31). It only requires A evaluated at $p=0$. So we can examine it directly. In this limit, we have

$$I_\Delta(p=0,K) = -\frac{4\pi(\Delta^2 - K^2)}{(\Delta^2 + K^2)^2}. \tag{35}$$

The maximum K in (35) runs from $\sim\sqrt{\alpha/\beta}$, to $\sim\alpha$, which in the ''intermediate'' regime $\alpha/\epsilon \sim \sqrt{\beta\alpha} \gtrsim 1$ means that K is constrained to lie within an order of magnitude of α and that $\mathscr{M}$ in (26) will be zero. Recalling that Δ^2 is also of order α^2, a rough estimate for $A(p=0,\,\beta)$ is provided by

$$A(p=0,\beta) \simeq 1 \frac{1}{4N\pi} \int_{\sqrt{\alpha/\beta}}^{\alpha} dK \, \frac{1}{K} \tanh\left(\beta \frac{K}{2}\right)$$
$$= 1 - \frac{1}{4N\pi} \int_{\sqrt{\alpha\beta/2}}^{\alpha\beta/2} \frac{dx}{x} \tanh x. \tag{36}$$

If $\beta\alpha$ were $\gg 1$ (the very-low-temperature limit), we could replace the ''tanh'' function in (36) by unity and deduce

$$A(p=0,\beta) \simeq 1 - \frac{1}{8N\pi} \ln(\alpha\beta). \tag{37}$$

Then the resistivity, which formally is given by the imaginary part of the inverse of (31) as $p \to 0$, would exhibit the temperature dependence [resummed up to $O(1/N)$]

$$\rho \propto O(T^{1-1/4N\pi}), \tag{38}$$

where we have taken $n=1$ as in Ref. 14. We cannot, in any case, take the precise value of the exponent in (38) seriously in view of the rough approximations made along the way.

However, the region $\beta\alpha \gg 1$ is, in fact, that of dynamical mass generation, rather than the intermediate region $\beta\alpha \gtrsim 1$ in which we expect the quasifixed-point structure to play a role. For $\beta\alpha \gtrsim 1$ the integral of the right-hand side of (36) has to be evaluated numerically. One finds that for $\beta\alpha \gtrsim 5$ the result is within 10% of (37) and that (37) is virtually exact for $\beta\alpha \gtrsim 10$. Thus we can conclude that for a wide range of temperature below α, but not so low that the symmetry-breaking phase is entered, the resistivity should have the form (38), where the precise coefficient of the $1/N$ power is not known accurately from the above analysis.

The main point, then, is the ''stability'' of this T dependence, which correlates remarkably with the quasifixed-point structure of Sec. II.

IV. BRIEF COMMENTS ON REALISTIC MODELS OF HOLONS AND SPINONS FOR PLANAR-DOPED ANTIFERROMAGNETS

A. Microscopic models and their (naive) continuum limit

Above, we have argued that the gauge-fermion interactions in planar QED_3 are responsible for non-Fermi-liquid behavior in the sense of exhibiting a nontrivial fixed-point structure of the RG at relatively low energies, below the scale set by the dimensionful coupling constant in three space-time dimensions.

The scope of this section is to connect the above results to realistic models of holons and spinons interacting magnetically via spin-spin interactions in models believed to simulate the physics of the recently discovered high-T_c materials. We shall be brief and concentrate only on some heuristic argumentation. Details can be found in the literature.[36–38,3]

First, we shall identify the roles of the various excitations of these materials in connection with the various fields appearing in QED_3 models described above. To this end, we note that in condensed-matter systems, relevant for high-T_c superconductivity, the basic excitations are electron fields with momenta lying close to the Fermi surface. Optical experiments have shown the existence of a large Fermi surface in these materials. At first sight, this implies that our model of Sec. II, based on Dirac fermions, is inadequate. However, as we remarked earlier, the most important interactions for fermions, in both the superconducting and normal phases, are those which are local on the Fermi surface, and as such an expansion of the effective theory about a single point on this surface would be adequate. This has been done in Ref. 6, with the result that under the *assumption* of spin-charge separation one arrives at an effective low-energy theory which resembles a variant of QED_3, with the Dirac fermions playing the role of holon excitations.

