Critical fluctuations and lowest-Landau-level scaling of the specific heat of high-temperature superconductors

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The specific heat data of various high-temperature superconducting materials at finite magnetic field and temperatures near the transition temperature are studied. It is found that lowest-Landau-level (LLL) scaling is a good description of the data for fields larger than a characteristic field $H^{*}\sim O(1\ \mathrm{T})$ and that it breaks down for fields less than H^{*}. The scaled data agrees very well with the calculated scaling functions, which provides further evidence for the validity of LLL theory at fields $H \geqslant H^{*}$. The dimensionality of the fluctuations is also studied and it is found that they are predominantly three dimensional near the transition with indications of crossover to two-dimensional behavior farther away.

I. INTRODUCTION

Thermal fluctuations play a significant role in the physics of high-temperature superconductors (HTSC's). The quantity which determines the importance of fluctuations is the Ginzburg number, θ.[1] The product θT_c, where T_c is the superconducting transition temperature, determines the width of the temperature window in which fluctuations are important. In conventional superconductors, this width is typically of $O(\mu\mathrm{K})$. In the HTSC's, however, there are three properties which make this temperature window several orders of magnitude larger than in conventional superconductors and readily observable. These properties are the very small correlation lengths $\xi\sim 10$ Å, the high transition temperature $T_c\sim 100$ K, and the layered structure which effectively reduces the dimensionality. As a result, the generally accepted estimate for θT_c (confirmed by the specific heat results, as we shall see below) is of $O(\mathrm{K})$, although the dimensionless factors[2] in the Ginzburg criterion formula for θ do not conclusively rule out a smaller value.

Fluctuations in the HTSC's have been studied and observed in magnetization, conductivity, current-voltage, and specific heat measurements among others. The theory which describes the fluctuations in the first three measurements is fairly well understood. In specific heat measurements, however, the nature of the fluctuations is still unestablished, although there is strong evidence that the fluctuations are described by either three-dimensional (3D) XY model[3] at small fields or lowest Landau level[4] (LLL) scaling at larger fields. Evidence for 3D XY scaling[5,6] and LLL scaling[7–10] has been presented for many types of HTSC's. Nonetheless, the relevance of LLL scaling to specific heat measurements has been questioned.[5,11]

In this paper, we present strong evidence for LLL scaling of specific heat data of various HTSC's for fields $H \geqslant 1-3$ tesla (T). The LLL-scaled data is compared with the analytic scaling function[12,13] and excellent agreement is found, giving further credence to the validity of LLL theory at larger fields. Another objective here is to investigate the value of the field at which the LLL scaling starts to become valid and where a description based on the zero-field critical behavior must break down. Hence we obtain H_{LLL}, defined as the field below which LLL scaling begins to break down, from each of the data sets studied here. Finally, we explore the dimensionality of the fluctuations and various relevant quantities for the different samples. We find that the fluctuations are three-dimensional near the transition with a possible crossover to two-dimensional (2D) away from the transition.

II. LLL SCALING

In this section we will analyze the data on various samples from different groups. Because results vary somewhat from sample to sample, it is important that one draw conclusions based on all of these samples. We will place an emphasis on the LLL scaling formalism.

For sufficiently small applied magnetic fields, the fluctuating BCS pairs occupy many Landau levels (LL's) and there are significant intra- and inter-LL interactions. At higher fields, the fluctuating pairs occupy a small number of the lower LL's and there is less inter-level mixing. For sufficiently large fields, only the lowest Landau level is occupied and only the intralevel interactions are important. In this region, one can use the lowest Landau level approximation. Tešanović and co-workers,[12,13] however, have shown that, even at smaller fields, where only a few Landau levels are occupied and the inter-Landau level interactions are negligible, one can use the LLL approximation by renormalizing the parameters. This is clear, since in that case the partition function reduces to a product over a few independent LL manifolds. These considerations extend the region in which LLL scaling is expected to be valid to $H \geqslant H^*$, with $H^* \cong (\theta/16)(T/T_{c0})H_{c2}(0)$,[13] where T_{c0} is the $H=0$, mean-field transition temperature. For typical HTSC's, $H^* \simeq 1$ T. At smaller fields, inter-LL interactions must be considered.[14] Theoretically, 3D XY scaling is expected to apply for infinitesimal fields but in practice it should apply for a range of fields near the $H=0$ critical point.

