# Concentration dependence of optical phonons in the  $TiO<sub>2</sub>$ -SnO<sub>2</sub> system

T. Hirata, K. Ishioka, M. Kitajima, and H. Doi

*National Research Institute for Metals, 1-2-1, Sengen, Tsukuba, Ibaraki 305, Japan*

(Received 17 August 1995)

The concentration dependence of optical phonons in the  $TiO<sub>2</sub>-SnO<sub>2</sub>$  system whose phase diagram exhibits a miscibility gap, has been studied by Raman scattering and Fourier-transform infrared spectroscopy. X-ray diffraction confirmed that the mixed oxides  $Ti_{1-x}Sn_xO_2$  with intervals  $x=0.1$  are a single uniform phase (tetragonal) except some *x*'s; no phase separation by spinodal decomposition could be avoided for  $x=0.3, 0.4$ , 0.5, and 0.6, characterizing x-ray-diffraction profiles by sidebands. The 449 cm<sup>-1</sup>  $A_{1g}$  and 610 cm<sup>-1</sup>  $E_{g}$ Raman modes for TiO<sub>2</sub> never changed linearly in frequency and linewidth with  $x$ . It is stressed that the  $x$ dependence of optical phonons is predominated by the octahedral distortion in  $Ti_{1-x}Sn_xO_2$ , where different/ anisotropic compressibilities and thermal expansivities of each end member play a crucial role. Infraredreflectance spectra consisting of the infrared-active  $A_{2u}$  and  $E_u$  modes between 50–900 cm<sup>-1</sup>, also changed systematically with *x* in  $Ti_{1-x}Sn_xO_2$ . The concentration dependence of optical phonons in  $Ti_{1-x}Sn_xO_2$  is discussed, from the viewpoint of the mode behavior of mixed crystals.

### **I. INTRODUCTION**

Both oxides  $TiO<sub>2</sub>$  and  $SnO<sub>2</sub>$  belong to the same crystal symmetry (tetragonal) with the space group  $D_{4h}^{14}$  $(P4<sub>2</sub>/mm)$  and two molecular units per primitive unit cell  $(Z=2)$ . Only lattice parameters are slightly different  $[a=4.732 \text{ Å} \text{ and } c=3.1871 \text{ Å} \text{ for } \text{SnO}_2; \frac{1}{4}a=4.594 \text{ Å} \text{ and}$  $c=2.956$  Å for TiO<sub>2</sub> (Ref. 2)], because of different ionic radii  $(Sn^{4+}=0.71 \text{ Å}; T1^{4+}=0.68 \text{ Å}).$ <sup>3</sup> The isostructure of both oxides but different lattice parameters provides the  $TiO<sub>2</sub>-SnO<sub>2</sub>$  phase diagram, exhibiting a miscibility gap with a critical temperature  $T_c = 1430 \degree C$  at about 50 mol %  $TiO<sub>2</sub>$ .<sup>4,5</sup>

We have measured the infrared and Raman spectra of  $Ti_{1-x}Zr_xO_2$ , in order to investigate the concentration dependence of optical phonons in the  $TiO<sub>2</sub>-ZrO<sub>2</sub>$  system.<sup>6</sup> However, the Zr substitution in  $TiO<sub>2</sub>$  was restricted to a low level since the second phase  $(ZrTi)_{0.5}O_2$  (orthorhombic) emerges for  $x \ge 0.1$  in Ti<sub>1-x</sub>Zr<sub>x</sub>O<sub>2</sub>. No doubt, it is important to study with the mixed oxides over a wide concentration region, in which metal cations are replaced by any others with different radius and/or charge. The concentration dependence of optical phonons sheds light on the change in bond distances/ angles or force constants. In this context, the  $TiO<sub>2</sub>-SnO<sub>2</sub>$  system is rather appealing because the substitutional solid solution  $Ti_{1-x}Sn_xO_2$  is formed between  $0 \le x \le 1$ .

Factor-group analysis predicts the following optical phonons at the wave vector  $k=0$  for tetragonal SnO<sub>2</sub> and TiO<sub>2</sub> with  $D_{4h}^{14}$ <sup>7,8</sup>

$$
\Gamma = A_{1g}(R) + A_{2g} + A_{2u}(\text{IR}) + B_{1g}(R) + B_{2g}(R) + 2B_{1u}
$$
  
+  $E_g(R) + 3E_u(\text{IR})$ .