To understand this point, which is our crucial difference from the approach of Refs. 4 and 3 using spinons only, we remark that the basic fields are electrons with both spin and charge described by a creation operator $C^{\,i}_{\alpha}$, with i a spatial lattice index and $\alpha=1,\ldots,M$ a spin SU(M) index. Realistic models have $M=2$. Spin-charge separation can be implemented by making the *Ansatz*[29,6]

$$C^i_\alpha = \psi^{\dagger,i} z^i_\alpha , \tag{39}$$

where $\psi^{\dagger,i}$ is a Grassmann field that represents the creation of a holon and z_α is a CP^{M-1} multiplet, representing a spinon excitation (magnon).

At this point we note that in condensed-matter physics one uses[3,4] an alternative *Ansatz*

$$C^i_\alpha = f^i_\alpha b^\dagger_i , \tag{40}$$

where the fermion fields f carry the spin index and thus represent the spinon excitations, carrying no electric charge, while the Bose fields $b^\dagger$ are spinless and are electrically charged. This is the description followed by Refs. 3 and 4, which treats the spin excitations as fermion fields in the effective Lagrangian approach. This description is related to the previous one (39) by bosonization techniques and may be viewed as a "gauge"-fixing choice.[39]

The gauge symmetry in both descriptions can be found by performing *local* phase rotations of the constituents in (39) and (40). Since for our purposes we shall follow the *Ansatz* (39), we concentrate on it from now on. The Abelian gauge symmetry that leaves the electron field invariant in (39) is

$$\psi^j \to e^{i\theta(j)}\psi^j, \quad z^j_\alpha \to e^{i\theta(j)} z^j_\alpha \tag{41}$$

and is valid beyond half-filling. This gauge symmetry refers to spatial indices only and can be expressed in an effective theory formalism via link variables in a Hartree-Fock approximation,[36,6]

$$\sum_{\langle ij\rangle} \psi^{i,\dagger}\psi^j \langle z^{i,\dagger}_\alpha z^{\alpha,j}\rangle \equiv \sum_{\langle ij\rangle} \Delta_{ij}\psi^{i,\dagger}\psi^j, \tag{42}$$

where the sums extend over appropriately defined nearest-neighbor sites to be specified below. The gauge symmetry is discovered by freezing the amplitude of the Hartree-Fock field $|\Delta_{ij}|\simeq$const, while letting its phase fluctuate $\exp(\int_i^j a\,dl)$ with a_i the spatial components of an Abelian [U(1)] gauge field.

In large-spin approximations[37] of doped antiferromagnets with a bipartite lattice structure, intrasublattice hopping is suppressed by terms of $O(1/S)$, where $S\gg 1$ is the effective spin of the excitations. In this case, the fermion fields in (39), ψ^j, may be assigned an internal "color" quantum number, labeling the sublattice they lie on. In such a case the nearest-neighbor sites in (42) lie on this sublattice and from the point of view of the bipartite lattice are next-to-nearest neighbors. The advantage of introducing this bipartite lattice structure lies in the fact that the dynamically generated gap through the gauge interactions (42) is parity conserving, due to energetics in the case of even-flavor fermion numbers.[40,6,41,10] Thus one seems to have a natural explanation of the absence of P,T violation in these materials, despite the fact that the superconducting (binding) forces are unconventional (magnetic) in origin.

The temporal component of the gauge field can be inserted by invoking the Gutzwyler projection operator ensuring the absence of double occupancy in these materials. This imposes the restriction of *at most one electron per site*, which formally can be expressed via

$$\psi^{i,\dagger}\psi^i + z^{i,\dagger}_\alpha z^i_\alpha = 1 \quad \text{no sum over } i. \tag{43}$$

In a path-integral approach to quantum-doped antiferromagnets, the above constraint (43) may be implemented by a Lagrange multiplier field a_0, playing the role of the temporal component of the gauge field. Alternatively, one may work in the $a_0=0$ axial gauge, appropriately for a Hamiltonian formulation,[6] in which case one has to use the constraint explicitly to arrive at an effective Lagrangian with the correct number of independent degrees of freedom.