According to LLL theory,[4] the specific heat should scale as

$$\frac{C(H,T)}{C_{\mathrm{MF}}(T)} = g_d\left(\frac{T-T_c(H)}{\Delta T_d(T,H)}\right), \tag{1}$$

where $C(H,T)$ is the measured specific heat minus the background specific heat $C_b(T)$, d is the dimensionality, $T_c(H)$ is the *mean-field* transition temperature as a function of applied field (which should be distinguished from the actual transition temperature), g_d is a dimensionality-dependent scaling function, and $C_{\mathrm{MF}}(T)$ is the mean-field contribution. Frequently, the left-hand side of Eq. (1) is written as $C(H,T)/[\alpha'^2 T/\beta]$ (where the correspondence $C_{\mathrm{MF}}(T) = [\alpha'^2 T/\beta]$ is a standard mean-field result). For $d=2$, $\Delta T_{2\mathrm{D}} = (HT/[C_{\mathrm{MF}}(T)/2k_B T]\phi_0\delta)^{1/2}$ where k_B is the Boltzmann constant, ϕ_0 is the superconducting flux quantum, and δ is a length in the c direction (i.e., the direction perpendicular to the copper-oxide planes). For 3D systems, $\Delta T_{3\mathrm{D}} = (HT/[4C_{\mathrm{MF}}(T)/k_B T]\phi_0\xi_c)^{2/3}$, where ξ_c is the zero-temperature correlation length in the c direction. For temperatures near T_c, the predominant field and temperature dependence of $\Delta T_d(T,H)$ are in the numerators.

In order to scale the data according to Eq. (1), we must extract $C_b(T)$, $C_{\mathrm{MF}}(T)$, and $T_c(H)$ from the raw data. The first two quantities are determined in a standard way[7,15,16] where one fits the zero-field data to a quadratic background, a BCS mean-field term $C_{\mathrm{MF}}(T) = \gamma T[1 + b(T/T_{c0} - 1)]$, and a Gaussian fluctuation term which should be a good approximation far away from the transition temperature where the fluctuations are weak. This simple approach is sufficient empirically for determining these quantities.

$T_c(H)$, the mean-field transition temperature, cannot be measured directly and is usually taken to be the inflection point of $C(H,T)$. Here we use a different and, we believe, more physical approach which we call the "crossing point technique." At $T = T_c(H)$, the quantity $C[H, T = T_c(H)]/C_{\mathrm{MF}}[T_c(H)] = g_d(0)$ [Eq. (1)] is a constant independent of field and is calculated in Refs. 12,13. Therefore, if one plots $C(H,T)/C_{\mathrm{MF}}(T)$ versus $x \equiv T - T_c(H)$ (i.e., only the numerator of the argument of the function g_d), one would find that the curves of each data set cross at $x=0$ where they have the same value. Our technique is to choose $T_c(H)$ for each data set to ensure this crossing at $x=0$ with the value of $g_d(0)$ being then the value calculated in Refs. 12,13. Of course, if this were not possible it would mean that the data cannot satisfy LLL scaling, but we find that the procedure works well. Typically the value of the mean-field transition $T_c(H)$ as determined by this technique is slightly larger than if that quantity were defined by the inflection point. The values of dT_c/dH as calculated by the two methods are within 10% of each other.

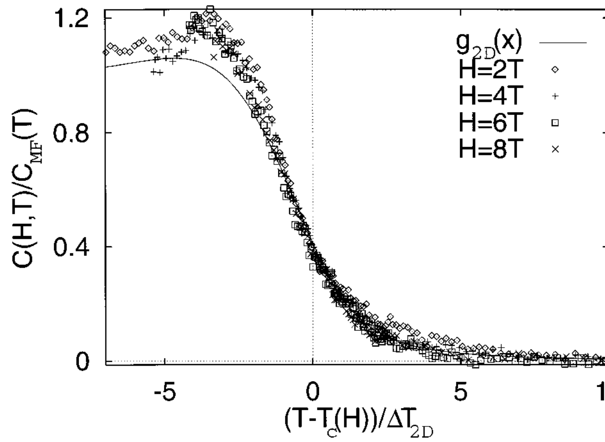

FIG. 1. The data of Ref. 5 scaled with 2D LLL theory. [$\Delta T_{2\mathrm{D}} = (HT^2/[C_{\mathrm{MF}}(T)\delta] \times 2.21 \times 10^{-10}(\mathrm{mJ\ cm/gKT}))^{1/2}$ where the mass density has been taken to be 6.4g/cm^3 for YBCO.] Also plotted in the analytical scaling function Eq. (2) as derived in Refs. 12,13. The agreement between the scaled data and the scaling function is very good.

