Quantitative analysis of Josephson-quasiparticle current in superconducting single-electron transistors

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We have investigated Josephson-quasiparticle (JQP) current in superconducting single-electron transistors in which charging energy E_C was larger than superconducting gap energy Δ and junction resistances were much larger than $R_Q \equiv h/4e^2$. We found that not only the shapes of the JQP peaks but also their absolute height were reproduced quantitatively with a theory by Averin and Aleshkin using a Josephson energy of Ambegaokar-Baratoff's value.

Recently "mesoscopic superconductivity" has attracted much attention particularly to the competition between Josephson energy and Coulomb energy which causes many intriguing phenomena due to the mutually conjugate nature of phase and charge.[1] For a single Josephson junction, there have been several theoretical studies[2–4] on modifications of the Cooper pair tunneling Hamiltonian from the effective adiabatic one $-E_J^{AB}\cos\phi$ because of the charging energy, where E_J^{AB} is the Josephson coupling energy calculated by Ambegaokar and Baratoff,[5] and ϕ is the phase difference at the junction. Matveev *et al.*[6] also predicted an enhancement of Josephson energy E_J in a voltage biased superconducting single-electron transistor (S-SET) [Fig. 1(a)] due to the charging energy $E_C \equiv e^2/2C_\Sigma$, where $C_\Sigma \equiv C_1 + C_2 + C_g$. However, there have been neither experimental observations of such effects, nor any quantitative evaluations of E_J from experiments yet, which is an ultimate aim of this work.

In a voltage-biased S-SET, Cooper pair tunneling causes several current transport mechanisms. The most extensively studied one is the "supercurrent" around zero bias voltage, in which Cooper pairs tunnel through both junctions. In spite of a theoretical prediction that gives maximum supercurrent $I_c^{max} = eE_J/\hbar$ for $E_J \ll E_C$,[7] experimentally observed ones were smaller than that value.[8,9] It is necessary to assume a proper environmental parameter to explain this descrepancy,[8] which makes it hard to evaluate E_J quantitatively. Moreover, sometimes they had finite resistance at zero bias voltage and appeared as a current peak at small but finite voltage.[10,11] Small impedance on the leads and finite temperature may easily make Cooper pair tunneling incoherent and degrade the supercurrent as predicted for the case of single junctions.[12,13] In addition, it has been noticed that an unwanted quasiparticle tunneling into the island poisons the supercurrent in S-SET severely.[6,8] For that reason, supercurrent of a order of I_c^{max} can be expected only for S-SET with $E_C < \Delta$. So in general, for S-SET with large E_C it is difficult to study the Josephson current at zero bias voltage.

For this purpose, it is better to focus on the other current channel that involving Cooper pair tunneling, namely Josephson-quasiparticle (JQP) current.[14–18] JQP current is carried by a cycle which consists of one Cooper pair tunneling through one junction and two quasiparticle tunneling through the other junction [Fig. 1(b)]. Due to a resonantlike nature of Cooper pair tunneling, the JQP cycle produces current peaks in the current versus voltage (I-V) and the current versus gate voltage (I-V_g) characteristics.

In the bias configuration as in Fig. 1, the resonant condition for Cooper pair tunneling at the left junction is fulfilled if

$$eV = \frac{1}{2}[E(n) - E(n-2)] = \frac{e}{C_\Sigma}[-Q_0 + (n-1)e], \quad (1)$$