In both formulations, the presence of the gauge field indicates the existence of redundant degrees of freedom which are unphysical.

The effective Lagrangian, describing the physically relevant degrees of freedom that lie close to a single point on the Fermi surface, can, then, be shown to acquire the form of a CP^1 σ model, describing the spin excitations of the system, coupled via a statistical Abelian gauge field to a system of electrically charged Dirac fermions in a spin-charge-separated environment,

$$\frac{1}{\gamma_0}\int d^3x |(\partial_\mu - a_\mu)z|^2 + \sum_{i=1}^N \int d^3x\, \overline{\Psi^i}(x) \times [i\partial\!\!\!/ + a\!\!\!/\,\tau_3 - (e/c)A\!\!\!/\,]\Psi^i(x), \tag{44}$$

where the constraint (43) becomes effectively[38] $z^\dagger z\simeq 1$. The quantity γ_0 is the antiferromagnetic interaction coupling constant of the σ model,[6] c is the light velocity in units of the Fermi velocity of holes, a_μ is the statistical gauge field, representing magnetic interactions, and A_μ is the electromagnetic field. [For simplicity we assumed that the Fermi velocity of holes is approximately equal to the velocity of magnons v_S occurring in the CP^1 sector. The realistic case is when the two velocities are different, which spoils the relativistic form of (44). However, this will not be important for our qualitative treatment in this article. For more comments on this point, see Ref. 6.] The fermion fields Ψ are color doublets with respect to the sublattice degree of freedom; the τ_3 structure, which acts in this color space, indicates the opposite spin of the antiferromagnetic (bipartite) lattice structure of the underlying lattice. This doublet structure should not be confused with the $i=1,2,\ldots,N$ flavor degree of freedom of the fields Ψ. As we have mentioned in the Introduction, this "flavor number" represents internal degrees of freedom, associated with the orientation of the momentum vectors of the quasiparticle excitations[9] in expansions about a certain point of a finite-size Fermi surface. For large Fermi surfaces and low-lying (infrared) excitations, where the cutoff Λ effectively collapses to zero, as compared with the radius k_F of the Fermi surface, a controlled large-$N(\Lambda)$ expansion is then applicable.

In condensed-matter-inspired models,[6,13] one may argue that the spontaneous scale α, above which nothing interesting happens in QED_3,[10] plays the role of the ultraviolet cutoff Λ of Ref. 2. Hence, after cell division of angular space, we have effectively[2] $N\sim\alpha/k_F$ [see (3) and following remarks]. In this interpretation of the flavor number, which in fact is essential for a consistent RG approach to the theory of the Fermi surface,[9] one has an effective running of the fermion flavor number with the RG scale, which is precisely the case of our running $g\propto 1/N$ discussed in Sec. II.

To form an estimate of this effective N, we use the phenomenological formula[6,13,42]