In Fig. 1, we show the data of the Leeds group[5] on $YBa_2Cu_3O_{7-y}$ (YBCO) scaled with 2D LLL theory. The different symbols represent the data at different fields. The solid line is the theoretical scaling function which will be discussed below. LLL scaling had not previously been shown to work for this data,[5,9] but one can see from our analysis that the scaling is good down to $H = 2-4$T. For smaller fields, the scaling is, as expected, poorer. The parameter values used in the scaling are shown in Table I. We have also scaled the Illinois data for a YBCO single crystal[15] (see Fig. 2) with similar and equally good results. This is also the case for the Geneva data on a YBCO porous single crystal[17] and on a YBCO ceramic data.[18] [Since C_{MF} becomes negative approximately 10 K below T_c for all of the samples, the left-hand side of Eq. (1) diverges in that vicinity. Because this result is not physical, we have cut off the data where it starts to manifest this artifact.] This cumulative evidence for LLL scaling of these four different samples, along with previous evidence of LLL scaling for the $LuBa_2Cu_3O_{7-y}$ (LBCO) single crystal of the Minnesota group,[7] and for the $(Bi,Pb)_2Sr_2Ca_2Cu_3O_y$ (BPSCCO) c-axis aligned sample of the Sendai, Japan group,[10] presents a strong case for describing the medium and high field data for HTSC's by LLL theory away from T_c. The Geneva group has shown[19] that their data agrees very well with LLL scaling if one analyzes it in terms of the derivative with respect to the field H. This agrees with our conclusions but we caution the reader that the derivative analysis neglects a contribution proportional to $dT_c(H)/dH$.

We now turn to the question of dimensionality crossover. In Fig. 3, we show the data of Ref. 5 scaled with 3D LLL theory. The scaling is observed to work down to fields $H = 2-3$ T, providing more support for the validity of LLL theory. We have found equally good 3D LLL scaling for the Minnesota LBCO sample,[7] the Illinois[15] YBCO sample,[9] and the Geneva[17] YBCO porous single crystal. The 2D LLL and 3D LLL scaling of this data will be contrasted in the paragraphs below.

The scaling of data with a certain theory is evidence for the validity of the application of that theory to a particular

TABLE I. Summary of various quantity values as calculated for different samples. H_{LLL} is the field above which it is found that LLL scaling describes the data. $H_{3\mathrm{D}XY}$ is the field up to which 3D XY scaling appears to work. The upper limit of this quantity is set by experimental constraints and not by an observed breakdown of the scaling. dT_c/dH, b, and δ are discussed in the text.

	Theory	YBCO[a]	YBCO[b]	YBCO[c]	LBCO[d]	YBCO[a]	(BP)SCCO[f]
H_{LLL} (T) $\geqslant$	1	2-3	3	0.5	2	1	1
$H_{3\mathrm{D}\,XY}$ (T) $\leqslant$		7	14	7	5	14	
dT_c/dH (K/T)		-0.17	-0.12	-0.58	-0.54	-0.51	-0.50
γ (mJ/gKT^2)		0.041	0.070	0.058	0.67	0.050	
T_c (K)		91.9	93.3	90.6	92.6	92.6	107.5
T_{c0} (K)		92.4	93.5	91.0	93.0	93.3	114.7
b		9.3	11.2	4.0	6.1	10.7	5.0
δ (Å)		400	290	55	89	77	
ξ_c (Å)		5	2.7	0.75	1.7	1.5	

[a]Derived from the data of Ref. 5 on a single crystal.
[b]Derived from the data of Ref. 18 on a porous ceramic.
[c]Derived from the data of Ref. 15 on a single crystal.
[d]Derived from the data of Ref. 7 on an untwinned single crystal.
[e]Derived from the digitized data of Ref. 17 on a porous single crystal.
[f]Taken from Ref. 10 on a c-axis aligned bulk sample.