The three modes  $A_{2g}$  and  $2B_{1u}$  are neither Raman active nor infrared active; the modes of symmetry  $A_{2u}$  and  $E_u$  are infrared active whereas the remaining  $A_{1g}$ ,  $B_{1g}$ ,  $B_{2g}$ , and  $E_{g}$ modes are Raman active. We could expect that the Raman and infrared spectra of  $SnO<sub>2</sub>$  and  $TiO<sub>2</sub>$  are similar to each other because of their isostructure (rutile type), and that these spectra of  $Ti_{1-x}Sn_xO_2$  would show any concentration dependence.

The objective of the present work is to investigate the concentration dependence of the Raman- and infrared-active modes for  $Ti_{1-x}Sn_xO_2$  ( $0 \le x \le 1$ ). A few models have been proposed to account for the mode behavior in mixed crystals.<sup>9</sup> Infrared-reflectance spectra of  $Zr_{1-x}Hf_xO_2$  $(0 \le x \le 1)$  were measured,<sup>10</sup> and it is concluded that the concentration dependence of the infrared-active modes with respect to frequency, linewidth, and intensity can be reconciled with two-mode behavior. It is another issue of interest to understand the concentration dependence of optical phonons in the  $TiO<sub>2</sub>-SnO<sub>2</sub>$  system from the viewpoint of the mode behavior of mixed crystals.

# **II. EXPERIMENT**

Mixed oxides of  $Ti_{1-x}Sn_xO_2$  ( $0 \le x \le 1$ ) were prepared from  $TiO<sub>2</sub>$  and  $SnO<sub>2</sub>$  by the solid-state reaction technique. Both end member oxides  $(99.99%)$  were weighed out so as to give *x*'s with intervals  $x=0.1$  in Ti<sub>1-x</sub>Sn<sub>x</sub>O<sub>2</sub>, and blended thoroughly in a mortar. Subsequently, the mixture was subjected to cold isostatic pressing at  $1500 \text{ kg/cm}^2$  and precalcined several times in air at 1450 °C for 7 h to ensure homogeneity. Final sintering was carried out in air at 1500 °C for 5 h and followed by rapid cooling to room temperature  $(RT)$ ; the resultant products were cut into pellets with approximately 10 mm  $\phi$  and 1 mm in thickness by a diamond saw, and they were used for x-ray diffraction, Fourier-transform infrared spectroscopy, and Raman scattering.

X-ray-diffraction profiles were recorded at RT by an x-ray diffractometer (Rigaku Rint2500) with Cu  $K\alpha$ . Lattice parameters were determined by an iterative least-squares procedure, using at least seven Bragg reflections with  $2\theta$  values in the range 25–70°. Infrared-reflectance spectra were measured in the wave-number region  $50-4000$  cm<sup>-1</sup>, by a Fourier-transform infrared spectrometer (JEOR JIR100). Resolution was 2 cm<sup>-1</sup> in the far-infrared region 50–550



FIG. 1. The x-ray-diffraction profiles of  $Ti_{1-x}Sn_xO_2$  with various  $x$ 's and the end member oxides  $TiO<sub>2</sub>$  and  $SnO<sub>2</sub>$ , for comparison.

 $cm^{-1}$  and 4  $cm^{-1}$  in the mid-infrared region 400–4000  $\text{cm}^{-1}$ . A different beam splitter, KBr or Mylar, was used with a detector of triglycine sulfate for each infrared region; a mirror of aluminum-evaporated thin film was used as the reference, to express the infrared-reflectance spectra as the sample to reference intensity ratio. Raman scattering was performed by a double monochrometer (JASCO TRS660) with a spectrometric multichannel analyzer (Princeton Inc. DRS700); the 514.5 nm  $Ar^+$  laser was used for an exciting source with approximately 20 mW power, and all Raman spectra were recorded at RT in a backscattering geometry.

#### **III. RESULTS**

#### **A. Lattice parameters**

Figure 1 shows the x-ray-diffraction profiles of  $Ti_{1-x}Sn_xO_2$  with various *x*'s and the end members TiO<sub>2</sub> and  $SnO<sub>2</sub>$ , for comparison. It is evident that all Bragg reflections assigned to tetragonal TiO<sub>2</sub> shift to lower 2 $\theta$  values with increasing *x* in  $Ti_{1-x}Sn_xO_2$ , indicating the lattice expansion; this is evidenced by a down shift of the  $(211)$  reflection at about  $2\theta = 54^{\circ}$  for TiO<sub>2</sub>, for example. In addition, it should be noted that the x-ray-diffraction profiles for  $x=0.3, 0.4$ , 0.5, and 0.6 reveal the sidebands flanked with Bragg reflections that belong to the tetragonal unit cell.