$$\alpha=\hbar v_F/(a\,\eta_{\max})\sim t'(\eta)^{1/2}/\eta_{\max}, \tag{45}$$

where a is the lattice spacing, v_F is the Fermi velocity of holes, t' is a hoping parameter for holes (on the same sublattice), and η ($\eta_{\max}$) denotes the average (maximum for superconductivity) number density of holes (doping concentration). In realistic models the various parameters entering (45) depend on temperature T. For our angular cell division, however, we shall use the k_F of a zero-temperature theory. A typical scale for the Fermi surface radius, which is a typical energy of electronic excitations, is thus of $O[1\text{ eV}]$. For the values of temperature and doping concentration relevant for superconductivity, a typical value of α is of order of 1 eV.[13] As argued in Sec. III, in the normal phase $T>T_c\sim O[100\text{ K}]$, one may replace the Fermi velocity by an effective one $v_F\propto T^{-1}$, and hence the corresponding $\alpha(T)$ gets considerably smaller, as compared to k_F, thereby shifting the effective scales towards the infrared or, equivalently, pushing the infrared cutoff to higher values. It is therefore not unreasonable to argue that the conditions for large $N\propto k_F/\alpha(T)\gg 1$ may be satisfied for the range of temperatures and (large) Fermi momenta characterizing the normal phase of these materials. Of course, it is understood that all such estimates are only qualitative. Any attempt to present quantitatively meaningful considerations would require working directly with microscopic models, which falls beyond the scope of the present work.

Note that for the superconducting phase of the model the sublattice structure is important in that the fermion condensate responsible for the spontaneous breaking of the electromagnetic gauge invariance $U(1)_{\rm em}$ associated with the A field in (44) occurs between fermions (holes) of opposite sublattice each of electric charge e. For the normal-phase analysis, however, which we are interested in for the purposes of the present work, the sublattice structure is irrelevant. From now on, therefore, we concentrate on a single sublattice, ignoring the τ_3 color structure of the fermions. Whenever the latter becomes important, it will be stated explicitly.

From this point of view, the statistical gauge interaction in (44) plays exactly the role of the fermion-gauge interaction of Sec. II, which leads to a nontrivial fixed-point structure at momenta $p\lesssim O[\alpha]$, where α is the dimensionful scale set by the statistical gauge-interaction coupling constant. To understand this point it is sufficient to remark that integrating out the magnon degrees of freedom, which are massive of mass m_z in the phase where long-range antiferromagnetic order has been destroyed, one obtains at low energies (much lower than the mass m_z scale) a Maxwell-like term for the gauge field a in (44), which thus becomes dynamical.[43,44] In this sense, the situation for the statistical gauge interaction becomes similar to the QED_3 case discussed previously.

B. Absence of charge- or antiferromagnetic-density-wave instabilities

An interesting question that arises in connection with the low-energy behavior of such systems concerns the existence of other type of instabilities which, from the point of view of an effective Lagrangian, would manifest themselves as marginal or relevant operators. The obvious class of candidate interactions, which in fact is the only one in these models by simple power counting in large-N treatments, would be four-fermion operators. Since our effective Lagrangian (44) has only trilinear gauge-fermion couplings, such effective operators could be shown to arise as a result of ladder (or cross ladder) graphs involving the exchange of gauge particles (cf. Fig. 2). If an operator of this sort is *exactly marginal*, then its scaling would be the same as the tree-level scaling of the effective gauge-fermion vertex. Exactly marginal deformations do not cause the appearance of a gap in the fermion spectrum. We shall argue below that this is what happens in our case in the infrared regime of momenta.

Interesting effects can be examined in this framework in association with the electromagnetic or statistical gauge interaction that could lead to antiferromagnetic instabilities in the normal phase, associated with the formation of electrically neutral spin- or charge-density waves (SDW's or CDW's), which could be described by fermion-antifermion condensates. In our formalism, since the Grassman variables ψ^j in (39) are spinless, the formation of fermion condensates on a single sublattice would then be appropriate for a description of CDW instabilities. What we shall show below is that in our model such CDW instabilities cannot occur as a result of the electromagnetic interaction. Notice that because of the τ_3 structure of our model (44), the fermion lines in these graphs can all lie on the same sublattice only if the exchanged gauge particle is the electromagnetic photon. Graphs in which the exchanged particle is the statistical gauge boson, and hence the fermion lines necessarily belong to different sublatices, are known[6] to lead at low momenta to superconducting mass generation and will not be of interest to us here. In the normal phase, such instabilities are absent.