system. It is however not conclusive, particularly when empirical scaling can be obtained from other theories with very different physical content. Much stronger evidence can be obtained if one knows the scaling function. Tešanović and co-workers[12,13] have calculated the LLL scaling functions $g_{2\mathrm{D}}$, $g_{3\mathrm{D}}$, and g_{Q2D}, the third function being for a quasi-two-dimensional, or layered system. The function for the two-dimensional case is given by

$$g_{2\mathrm{D}}(x)=\frac{1}{2}\left(1-\frac{u(x)x}{\sqrt{x^2u^2(x)+2}}\right)\left[u^2(x)+\left(\sqrt{x^2u^2(x)+2}-xu(x)\right)\left|\frac{du}{dx}\right|\right], \tag{2}$$

where $x=[T-T_c(H)]/(HT/[C_{\mathrm{MF}}(T)/2k_BT]\phi_0\delta)^{1/2}$ and

$$u(x)=0.818-0.11\tanh\left(\frac{x+\sqrt{2}}{2\sqrt{2}}\right). \tag{3}$$

In Figs. 1 and 2 we have plotted, together with the data points, a line representing Eq. (2).[20] One can see that for both samples the scaling function agrees reasonably well with the scaled data, providing convincing evidence for this theory. However, near the peak the agreement breaks down indicating that the fluctuations are effectively 3D. Similar results were obtained for the other data.

The scaling function $g_{3\mathrm{D}}[(T-T_c(H))/\Delta T_{3\mathrm{D}}]$ (Ref. 13) is complicated due to the appearance of a new length $\Lambda(T,H)$ which regulates the bending of vortex lines along the field direction.[21] A simple and accurate approximation to $g_{3\mathrm{D}}(x)$ is obtained by assuming that Λ does not change rap-

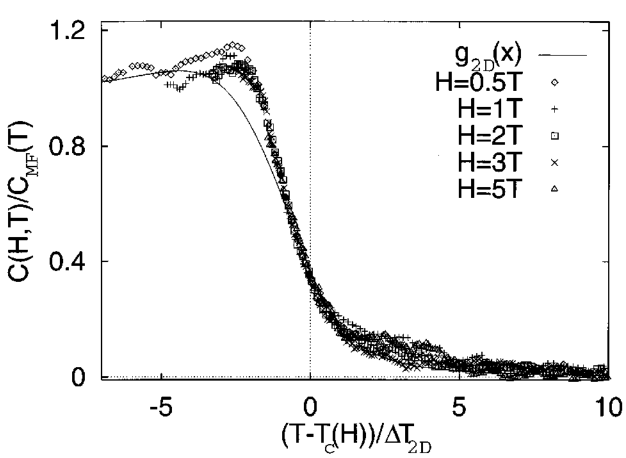

FIG. 2. The specific heat data of a YBCO sample (Ref. 15) scaled with Eq. (1), the 2D LLL scaling form. The solid line is the 2D LLL scaling function (Refs. 12,13), Eq. (2).

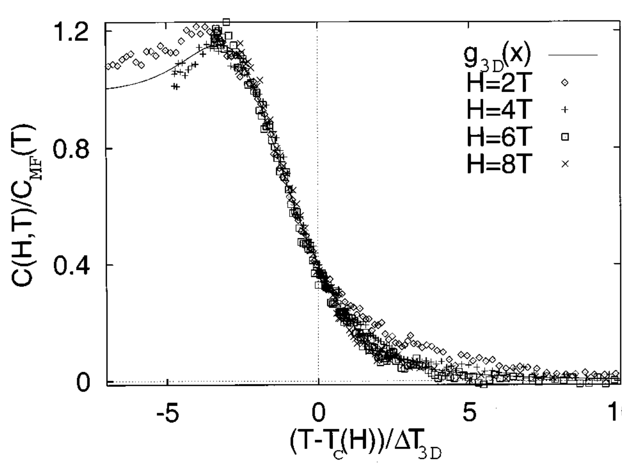

FIG. 3. 3D LLL scaling of the specific heat data of a YBCO single crystal (Ref. 5). [We used $\Delta T_{3\mathrm{D}}=(HT^2/[T_c^{1/2}C_{\mathrm{MF}}(T)\xi_c]\times 2.61\times 10^{-11}(\mathrm{mJ\ cm/gKT}))^{2/3}$.] Also plotted (solid line) is the 3D LLL scaling function (Ref. 13) which coincides with the scaled data very well near T_c.

