Phase-coherent conductance of a superconductor–normal-metal quantum interferometer

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(Received 13 January 1995; revised manuscript received 6 October 1995)

A theory of the subgap conductance of a hybrid superconductor–normal-metal (S/N) quantum interferometer consisting of two tunnel junctions (S/N) in contact with an N layer is developed. The oscillatory dependence of the conductance of the system G on the phase difference between superconductors φ is shown to be due to the proximity effect, i.e., to the penetration of the condensate into the N region. The $G(\varphi)$ dependence is found for the cases of both weak and strong proximity effects. This dependence can radically change its shape form qualitatively when the voltage is varied. The amplitude of the conductance oscillations appears to be large in the case of strong proximity effect. Different layouts are considered including the cases when there is the third tunnel junction or point contact between the N conductor and the N electrode.

I. INTRODUCTION

In recent years there has been considerable interest in the conductance of mesoscopic "hybrid" systems with more than one superconductor–normal-metal (S-N) contact.[1–8] Many recent experiments were also devoted to the measurement of a single S-N contact conductance at low temperatures and voltages.[9–12] In most of these studies, a highly doped semiconductor with a two-dimensional (2D) or 3D electron gas was used as a normal conductor. Generally, a semiconductor-superconductor interface exhibits a low transmittance, which is determined by the Schottky barrier and by the differences in the electronic parameters of the contacting materials. Because of the presence of the barrier, the superconductor-semiconductor contacts are similar to superconductor–insulator–normal-metal (S-I-N) tunnel junctions. In the lowest approximation in the barrier transmittance, a theory based on the tunnel Hamiltonian method[13] predicts that at low temperatures and voltages ($eV,T\ll\Delta$, Δ being the energy gap of the superconductor) the conductance of a S-I-N contact, G, is much smaller than the conductance in the normal state, G_n. However, the experiments[9,10] revealed that at low temperatures the conductance of the investigated junctions exhibits a peak at $V=0$ (the so-called "zero-bias anomaly"), the magnitude of which can be comparable with G_n. The phenomenon of the subgap conductance enhancement has been studied theoretically.[14–19,5–7] The authors of Refs. 5,6,16–18 studied the S-I-N contact conductance for some particular cases (low transmittance of the barrier,[6] or low voltage[6,16–18]) when the deviation of the quasiparticle distribution from the equilibrium distribution may be ignored on both sides of the barrier. In this case the problem is reduced to the calculation of the probability of two-electron tunneling through the barrier. Note that a perturbation approach to the calculation of this probability[4] does not allow one to establish the actual relation[15] between the proximity effect, i.e., the penetration of the condensate into the N region, and the zero-bias anomaly. It should also be noted that several questions were left unsolved by the authors of Refs. 4,5,16–18; in particular, it is not clear how the conductance depends on the normal lead resistance if the latter is comparable with or larger than the resistance of the barrier and what is the limiting value of the conductance in the case of very low temperatures when the length of the N leads and its coherence length $\xi_N(T)\sim\sqrt{D/\max(T,\gamma)}$ ($D=v_F l/3$ is the electron diffusion constant, l is the elastic mean free path, and γ is the pair-breaking rate) become comparable.

All these questions are readily solved by using another approach employed in Refs. 6,14,15,19. It is based on the well elaborated technique of quasiclassical Green's functions integrated over the variable $\zeta=v_F(p-p_F)$.[20,21] Microscopic equations for the Green's functions, together with the boundary conditions,[22] can be written in a compact form with the use of a 4×4 matrix $\check{g}$ (Ref. 21) in Keldysh space,

$$\check{g}=\begin{pmatrix}\hat{g}^R & \hat{g}\\ \hat{0} & \hat{g}^A\end{pmatrix}. \quad (1)$$

It consists of retarded ($\hat{g}^R$), advanced ($\hat{g}^A$), and Keldysh ($\hat{g}$) Green's functions. Each of these matrices in turn is a 2×2 matrix in Nambu space composed of ordinary and anomalous Green functions,

$$\hat{g}^{(R,A)}=\begin{pmatrix}g^{(R,A)} & f^{(R,A)}\\ -f^{\dagger(R,A)} & \bar{g}^{(R,A)}\end{pmatrix}, \quad (2)$$

where the Keldysh matrix

$$\hat{g}=\hat{g}^R(f\hat{1}+f_z\hat{\tau}_z)-(f\hat{1}+f_z\hat{\tau}_z)\hat{g}^A \quad (3)$$

allows one to take into account the deviation of the quasiparticle distribution functions f and f_z from their equilibrium values,[21] and $\hat{\tau}_z$ is the Pauli matrix. Impurity averaging is taken into account in these equations from the very beginning. [Interference phenomena appearing due to impurity scattering[23] are not taken into account in these equations; it is true in the main approximation with respect to the small parameter $(lp_F)^{-1}$.] The tunneling of quasiparticles through the interface of an arbitrary transmittance is determined by the boundary conditions[22] which are significantly simplified[24] in the dirty limit when the impurity mean free path is very short: $l\ll\xi_N(T)$. The conductance of different superconducting systems with a barrier at the S-N interface was studied on the basis of this method in Refs. 6,14,15,19.

In particular, it was shown that the expression for the zero-temperature subgap differential conductance of a S-I-N contact whose barrier resistance exceeds the N-electrode resistance may be written in the form

$$\frac{G(V)}{G_n}=\mathrm{Im}[f_s^R(\epsilon)]\mathrm{Im}[f^R(\epsilon)]|_{\epsilon=eV}, \tag{4}$$

where $G(V)=dI/dV$, and $f_s^R(\epsilon)$ and $f^R(\epsilon)$ are retarded condensate Green's functions in the S and N electrodes (at the interface), respectively. Formally, this or a more general [see Eq. (21)] expression may be obtained from the component of the current known in the theory of the Josephson effect in S-I-S' junctions as the so-called "interference current." But in our case, the physics described by this term is different. First, the condensate function f^R in the N electrode differs from zero in S-I-N contacts due only to the proximity effect, and it may not be small at low energies (i.e., at low temperatures and voltages). Second, whereas a voltage in a S-I-S' junction leads to a time-dependent phase difference φ between superconducting electrodes (the voltage and φ are coupled by the Josephson relation $\partial\varphi/\partial t=2$ eV), the phase difference between the condensate functions in S and N electrodes in S-I-N contacts is time independent and may be zero in spite of the presence of the voltage.

Interesting phenomena, related to the wave nature of quasiparticles, occur in mesoscopic systems that include a normal conductor in contact with more than one superconductor. In the presence of a phase difference φ between the order parameters of the superconductors, controlled by a supercurrent or a magnetic field, the wave nature of quasiparticles manifests itself in oscillatory dependences on φ of the conductance of such mesoscopic systems. This oscillatory dependence of the conductance has been studied theoretically in Refs. 4,6 for some mesoscopic systems. Recently the phase-coherent conductance of a "hybrid" S/N quantum interferometer with two S-I-N contacts [shown in Fig. 1(a)] has been experimentally investigated.[7] We intend to study the conductance of this interferometer and another multicontact hybrid interferometer of a different type (see Fig. 1). Our results for the structure studied in Ref. 7 are obtained for weak and strong proximity effects. In the first limit our formulas are valid for a wider range of parameters than expressions (based on the theory developed in Ref. 4) presented in Ref. 7.

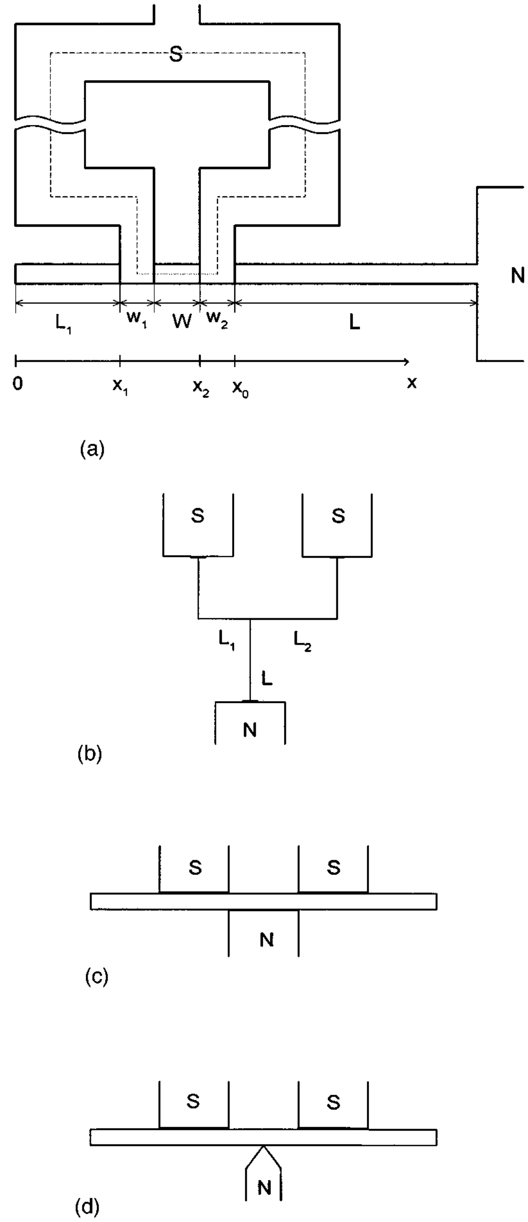

FIG. 1. Schematic diagram of the system under consideration. (a) Top view of the system studied experimentally in Ref. 7. The distribution functions are supposed to have the equilibrium form in superconductors S and in the wide part of the N layer. (b) A one-dimensional conductor N connecting two superconductors S and a normal metal N. (c) and (d) A thin N layer sandwiched between two superconductors S and a normal-metal N (or normal metal point contact). The thickened lines between S and N denote barriers at the S/N interfaces.