Following Ref. 3, we consider the ladder and cross-ladder graphs of Fig. 2, where the external legs are set to zero momentum and the propagators of the electromagnetic (gauge) and fermion fields are dressed in a Schwinger-Dyson fashion. The important point for the electromagnetic photon is that in three dimensions its kinetic term acquires the modified Coulomb form (1), in all ranges of momenta; this form implies that the relevant propagator scales like $1/q$, where q is the momentum transfer circulating around the loop of Fig. 2, for zero external momenta of the fermion legs. In the phase where there is no gap for the fermion propagators, the latter scales with momenta like $1/[A(p)/\not{p}]$, where $A(p)$ is the wave-function renormalization. This is also the same

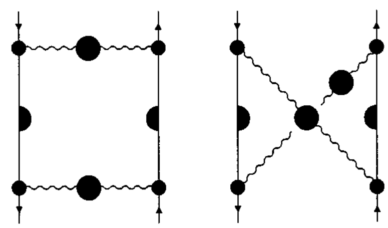

FIG. 2. Ladder and cross-ladder (resummed) one-loop graphs in QED_3. The soft Coulombic form of the infrared gauge-boson propagator results in the exactly marginal character of these (four-fermion) interactions: The scaling is that of tree level. This leads to the absence of the respective instabilities.

scaling as the one in the region of momenta $M \ll p \ll \alpha$, where dynamical gap generation could occur. Hence for our purposes we shall adopt this Feynman rule for the momentum-space scaling of the dressed fermion propagator. The vertex function is assumed to scale like $A(p)^n \gamma_\mu$ according to the *Ansatz* (5) even for the case of electromagnetic interactions. The result of the one-loop integral of the ladder and cross-ladder graphs, then, scales like

$$\int d^3q \, \frac{1}{[q]^4} A^{2n}(q) A^{2(n-1)}. \tag{46}$$

Thus, by choosing the Pennington-Webb vertex *Ansatz* $n=1$, dictated by gauge invariance,[14] we observe that the gauge interaction becomes *exactly marginal*, since the scaling behavior of the ladder and cross-ladder graphs of Fig. 2, Eq. (46), is similar to the tree-level scaling, at least in the region of momenta where dynamical gap generation could occur.

This implies the absence of charge-density waves of these systems caused by the electromagnetic interactions, in agreement with more rigorous condensed-matter models.[1,3] It should be remarked that the above marginal character of the interaction refers to four-fermion graphs, which from an effective Lagrangian point of view simply denotes the absence of the pertinent instability caused by such four fermion interactions. It should not be confused with the fermion-gauge trilinear interaction causing a mass gap, which exists anyhow at low momenta as a result of the gauge interactions.[10,6]

An additional type of instability of such systems is that of an antiferromagnetic spin-density wave. To study SDW's in the present formalism one should examine the CP part of the effective action (44). An easier way, which is closer to the present context, would be to pass to the alternative spin-charge separation *Ansatz* (40), by fermionizing the spin excitations. In such a case, the sublattice structure would be totally irrelevant, and one should consider the spin degrees of freedom as fermions interacting with a statistical gauge field of QED_3 type. The low-energy behavior of the system would be described again by a modified photon propagator of $1/p$ form, as a result of fermion vacuum polarization,[10,20] which would yield exactly marginal four-fermion interactions as in (46). Hence one finds again that such gauge systems exhibit no antiferromagnetic instability.[3]

The masslessness of the gauge particle was important for the above marginal scaling behavior, as was the modified $1/p$ scaling behavior of the dressed gauge propagator, which itself was a result of the fermion vacuum polarization or (in the case electromagnetic interactions) the projection from four to three dimensions.[6] The fact that the gauge invariance dictates the value $n=1$ in the *Ansatz* (5) of the gauge-fermion vertex, leading to the above marginal behavior of the gauge interaction in the ladder graphs of Fig. 2, implies that the absence of charge-density waves in the present model, or antiferromagnetic instabilities in the case of spinon systems, can be considered as a clear-cut prediction of the gauge nature of the interactions among the fermionic quasiparticle excitations.