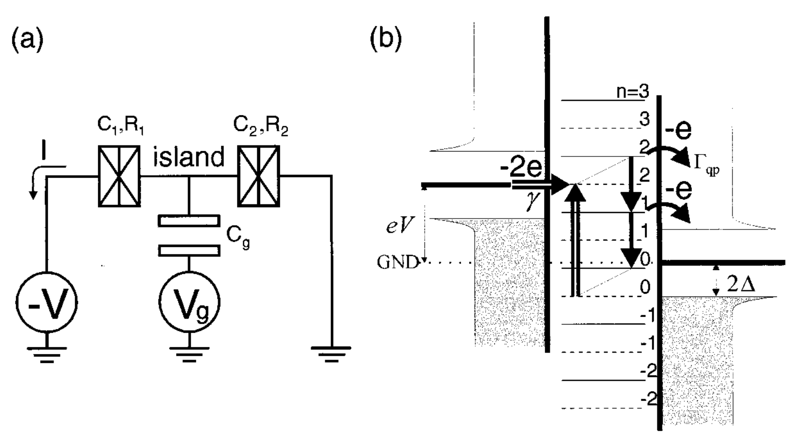

FIG. 1. (a) A schematic of S-SET. (b) An energy diagram of the JQP cycle. The arrows indicate quasiparticle tunneling and the double arrow represents Cooper pair tunneling.

where $E(n)$ is the electrostatic energy of the island with n excess electrons and $Q_0 \equiv -C_1 V + C_g V_g$ is the polarization charge of the island. For Cooper pair tunneling at the right junction, eV at the left-hand side of (1) should be replaced with 0. The right-hand side of (1) is the increase of the electrostatic energy of the island due to the Cooper pair tunneling divided by 2 in order to normalize for one tunneling electron, which we drew in Fig. 1(b) as dashed levels. On the other hand, a quasiparticle can tunnel, for example, through the right junction from the island to the right electrode at $T=0$ if

$$2\Delta < E(n) - E(n-1) = \frac{e}{C_\Sigma}\left[-Q_0 + \left(n - \frac{1}{2}\right)e\right]. \qquad (2)$$

The right-hand side is also drawn as solid levels in Fig. 1(b). Note that they differ from those for Cooper pair tunneling by E_C. The 2Δ at the left-hand side is the sum of the superconducting gaps in the density of states both in the island and the right electrode, and shown in Fig. 1(b) as the gap energy of convoluted density of states. We can easily understand from Fig. 1(b) that the JQP cycle occurs when the condition (1) is fulfilled for bias voltages $(2\Delta + E_C)/e < V < (2\Delta + 3E_C)/e$.[14] For the case of $E_C > \frac{2}{3}\Delta$, the JQP cycle also occurs for $V > (2\Delta + 3E_C)/e$, coexisting with quasiparticle current.[18]

Theoretically the JQP cycle was treated quantitatively by Averin and Aleshkin.[15] They deduced master equations to describe the transport process in S-SET from the density matrix approach under assumptions of $E_J \ll E_C$, $R_N \gg R_Q$, where R_N is the normal resistance of the junctions and $R_Q \equiv h/4e^2 \simeq 6.45$ kΩ. According to their theory and at $T=0$, the Cooper pair tunneling rate γ is determined as

$$\gamma = \frac{\Gamma_{\rm qp} E_J^2}{4\delta^2 + (\hbar\Gamma_{\rm qp})^2}, \qquad (3)$$

where $\Gamma_{\rm qp}$ $[\simeq (eV+E_C)/e^2 R_2]$ is the rate of the first quasipariticle tunneling in Fig. 1(b) which dephases the coherence of the Cooper pair tunneling and δ is the energy difference of the system before and after the Cooper pair tunneling [$\delta=0$ in Fig. 1(b)]. Solving the master equations, we can calculate the JQP current. Especially in the limit of $\gamma \ll \Gamma_{\rm qp}$, the JQP cycle is bottlenecked by the Cooper pair tunneling process and gives JQP peak current $\sim 2eE_J^2/\hbar^2\Gamma_{\rm qp}$ and the peak width $\sim \hbar\Gamma_{\rm qp}$ in energy scale. Moreover, it can be expected that, in contrast to the nearly coherent supercurrent, JQP current is not sensitive to further decoherencing factors such as small environmental impedance and finite temperature because it is already dephased much by $\Gamma_{\rm qp}$ and $\Gamma_{\rm qp}$ itself (and additonal tunneling terms at finite temperatures) does not vary so much at low temperatures ($\ll \Delta/k_B$).