Figure 2 depicts an example of x-ray-diffraction profile of  $Ti_{1-x}Sn_xO_2$  with  $x=0.6$ , where the sideband is flanked with the (211) reflection at about  $2\theta = 54^\circ$ . It is most probable that this extra peak is originated from the phase separation by spinodal decomposition in the TiO<sub>2</sub>-SnO<sub>2</sub> system.<sup>11,12</sup> In general, thermodynamically unstable solutions phase separated



FIG. 2. The x-ray-diffraction profile of  $Ti_{1-x}Sn_xO_2$  with  $x=0.6$ between  $2\theta = 51-55^{\circ}$ , to demonstrate a sideband flanked with the  $(211)$  Bragg reflection that belongs to the tetragonal unit cell; this sideband is originated from the phase separation by spinodal decomposition.

by spinodal decomposition and the sidebands are observed in x-ray-diffraction profiles as shown in Fig. 2; we notice that no rapid cooling from 1500 °C to RT was sufficient to prevent the decomposition of  $Ti_{1-x}Sn_xO_2$  with a few *x*'s into a two-phase system.

In Fig. 3, the lattice parameters *a* and *c* are plotted as a function of *x* for  $Ti_{1-x}Sn_xO_2$ ; the axial ratio *c/a* (tetragonality) and the unit-cell volume  $V = a^2c$  are shown as well. The lattice parameters of  $TiO<sub>2</sub>$  and  $SnO<sub>2</sub>$  coincide with the previous data.2,3 However, it seems difficult to connect the lattice parameters of each end member by a linear relationship, even if the errors in  $c$  and  $a$  (no more than 0.5%) are taken into consideration. Rather, there are some deviations from Vegard's law. In fact, it has been found that there are positive deviations from Vegard's law in the  $TiO<sub>2</sub>-SnO<sub>2</sub>$  system, which can be predicted using a theory based on nonlinear second-order elasticity.<sup>4</sup>

It is evident that the data for  $x=0.3$ , 0.4, 0.5, and 0.6 are responsible for the noticeable deviations from Vegard's law. It is likely that the phase separation by spinodal decomposition affects the lattice paraemters at these *x*'s. The axial ratio *c*/*a* also exhibits anomalies over the concentration region  $x=0.3-0.6$  reflecting the scattered lattice parameters at these *x*'s.

The unit-cell volume of  $Ti_{1-x}Sn_xO_2$  increases monotonously with *x*. The monotonous increase of *V* indicates that the lattice of  $Ti_{1-x}Sn_xO_2$  would relax as  $Sn^{4+}$  with a larger ionic radius is substituted for  $Ti^{4+}$  in TiO<sub>2</sub>.

### **B. Infrared-reflectance spectra**

Figure 4 displays the infrared-reflectance spectra of  $Ti_{1-x}Sn_xO_2$  in the two wave-number regions that partly overlap. The infrared-reflectance spectrum of  $TiO<sub>2</sub>$ , which is characterized by the two  $E_u$  modes at 398/516 cm<sup>-1</sup> and the  $A_{2u}$  mode (broad) over 200–300 cm<sup>-1</sup>, is in close agreement with the literature;<sup>13–16</sup> the remaining  $E_u$  mode coexists with the  $A_{2u}$  mode over 200–300 cm<sup>-1</sup>. Numerous spiky features over  $50-300$  cm<sup>-1</sup> are due to water vapor and a sharp band at 668 cm<sup>-1</sup> is attributed to  $CO<sub>2</sub>$  in ambient atmosphere. As for the infrared-reflectance spectrum of  $SnO<sub>2</sub>$  which is comparable to the literature,<sup>8</sup> a small band around  $450 \text{ cm}^{-1}$  is identified with the  $A_{2u}$  mode. A hump at about 600 cm<sup>-1</sup> can be assigned to the infrared-active mode of  $E<sub>u</sub>$  symmetry. The



FIG. 3. The lattice parameters *a* and *c* as a function of *x* for  $Ti_{1-x}Sn_xO_2$ ; the axial ratio *c/a* (tetragonality) and the unit-cell volume  $V = a^2c$  are shown as well; note that the lines connecting data points are only a guide to eyes, and the errors in  $a$  and  $c$  (no more than 0.5%) approximately correspond to three times the size of symbols.

polarized infrared-reflectance spectra of single  $SnO<sub>2</sub>$  crystals allow to reveal the  $A_{2u}$  mode with the electric vector of the radiation parallel to the *c* axis  $(E_{\text{parallel}})$  and the three  $E_u$ modes with the electric vector perpendicular to the *c* axis  $(E_{perpend})$ . In fact, the imaginary part  $\varepsilon''$  of the dielectric parameters, which were obtained from a Kramers-Kronig analysis of the polarized infrared-reflectance spectra of single  $\text{SnO}_2$  crystals,<sup>8</sup> show three resonances at 244, 293, and  $618 \text{ cm}^{-1}$  for  $E_{\text{perpend}}$  and a strong resonance at 477 cm<sup>-1</sup> for  $E_{\text{parallel}}$ . In view of these results, it seems that the remaining two  $E_u$  modes appear around 200 and 300 cm<sup>-1</sup>, respectively, in our infrared-reflectance spectrum of  $SnO<sub>2</sub>$  (polycrystalline).