II. CONDUCTANCE OF A HYBRID SUPERCONDUCTING TWO-CONTACT INTERFEROMETER

Consider first an interferometer with two planar S-I-N contacts and the N film of a small thickness $d_N \ll \xi_N$. Note that a planar single S-I-N contact with a width $w \gg \xi_N$ was analyzed in Ref. 19. As is well known,[21] in the dirty limit the matrix Green's function $\check{g}(\epsilon,\mathbf{r},\mathbf{v}_F)$ may be presented as a sum of two terms $\check{g}(\epsilon,\mathbf{r},\mathbf{v}_F)=\check{g}(\epsilon,\mathbf{r})+\check{g}_1(\epsilon,\mathbf{r},\mathbf{v}_F)$, where $\check{g}(\epsilon,\mathbf{r})$ is the isotropic matrix which we are interested in and $\check{g}_1(\epsilon,\mathbf{r},\mathbf{v}_F)$ is a small matrix depending on the direction of $\mathbf{v}_F$; it is expressed through the matrix $\check{g}(\epsilon,\mathbf{r}) \equiv \check{g}:\check{g}_1=-l(\mathbf{v}_F/v_F)\check{g}\partial_{\mathbf{r}}\check{g}$. The equation for $\check{g}$ has the form[21] $(\hbar=1)$

$$iD\partial_{\mathbf{r}}(\check{g}\partial_{\mathbf{r}}\check{g})-[\epsilon\check{\tau}_z+(i/2)\gamma\check{\tau}_z\check{g}\check{\tau}_z,\check{g}]=\check{I}_{\mathrm{in}}. \tag{5}$$

Here

$$\check{\tau}_z=\begin{pmatrix}\hat{\tau}_z & \hat{0}\\ \hat{0} & \hat{\tau}_z\end{pmatrix},$$

and γ is determined by the sum of pair-breaking rates resulting from magnetic impurities and magnetic field. We suppose that the electron-phonon coupling constant $\lambda_{e\text{-ph}}$ in the N

film is small so that the order parameter Δ_N can be neglected: $\Delta_N \simeq 0$. The term on the right of Eq. (5) describes inelastic scattering. The magnitude of $\check{I}_{\rm in}$ is of the order $\check{g}/\tau_{\rm in}$, where $\tau_{\rm in}$ is the inelastic scattering time. We consider mesoscopic structures only; this means that the inequality

$$D/L^2 \ll 1/\tau_{\rm in} \tag{6}$$

should be fulfilled (L is the length of the system). We can therefore neglect $\check{I}_{\rm in}$ in comparison with the first term in Eq. (5). This equation must be supplemented by boundary conditions. Such conditions were obtained in Ref. 22 for a general case and were simplified in Ref. 24 for the dirty case. They have the form

$$(\check{g}\partial_y\check{g})(\mathbf{r}_b) = \kappa_j[\check{g}(\mathbf{r}_b),\check{g}_{sj}]. \tag{7}$$

Here $\kappa_j = D/2\sigma_N\tilde{R}_{bj}, \sigma_N$ is the specific conductivity of the N film, $\tilde{R}_{bj}$ is the S_j/N interface resistance per unit area, $\check{g}_{sj}$ is the Green's function of the superconductor at the jth interface, and the y axis is directed normally to the film plane. We assume that the thickness of the N film is small: $d_N \ll \sqrt{D/\epsilon_*}$, where $\epsilon_* = \max\{T,eV,\gamma\}$. We can therefore average Eq. (5) over the thickness d_N, taking into account Eq. (7) and regarding $\check{g}$ as a nearly y-independent function. Then we arrive at the equation for $\check{g}(\epsilon,x)$,

$$iD\partial_x(\check{g}\partial_x\check{g}) - [\epsilon\check{\tau}_z + (i/2)\gamma\check{\tau}_z\check{g}\check{\tau}_z,\check{g}] = \check{I}_b, \tag{8}$$

where $\check{I}_b = i[\gamma_{b1}\Theta_1(x)\check{g}_{s1} + \gamma_{b2}\Theta_2(x)\check{g}_{s2},\check{g}]$, the tunneling rates $\gamma_{bj} = D/2\sigma_N d_N\tilde{R}_{bj}$, and $\Theta_j(x)$ is a step function which is equal to 1 (0) for x inside (outside) the interval corresponding the S_j/N interface. In the equations for $\hat{g}^{R,A}$ following from Eq. (8) the components $\hat{I}_b^{R,A}$ describe the proximity effect, and in the equation for $\hat{g}$ the component $\hat{I}_b$ describes the tunneling of quasiparticles through the S_j/N interfaces. In addition, one should take into account that the matrix $\check{g}$ obeys the normalization condition[21]

$$\check{g}^2 = \check{1}. \tag{9}$$

Note that we included the vector potential of the magnetic field $\mathbf{A}$ in the definition of the phase of Green's functions $\varphi(\mathbf{r})$ which is determined by the relation $\partial_{\mathbf{r}}\varphi = \mathbf{p}_s = \partial_{\mathbf{r}}\chi - (2e/c)\mathbf{A}$, where $\mathbf{p}_s$ and χ are the momentum and the phase of superconducting condensate, respectively. Supposing that the widths of the tunnel junctions, w_j, are small with respect to the London penetration length λ, we will ignore the x dependence of phases at the interfaces in the superconductor; therefore at the jth junction $\varphi(x_j) \equiv \varphi_j$, where for the phase difference $\varphi = \varphi_1 - \varphi_2$ we have (see, e.g., Ref. 4)

$$\varphi = 2\pi\Phi/\Phi_0 + \int_{C_S}\mathbf{p}_s d\mathbf{r} = 2\pi\Phi/\Phi_0.$$

Here Φ is the flux penetrating the closed loop formed by the curves $C_S + C_N$ [shown by dashed and dotted lines in Fig. 1(a)], where C_S and C_N are the curves connecting the junctions 1 and 2 in the superconductor and the normal metal, respectively, and C_S is chosen to satisfy the requirement that $\mathbf{p}_s = \mathbf{0}$ along this curve, and $\Phi_0 = hc/2e$ is the flux quantum. The matrices at the jth interface in the superconductor $\check{g}_{sj}$ are expressed as follows:

$$\check{g}_{sj} = \check{S}_j\check{g}_s\check{S}_j^\dagger, \tag{10}$$

where $\check{S}_j = \cos(\varphi_j/2) + i\check{\tau}_z\sin(\varphi_j/2)$, and $\check{g}_s$ is the Green's function of a homogeneous superconductor, whose components have a form

$$\hat{g}_s^{R(A)} = g_s^{R(A)}\hat{\tau}_z + f_s^{R(A)}i\hat{\tau}_y, \quad \hat{g}_s = (\hat{g}_s^R - \hat{g}_s^A)\tanh(\epsilon/2T).$$

We neglect the influence of the N film on the superconductor, assuming that either the interface transmittance is small ($D_S/\sigma_S d_S\tilde{R}_{bj} \ll \Delta$; here the subscript S denotes the superconducting electrodes) or the cross-section area of the N electrode is much smaller than the cross section area of the superconductors [as it is shown in Fig. 1(b)]. In addition, we take into account the boundary conditions at $x=0$ and $x = x_0 + L$:

$$\partial_x\check{g}(0) = 0, \quad \check{g}(x_0+L) = \check{g}_{\rm eq}, \tag{11}$$

where $\check{g}_{\rm eq}$ is the equilibrium Green's function of the N electrode.

One can readily see from Eq. (8) that the density of the current in the x direction, $j(x)$, determined by the expression

$$j(x) = \frac{\sigma_N}{4}\int_0^\infty d\epsilon\ {\rm Tr}\hat{\tau}_z(\hat{g}^R\partial_x\hat{g} + \hat{g}\partial_x\hat{g}^A)(\epsilon,x), \tag{12}$$

varies from zero (at $x < x_1$) to an a x-independent value j at $x > x_0$. Therefore after the integration of Eq. (8) over the interval $x_1 < x < x_0$, the continuity equation for the current is found to be

$$I = \int_0^\infty d\epsilon[G_{b1}F_1(\epsilon) + G_{b2}F_2(\epsilon)]f_z(\epsilon), \tag{13}$$

where

$$F_j(\epsilon) = \left\langle\left[\nu\nu_s + \frac{1}{8}{\rm Tr}(\hat{f}^R + \hat{f}^A)(\hat{f}_{sj}^R + \hat{f}_{sj}^A)\right](\epsilon,x)\right\rangle_j. \tag{13$'$}$$

Here $\nu_{(s)} = {\rm Re}g_{(s)}^R$ is the normalized density of states, $\hat{f}_{sj}^a = if_s^a[\cos(\varphi_j)\hat{\tau}_y + \sin(\varphi_j)\hat{\tau}_x]$, $\hat{f}^a = f_y^a i\hat{\tau}_y + f_x^a\hat{\tau}_x$, $a = A,R$, $\langle(\cdots)\rangle_j$ denotes averaging over the width of the jth barrier, w_j, $G_{bj} = S_j/\tilde{R}_{bj}$, and S_j are the barrier conductance and area, respectively, and $f_z(\epsilon)$ is the distribution function in the interval $x_1 < x < x_0$, where its x dependence can be ignored if the length of this interval, $\tilde{W} = W + w_1 + w_2$, is not too large, i.e.,

$$\tilde{W} \ll (R_{bj}/\rho_N). \tag{14}$$

Note that the condition (14) means that the voltage drop across the length $\tilde{W}$ is negligible with respect to the voltage drop across the barrier. When obtaining Eq. (13), we also took into account the relations following from Eq. (8) ($a = A,R$),

$$G_{b1}\langle{\rm Tr}\hat{f}^a\hat{f}_{s1}^a\hat{\tau}_z\rangle_1 + G_{b2}\langle{\rm Tr}\hat{f}^a\hat{f}_{s2}^a\hat{\tau}_z\rangle_2 = 0,$$

which are due to the continuity of the supercurrent flowing between the two junctions.