Under the above limit of incoherence, it resembles the perturbative theory of the current through a voltage-biased single Josephson junction with resistive leads.[12] For a large environmental lead resistance ($\gg R_Q$) the current peak due to incoherent process can be treated properly with perturbative approach, giving a current peak also proportional to E_J^2. However, an experimental investigation on such devices has given an inconclusive result on E_J.[19] On the other hand, in the JQP process, one of the tunnel junctions works as a resistor which we can characterize very precisely from experiment even though it has tunneling nature and nonlinearity. We could fit the experimental results without fitting parameters.

Another advantage in studying JQP peaks against supercurrent measurement is that the JQP peaks survive even for $E_C > \Delta$. This is from the fact that, JQP cycle is essentially a nonequilibrium process. It is tough against any unwanted quasiparticle tunneling, for example, due to the nonequilibrium quasiparticles in the superconducting lead. The quasiparticle will be disposed to the drain immediately, which is in contrast to the case of equilibrium supercurrent where the quasiparticle will be trapped in the island destroying supercurrent.

Taking these advantages, we report the first measurement and the detail analysis of JQP current in S-SET's with $E_C > \Delta$. Comparing the experimental results with the theory,[15] we got a quantitative agreement between them assuming $E_J = E_J^{\rm AB}$.

Al-based single-electron transistors were made with e-beam lithography and the standard two-angle evaporation method. Samples had asymmetric double barrier structure with a gate electrode, replacing one of the junction with superconducting quantum interference device (SQUID)-type parallel junctions, though here we focus on the zero magnetic field characteristics where we can consider the SQUID as a single junction. Four-probe voltage-biased dc transport measurements were performed in a dilution refrigerator at a temperature of 20 mK. In our cryostat measurement leads consisted of constantan twist-pair cables (total resistance $\sim 200\ \Omega$) with RC filters at room temperature. Samples were biased and measured with custom-made battery-powered analog circuits.

Most of the sample parameters were obtained directly from measurement. For example for sample 1 we got $C_1 = 210$ aF, $C_2 = 117$ aF, $C_g = 3.15$ aF, $R_1 = 105$ kΩ, $R_2 = 135$ kΩ, $E_C = 240$ μeV, and $\Delta = 198$ μeV. We could know $R_1 + R_2$ from the total normal resistance at high bias voltage above 10 mV, e/C_g from the period of gate modulation, C_g/C_1 and $C_g/(C_2 + C_g)$ from the modulation of the Coulomb blockade threshold, and 4Δ from the minimum quasiparticle tunneling gap voltage. Only the ratio of the two resistances of serial tunnel junctions R_1/R_2, which was difficult to know accurately from any features, was determined by fitting as we will mention below. For each sample E_C was larger than Δ. $E_{Ji}^{\rm AB}$, E_C, and R_N as well as $\Gamma_{\rm qp}$ fulfilled the requirements to use the Averin-Aleshkin theory.

In Fig. 2(a) we showed the I-V_g curves for sample 1. Each curves were taken for the various bias voltages and shifted by 100 pA each other for clarity. All the structures observed were e periodic, i.e., had the same period as the Coulomb oscillation in the normal state. We could see that small peaks make lines in the gap region, which were attributed to JQP ones. We also plotted the peak positions on a bias-voltage versus gate voltage (V-V_g) plane in Fig. 2(b). The peaks were on two groups of parallel lines, which can be considered as the resonant conditions for Cooper pair tunneling through either junctions [Eq. (1)] and be consistent with the capacitances estimated from the gate modulations of quasiparticle current. The bias voltages at which these lines cross each other corresponded to $2nE_C/e$ where n is integer.

