# Electronic band structure in a periodic magnetic field

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We analyze the energy band structure of a two-dimensional electron gas in a periodic magnetic field of a longitudinal antiferromagnet by considering a simple exactly solvable model. Two types of states appear: with a finite and infinitesimal longitudinal mobility. Both types of states are present at a generic Fermi surface. The system exhibits a transition to an insulating regime with respect to the longitudinal current, if the electron density is sufficiently low.

The interest in the magnetoconductance properties of the two-dimensional electron gas in spatially periodic lateral magnetic fields has been further stimulated by the recent experimental availability of such systems.[1,2] In the work of Carmona *et al.*[1] spatial modulation of a magnetic field was produced by means of equidistantly located superconducting stripes where magnetic vortices were trapped by impurities resulting in periodic inhomogeneity of the external magnetic field, while in the work of Ye *et al.*[2] it was produced by deposition of ferromagnetic microstructures on top of the high-mobility two-dimensional (2D) electron gas. Vast theoretical efforts on a 2D electron gas in an inhomogeneous external magnetic field range from the theory of momentum-dependent tunneling through a magnetic barrier[3] to properties of electronic states and transport in a weakly spatially modulated magnetic field.[4–7] In this paper we will be concerned with the one-electron energy band structure of the 2D electron gas under a periodic lateral magnetic field of an antiferromagnet, which is a limiting case of a strong periodic modulation. We will show that two types of states appear: with a finite and infinitesimal longitudinal mobility. Both types of states are present at a generic Fermi surface. The system exhibits a transition to an insulating regime with respect to the longitudinal current if the electron density is sufficiently low.

The effect of a uniform magnetic field on energy bands produced by the periodic (electric) potential is well known.[8] The impact of the slightly inhomogeneous magnetic field on the Landau levels of a free electron was considered by Müller.[4] He showed that the energy bands exhibit a pronounced asymmetry in the lateral direction. For a spatially modulated magnetic field a common theoretical model[5] employs a magnetic field perpendicular to the plane of the two-dimensional electron gas which has a ‘‘carrier’’ field $B_0$ with a periodic modulation on top of it:

$$\mathbf{B}=(B_0+B\cos Ky)\hat{\mathbf{z}}. \tag{1}$$

In the work of Peeters and Vasilopoulos[5] the effect of a periodic electric and weakly modulated magnetic field ($B \ll B_0$) was considered. They showed that the broadening of the Landau levels is roughly proportional to the modulation amplitude $B$. The ‘‘Hofstadter-like’’ spectrum was obtained by Wu and Ulloa,[6] and collective excitations were analyzed in Ref. 7 by the same authors.

In this work we will deal with an electron gas confined to a plane in a perpendicular periodic magnetic field without a ‘‘carrier’’ field. In other words, in (1) we take $B_0=0$. This corresponds to the extreme case of the other limit, $B_0 \ll B_m$. Such a periodic field will create an energy band structure of its own. We see this type of arrangement experimentally realizable by bringing a two-dimensional electron gas in close contact with a mesoscopic longitudinal antiferromagnetic (sandwich) structure, without an external magnetic field. When dealing with the one-electron spectrum it is useful to have some exactly solvable models (potentials), as they elucidate the whole structure of the energy bands.[9] Below we show that the energy bands can be obtained exactly in a simple way for a reasonably idealized periodic magnetic field. We present a full band picture for both spin and spinless electrons, and discuss the topology of the Fermi surface.

The Hamiltonian for a free spinless electron in a magnetic field is

$$\hat{H}=\frac{1}{2m}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2. \tag{2}$$

In our case the electron is confined to a plane, and a periodic magnetic field of lateral antiferromagnet is superimposed. The magnetic field can be modeled as (see Fig. 1)

$$\mathbf{B}=Ba\sum_{n=0,\pm 1,\ldots}[\delta(y+c+an)-\delta(y+an)]\hat{\mathbf{z}}. \tag{3}$$

For such a magnetic field the vector potential takes a form

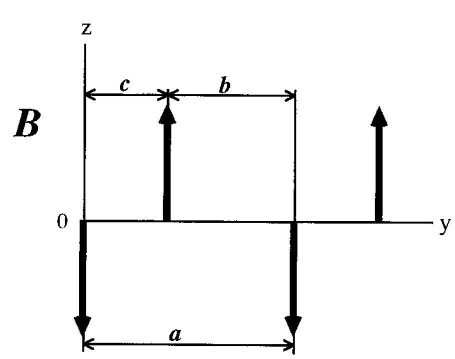


FIG. 1. The external periodic magnetic field as modeled by (3).