C. Electromagnetic effects

A final comment concerns the effects of the electromagnetic-field–fermion coupling on the deviation from Fermi-liquid behavior in the infrared. The effect is known to occur in four space-time dimensions,[5] with the result that the presence of the vector potential in nonrelativistic condensed-matter systems causes deviations from the Fermi-liquid behavior at low temperatures, which, however, are suppressed by terms of $O[v_F^2/c^2]$.

In three space-time dimensions, in the presence of statistical interactions, the situation is quite different if one restricts one's attention in a given sublattice in these antiferromagnetic oxides. As we shall show below, the electromagnetic-field–fermion interactions become irrelevant in the presence of the electron-electron interactions caused by the statistical gauge field. This is easily demonstrated by first integrating out the auxiliary gauge field a_μ in (44). We concentrate on the effects of fermions within each sublattice. In the normal phase, where no mass is generated, integrating out the fermions of the other sublattice just produces Maxwell terms for the statistical gauge field, which due to the vacuum polarization acquire the form

$$\mathcal{L}^{\mathrm{kin}} = \frac{1}{g^2} f_{\mu\nu}^2 + f_{\mu\nu} \frac{1}{\sqrt{\partial^2}} f^{\mu\nu} + \cdots . \tag{47}$$

Such terms are irrelevant operators in the infrared, as compared with the nonderivative a terms in the CP^{N-1} part of the action (44). Indeed, after a-field integration in the sublattice, one would get current-current terms multiplying the inverse of the operator

$$\mathcal{D}_{\mu\nu} = \delta_{\mu\nu} - \frac{\partial^2 \delta_{\mu\nu} - \partial_\mu \partial_\nu}{\sqrt{\partial^2}},$$

appearing in (47). Only the nonderivative part of such an inverse is relevant in the infrared. Thus reconstructing the electron operators χ out of the spin-charge constituents as[45]

$$\chi_\alpha = z_\alpha^\dagger \psi \tag{48}$$

and integrating out the a field in (44) yields a Thirring interaction between the electrically charged electron fields,[45]

$$S^{\mathrm{eff}} = \int d^3x \left[i\bar{\chi} \, \partial\!\!\!/ \chi - \frac{\gamma_0}{4} (\bar{\chi}\gamma_\mu \chi)^2 + \frac{e}{c} A_\mu \bar{\chi} \gamma^\mu + \cdots \right]. \tag{49}$$

In the infrared, the electron kinetic terms become irrelevant operators, as compared with the Thirring contact interactions, and from now on we shall omit them. Assuming conservation of the fermion number in each sublattice, as a result of the assumed suppression of intrasublattice and interplanar hopping, we may represent in three dimensions the conserved sublattice fermion current as a curl of a vector field V_μ,

$$\bar{\chi}\gamma_\mu\chi=\epsilon_{\mu\nu\rho}\partial_\nu V_\rho . \tag{50}$$

(Spontaneous breaking of the fermion number occurs in the superconducting phase, as a result of one-loop anomalies due to gap generation.[6] In the normal phase, which we are interested in, such phenomena are absent and the fermion current is assumed to be conserved at a quantum level.) In this case the effective low-energy action (49) can be written in the form

$$S^{\rm eff}=\int d^3x\, \frac{e}{c}\, A_\mu \epsilon^{\mu\nu\rho}\partial_\nu V_\rho - \frac{\gamma_0}{4}\, F_{\mu\nu}(V)^2 - \frac{\gamma_0}{4}\,(\partial_\mu V_\mu)^2+\cdots , \tag{51}$$

where the ellipsis indicates terms that are more irrelevant, in a RG sense, in the infrared, than the terms kept. The last term in (51) is viewed as a gauge-fixing term. Our aim is to examine whether the electromagnetic field interactions are capable of driving the theory to a nontrivial fixed point, away from the free-electron (Landau) fixed point. We are thus interested in the behavior of the mixed Chern-Simons term $A\ dV$ in the presence of a weak Thirring interaction [i.e., close to the free-electron (bare) interactions]. This is equivalent to a strong-coupling problem for the gauge field V, which allows a heuristic proof of the irrelevant character of the $A\ dV$ interaction, as follows: First, we represent the mixed Chern-Simons term, in the infrared, as a heavy-fermion-gauge interaction,

$$A\ dV \propto \overline{\Psi}\left(i\partial\!\!\!/ + V\!\!\!/ \tau_3 + \frac{e}{c}\, A\!\!\!/ \right)\Psi + M\overline{\Psi}\Psi, \quad M\rightarrow\infty. \tag{52}$$