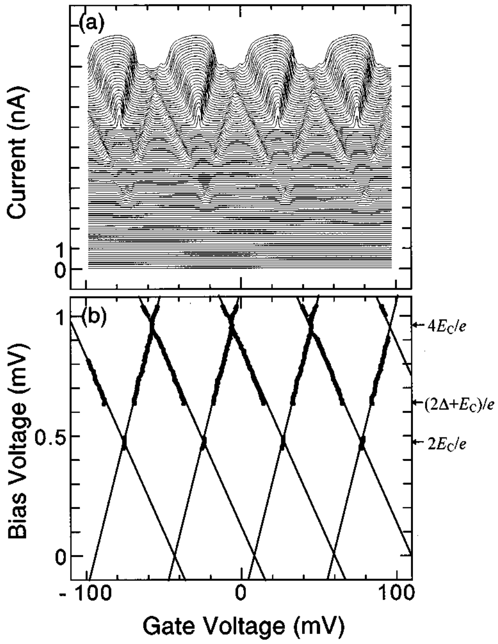

FIG. 2. (a) The experimental I-V_g curves of sample 1 for various bias voltages. (b) The JQP peak positions on V-V_g plane. Solid lines show the resonance conditions for Cooper pair tunneling.

Sudden appearance of the peaks above $V \simeq 640$ μV corresponded to the condition to complete JQP cycle, that is, $2\Delta + E_C < V$. Those arrays of peaks extended to $V = (2\Delta + 3E_C)/e$, above where large quasiparticle current coexisted with them.[18]

In Fig. 3, we plotted two I-V_g curves of sample 1 at bias voltages of (a) $V = 650$ μV and (b) $V = 750$ μV. The dots representing experimental results have two JQP peaks for each period of $\Delta V = e/C_g \simeq 51$ mV. The different peak heights of the two peaks were due to the asymmetry of the two junctions. As the solid lines, we calculated the I-V_g curves for the device according to the theory by Averin and Aleshkin.[15] In the calculation, we assumed an ideal BCS gap structure, $T = 0$, and $E_J = E_J^{\mathrm{AB}}$ $[=(\Delta/2)(R_Q/R_N)]$ and used the parameters above. The only adjustable fitting parameter was the ratio of the tunnel resistances in series R_1/R_2 to fit the ratio of the peak heights. We have added to the calculated values experimentally evaluated background current of 6 and 20 pA for $V = 650$ and 750 μV, respectively. The observed shapes of JQP peaks were reproduced very well by the calculations, indicating that the theory works well. For example, the small asymmetry in the observed peaks in Fig. 3(a) could be understood as a reflection of the BCS gap edge. This feature occurred because the second quasiparticle transition in Fig. 1(b) was suddenly forbidden when the edge of the density of states became higher than the energy level in the island as we sweeped the gate voltage. For $V = 750$ μV, where the BCS gap edge only affected the current far away from the JQP peak, the JQP peaks became more symmetric. The small bumps that appeared between two JQP peaks might be due to higher-order processes though we have not analyzed them in detail.

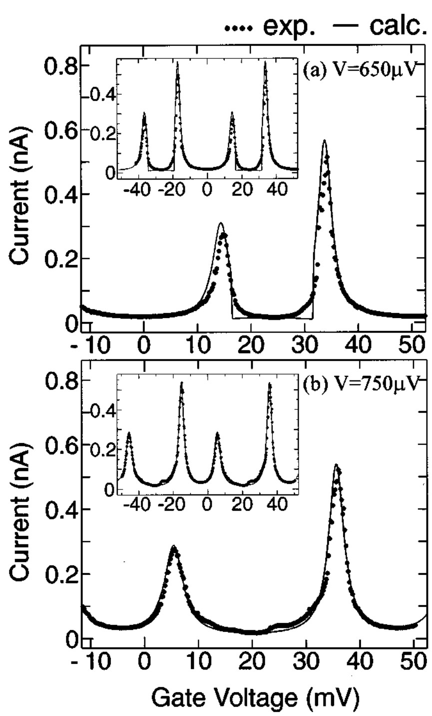

FIG. 3. The I-V_g curves of sample 1 for (a) $V = 650$ μV and (b) $V = 750$ μV. Solid curves shows the calculated results. Insets show the curves for wider gate voltage ranges.