idly through the critical region around $T_c(H)$: $\Lambda(T,H) \sim \xi_c [T_{c0} H_{c2}(0)/\theta TH]^{1/3}$. With this simplification the dominant contribution to $g_{3D}(x)$ takes the same form as Eq. (2) but with x now being the 3D scaling variable $x=(T-T_c(H))/\Delta T_{3D}$. In 3D, it is conveniently assumed that $T_c(H)$ in x already contains renormalizations from the vortex line bending along the field and is directly determined by our crossing point technique, while ξ_c incorporates a factor of order unity which accounts for the difference between the superconducting (ξ_c) and vortex positional correlations (Λ). Also, $u(x)$ is different in 3D and is given by $u(x) \simeq 0.818 - 0.11 \tanh[(G(x)+K)/M]$ where $G(x)$ is defined in Ref. 13 and K and M are fitting parameters. [See Eqs. (21) and (25) and accompanying discussion in Ref. 13.] We have plotted this form of the function $g_{3D}(x)$ in Fig. 3 using $M=2.3$, $K=1.9$, and $\xi_c = 5.0$ Å. (Because of the fitting parameters M and K, ξ_c can only be determined to within $\sim 20\%$.) The agreement between the scaled data and the scaling function is very good. We have also fit $g_{3D}(x)$ to the data of Refs. 7,15,17,18 consistently finding good agreement between the scaled data and the scaling function, especially near the peak. Note that the obtained value for ξ_c is within factors of order unity of the superconducting coherence length along the c axis ($\sim 2-3$ Å) (see Table I) just as expected from the theory.

The quantity δ in ΔT_{2D} describes a length scale in the c direction. It is frequently taken to be the thickness of the superconducting layer within a unit cell, but in layered systems, it is more accurately associated with Λ, the distance over which a vortex line which threads perpendicular to the layers does not bend. δ for the various samples, as shown in Table I, is typically large $O(100$ Å). In a quasi-two-dimensional regime, where the effective interlayer coupling is weak, one expects δ to be of the order of a few times the interlayer spacing. The fact that our δ appears to be longer is a strong indication that YBCO behaves as an homogeneous anisotropic superconductor in this range of fields and temperatures and that the 3D LLL scaling is more appropriate. This is further supported by the high quality of the 3D LLL scaling and good agreement with the 3D scaling function in Fig. 3. It should also be noted, however, that our values of δ are consistent with those found in scaling magnetization studies in thallium compounds,[22] which indicates that our values of δ do not exclude the possibility of crossover to two-dimensional behavior away from the peak.

In Ref. 7, it was shown that, away from the peak, the specific heat data for the sample studied there scaled better in terms of the 2D LLL variable, while near the peak the data scaled better with the 3D LLL variable. This phenomenon was presented not only as evidence for the validity of LLL scaling, but also for dimensional crossover—from 2D behavior away from the peak to 3D near it. That picture was further supported by comparison of the scaled data with the scaling functions. It was found that the 3D scaling function better described the peak area in the 3D LLL-scaled data and that the 2D scaling function better described the 2D LLL-scaled data away from the peak. Here we present further indications for a temperature-dependent crossover at higher fields (i.e., fields sufficiently large that the system is described by LLL theory). Comparing Figs. 1 and 3 we see that in the peak region the agreement between the 3D scaling function and the scaled data is striking. The predicted asymmetric shape of the scaling function peak is very well reproduced by the experiment. Overall, both the scaling and the fit to the scaling function are, as a whole, clearly better for the 3D case than for 2D. Nevertheless, there is some evidence for 2D behavior away from the transition. Thus the scaling behavior on the high temperature side is a little better for 2D, and the 2D scaling function is a somewhat better fit to the data in the low temperature region to the left of the peak. Very similar considerations apply if one compares the YBCO single crystal data of Fig. 2 with the 3D scaling of the same data shown in Fig. 1 of Ref. 9. Therefore, the overall conclusion is that YBCO is in the regime where the effective interlayer coupling is sufficiently strong to validate description in terms of the 3D anisotropic homogeneous superconductor. Still, the vestiges of its microscopic layered structure are present and visible in the data away from the peak where $\Lambda(T,H)$ changes very slowly and $\Lambda \sim \delta$.