The striking band at 770  $\text{cm}^{-1}$  in the infrared-reflectance spectrum of  $SnO<sub>2</sub>$  is regarded as the longitudinal infrared mode of  $E_u$  symmetry. Another spectral feature of interest is that the two dips between the  $E_u$  modes at 516 and 398  $\text{cm}^{-1}$ , and the 398  $\text{cm}^{-1}$   $E_u$  mode and the  $A_{2u}$  mode for TiO<sub>2</sub> become invisible and shift downwards with *x* in  $Ti_{1-x}Sn_xO_2$ . Besides, the spectral weight totally shifts to low wave numbers with increasing *x* since the lattice of  $Ti_{1-x}Sn_xO_2$  expands with *x*, hence the bond strength lessens.



FIG. 4. The infrared-reflectance spectra of  $Ti_{1-x}Sn_xO_2$  with various *x*'s in two wave-number regions that partly overlap; 380– 980 cm<sup>-1</sup> (a) and 60–550 cm<sup>-1</sup> (b).

No attempt has been made to extract dispersion parameters such as the oscillator strength, the damping factor and the frequency of each infrared-active mode, by a Kramers-Kronig analysis of our infrared-reflectance spectra. It might be expected that these dispersion parameters depend on *x*. However, the systematic spectral changes of  $Ti_{1-x}Sn_xO_2$ 



FIG. 5. The Raman spectra of  $Ti_{1-x}Sn_xO_2$  with various *x*'s in the frequency region  $100-820$  cm<sup>-1</sup>.

with *x* are fairly informative concerning the concentration dependence of the infrared-active modes for  $Ti_{1-x}Sn_xO_2$ .

## **C. Raman spectra**

Figure 5 shows the Raman spectra of  $Ti_{1-x}Sn_xO_2$  in the frequency region  $100-820$  cm<sup>-1</sup>. The Raman spectrum of  $TiO<sub>2</sub>$  agrees with the literature<sup>13,14,17–23</sup> except that the 143  $\text{cm}^{-1}$  *B*<sub>1*g*</sub> mode is not detected here. The broad band at 235  $cm^{-1}$ , which is explained in terms of disorder-induced effect or second-order Raman scattering, $14,22-25$  is not well resolved either; besides, the 826 cm<sup>-1</sup>  $B_{2g}$  mode is invisible because of its weak intensity. In the present work, the Raman-active modes of symmetry  $E_g$  and  $A_{1g}$  are observed at 449 and 610  $cm^{-1}$ , respectively.

On the other hand, the Raman spectrum of  $SnO<sub>2</sub>$  exhibits the 631 cm<sup>-1</sup>  $A_{1g}$  mode and the 800 cm<sup>-1</sup>  $B_{2g}$  mode, in agreement with the literature,<sup>25–27</sup> whereas the  $123^{\circ}$  cm<sup>-1</sup>  $B_{1g}$ mode is not observed and the 475 cm<sup>-1</sup>  $E<sub>g</sub>$  mode is detected only slightly. The intensity of the  $B_{1g}$  mode  $I(B_{1g})$  is too low *vis-a-vis* the  $A_{1g}$  mode intensity  $I(A_{1g})$ , i.e.,  $I(B_{1g})$  <  $10^{-3}I(A_{1g})$ , <sup>26</sup> this makes its detection difficult. It is obvious that the Raman spectra of  $Ti_{1-x}Sn_xO_2$  vary with *x*. The 800 cm<sup>-1</sup>  $B_{2g}$  mode gets observable for  $x > 0.3$  and increases in intensity with  $x$ . It also appears that the  $610$  $cm^{-1} A_{1g}$  mode and the 449 cm<sup>-1</sup>  $E_g$  mode for TiO<sub>2</sub> change in frequency and linewidth with  $x$ ; another striking feature is that the 420 cm<sup>-1</sup>  $E_g$  mode splits for  $x \ge 0.7$ .