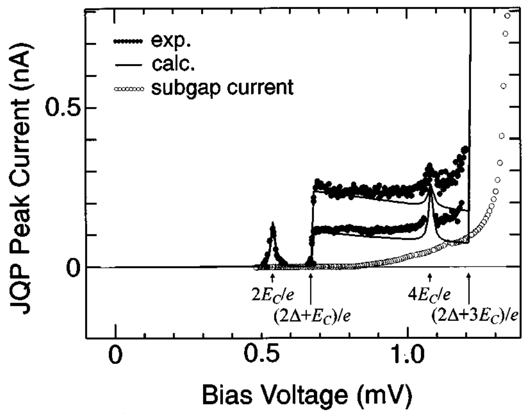

FIG. 4. The JQP peak height of sample 2 along the Cooper pair resonant conditions as a function of bias voltage. Solid curves are the calculated ones. We also plotted minimum current in the I-V_g characteristics as a guide for subgap leakage current.

Figure 4 shows the bias-voltage dependence of JQP peak current along the resonant conditions of Cooper pair tunneling such as shown in Fig. 2(b) for sample 2 with $E_C = 270$ μeV and $\Delta = 202$ μeV . Two curves corresponds to each junction where the resonance occurs, respectively. Again the theoretical curves fitted the experimental data very well, particular for the lower bias voltages. Also in this sample the JQP peaks with $2e$-e-e process existed for $V > (2\Delta + E_C)/e$. The peak at $V = 4E_C/e$ was due to the simultaneous fulfilment of the resonant conditions for both junctions, while another peak at $V = 2E_C/e$ was due to a process called the $3e$ process where alternative transitions of $2e$ and e carried the current.[16]

Except those singular points, the theory predicted a decreasing JQP peak height as increasing bias voltage, that is,

$I_{\rm JQP} \propto 1/\Gamma_{\rm qp} \propto 1/(V + E_C/e)$. Although the decreasing behavior of $I_{\rm JQP}$ was not clear in Fig. 4, we believe that the increasing background subgap current obscured the dependence. Indeed from the magnetic modulation of $I_{\rm JQP}$ we found that the precise JQP contribution decreased with increasing bias voltage (data not shown)[20] and further confirmed the theoretical prediction.

In the comparisons betweeen the experimental and the theoretical results above, we assumed $E_J = E_J^{\rm AB}$ and used no scaling factor. The good theoretical fitting indicates that the assumption was rather good. It is in contrast to the case of the supercurrent where the observed ones were smaller than the expected value. Perhaps the insensitivity of the JQP cycle to the external electromagnetic environment and thermal excitation as well as to the unwanted quasiparticle tunneling enabled us to observe the current close to its intrinsic value. It is also worth pointing out that in our samples $E_C > \Delta$ and $R_N \gg R_Q$. Thus E_J and $\Gamma_{\rm qp}$ were much smaller than E_C, which was desirable for the theoretical assumption.

Finally we would like to mention the predicted enhancement of E_J [Eq. (3) in Ref. 6], where the charging effect reduces the energy of the intermediate state of a Cooper pair tunneling by E_C and thus increases E_J by a factor F related to the ratio E_C/Δ. Even though Matveev *et al.*[6] applied the concept to the case of $V=0$, we think that it can be also useful in the analysis of JQP current where Cooper pair tunneling occurs between two degenerate charge states in spite of the finite total voltage drop. We have measured six samples with various E_C/Δ ranging from 1.03 to 1.58, where F should be increased about 1.3 to 1.7. However, if we considered E_J as a fitting parameter for the JQP peak height, $F = E_J/E_J^{\rm AB}$ sat within 30% of unity and showed no increasing behavior with E_C/Δ. In addition, for the case of $\delta \neq 0$, the factor F should be dependent on V_g, which would modify the shape of the JQP peaks. However, our experiment showed good agreement in the peak shape with the calculation with constant E_J. In this experiment any evidence of E_J enhancement could not be observed, though the reason is not yet clear.

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