This yields the following two-point function for the field $\tilde{V}\equiv\epsilon_{\mu\nu\rho}\partial_\nu V_\rho$:

$$K_{\mu\nu}\propto \int d^3x e^{ip.x}\langle T\tilde{V}(x)\tilde{V}(0)\rangle = \left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{p^2}{p^2+e^2p^2I(p)}, \tag{53}$$

where

$$I(p)\propto \frac{1}{4\pi}\left(\frac{4M^2}{p^2}\right)^{1/2} \tan^{-1}\left[\left(\frac{4M^2}{p^2}\right)^{1/2}\right], \tag{54}$$

with $M\rightarrow\infty$ the auxiliary (massive) fermion mass.

The scaling of the electromagnetic photon two-point function is not affected by the Ψ fermions in this limit, and hence it is given by $1/p$, due to (1) in three space-time dimensions. Thus we observe that in the infrared the fermion-current term $\bar{\chi}\gamma_\mu\chi$ is marginal in the sense that it does not scale with momenta. On the other hand, the electromagnetic gauge field scales like $p^{-1/2}$, implying the RG irrelevant nature of the electromagnetic-field–fermion vertex.

This means that, in the models examined above, with suppressed intrasublattice hopping, in each sublattice the only dominant deviations from the Fermi-liquid behavior can be induced by the statistical gauge interactions at energy scales close to α. This result might be subject to experimental test, provided that accurate enough experiments can be made so as to obtain data within one sublattice only. It goes without saying that intrasublattice hopping, which increases with increasing doping concentration,[46] affects the above result.

V. CONCLUSIONS AND OUTLOOK

In this article we have examined certain interesting effects of the wave-function renormalization in (a variant of) QED_3, which is believed to be a qualitatively correct continuum limit of semirealistic condensed-matter systems simulating (planar) high-temperature superconducting cuprates.

Based on an (approximate) Schwinger-Dyson-improved renormalization-group analysis, we have argued for the existence of an (intermediate) regime of momenta, where the running of the renormalized dimensionless coupling of multiflavor QED_3, which is nothing other than the inverse of the flavor number, is considerably slowed down, exhibiting a behavior similar to that of "walking technicolor" models of particle physics. This slow running, or (quasi)fixed-point structure, has been argued to be responsible for an increase of the chiral-symmetry-breaking (superconducting) fermion condensate of the model, as well as for a (marginal) deviation from the Landau Fermi-liquid fixed point. In connection with the latter property, we have argued that the large-N expansion is fully justified from a rather rigorous renormalization-group approach to low-energy interacting fermionic systems with large Fermi surfaces. Some experimentally observable consequences of this (marginal) non-Fermi-liquid behavior, including logarithmic temperature-dependent corrections to the linear resistivity, have been pointed out, which could be relevant for an explanation of the abnormal normal-state properties of the high-T_c cuprates.

The above RG-SD analysis was, however, only approximately performed at present. To fully justify the above consideration and to make sure that the above-mentioned effects are not washed out in an exact treatment, one has to perform lattice simulations of the above models. Given that this might not be feasible yet, due to the restricted capacities of the existing computer devices, an intermediate step would be to perform a more complete analytic RG treatment of the relevant large-N SD equations at finite temperatures. Such a treatment is not easy, however, due to the mathematical complexity of the involved equations. In addition, finite-temperature field theory is known to exhibit unresolved ambiguities concerning the low-momentum limit, which complicates the situation. Some of these issues constitute the object of intensive research effort of our group at present, and we hope to be able to reach some useful conclusions soon.

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