In order to demonstrate these spectral features, the Raman-active  $A_{1g}$  and  $E_g$  modes are blown up in Fig. 6. The frequency and linewidth of these modes were determined



FIG. 6. Blowup of the Raman-active  $E<sub>g</sub>$  and  $A<sub>1g</sub>$  modes for  $Ti_{1-x}Sn_xO_2$  with various *x*'s.

by fitting the data to Lorenzian curve  $L(\omega) = h_0 + h_1/[(\omega - \omega_0)^2 + \Gamma^2]$ , where  $h_0$  is the background intensity,  $h_1$  the intensity scale factor,  $\omega_0$  is the mode frequency, and  $\Gamma$  is the linewidth. It should be emphasized that fitting was nicely achieved, implying that the *x* dependence of the Raman-active modes with respect to frequency and linewidth is of great significance.

Figure 7 shows the *x* dependence of frequency and linewidth for the two Raman-active modes in question. No significant change in frequency occurs up to  $x=0.6$  for the  $A_{1g}$ 



FIG. 7. The frequency  $(a)$  and linewidth  $(b)$  of the two Ramanactive  $E_g$  and  $A_{1g}$  modes as a function of *x* for  $Ti_{1-x}Sn_xO_2$ .

TABLE I. Frequencies of the Raman- and infrared-active modes for  $SnO<sub>2</sub>$  and  $TiO<sub>2</sub>$ . The figures in parentheses represent the frequencies of the longitudinal infrared modes. The data of the Ramanand infrared-active modes are from Refs. 23 and 8 for  $SnO<sub>2</sub>$ , Refs. 26 and 30 for  $TiO<sub>2</sub>$ , respectively; note the data in Refs. 30 and 15 are comparable to each other.

			$B_{1g}$ $E_g$ $A_{1g}$ $B_{2g}$ $E_u$ $E_u$ $E_u$ $A_{2u}$	
			$SnO2$ 123 475 634 776 244 293 618 477	
			$(276)$ $(366)$ $(770)$ $(705)$	
			$TiO_2$ 143 447 612 826 183 388 500 167	
			$(373)$ $(458)$ $(806)$ $(811)$	

mode, then it considerably increases toward the  $A_{1g}$  mode frequency for  $\text{SnO}_2$  (628 cm<sup>-1</sup>). On the other hand, the  $E_g$ mode somewhat softens up to  $x=0.4$ , whereupon it increases slightly and splits for  $x \ge 0.7$ . In Fig. 7, each frequency of the split  $E_g$  modes is plotted as well.

The linewidth of both Raman-active modes increases up to  $x=0.6$ , and it subsequently decreases through a maximum at about  $x=0.5$ ; note that the linewidth of the  $E<sub>g</sub>$  mode is represented as the sum of each linewidth of the split  $E<sub>g</sub>$ modes for  $x \ge 0.7$ . It was remarked that the phase separation by spinodal decomposition takes place at some *x*'s before and after  $x=0.5$  (the spinodal region). Interestingly enough, the linewidth vs *x* curve for both Raman-active modes is similar to the phase diagram, which exhibits a miscibility gap with the maximum at about 50 mol %  $TiO<sub>2</sub>$  in the  $TiO<sub>2</sub>-SnO<sub>2</sub> system.<sup>4,5</sup>$ 

### **IV. DISCUSSION**

Setting aside a question whether the *x* dependence of the Raman-active modes for  $Ti_{1-x}Sn_xO_2$  is linear or nonlinear from the viewpoint of the mode behavior of mixed crystals, it is first surprising that the  $A_{1g}$  mode for  $SnO_2$  shows its higher frequency as compared with TiO<sub>2</sub>, in spite of  $SnO<sub>2</sub>$ being less compact than  $TiO<sub>2</sub>$ ; notice that the same situation applies to the  $E<sub>g</sub>$  mode too. This is at variance with the conception that the bond strength lessens and optical phonons soften, $^{29}$  if the lattice becomes more loose. In accordance to this conception, the Raman-active modes of symmetries  $B_{1g}$  and  $B_{2g}$  for  $SnQ_2$  are actually softer than the corresponding modes for  $TiO<sub>2</sub>$ .<sup>27,28</sup> Table I compares the frequencies of the Raman- and infrared-active modes for  $SnO<sub>2</sub>$ and  $TiO<sub>2</sub>$ . It obviously depends on the Raman- and infraredactive modes whether optical phonons soften or stiffen.