III. SUMMARY AND DISCUSSION

We have investigated the validity of LLL theory as a description of the specific heat in the critical region of the HTSC's in the high-field regime, and the value of the field H^* which marks the onset of this "high-field" regime. We have found that LLL scaling works for a number of different materials and samples at fields $H \geqslant \sim 2$ T. At field values lower than H^*, LLL scaling is expected to break down and we have observed this. In the first row of Table I is H_{LLL}, the field above which we find that LLL scaling works for that set of experimental data. The values as one moves from sample to sample are quite consistent, $H_{\mathrm{LLL}} \simeq 2$ T. One must clearly identify the theoretical quantity H^* with H_{LLL}, and we indeed find quantitative agreement: $H^* \simeq H_{\mathrm{LLL}}$. We have also found that the scaled data agrees very well with the scaling functions for this system,[12,13] providing particularly strong evidence for the validity of LLL theory. Mere scaling, in the absence of a calculated scaling function with which to compare the data, is weaker evidence for a theory.

The next row in Table I is $H_{3D\,XY}$, the field up to which the data is found to scale with 3D XY theory. To date, a clear breakdown of 3D XY scaling has not been observed and so $H_{3D\,XY}$ is larger than H_{LLL} and is only limited by the largest applied magnetic fields available to the different groups. In other words, the fields for which LLL scaling and 3D XY scaling appear to be valid overlap. One implication of this is that the crossover from one regime to another is gradual and that present measurements have not gone to large enough fields to observe the breakdown of 3D XY scaling. Another possibility is that the scaling of the data with 3D XY theory is a coincidence or an artifact. Scaling in the 3D XY model arises simply from a hypothesis on the scaling variable, and there is no theoretically derived scaling function. As a result, the validity of that theory is less convincing, as explained above.

We expect on general grounds that the regimes of the LLL and the physical XY scaling do not overlap. This is so because the former arises from a high field theory whereas the latter is inherently tied to the zero- and low-field behavior.[14] 3D XY scaling with zero-field critical exponents, in principle, is expected to be valid only at an infinitesimally small field; at low but finite fields a different, finite-field critical behavior might set in.[14] At higher fields, the formation of LLL manifolds begins. The nature of the dominant fluctuation modes in the XY and LLL cases is very different: As the formation of LLL manifolds takes place, many degrees of freedom of a complex field $\Psi(\mathbf{r})$ obtain a mass (cyclotron) gap. These degrees of freedom then become irrelevant, drastically changing the fluctuation behavior.

We have also investigated the values of other quantities for the various samples which are listed in Table I, including dT_c/dH, δ, and b. These quantities vary significantly from sample to sample, as one would expect from the different compositions and preparation methods. Despite these variations, there does seem to be some overall trends among the various quantities. For example, for the samples where dT_c/dH is the smallest, b, δ, and ξ_c are larger. Indeed the correspondence of the values for δ and ξ_c is particularly convincing evidence for LLL theory and the results of Ref. 12,13 because these quantities have the most physical significance and are in agreement with the expected values. Their values are also indicative of the 3D nature of the YBCO materials. It should also be noted that for all of the samples, $H_{\rm LLL}$ is remarkably consistent.

In conclusion, we have studied LLL scaling for the field dependent specific heat in the context of data obtained in different HTSC materials by several groups. We find excellent quantitative agreement with the calculated LLL scaling functions. The scaling is clearly three-dimensional near the peak, and there are indications that it may cross over to two-dimensional elsewhere. Finally, we see that in all cases without exception one finds a transition region with a width of O(K), confirming that the commonly accepted interpretation of the Ginzburg criterion is correct and that the transition region is very accessible in HTSC's.

After this paper was submitted for publication, we received a preprint by Jeandupeux *et al.*[23] in which the low-field 3D XY scaling is tested on specific heat and magnetization data of a YBCO sample. These authors find that the predicted form of the XY scaling, including the zero-field correlation length exponent, breaks down at fields above 1 T. Their results are in general agreement with ours: The field at which one crosses over from the low- to the high-field limit of fluctuation behavior is approximately 1 T for YBCO.

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