The distribution function $f_z(\epsilon)$ in the N electrode may differ from the equilibrium one if the barrier resistances are comparable to, or less than, the resistance of the film. The distribution function f_z obeys a kinetic equation which may be easily obtained from the (12) element of Eq. (8) by multiplying it by $\hat{\tau}_z$ and calculating the trace. Generally one should find $f_z(\epsilon)$ from the kinetic equation at $x>x_0$

$$\partial_x(M_z\partial_x f_z)=0\,, \tag{15}$$

where $M_z=\mathrm{Tr}(\hat{1}\text{-}\hat{g}^R\hat{\tau}_z\hat{g}^A\hat{\tau}_z)/8$. One should solve this equation for f_z taking into account the boundary conditions, one of which is Eq. (13), and the second one corresponds to the point x_0+L, where the distribution function has the equilibrium form:

$$f_z(\epsilon,x_0+L)=[\tanh\beta(\epsilon+eV)-\tanh\beta(\epsilon-eV)]/2\equiv F_{\mathrm{eq}}(\epsilon), \tag{16}$$

where $\beta=1/2T$ and V is the voltage across the system. It follows from Eq. (15) that $M_z\partial_x f_z=J(\epsilon)$. Therefore taking into account Eqs. (13),(16), we obtain for the differential conductance $G=dI/dV$ the expression

$$G=\int_0^\infty d\epsilon\,\mathscr{G}(\epsilon)\partial_V F_{\mathrm{eq}}(\epsilon), \tag{17}$$

where

$$\mathscr{G}(\epsilon)=\frac{1}{[G_{b1}F_1(\epsilon)+G_{b2}F_2(\epsilon)]^{-1}+R_{NL}\langle 1/M_z(\epsilon,x)\rangle_L}.$$

Here $R_{NL}=\rho_N L$, ρ_N being the resistance per unit length of the film; here $\langle 1/M_z(\epsilon,x)\rangle_L$ denotes averaging over the interval $(x_0<x<x_2)$ of length L, which is supposed to be much larger than $\tilde{W}$. Note that in the limit of small w_j this expression coincides with that found in Ref. 6 for the system shown in Fig. 1(b).

A. Weak proximity effect

Let us turn to the case of barriers with relatively large resistances,

$$R_{bj}\gg\rho_N\{\tilde{W}+\min[L,\xi_N(\epsilon_\star)]\}, \tag{18}$$

where $\epsilon_\star=\max(eV,T)$. Under this condition the proximity effect appears to be weak for arbitrary energies (if $\gamma>D/L^2$) or in the range of interest, $\epsilon\sim\epsilon_\star$; i.e., the condensate functions in the N film, $\hat{f}^{R(A)}$, are small. Note that inequality (18) holds in a wider range of the barrier resistances than the inequalities

$$R_{bj}\gg\rho_N(L+\tilde{W}),\quad \gamma\gg\gamma_b\,,\;\; D/L^2, \tag{19}$$

which were satisfied in the experiment[7] (whose data for $\tilde{R}_{bj}$, σ_N, and d_N allow an estimation of the characteristic tunneling rate $\gamma_b\approx 10^8$ 1/s$\ll\gamma\approx 10^{10}$ 1/s). Under the conditions (19) the proximity effect is weak at any energies and, in addition, the resistance of the structure is determined by the barriers; i.e., the conductance in the normal state is equal to $G_n=G_{b1}+G_{b2}$.

The Green's function $\hat{f}^R$ is supposed to be small and can be found from the equation

$$[\partial_x^2-(k_\epsilon^R)^2]\hat{f}^R\equiv\Upsilon_x^R(\epsilon)\hat{f}^R=[b_1\hat{f}_{s1}^R\Theta_1(x)+b_2\hat{f}_{s2}^R\Theta_2(x)], \tag{20}$$

supplemented by the boundary conditions (11). Here $(k_\epsilon^R)^2=2(-i\epsilon+\gamma)/D$, $b_j=2\gamma_{bj}/D$. This matrix equation is the (11) element of the supermatrix in Eq. (8). The solution of Eq. (20) is readily found, and, as a result, we obtain from Eqs. (13)

$$\mathscr{G}(\epsilon)=\frac{1}{8}\,\mathrm{Tr}[\langle(\hat{f}^R+\hat{f}^A)(\hat{f}_{s1}^R+\hat{f}_{s1}^A)\rangle_1+\langle(\hat{f}^R+\hat{f}^A)(\hat{f}_{s2}^R+\hat{f}_{s2}^A)\rangle_2]=\mathscr{G}_1(\epsilon)+\mathscr{G}_2(\epsilon)+\mathscr{G}_{\mathrm{int}}(\epsilon)\cos\varphi, \tag{21}$$

where, for energies $\epsilon<\Delta$ (we allow for simplicity $w_j=w$; $k_\epsilon^R\equiv k_\epsilon$)

$$\mathscr{G}_j(\epsilon)=\frac{\rho_N G_{bj}^2}{2w^2}(\mathrm{Im}f_{s\epsilon}^R)^2\mathrm{Re}\frac{1}{k_\epsilon^3}\left[wk_\epsilon-\sinh(k_\epsilon w)+\frac{4\,\sinh^2(k_\epsilon w/2)}{\cosh(k_\epsilon\mathscr{L})}\sinh[k_\epsilon(\mathscr{L}-\tilde{L}_1)]\cosh(k_\epsilon\tilde{L}_1)\right],$$

$$\mathscr{G}_{\mathrm{int}}(\epsilon)=\frac{4\rho_N G_{b1}G_{b2}}{w^2}(\mathrm{Im}f_{s\epsilon}^R)^2\mathrm{Re}\frac{\sinh^2(k_\epsilon w/2)}{k_\epsilon^3\cosh(k_\epsilon\mathscr{L})}\cosh(k_\epsilon\tilde{L}_1)\sinh[k_\epsilon(\mathscr{L}-\tilde{L}_2)], \tag{22}$$

where $\mathscr{L}$ is the length of the film, $\tilde{L}_j=L_j+w/2$, and L_j is the distance between the left edge of the film and the left edge of the jth barrier. Note that for the subgap region $\epsilon\ll\Delta$, which we are interested in, $(\mathrm{Im}f_{s\epsilon}^R)^2=1$. Equations (21) and (22) describe the conductance of the structure [Fig. 1(a)] with high barrier resistances. Such a system has been studied experimentally.[7] In particular, the temperature dependence of the conductance (see Fig. 2), computed from Eq. (22) for the geometrical parameters close to the experimental ones, is similar to the experimental data.

Note that the solution of Eq. (20) may be written with the use of the Green's function of the differential equation $\Upsilon_x^R(\epsilon)P_\epsilon^R(x,x')=\delta(x-x')$ with boundary conditions $\partial_x P_\epsilon^R(x,x')|_{x=0}=0$, $P_\epsilon^R(x_2,x')=0$. Then expressions for $\epsilon\ll\Delta$ can be written in the form

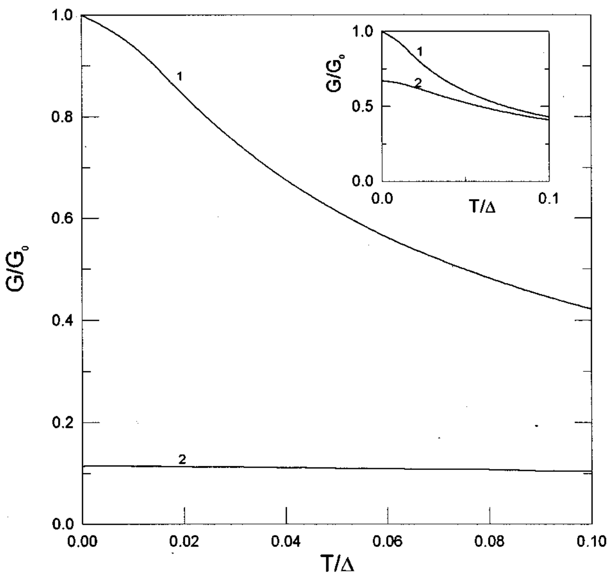

FIG. 2. Temperature dependence of the zero-bias normalized conductance G/G_0 (G_0 corresponds to zero temperature) of the system depicted on Fig. 1(a) for $\Phi=k\Phi_0$ (curves 1) and $\Phi=(k+1/2)\Phi_0$ (curves 2); the curves are computed for $L_1=0.5\xi_N(\Delta)$, $w_1=w_2=\xi_N(\Delta)$, $W=0.3\xi_N(\Delta)$, $L=20\xi_N(\Delta)$, and $\gamma/\Delta=0.05$; the inset shows the curves for $W=5\xi_N(\Delta)$ and the same values of other parameters.