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$$\mathbf{A}=(-Ba\epsilon(y),0,0), \tag{4}$$

where

$$\epsilon(y)=\epsilon(y+an), \tag{5}$$

$$\epsilon(y)=\begin{cases}-1/2 & 0<y<c,\\ 1/2 & c<y<a.\end{cases} \tag{6}$$

We proceed with solution of Eqs. (2)–(6) in a standard way. We look for the solution in a form

$$\psi(x,y)=e^{ik_x x}\chi(y). \tag{7}$$

Thus, in the gauge (4) the solution is a plane wave in the $x$ direction. For the $y$-dependent part of the wave function $\chi(y)$ we arrive at

$$\left[-\frac{1}{2}\frac{d^2}{dy^2}+k_x\gamma\epsilon(y)\right]\chi(y)=\left(E-\frac{k_x^2}{2}\right)\chi(y). \tag{8}$$

Here atomic units are adopted; $E$ is the energy up to an unimportant constant; $\gamma=a/a_B^2$, the dimensionless "magnetic length," is given by $a_B=\sqrt{\hbar c/eB}/a_0$; and $a_0$ is the Bohr radius. Equation (8) is precisely the Schrödinger equation for the Kronig-Penney model, and can be easily solved exactly. The resulting dispersion relation is given by

$$\cos k_y a=\frac{\beta^2-\alpha^2}{2\alpha\beta}\sinh\beta b\ \sin\alpha c+\cosh\beta b\ \cos\alpha c, \tag{9}$$

where

$$\alpha=\sqrt{k_x\gamma+2(E-k_x^2/2)}, \tag{10}$$

$$\beta=\sqrt{k_x\gamma-2(E-k_x^2/2)}; \tag{11}$$

$k_y$ is the quasimomentum in the longitudinal direction. The band structure for $B=0.1$ T, $a=1\,\mu$m, $c=a/3$, $b=2a/3$ is presented in Figs. 2 and 3. It is compressed in the (longitudinal) $y$ direction. As was pointed out in Ref. 4, the pronounced asymmetry along the $x$ direction is the signature of the energy spectrum in the inhomogeneuos magnetic field. In the upper quarter of Fig. 2 is the region where "broad" bands are formed. These bands have a finite width in the $y$ direction, and the particles occupying these states will have a finite mobility in the longitudinal direction. The other set of "narrow" bands occupies the left and right quarters of Fig. 2. From the point of view of the Kronig-Penney model (8) they correspond to the valence bands of the periodic potential. These bands are infinitesimally narrow in the $y$ direction, and electrons populating them would have a vanishingly small longitudinal mobility. Of course, in the transverse direction states in both types of bands would have some finite mobility. Figure 3 represents the same band structure on a bigger scale. The part of the spectrum shown in Fig. 2 corresponds to the area inside the box of Fig. 3. Dark areas of Fig. 3 represent regions of "broad" bands, while the parabolas represent "narrow" bands. In order not to overcomplicate the picture we show only every fourth of the latter. As we will see below, the peculiarity of the energy spectrum in a periodic magnetic field will appear in the fact that at the Fermi surface both types of states will appear.