Figure 8 shows the rutile structure, and atom displacements when viewing along the *c* axis for the Raman- and infrared-active modes in this structure. This figure demonstrates the significant role of the  $TiO<sub>6</sub>$  octahedron, which is a building unit of the rutile structure. Here, it should be recalled that the compressibilities  $\kappa$  and thermal expansivities  $\beta$  of SnO<sub>2</sub> and TiO<sub>2</sub> are different and anisotropic, and that the ratios  $\beta_a/\beta_c(\kappa_a/\kappa_c)$  for SnO<sub>2</sub> are reversed from those for  $TiO<sub>2</sub>,<sup>24,26</sup>$  as compiled in Table II. This implies that the structures of  $SnO<sub>2</sub>$  and  $TiO<sub>2</sub>$  deform in opposite directions with changes in either temperature or pressure. In other words, the  $c/a$  ratio decreases with increasing temperature for  $SnO<sub>2</sub>$ whereas it increases for TiO<sub>2</sub>; the  $c/a$  ratio increases with



FIG. 8. Rutile structure and atom displacements when viewing along the *c* axis for the Raman- and infrared-active modes in this structure after Refs. 24 and 30.

increasing pressure for  $SnO<sub>2</sub>$  but decreases for  $TiO<sub>2</sub>$ . Straightforwardly, this means that the ions move easily along the *c* axis than the *a* axis for  $TiO<sub>2</sub>$  whereas the situation is quite reverse in  $SnO<sub>2</sub>$ . Based on these facts, it is considered that the octahedron fairly distorts so as to reproduce the reversed ratios  $\beta_a/\beta_c(\kappa_a/\kappa_c)$  between SnO<sub>2</sub> and TiO<sub>2</sub>, as Sn is substituted for Ti to form  $Ti_{1-x}Sn_xO_2$ . We would incline to regard that the octahedral distortion of  $TiO<sub>6</sub>$  is closely related to either softening or stiffening of the Raman- and infraredactive modes in  $Ti_{1-x}Sn_xO_2$ .

The bulk modulus of the octahedron is given by  $K_{\text{octa}} = 7.5S^2 Z_c Z_a / \langle d \rangle^3$ , where  $Z_c$  and  $Z_a$  are cation and anion formal charge,  $\langle d \rangle$  is the mean cation-anion bond distance, and  $S<sup>2</sup>$  is an empirical ionicity term equal to 0.5 for oxides. $31$  The data of the bond distances<sup>31</sup> yield 4.0 and 4.9

TABLE II. Compressibilities  $\kappa_{a,c}$  and thermal expansivities  $\beta_{a,c}$ of SnO<sub>2</sub> and TiO<sub>2</sub> after Ref. 26; the ratios  $\kappa_a/\kappa_c(\beta_a/\beta_c)$  are given as well. The volume expansion (compressibility) is equal to  $\beta_v(\kappa_v) = 2\beta_a(2\kappa_a) + \beta_c(k_c)$ .  $\kappa_{a,c}$ : *T*=296 K;  $\beta_{a,c}$ : *T*>350 K.

	$SnO2$ <sup>a</sup>	$TiO2$ <sup>a</sup>
$\kappa_a (10^{-4}/\text{kbar})$	1.3	1.93
$\kappa_c$ (10 <sup>-4</sup> /kbar)	1.9	0.8
$\kappa_a/\kappa_c$	0.68	2.41
$\beta_a (10^{-6} / \text{K})$	4.0	8.5
$\beta_c$ (10 <sup>-6</sup> /K)	3.7	11.0
$\beta_a/\beta_c$	1.08	0.77
$\kappa_v (10^{-4}/\text{kbar})$	4.5	4.73
$\beta_v (10^{-6}$ /K)	11.0	28.0

a Reference 26.



FIG. 9. The dip's position ( $\sim$ 450 cm<sup>-1</sup> for TiO<sub>2</sub>) between the two  $E_u$  modes in the infrared-reflectance spectra of  $Ti_{1-x}Sn_xO_2$  is plotted as a function of *x*.

as  $K_{\text{octa}}$ 's of TiO<sub>2</sub> and SnO<sub>2</sub>, respectively. This indicates that the octahedron of  $SnO<sub>6</sub>$  is stiffer than TiO<sub>6</sub>, which may account for the higher frequency of some optical phonons for SnO<sub>2</sub> in reference to TiO<sub>2</sub>.

However, the change in the six-bond distances and/or bond angles for TiO<sub>6</sub> with *x* in Ti<sub>1-x</sub>Sn<sub>*x*</sub>O<sub>2</sub> is anisotropic or complex, reflecting the reversed ratios  $\beta_a/\beta_c(\kappa_a/\kappa_c)$  between  $\text{SnO}_2$  and  $\text{TiO}_2$  such that either softening or stiffening of optical phonons can be induced. Simply from a mass change of the metal-O bonds upon substitution, optical phonons are expected to soften because the frequency  $v(x)$ is given by  $v(x) = [f(x)/\mu(x)]^{1/2}$ , where  $f(x)$  is the force constant,  $\mu(x)$  is the reduced mass defined by  $\mu(x)^{-1} = (1-x)/m_{\text{Ti}} + x/m_{\text{Sn}}$ ; note  $m_{\text{Ti}} < m_{\text{Sn}}$  with  $m_{\text{Ti(Sn)}}$ being the atomic mass of Ti or Sn.