$$\mathcal{G}_j(\epsilon)=\rho_N G_{bj}^2 \mathrm{Re}\langle\langle P_\epsilon^R(x,x')\rangle\rangle_{jj},$$

$$\mathcal{G}_{\rm int}(\epsilon)=2\rho_N G_{b1}G_{b2}\mathrm{Re}\langle\langle P_\epsilon^R(x,x')\rangle\rangle_{12}. \qquad (23)$$

Thus, one can see that, being written in terms of the Green's function $P_\epsilon^R(x,x')$, our expression for the subgap conductance following from Eqs. (21) and (17) coincides with that obtained in the Ref. 4 if a correction to the coefficient in formulas (5) of Ref. 4(a) is taken into account. Note that, in contrast to the more general expression (17), Eqs. (21), (23) are valid only if the barrier transmittances are small and the proximity effect is weak.

Equation (22) is considerably simplified in the case where the lengths of both (right and left) segments of the film outside the barrier regions are large: $L_1, L-L_2-w \gg \xi_N(\epsilon)=\sqrt{D/(\epsilon^2+\gamma^2)^{1/2}}$. Then we get

$$\mathcal{G}_j(\epsilon)=\frac{\rho_N G_{bj}^2}{2w^2}\mathrm{Re}\frac{1}{k_\epsilon^3}[wk_\epsilon-\sinh(k_\epsilon w)+2\sinh^2(k_\epsilon w/2)],$$

$$\mathcal{G}_{\rm int}(\epsilon)=\frac{2\rho_N G_{b1}G_{b2}}{w^2}\mathrm{Re}\frac{\sinh^2(k_\epsilon w/2)}{k_\epsilon^3}\exp[-k_\epsilon(W+w)]. \qquad (24)$$

In the limit of small width, $w\ll\xi_N(\epsilon)$, i.e., ϵ, $\gamma\ll D/w^2$, Δ, the result becomes independent of w (except for the coefficients containing G_{jb}):

$$\mathcal{G}_j(\epsilon)=\frac{1}{2}\rho_N G_{bj}^2\mathrm{Re}\frac{\sinh[k_\epsilon(\mathcal{L}-L_j)]\cosh(k_\epsilon L_j)}{k_\epsilon\cosh(k_\epsilon\mathcal{L})},$$

$$\mathcal{G}_{\rm int}(\epsilon)=\rho_N G_{b1}G_{b2}\mathrm{Re}\frac{\cosh(k_\epsilon L_1)\sinh[k_\epsilon(\mathcal{L}-L_2)]}{k_\epsilon\cosh(k_\epsilon\mathcal{L})}.$$

For the system shown in Fig. 1(b) we find, analogously to Ref. 6, the following expression for the functions $\mathcal{G}_j(\epsilon)$ and $\mathcal{G}_{\rm int}(\epsilon)$ [they determine the conductance via Eq. (11)] in the limit $R_{bj}\gg\rho_N\{L_j+\min[L,\xi_N(\epsilon_\star)]\}$:

$$\mathcal{G}_j(\epsilon)=G_{bj}^2\rho_N g_{jj}(\epsilon), \quad \mathcal{G}_{\rm int}(\epsilon)=G_{b1}G_{b2}\rho_N g_{12}(\epsilon), \qquad (25)$$

where

$$g_{ij}(\epsilon)=\mathrm{Re}\frac{1}{2k_\epsilon}\left[\tanh(k_\epsilon L_j)\delta_{ij}+\frac{2-\delta_{ij}}{[\tanh(k_\epsilon L_1)+\tanh(k_\epsilon L_2)+\coth(k_\epsilon L)]\cosh(k_\epsilon L_i)\cosh(k_\epsilon L_j)}\right]. \qquad (25')$$

Note that in the symmetrical case ($G_{b1}=G_{b2}, L_1=L_2$), Eq. (25) is reduced to that found in Ref. 6. In the case of small $L_j, w, W\ll\xi_N(\epsilon_\star)L$ [where $\epsilon_\star=\max(eV,T)$], Eqs. (22), (25) result in the following expression for the conductance of both systems shown in Figs. 1(a) and 1(b):

$$G(V,\varphi)=R(V)(G_{b1}^2+G_{b2}^2+2G_{b1}G_{b2}\cos\varphi)$$

$$=R(V)(G_{b1}+G_{b2})^2[1-r_b\sin^2(\varphi/2)], \qquad (26)$$

where $r_b=4G_{b1}G_{b2}/(G_{b1}+G_{b2})^2$, and

$$R(V)=\frac{\rho_N}{2}\mathrm{Re}\int_0^\infty\frac{\tanh(k_\epsilon L)}{k_\epsilon}[\partial_V F_{\rm eq}(\epsilon)]d\epsilon$$

$$=\frac{\rho_N}{2}\begin{cases} L, & eV,\gamma,T\ll D/L^2,\\ c\,\xi_N(T), & eV, D/L^2\ll\min(\gamma,T),\\ \frac{1}{2}\xi_N(eV), & D/L^2,T,\gamma\ll eV.\end{cases}$$

In particular $c=(1-2\sqrt{2})\sqrt{\pi}\zeta(-1/2)\simeq 0.33$ at $\gamma\ll T$.

Thus in the considered limit, $\max G(\varphi)$ and $\min G(\varphi)$ correspond to $\varphi=2\pi k(\Phi/\Phi_0=k)$ and $\varphi=\pi(2k+1)[\Phi/\Phi_0=(k+1/2)]$, $k=0,1,\ldots$, respectively, and the normalized amplitude of the conductance oscillations,

$$\frac{\max G(\varphi)-\min G(\varphi)}{G(0)}=r,$$

is of the order of unity at $G_{1b}\sim G_{2b}$; i.e., the conductance oscillations are more pronounced in the case of the asymmetry of *S-I-N* barriers being small.

We make note of an interesting feature. If the length of the *N*-conductor between two barriers is comparable to or larger than $\xi_N(\epsilon_\star)$, then $\mathcal{G}_{\rm int}(\epsilon)$may change its sign with increasing energy, and as a result, the shape of the conductance versus φ curve may radically change with increasing voltage. In contrast to the case considered above, $\max G(\varphi)$

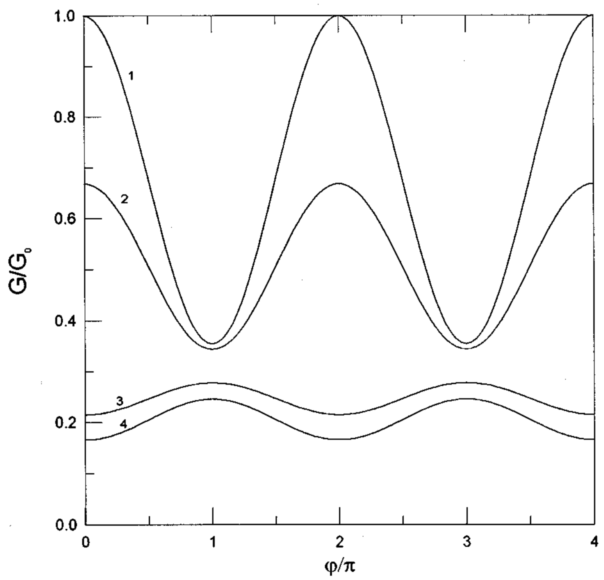

FIG. 3. Normalized zero-temperature conductance G/G_0 (here G_0 corresponds to a zero voltage) of the system depicted in Fig. 1(a) [for $w_1=w_2=0.1\xi_N(\Delta)$, $W=3\xi_N(\Delta)$, $L_1=0.05\xi_N(\Delta)$, $L=20\xi_N(\Delta)$, and $\gamma/\Delta=0.05$] versus phase difference determined by the magnetic flux, $\varphi/\pi=2\Phi/\Phi_0$, at different voltages $eV/\Delta=0$ (1), 0.05 (2), 0.2 (3), 0.3 (4).

and $\min G(\varphi)$ may correspond, respectively, to $\varphi=\pi(2k+1)$ and $\varphi=2\pi k$ (see Figs. 3 and 4). The interference part of the conductance will decrease more rapidly [at W or $L_{1,2}\sim\xi_N(\epsilon_\star)$] than the phase-independent part and may be an oscillating function of voltage. This is easily seen from Eqs. (22), (25) if one takes into account that $k_\epsilon=(1-i)/\xi_N(\epsilon)$ at $\epsilon\gg\gamma$. In particular, it follows from Eq.