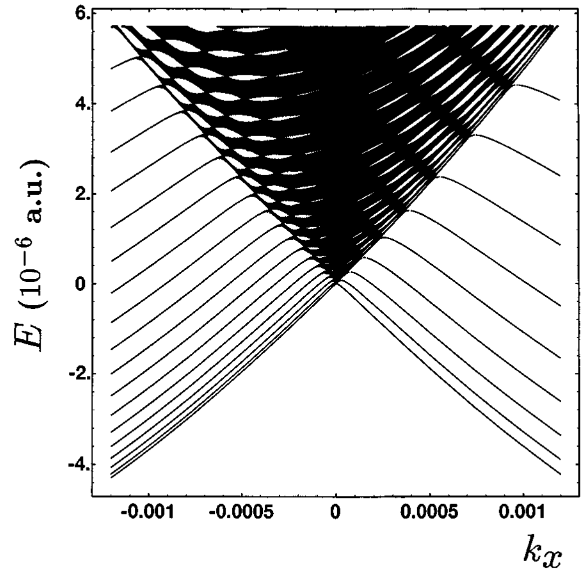


FIG. 2. The band structure for a spinless problem compressed in the (longitudinal) $y$ direction. $B=0.1$ T, $a=3000$ Å, $c=a/3$, $b=2a/3$. The states in the "broad" bands (upper quarter) have a finite longitudinal mobility, while the longitudinal mobility of the states in the "narrow" bands (left and right quarters) is infinitesimal.

In this model the problem of electrons with spin is equally easy to treat. This amounts to simply adding the spin-dependent term to the left-hand side of Eq. (8):

$$\left[-\frac{1}{2}\frac{d^2}{dy^2}+k_x\gamma\epsilon(y)+\frac{\gamma}{2}\sum_n\ [\delta(y+c+an)-\delta(y+an)]\right]\chi(y)=\left(E-\frac{k_x^2}{2}\right)\chi(y), \tag{12}$$

which results in a slightly modified dispersion relation

$$\cos k_y a=\frac{\beta^2+\gamma^2-\alpha^2}{2\alpha\beta}\sinh\beta b\ \sin\alpha a+\frac{\gamma}{\alpha}\cosh\beta b\ \sin\alpha c+\frac{\gamma}{\beta}\sinh\beta b\ \cos\alpha c+\cosh\beta b\ \cos\alpha c, \tag{13}$$

with the same $\alpha$ and $\beta$ as in Eqs. (10), (11). The detailed band structure for the same magnetic field as before is shown in Fig. 4. The main structure of the whole spectrum is still represented by Fig. 3. As compared to the spinless problem, the "broad" bands are characterized by wider gaps in the density of states, while "narrow" bands only slightly change their locations. As we fill the spin "up" and "down" states up to the Fermi level, the ground state exhibiting transverse oscillatory spin oscillations in the spirit of the ones discussed by Chudnovsky[10] may result.

At possible Fermi surface corresponding to cutting the energy manifold at $E_F=4\times10^{-6}$ a.u. is presented in Fig. 5. We show only a part of the first Brillouin zone. Shaded areas are populated by electrons. The curved lines of the Fermi surface (in the middle) correspond to states in the "broad" bands. As discussed above, these states have a finite mobility

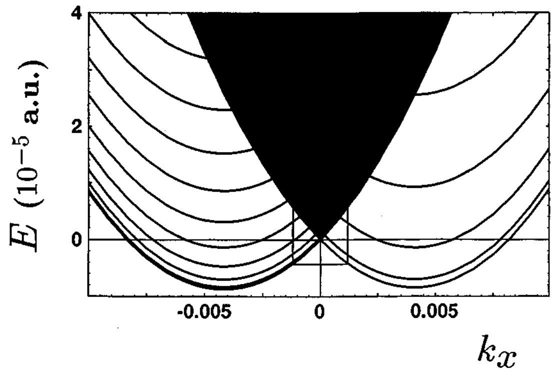


FIG. 3. The band structure on a bigger scale. Only every fourth of the ''narrow'' bands is shown. The box in the middle is magnified for the spinless problem (Fig. 2), and the problem with spin (Fig. 4).

in both directions and will always contribute to the conductivity of the sample. The vertical lines on the extreme right and left correspond to the sections of ''narrow'' bands which are flat in the longitudinal direction. Thus, these states will not contribute to the longitudinal current, while always contributing to the transverse one. If the electron gas is dilute enough so that the Fermi level drops below the $E=0$ level (see Fig. 3), the sample will not conduct in the longitudinal direction at all as all the states at the Fermi surface will have a vanishing mobility in the $y$ direction. It is easy to estimate the electron density for transition to an insulating regime by counting the states with $E<0$. In our range of parameters we can totally neglect the width of the ''narrow'' bands. The transition density is given by

$$n=\frac{1}{2\pi a}\left(\sum_{n}\sqrt{\gamma^2-(\pi+2\pi n)^2/c^2}+\sum_{n}\sqrt{\gamma^2-(\pi+2\pi n)^2/b^2}\right) \tag{14}$$