As for the *x* dependence of the infrared-active modes for  $Ti_{1-x}Sn_xO_2$ , the dip's position (~450 cm<sup>-1</sup> for TiO<sub>2</sub>) between the two  $E_u$  modes is plotted as a function of x in Fig. 9. It can be seen that the dip's position changes only slightly up to  $x=0.6$ , whereupon its significant reduction follows exhibiting a clear nonlinear dependence; the positional change of this dip can be regarded as the  $E_u$  mode shift in frequency with *x*.

As far as we are concerned with Figs. 7 and 9, the *x* dependence of optical phonons in the  $TiO<sub>2</sub>-SnO<sub>2</sub>$  system is not linear with respect to frequency or linewidth, ruling out one-mode behavior, in which we should predict a linear relationship between the mode frequencies of each end member. In the random-element-isodisplacement models,  $32-36$  it is assumed that the three force constants  $f_{ij}(x)$  in a mixed crystal  $AB_{1-x}C_x$  (*i* j = AB, AC, BC) have the same compositional dependence such that  $f_{ii}(x) = f_{ii}(0)[1 + \Theta x]$ , where  $f_{ii}(0)$  is the force constant at  $x=0$  and  $\Theta$  is a constant which describes the effect of lattice parameter change on the force constants. The lattice parameters change nonlinearly with *x* for  $Ti_{1-x}Sn_xO_2$ , supporting positive deviations from Vegard's law in the  $TiO<sub>2</sub>$ -SnO<sub>2</sub> system.<sup>4</sup> This urges us to consider that the constant  $\Theta$  depends on  $x$ , <sup>36,37</sup> and consequently the nonlinear *x* dependence of optical phonons is predicted through the *x*-dependent constant  $\Theta$ , ruling out one-mode behavior for  $Ti_{1-x}Sn_xO_2$ ; we have already commented that the nonlinear change in lattice parameters is due to the phase separation by spinodal decomposition.

Finally, we must give any probable cause for splitting of the  $E_g$  mode for  $x \ge 0.7$ . Inspection of Fig. 6 reveals that the 470 cm<sup>-1</sup> mode of the split  $E_g$  modes corresponds to the  $E_g$ mode for  $SnO<sub>2</sub>$  (subtly visible) in Fig. 6. This assures that  $TiO<sub>2</sub>$ - and  $SnO<sub>2</sub>$ -rich phases, which may exist microscopically in  $Ti_{1-x}Sn_xO_2$  with higher *x*'s, can be detected by Raman scattering.

# **V. CONCLUSIONS**

The concentration dependence of optical phonons in the  $TiO<sub>2</sub>-SnO<sub>2</sub>$  system has been studied by Raman scattering and Fourier-transform infrared spectroscopy. The 610 cm<sup>-1</sup>  $A_{1g}$ mode and the 449 cm<sup>-1</sup>  $E_g$  mode for TiO<sub>2</sub> exhibited a nonlinear *x* dependence with respect to frequency and linewidth. It is stressed that the octahedral distortion of  $TiO<sub>6</sub>$  predominates the *x* dependence of optical phonons in  $Ti_{1-x}Sn_xO_2$ , whose end members deform in opposite directions with changes in either pressure or temperature because of their different/anisotropic compressibilities and thermal expansivities. The infrared-reflectance spectra of  $Ti_{1-x}Sn_xO_2$  also changed systematically with *x*. The concentration dependence of optical phonons is discussed from the viewpoint of the mode behavior of mixed crystals, ruling out one-mode behavior for  $Ti_{1-x}Sn_xO_2$ . It is concluded that the nonlinear *x* dependence of optical phonons is predicted through the *x*-dependent term which describes the effect of lattice parameter change on the force constants. Actually, x-ray diffraction showed the nonlinear change in lattice parameters, supporting positive deviations from Vegard's law in the  $TiO<sub>2</sub>-SnO<sub>2</sub>$ system. It was also found that  $Ti_{1-x}Sn_xO_2$  with some *x*'s  $(x=0.3, 0.4, 0.5,$  and 0.6) could phase separate by spinodal decomposition, as evidenced by the sidebands flanked with Bragg reflections in x-ray-diffraction profiles. The present work somewhat highlights the effect of the phase separation on the *x* dependence of optical phonons for  $Ti_{1-x}Sn_xO_2$ .