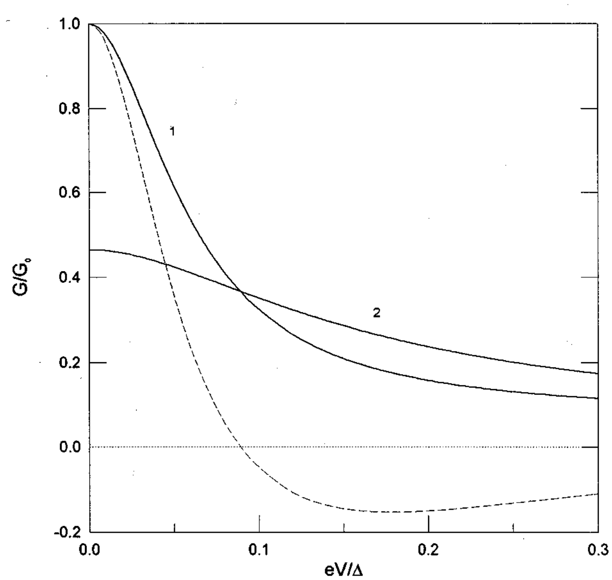

FIG. 4. Voltage dependence of the zero-temperature normalized conductance for the system depicted in Fig. 1(a) [for $w_1=w_2=\xi_N(\Delta)$, $W=3\xi_N(\Delta)$, $L_1=0.05\xi_N(\Delta)$, $L=20\xi_N(\Delta)$, and $\gamma/\Delta=0.05$] corresponding to $\Phi=k\Phi_0(1)$ and $\Phi=(k+1/2)\Phi_0(2)$; the dashed line is the voltage dependence of the normalized coefficient $G_{\rm int}(V)/G_{\rm int}(0)$ determining the interference part of the conductance $G(V)=G_1(V)+G_2(V)+G_{\rm int}(V)\cos\varphi$.

(24) that the zero-temperature conductance of the system depicted in Fig. 1(a) with identical high-resistance barriers is given by the expression

$$\frac{G(V,\varphi)}{G_n}=\frac{1}{16\sqrt{2}}\rho_N\xi_N(eV)G_n\left[3\sqrt{2}+\exp(-2q_V)\right.$$
$$\times\cos\left(2q_V+\frac{\pi}{4}\right)+2\,\exp(-q_V)$$
$$\left.\times\cos\left(q_V+\frac{\pi}{4}\right)\cos\varphi\right],$$

where $q_V=W/\xi_N(eV)$, and L_1 and w are assumed to be less than $\xi_N(eV)\ll\mathscr{L}$.

It is noteworthy that at large L $[L\gg\xi_N(\epsilon_\star)]$ the subgap conductance of the systems in the superconducting state becomes independent of length. Thus, under the condition (18), the resistance of the structure (below T_c) is determined by the barriers even if the total normal-state resistance of the film, or the wires, R_{NL}, is comparable with $R_b=(G_{b1}+G_{b2})^{-1}$. This is the case if the length $L\ll L_{\rm in}$. Otherwise R_{NL} may be ignored only if it is smaller than R_b. Under the latter condition the results of this section are valid at arbitrary ratios $L/L_{\rm in}$.

B. Strong proximity effect

We consider now the case when the resistances of the barriers are comparable with the resistance of the N conductor, R_{NL}. Then the Green's functions $\hat{f}^{R,A}$ are not small in the vicinity of the barriers, and therefore the results based on the solution of the linear equation (20) [in particular Eqs. (22), (25)] are not valid. In what follows we confine ourselves to the case of a sufficiently small length of the N conductor segment between the barriers $(\tilde{W}\ll\xi_N(\epsilon_\star)$, R_{bj}/ρ_N [Fig. 1(a)] and $L_j\ll\xi_N(\epsilon_\star)$, R_{bj}/ρ_N [Fig. 1(b)]) when interference effects are most strong.

Let us take into account that the retarded Green's function may be written in the form $\hat{g}^R=\cosh u^R\hat{\tau}_z+\sinh u^R i\hat{\tau}_y e^{i\chi\hat{\tau}_z}$, where the phase χ is independent of x at $x>x_0$ and $x<x_1$ [see Eq. (8)]. Therefore, χ may be put equal to zero, and u^R obeys the equation

$$D\partial_x^2u^R+2(i\epsilon-\gamma\cosh u^R)\sinh u^R=0\,, \qquad (27)$$

whose solution is determined also by the boundary conditions

$$\tilde{\kappa}(R_b/\rho_N)(\partial_xu^R)(x_0)=g_s^R\sinh u_0^R$$
$$-f_s^Rc_\varphi\cosh u_0^R,\quad u^R(x_2)=0\,, \qquad (28)$$

where $c_\varphi=[1-r_b\sin^2(\varphi/2)]^{1/2}$, and $R_b=1/(G_{1b}+G_{2b})$, $u^R(x_0)\equiv u_0^R$. For the systems in Figs. 1(a) and 1(b) at $L_j\ll\xi_N(\epsilon)$, we obtain $\tilde{\kappa}=1/2$; it can be shown that in the limit $L_1\gg\xi_N(\epsilon)\gg\tilde{W}$ for the system in Fig. 1(a), $\tilde{\kappa}=1$.

Consider first the case of small ϵ and γ: $\epsilon,\gamma\ll\epsilon_L=D/L^2$. Then the solution of Eq. (27) may be written in the form $u^R=ax+b$. Therefore carrying out calcula-

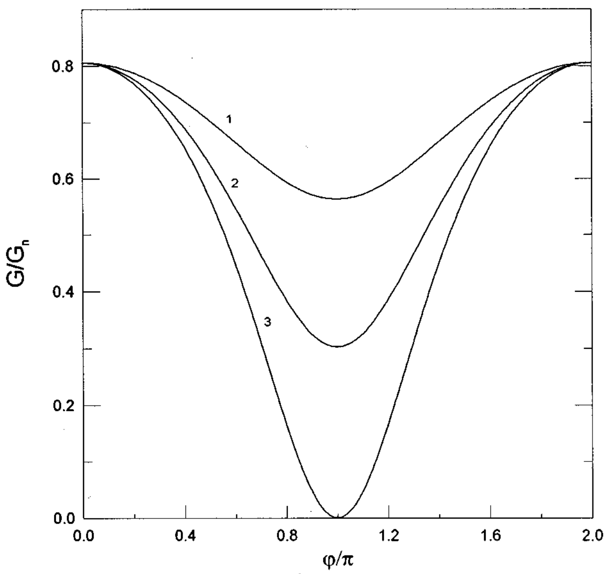

FIG. 5. Normalized zero-bias conductance G/G_n at $T=0$ versus phase difference $\varphi=2\pi\Phi/\Phi_0$ for systems shown in Figs. 1(a) and 1(b) in the case when the proximity effect is strong [the resistance of the N conductor R_{NL} is large, i.e., comparable with $R_b=1/(G_{b1}+G_{b2})$]; the curves are computed for the case $R_{NL}=R_b$ at different ratios G_{b1}/G_{b2}, i.e., at different values of the parameter $r_b=4G_{b1}G_{b2}/(G_{b1}+G_{b2})^2$: $1-r_b=1$ ($G_{b1}/G_{b2}=1$); $2-r_b=0.8$ ($G_{b1}/G_{b2}\simeq 2.9$ or 1/2.9); $3-r_b=0.5$ ($G_{b1}/G_{b2}\simeq 5.8$ or 1/5.8).

tions similar to those in Refs. 15,6, we obtain from Eqs. (27), (28), (17) the parametric expression

$$\frac{\mathcal{G}(\epsilon,\varphi)}{G_n}\equiv \tilde{G}_0(\varphi)=\frac{(1+r)\alpha_\varphi}{\alpha_\varphi+\cot\alpha_\varphi},\quad c_\varphi\cos\alpha_\varphi=r\alpha_\varphi,\qquad \epsilon,\gamma\ll\epsilon_L,\tag{29}$$

where $r=R_b/R_{NL}$. In the symmetrical case [$G_{b1}=G_{b2}$, $c_\varphi=\cos(\varphi/2)$] this is reduced to the formula found in Ref. 6. Note that the function $\tilde{G}_0(\varphi)$, which does not depend on T, is equal to the zero-bias normalized conductance at low temperatures: $T\ll\epsilon_L$. This is plotted for different r in Fig. 5. Thus, as was noted in Ref. 6, for the case when the resistances of the barriers and the N conductor are comparable with each other and the length of the N conductor segment between the barriers is comparable with or less than $\xi_N(\epsilon_\star)$, the amplitude of the conductance oscillations may be large, i.e., of order of G_n.