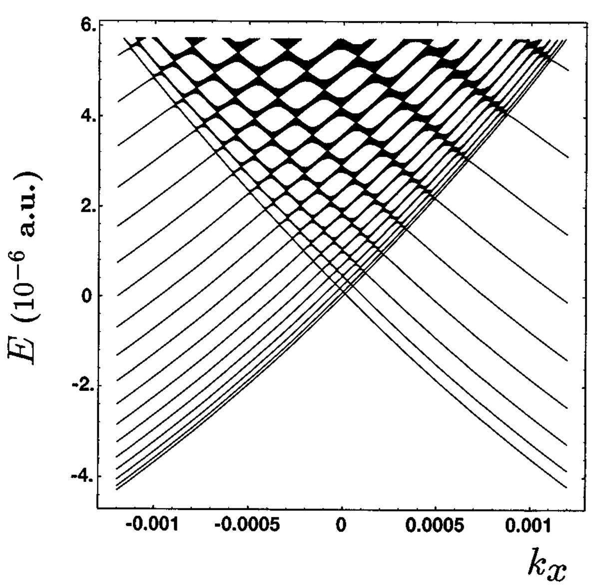


FIG. 4. Same as in Fig. 2 but for the problem with spin.

(all quantities are in atomic units). The summations are over all $n=0,1,\ldots,n_{\max}$ for which the radicals remain positive. For our values of $b$, $c$, and $B$, $n\approx 10^{11}$ cm$^{-2}$. If any of the summations turns out to be restricted to the first ''narrow'' band, one has to account for the bandwidth in the $y$ direction as well.

In a realistic experimental situation many-body effects will be present. The simplest of them is screening. Screening will ''smear'' the effective single-particle potential, which may result in suppressing smaller gaps predicted in the calculation. However, these effects do not change the overall structure of the spectrum.

In conclusion, we have presented a simple exactly solvable model of the electronic band structure in a spatially periodic magnetic field. All the conclusions derived from the model are not restricted to this particular model, but illustrate the general structure of the energy bands of the two-dimensional electron gas in a periodic lateral magnetic field. This type of system can be realized by imposing magnetic field of a longitudinal antiferromagnet on a high-mobility two-dimensional electron gas. The band structure exhibits a pronounced asymmetry in the lateral direction and consists of the two types of bands. The states in ''broad'' bands will have a finite mobility in the longitudinal direction while the longitudinal mobility of the electrons occupying states in ''narrow'' bands is infinitesimally small. A generic Fermi surface will contain both types of states. If the electron density is sufficiently low, only the ''narrow'' bands will be occupied resulting in vanishing of the longitudinal conductivity. The electron density for transition to the insulating regime has been estimated. An interesting extension of this work is to account for screening in such a system.

*Note added in proof.* Recently we learned about related work by Ibrahim and Peeters [Phys. Rev. B **52**, 17 321 (1995); Am. J. Phys. **63**, 171 (1995)].

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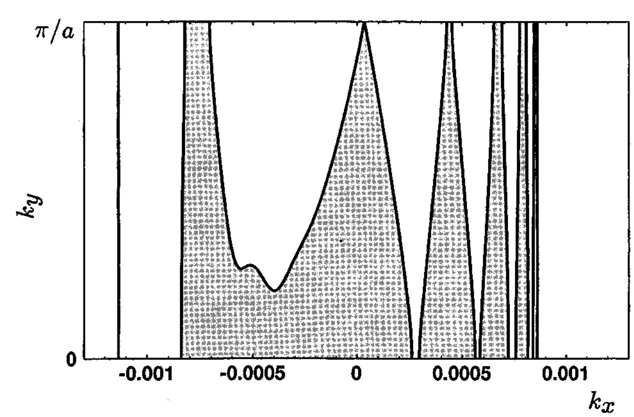


FIG. 5. Fermi surface for $E_F=4\times 10^{-6}$ a.u. Only a part of the first Brillouin zone is shown. Shaded areas are populated by electrons. The curved lines of the Fermi surface (in the middle) correspond to the states in the ''broad'' bands, while the vertical lines on the extreme right and left represent the states in the ''narrow'' bands.

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