- <sup>1</sup>G. McCarthy and J. Welton, J. Powder Diffraction 4, 156 (1989).
- $2$ W. H. Baur and A. A. Khan, Acta Crystallogr. B  $27$ , 2133 (1971). <sup>3</sup>*Handbook of Chemistry and Physics*, 53rd ed., edited by R. C.
- Weast (Chemical Rubber, Cleveland, 1972–1973).
- 4M. Park, T. E. Mitchell, and A. H. Heuer, J. Am. Ceram. Soc. **58**, 43 (1975).
- <sup>5</sup>D. Garcia and D. Speidel, J. Am. Ceram. Soc. 53, 322 (1972).
- <sup>6</sup>T. Hirata, M. Kitajima, K. G. Nakamura, and E. Asari, J. Phys. Chem. Solids 55, 349 (1994).
- <sup>7</sup> V. A. Maroni, J. Phys. Chem. Solids **49**, 307 (1988).
- 8R. S. Katiyar, P. Dawson, M. M. Hargreave, and G. R. Wilkinson, J. Phys. C 4, 2421 (1971).
- 9D. W. Taylor, in *Optical Properties of Mixed Crystals*, edited by R. J. Elliot and I. P. Ipatova (Elsevier Science, Amsterdam, 1988), p. 35.
- <sup>10</sup>T. Hirata, Phys. Rev. B **50**, 2874 (1994).
- 11V. S. Stubican and A. H. Schultz, J. Am. Ceram. Soc. **53**, 211  $(1970).$
- $12$  P. K. Gupta and A. R. Cooper, Philos. Mag. 21, 611 (1970).
- 13M. Ocana, J. V. Garcia-Ramos, and C. J. Serna, J. Am. Ceram. Soc. 75, 2010 (1972).
- 14M. Ocana, V. Fornes, J. V. Garcia Ramos, and C. J. Serna, J. Solid State Chem. **75**, 364 (1988).
- $^{15}$ F. Gervais and B. Piriou, Phys. Rev. B 10, 1642 (1974).
- 16W. G. Spitzer, R. C. Miller, D. A. Kleinman, and L. E. Howarth, Phys. Rev. 126, 1710 (1962).
- 17U. Balachandran and N. G. Eror, J. Solid State Chem. **42**, 276  $(1982).$
- <sup>18</sup> J. F. Mammone, S. K. Sharma, and M. Nicol, Solid State Commun. 34, 799 (1980).
- <sup>19</sup> Y. Hara and M. Nicol, Phys. Status Solidi B 94, 317 (1979).
- $^{20}$ H. Arashi, J. Phys. Chem. Solids **53**, 355 (1992).
- <sup>21</sup> J. F. Mammone, N. Nicol, and S. K. Sharma, J. Phys. Chem. Solids **42**, 379 (1981).
- 22R. J. Betsch, H. L. Park, and W. B. White, Mater. Res. Bull. **26**, 613 (1991).
- 23S. P. S. Porto, P. A. Fleury, and T. C. Damen, Phys. Rev. B **154**, 522 (1967).
- <sup>24</sup> G. A. Samara and P. S. Peercy, Phys. Rev. B 7, 1131 (1973).
- 25U. Balachandran and N. G. Eror, J. Solid State Chem. **42**, 276  $(1982).$
- <sup>26</sup>P. S. Peercy and B. Morosin, Phys. Rev. B 7, 2779 (1973).
- $27$  J. F. Scott, J. Chem. Phys. **53**, 852 (1970).
- 28Y. S. Hung and Fred H. Pollak, Solid State Commun. **43**, 921  $(1982).$
- <sup>29</sup> I. L. Botto, E. J. Baran, C. Cascales, I. Rasines, and R. Saez Puche, J. Phys. Chem. Solids **52**, 431 (1991).
- <sup>30</sup>D. M. Eagles, J. Phys. Chem. Solids **25**, 1243 (1964).
- 31R. M. Hazen and L. W. Finger, J. Phys. Chem. Solids **42**, 143  $(1981).$
- 32Y. S. Chen, W. Shockley, and G. L. Pearson, Phys. Rev. **151**, 648  $(1966).$
- <sup>33</sup>M. Ilegems and G. L. Pearson, Phys. Rev. B 1, 1576 (1970).
- 34G. Lucovsky, K. Y. Cheng, and G. L. Pearson, Phys. Rev. B **12**, 4135 (1975).
- 35G. Lucovsky, R. D. Burnham, and A. S. Alimonda, Phys. Rev. B **14**, 2503 (1976).
- 36D. L. Peterson, A. Petrou, W. Giriat, A. K. Ramdas, and S. Rodriguez, Phys. Rev. B 33, 1160 (1986).
- 37P. D. Lao, Yile Guo, G. G. Siu, and S. C. Shen, Phys. Rev. B **48**, 11 701 (1993).