Consider now the case of energies $\epsilon\gg\epsilon_L,\gamma$. Then the solution of Eq. (27) at $x>x_0$ is

$$\tanh\frac{u^R}{4}=\tanh\left(\frac{u_0^R}{4}\right)\exp[-k_\epsilon(x-x_0)].\tag{30}$$

Therefore from Eq. (28) we obtain at $\epsilon\ll\Delta$

$$\sinh(u_0^R/2)\equiv -z_\epsilon=(\sqrt{\beta_\epsilon^2+2c_\varphi^2}-\beta_\epsilon)i/2c_\varphi,\tag{31}$$

where $\beta_\epsilon=\tilde{\kappa}rk_\epsilon$. With the use of Eq. (17) we find

$$\frac{\mathcal{G}(\epsilon,\varphi)}{G_n}=(1+r)\left[1+\frac{r}{2c_\varphi\mathrm{Im}(z_\epsilon\sqrt{1+z_\epsilon^2})}-\sqrt{\frac{\epsilon_L}{2\epsilon}}\int_0^\infty\frac{q(z_\epsilon,y)dy}{[1+q(z_\epsilon,y)]^2}\right]^{-1},\tag{32}$$

where $q(z,y)=4e^{-2y}[\mathrm{Re}\sqrt{1+z^2}e^{-iy}/z]^2[|1+z^2|e^{-2y}/|z|^2+1]^{-2}$. At energies $(\epsilon/\epsilon_L)\gg 1$, $(c_\varphi/r)^2$ only the first two terms in the square brackets of Eq. (32) are significant and it is reduced in the main approximation to the simple form

$$\frac{\mathcal{G}(\epsilon,\varphi)}{G_n}=\frac{(1+r)c_\varphi^2}{c_\varphi^2+2r^2\tilde{\kappa}\sqrt{\epsilon/\epsilon_L}}\equiv\frac{(1+r)c_\varphi^2}{c_\varphi^2+\sqrt{\epsilon/\tilde{\epsilon}}},\tag{33}$$

where $\tilde{\epsilon}=\epsilon_L/4r^4\tilde{\kappa}^2$. Thus, if $R_{NL}\gg R_b$ $(r\ll 1)$, even at not too small temperatures and voltages $(\epsilon_\star\gg\epsilon_L)$ the amplitude of the conductance oscillations may be comparable with G_n. Note that for $\epsilon_\star\gg\tilde{\epsilon}c_\varphi^4$, we obtain from Eq. (33) an expression that coincides with Eq. (26), in the limit of infinite L. In this case the conductance, which does not depend on L, is small with respect to G_n.

Consider now the case when the pair-breaking rate is relatively high: $\epsilon_L\ll\gamma\ll D/\tilde{W}^2$. Then at low energies $(\epsilon\ll\gamma)$, one can readily find the solution of Eq. (27) and obtain from Eqs. (28), (17) the expression

$$\frac{\mathcal{G}(\epsilon,\varphi)}{G_n}=\frac{(1+r)c_\varphi^2}{c_\varphi^2+r\sqrt{\tilde{r}^2+c_\varphi^2}}\equiv\tilde{G}_0(\varphi),\quad \epsilon,\epsilon_L\ll\gamma,$$

where $\tilde{r}=2\tilde{\kappa}R_b/\rho_N\sqrt{D/\gamma}\gg r$. Thus, like the case considered above [see Eq. (29)], the zero-bias conductance is temperature independent at sufficiently low temperatures $(T\ll\gamma)$ and the amplitude of the conductance oscillations may be comparable with G_n (if the parameter $\tilde{r}$ is not large).

III. CONDUCTANCE OF A THREE-CONTACT INTERFEROMETER

The amplitude of the conductance oscillations can also be large in the case when there is a third barrier (with a resistance R_{b3}) between the N electrode and the N conductor. For the system in Fig. 1(b) this case was analyzed in Ref. 6. Consider one of the possible cases when the resistance of the system is determined by the barriers so that the resistance in the normal state $R_n=[(G_{b1}+G_{b2})^{-1}+R_{b3}]$. Then, for the case $\mathcal{L}\ll\xi_N(\epsilon_\star)$ [$\mathcal{L}$ being the total length of the wire $\mathcal{L}=L_1+L_2+L$, or the film; see Fig. 1(b) or 1(c)], i.e., $\epsilon_\star\ll D/\mathcal{L}^2$, the spatial variation of the matrix Green's function in the N conductor is small, and therefore $\check{g}$ can be found from the equation

$$[\epsilon\check{\tau}_z+i\gamma_{b1}\check{g}_{s1}+i\gamma_{b2}\check{g}_{s2}+i\gamma_{b3}\check{g}_N+(i/2)\gamma\check{\tau}_z\check{g}\check{\tau}_z,\check{g}]=\check{I}_{\rm in},\tag{34}$$

which is obtained from Eq. (5). Here $\gamma_{bj}=\mathcal{D}_{bj}v_F/4\mathcal{L}$ for a system with three barriers shown in Fig. 1(b) and $\gamma_{bj}=\mathcal{D}_{bj}v_Fw_j/4d_N\mathcal{L}$ for the system in Fig. 1(c); $\mathcal{D}_{bj}$ is the averaged (over momentum direction) transmittance of the barriers which is related to their resistances $\tilde{R}_{bj}$.[24,14,15] Suppose that the tunneling rates exceed the pair-breaking and energy relaxation rates, $\gamma_{bj}\gg\gamma_{(\rm in)}$; i.e., the strong proximity effect can occur for the energies $\epsilon \leqslant \max(\gamma_{b1},\gamma_{b2})$. Then with the use of Eq. (9) we find the solution of Eq. (34) and obtain from Eq. (17)

$$\frac{\mathcal{G}(\epsilon,\varphi)}{G_n}=U\left(c_\varphi^2,\frac{(\gamma_{b3}-i\epsilon)}{(\gamma_{b1}+\gamma_{b2})}\right),\quad \epsilon,\gamma_{bj}\ll\Delta, \qquad (35)$$

where $U(y,z)=(1+\tilde{r}_b)y[1/\mathrm{Re}(1/\sqrt{y+z^2})+y\tilde{r}_b/\mathrm{Re}(z/\sqrt{y+z^2})]^{-1}$, and $\tilde{r}_b=R_{b3}(G_{b1}+G_{b2})=(\gamma_{b1}+\gamma_{b2})/\gamma_{b3}$. For symmetrical barriers ($G_{b1}=G_{b2}$), Eq. (35) is reduced to the expression obtained in Ref. 6. It follows from Eq. (35) that in the case when the resistance of the structure R_n is determined by the third barrier, i.e., $R_n \simeq R_{b3} \gg R_{b1},R_{b2}$ ($\tilde{r}_b \gg 1$),

$$\frac{\mathcal{G}(\epsilon,\varphi)}{G_n}=\mathrm{Re}\frac{(\epsilon+i\gamma_{b3})}{\sqrt{(\epsilon+i\gamma_{b3})^2-\epsilon_g^2(\varphi)}}, \qquad (36)$$

where $\epsilon_g(\varphi)=(\gamma_{b1}+\gamma_{b2})c_\varphi$ is supposed to be large as compared with γ_{b3} ($c_\varphi \gg 1/\tilde{r}_b$). Thus in this case the conductance is determined by the density of states in the N film (wire) for which $\epsilon_g(\varphi)$ may be called an energy gap; note that for the asymmetry of S-I-N barriers determined by the inequality $|G_{b1}-G_{b2}| \gg G_{b3}$ (where $G_{b3} \ll G_{b1},G_{b2}$), Eq. (36) is valid at arbitrary φ. In the opposite limit $R_{b3} \ll R_{b1},R_{b2}$ a pair-breaking rate due to the tunneling of electrons into the N electrode is large: $\gamma_{b3} \gg \gamma_{b1}+\gamma_{b2}$; therefore we obtain from Eq. (35)

$$\frac{\mathcal{G}(\epsilon,\varphi)}{G_n}=\frac{\gamma_{b3}(\gamma_{b1}+\gamma_{b2})c_\varphi^2}{\gamma_{b3}^2+\epsilon^2}; \qquad (37)$$

i.e., like the case considered in the previous section, the phase dependence of the conductance is determined for strong pair breaking by a $\cos\varphi$ term, and the conductance decreases with increasing voltage or temperature.

At low temperatures $T \ll \gamma_{bj}$ one can obtain from Eq. (35) the following simple expression for the zero-bias normalized conductance:

$$\tilde{G}_0(\varphi)=\frac{(1+\tilde{r}_b)\tilde{r}_b c_\varphi^2}{(1+\tilde{r}_b^2 c_\varphi^2)^{3/2}}=\begin{cases}1/\tilde{r}_b c_\varphi, & \tilde{r}_b \gg 1/c_\varphi,\\ \tilde{r}_b c_\varphi^2, & \tilde{r}_b \ll 1.\end{cases} \qquad (38)$$

Plots of the conductance found from Eqs. (38), (17) are shown in Figs. 6 and 7. One can see that the minima of the conductance may occur at both $\varphi/2\pi=\Phi/\Phi_0=k$ and $\varphi/2\pi=(k+1/2)$ ($k=0,1,\ldots$) depending on the particular value of the parameter $\tilde{r}_b$. Also we see that for nonidentical resistances R_{b1} and R_{b2} the maximum of the conductance may correspond to $\varphi/2\pi=(k+1/2)$ even at zero bias, unlike the case considered in the previous section, where this can occur only at nonzero voltages.

Consider now the system [Fig. 1(d)] with a small point contact (without a barrier) between the N electrode and the N film whose characteristic size a is small, $a \ll d_N \ll \xi_N$. A similar system with only one superconducting electrode, where interference effects do not arise, was analyzed in Ref. 25. Let us assume that the resistance of the point contact, R_c, is large, $R_c \gg R_b=1/(G_{b1}+G_{b2})$. Then one can neglect both the voltage drop across the barriers with respect to V and the influence of the N electrode on the retarded function

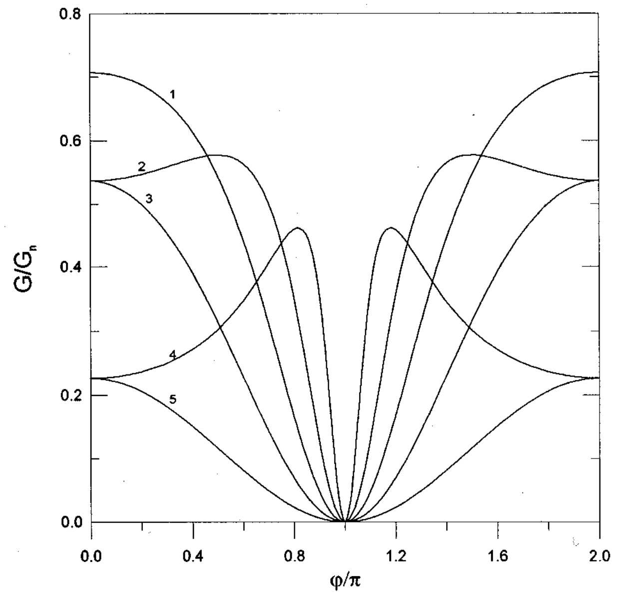

FIG. 6. Normalized zero-bias conductance G/G_n at $T=0$ versus phase difference $\varphi=2\pi\Phi/\Phi_0$ for systems with three barriers [in Figs. 1(b) and 1(c)] whose resistances dominate. The curves are computed for different values of the parameter $\tilde{r}_b=R_{b3}(G_{b1}+G_{b2})$ for the case $G_{b1}=G_{b2}$; $\tilde{r}_b=1$ (1), 2 (2), 0.5 (3), 4 (5), 0.2 (5).

$\hat{g}^R$ in the N film. Therefore, using the results of Refs. 26 and 27, where the conductance of point contacts S-c-N (c denotes a constriction) has been calculated for superconducting point contacts with different relations between their size and the mean free path, we obtain

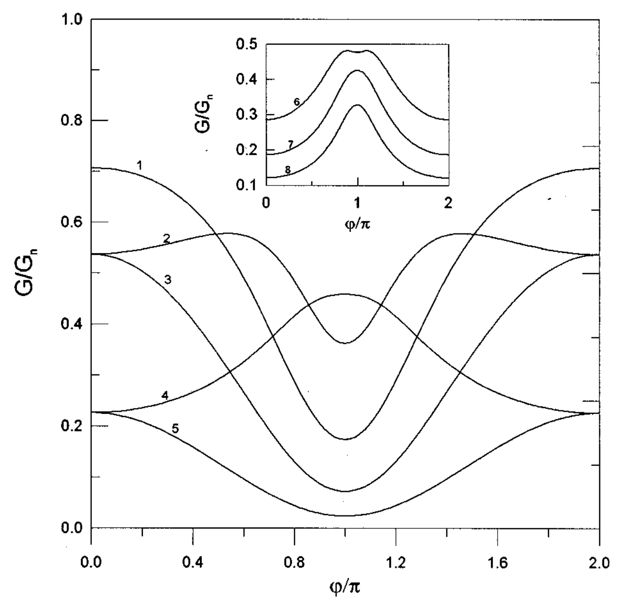

FIG. 7. Normalized zero-bias conductance G/G_n at $T=0$ versus the phase difference $\varphi=2\pi\Phi/\Phi_0$ for systems with three barriers [in Figs. 1(b) and 1(c)] whose resistances dominate. The curves are computed for different values of $\tilde{r}_b=R_{b3}(G_{b1}+G_{b2})$ and for $r_b=4G_{b1}G_{b2}/(G_{b1}+G_{b2})^2=0.9$ ($G_{b1}/G_{b2}=1.9$ or 1/1.9) $\tilde{r}_b=1$ (1), 2 (2), 0.5 (3), 4 (4), 0.2 (5); the insets show the curves (with maximum corresponding to $\varphi/2\pi=\Phi/\Phi_0=k+1/2$) computed for $\tilde{r}_b=4$ (6), 6 (7), 9 (8).

$$\frac{\mathscr{G}(\epsilon,\varphi)}{G_n}=\begin{cases}(a)\quad \mathrm{Re}(a\ \sinh f^R_\epsilon)\mathrm{Re}g^R_\epsilon/\ \mathrm{Re}f^R_\epsilon, & l\ll a,\\ (b)\quad 1+|f^R_\epsilon|^2/|1+g^R_\epsilon|^2, & a\ll l,\end{cases} \tag{39}$$

where g^R_ϵ, f^R_ϵ are the Green's functions at the point contact position. Note that the cases (a) and (b) correspond to diffusive and ballistic passage of electrons through the contact region, respectively. If the length of the film is small, $\mathscr{L}\ll\xi_N(\epsilon_\star)$, then for energies $\epsilon\sim\epsilon_\star$ we obtain the equation for $\hat{g}^R$ putting $\gamma_{b3}=0$ in Eq. (35). Therefore, taking into account Eq. (9) we find $g^R_\epsilon=\epsilon^R/\xi^R_\epsilon$, $f^R_\epsilon=\Delta^R_\epsilon/\xi^R_\epsilon$, where $(\gamma_{b1}+\gamma_{b2}\equiv\gamma_b)$

$$\epsilon^R=\epsilon+i\gamma_b g^R_{s\epsilon}+i\gamma g^R_\epsilon,\quad \Delta^R_\epsilon=if^R_{s\epsilon}\gamma_b c_\varphi,$$

$$\xi^R_\epsilon=\sqrt{(\epsilon^R)^2-(\Delta^R_\epsilon)^2}. \tag{40}$$

If the pair-breaking rate is small, $\gamma\ll\gamma_b$, then a phase-dependent energy gap $\epsilon_g<\Delta$ is induced in the N film which is determined by the equation

$$\epsilon_g=\frac{\Delta\gamma_b c_\varphi}{\sqrt{\Delta^2-\epsilon_g^2}+\gamma_b}\rightarrow c_\varphi\gamma_b,\quad \gamma_b\ll\Delta.$$

In particular, at $\epsilon\ll\Delta$ and $\gamma\ll\gamma_b\ll\Delta$ we obtain from Eq. (39)

$$\frac{\mathscr{G}(\epsilon,\varphi)}{G_n}=\tilde{G}\left(\frac{\epsilon}{c_\varphi\gamma_b}\right), \tag{41}$$

where

$$\tilde{G}(x)=\begin{cases}(a)\quad \frac{1}{2}[x^{-1}\theta(1-x)+x\theta(x-1)]\ln\left|\frac{1+x}{1-x}\right|,\\ (b)\quad 2\theta(1-x)+\theta(x-1)[1+(x+\sqrt{x^2-1})^{-2}].\end{cases}$$

As follows from Eq. (41), the conductance of a system with a large point-contact resistance may exceed G_n (in contrast to the previous cases), and its phase dependence may exhibit both one [case (a)] and two (b) sequences of minima corresponding to $\varphi/2\pi=k$ and $\varphi/2\pi=k+1/2$, respectively (see Fig. 8).

Thus, one can use the third tunnel *N-I-N* or point *N-c-N* contacts (with higher resistances than the ones of *S-I-N* junctions) which prevent the N film from the pair-breaking influence of the N electrode. Then at sufficiently small thickness of the N film or at relatively transparent *S-I-N* barriers $(\gamma_b>\gamma)$ the strong proximity effect can be realized and the amplitude of the conductance oscillations may become large, i.e., comparable with the conductance of the structure in the normal state.

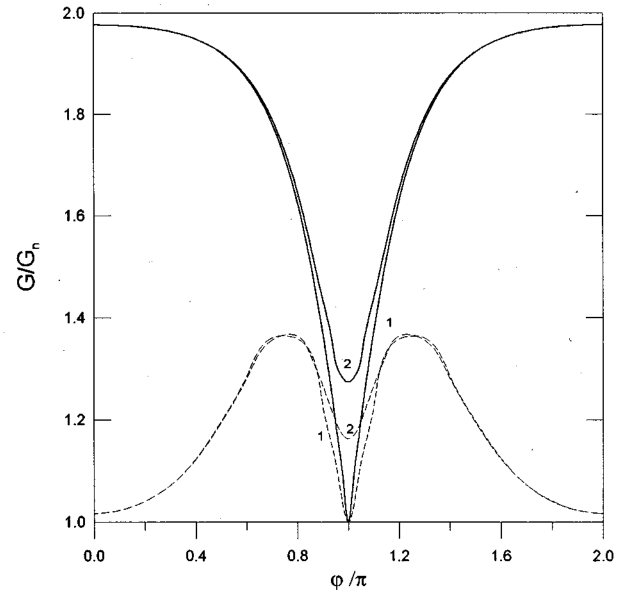

FIG. 8. Normalized zero-bias conductance G/G_n at low temperature $T=0.025\Delta$ versus phase difference φ for systems [Fig. 1(d)] with a small point contact between the N film and the N electrode, whose resistance R_c is large as compared with $R_b=1/(G_{b1}+G_{b2})$. The curves are computed for different ratios $G_{b1}/G_{b2}=1$ (curves 1) and 0.8 or 1.25 (curves 2) for the case of ballistic (solid line) and diffusive (dashed line) passage of electrons through the contact; the sum of tunneling rates $\gamma_{b1}+\gamma_{b2}$ is put equal to 0.1Δ.

IV. CONCLUSIONS

We have calculated the conductance of a mesoscopic interferometer including two *S-I-N* contacts for different configurations with, and without, the third barrier between the N film (or the wire) and the bulk N electrode (see Fig. 1). For the geometry depicted in Fig. 1(a), it was essential to assume that the barrier transmittances are sufficiently small (i.e., the barrier sheet resistances are high: $\tilde{R}_{bj}\gg D_S/\sigma_S d_S\Delta$). In this case the order parameter in the S electrodes is not disturbed. But even in the case of small transmittances, the proximity effect (i.e., the penetration of the condensate into the N region) may be weak or strong. The first limiting case is realized if the resistances are large: $R_{bj}\gg\rho_N\min\{L,\xi_N(\epsilon_\star)\}$, where $\epsilon_\star=\max\{eV,T\}$ is a scale of energy of quasiparticles. Then the condensate Green's function f^Rsatisfies the linear equation (20) and can easily be found for different geometries. Precisely this case was analyzed by Hekking and Nazarov[4] and studied experimentally in Ref. 7, where the theoretical analysis, based on the theory of Ref. 4, was also presented. If the interface resistance is sufficiently small $[R_{bj}\ll\rho_N\min\{L,\xi_N(\epsilon_\star)\}]$, the strong proximity effect in the N conductor takes place. This means that the condensate function f^R in the N conductor, arising due to the proximity effect, is not small and satisfies the nonlinear equation (27). Both cases (weak and strong proximity effects) were analyzed for particular parameters and geometries of hybrid S/N interferometer by one of the authors.[6] In the present work and in Ref. 6, it was shown that the conductance of different systems is a periodic function of the phase difference φ (at sufficiently low magnetic field when one may ignore its effect on the pair-breaking rate); i.e., it can be represented in the form

$$G(V,\varphi)=G_n\sum_{k=0}^{\infty}g_k(V)\cos(k\varphi), \tag{42}$$

where coefficients $g_k(V)$ may be comparable with unity; i.e., subgap conductance is comparable with G_n. This is due to the fact that the conductance of the *S-I-N* contact is proportional to the product of the condensate functions of the superconductor, f^R_s, and of the N conductor, f^R (arising due to the proximity effect), where the latter may be of the order of unity at small energies $\epsilon \sim (eV,T) \ll \Delta$ (anomalous proximity effect). In the case when the proximity effect is weak, only the first two terms are essential in the expression (34) for the conductance which are determined by the coefficients $g_0(V), g_1(V) \ll 1$. They are much larger than the contribution to the conductance determined by quasiparticles with $\epsilon > \Delta$ [the latter was ignored in this paper since it contains a small factor $\exp(-\Delta/T)$]. Note that in the case of a weak proximity effect, the conductance dependence is similar to that arising in the well-known quantum-mechanical problem of the interference of two amplitudes A_1 and $A_2\exp(i\varphi)$, in which the resulting probability of a process is $|A_1 + A_2\exp(i\varphi)|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\varphi$. This fact is not surprising since we are dealing essentially with the same quantum-mechanical problem. We have shown that even in the case of the weak proximity effect, the conductance appears to exhibit many interesting features (see Figs. 2–4). One feature appears in systems where the distance between the S-I-N contacts [W in Fig. 1(a) or $(L_1 + L_2)$ in Fig. 1(b)] is comparable with, or larger than, $\xi_N(\epsilon_\star)$. The interference conductance of such systems may change its sign with increasing voltage, and, as a result, the total conductance may exhibit a sequence of maxima (unlike the zero-bias case) at Φ corresponding to $\Phi_0(k+1/2)$, $k = 0,1,2 \ldots$. Note that similar change of the sign of the phase-dependent conductance has been observed in Ref. 8. The phase dependences of the conductance turn out to be more complicated in the case of strong proximity effect under the condition when the distance between the S-I-N contacts is smaller or comparable with $\xi_N(\epsilon_\star)$. In order to realize this case, one can use the third tunnel N-I-N or point N-c-N contact with a high resistance which prevents the N film from the pair-breaking influence of the N electrode. We have shown that different forms of $G(\varphi)$ dependences may occur in the case of a strong proximity effect which depends on the relationship between the S-I-N contact resistances and on the way by which the N film is connected with the N electrode (see Figs. 4–8).

Finally let us note works[28,29] which have recently appeared. In Ref. 28 the conductance oscillations in Au/Nb systems have been observed in the presence of a phase difference created by a supercurrent. We believe that the observed oscillations are related to the proximity effect. In the theoretical work[29] an interferometer with three tunnel junctions has been considered for the case of symmetrical barriers at the S-N interfaces. An expression for the conductance obtained in this work is valid in the case of small voltages, $eV \ll D/L^2, \gamma_{bj}$. In this limit our more general expressions (obtained in this work and in Ref. 6) are reduced to the result found in Ref. 29.

ACKNOWLEDGMENTS

This work was supported by the Russian National Fund for Basic Research, by the International Science Foundation (Grant No. MRC300), and by INTAS (Grant Nos. 1010-CT-93-0051, 94-3862). The authors are grateful to R. Hogg for reading the manuscript.

[1] *Transport Phenomena in Mesoscopic Systems*, Springer Series in Solid State Sciences, edited by H. Fukuyama and T. Ando (Springer, Berlin, 1992).

[2] V.T. Petrashov *et al.*, Phys. Rev. Lett. **70**, 347 (1993).

[3] H. Nakano and H. Takayanagi, Phys. Rev. B **47**, 7986 (1993).

[4] F. W. Hekking, and Y.V. Nazarov, (a) Phys. Rev. Lett. **71**, 1625 (1993); (b) Phys. Rev. B **49**, 6847 (1994).

[5] C.J. Lambert, J. Phys. Condens. Matter **5**, 707 (1993).

[6] A. V. Zaitsev, Phys. Lett. A **194**, 315 (1994); Physica B **203**, 274 (1994).

[7] H. Pothier *et al.*, Phys. Rev. Lett. **73**, 2488 (1994).

[8] A. Dimoulas *et al.*, Phys. Rev. Lett. **74**, 602 (1995).

[9] A. Kastalsky *et al.*, Phys. Rev. Lett. **67**, 3026 (1991); A. Kleinasser and A. Kastalsky, Phys. Rev. B **47**, 8361 (1993).

[10] S.J.M. Bakker *et al.*, Phys. Rev. B **48**, 8361 (1993).

[11] C. Nguen, H. Kroemer, and E. Hu, Phys. Rev. Lett. **69**, 2847 (1992).

[12] P. Xiong, G. Xiao, and R.B. Laibowitz, Phys. Rev. Lett. **71**, 1907 (1993).

[13] J.R. Schrieffer, *Theory of Superconductivity* (Addison-Wesley, Reading, MA, 1983).

[14] A.V. Zaitsev, Pis'ma Zh. Éksp. Teor. Fiz. **51,** 35 (1990) [JETP Lett. **51**, 41 (1990)]; Physica C **185-189**, 2539 (1991); A.V. Zaitsev and G.A. Ovsyannikov (unpublished).

[15] A.F. Volkov, A.V. Zaitsev, and T.M. Klapwijk, Physica C **210**, 21 (1993).

[16] C.W.J. Beenakker, Phys. Rev. B **46**, 12 841 (1992).

[17] I.J. Makmorkos, C.W.J. Beenakker, and R.J. Jalabert, Phys. Rev. B **48**, 2811 (1993).

[18] B.J. van Wees, P. de Vries, P. Magnee, and T.M. Klapwijk, Phys. Rev. Lett. **69**, 510 (1993).

[19] A.F. Volkov, Phys. Lett. A **174**, 144 (1993); Physica B **203**, 267 (1994); Pis'ma Zh. Éksp. Teor. Fiz. **55**, 713 (1992); [JETP Lett. **55**, 746 (1992)]; A.F. Volkov, and T.M. Klapwijk, Phys. Lett. A **168**, 217 (1992).

[20] G.M. Eliashberg, Zh. Éksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1971)].

[21] A.I. Larkin and Yu.N. Ovchinnikov, Zh. Éksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP, **46** , 155 (1977)].

[22] A.V. Zaitsev, Zh. Éksp. Teor. Fiz. **86**, 1742 (1984) [Sov. Phys. JETP **59**, 1015 (1984)].

[23] A.G. Aronov and Yu.V. Sharvin, Rev. Mod. Phys. **59**, 755 (1987).

[24] M.Yu. Kuprianov and V.F. Lukichev, Zh. Éksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].

[25] A.F. Volkov, Phys. Lett. A **187**, 404 (1994).

[26] S.N. Artemenko, A.F. Volkov, and A.V. Zaitsev, Zh. Éksp. Teor. Fiz. **76**, 1816 (1979) [Sov. Phys. JETP, **49**, 924 (1979)]; Solid State Commun. **30**, 771 (1979).

[27] A.V. Zaitsev, Zh. Éksp. Teor. Fiz. **78**, 221 (1980) [Sov. Phys. JETP **51**, 111 (1980)]; **52**, 1018(E) (1980).

[28] P.G.N. de Vegvar , Phys. Rev. Lett. **73**, 1417 (1994).

[29] Yu.V. Nazarov, Phys. Rev. Lett. **73**, 1420 (1994).