# Green's-function theory for row and periodic defect arrays in photonic band structures

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The defect modes associated with a row or a periodic array of defects in a two-dimensional photonic band structure are studied using exact Green's-function methods. Specifically, we consider the above-described defect problems created in a pure photonic band-structure system that models an experimental system recently investigated by McCall *et al.* This is a square lattice array of cylindrical rods formed of linear dielectric material with $\epsilon=9$ surrounded by vacuum and of filling fraction $f=0.4488$. In one study a row of rods is replaced in the photonic band structure by a row of impurity rods that may be of either linear or nonlinear dielectric material, and conditions are determined for the existence of impurity modes, associated with the impurity array, in the photonic band gap. The electric fields of the impurity modes are computed for impurity modes in the gaps. In a second study, a two-dimensionally periodic array of defects is introduced into the photonic band structure such that the periodicity of the defect array may or may not coincide with the periodicity of the original photonic band structure. The defect rods are treated in the cases for which the impurities are formed from both linear and nonlinear dielectric materials, and the conditions necessary for defect modes to exist in the photonic band gaps are determined.

Recently there has been considerable interest in defects in photonic band structures as related to laser or optical circuit applications, and a number of different defect types have been studied in these regards including single-site defects, multiple-site defects, rows of defects, and surfaces.[1–9] The goal of these studies has been the determination of the conditions under which the impurity modes exist for frequencies in the band gaps of photonic band structures and the examination of practical uses which can be made of such impurity structures. Calculations of impurity modes have been performed using supercell methods[1–3] which are based on computer simulation techniques and exact Green's-functions methods,[5–8] which yield solutions of impurity problems in terms of the eigenvalues and eigenfunctions of the photonic band structure in the absence of impurity dielectric material. Most of the early calculations of impurity modes have been performed using supercell methods[1–4] with exact Green's function calculations[5–8] of the type discussed in this paper only appearing in the last two to three years. While exact Green's-function methods have been applied to single and finite clusters of defects, these techniques have not been applied to infinite clusters of defects, and in this paper we present results for the exact Green's-function method applied to rows of defects and to periodic arrays of defects in two-dimensional (2D) photonic band structures formed from an array of cylindrical rods. Both of these defect systems then involve an infinite number of impurity rods. The conditions needed for defect modes to exist at frequencies in the band gaps of the photonic band structures are determined and expressions for the electric fields of these defect modes are given. In both of these studies, impurities formed from both linear and nonlinear dielectric media will be treated.

The application of exact Green's-function methods to the study of defect modes in photonic band structures was given by Maradudin and McGurn[5] for a two-dimensional photonic band structure formed from an array of cylindrical dielectric rods. In this work a calculation was given for the particular single-site defect structure, studied experimentally by Smith *et al.*,[4] in which a single rod was removed from an otherwise square lattice array of dielectric rods. Only defect modes with the electric field polarized parallel to the axes of the rods were considered and good agreement between theory and experiment was found. In Algul *et al.*[7] the general problem of a single dielectric impurity rod or a cluster of five dielectric impurity rods with general impurity dielectric constant in a square lattice truncated photonic band structure was considered. (The system is truncated as the photonic band structure is bounded between two perfectly conducting plates which perpendicularly intersect the axes of the dielectric rods forming a square lattice array.) A method was then given by Algul *et al.* which allowed for the determination of the impurity dielectric constants needed to observe impurity modes of a given frequency in the band gaps of the photonic band structure. In general it was found that for a fixed impurity mode frequency, the solution for the impurity mode dielectric contrast was multiple valued. This represented an advancement over other non-Green's-function-based methods for the study of the single impurity problem as non-Green's-function-based methods do not readily yield the multiple-valued impurity dielectric constants corresponding to a given impurity mode frequency. More recently, in McGurn and Khazhinsky[8] it was shown how the single impurity problem solutions for the impurity dielectric constant determined as a function of the impurity mode frequency for one-dimensional (slabs) and two-dimensional (square lattice array of dielectric rods) photonic band structures could be used to solve the problem of a single impurity formed from frequency-dependent dielectric material in photonic band structures otherwise composed from frequency-independent dielectric materials. In particular, results were presented for GaAs impurities in photonic band structures formed from air and aluminum composites. One feature common to all of these single impurities and finite clusters of impurities is that the impurity modes at frequencies in the band gaps are lo-

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calized about the impurity media with fields that decay quickly as one leaves the site of the impurity media.

In the work discussed below we give an exact Green's-function treatment of a new type of impurity problem. Whereas past Green's-function works considered impurities obtained by replacing one or five dielectric rods in a two-dimensional photonic band structure by impurity rods, in the work presented below we replace an infinite number of rods in a two-dimensional photonic band structure by impurity rods. In particular, in one problem we replace a row of rods in the two-dimensional photonic band structure by a row of impurity rods. The row of impurity rods is found to act as a type of wave guide which propagates waves through the system at frequencies which are in the stop bands of the photonic band structure in the absence of impurities. We solve for the impurity dielectric constant as a function of impurity mode frequency and wave vector, obtaining multiple-valued solutions not previously obtained by other means. Results for the electric field are also presented. In a second problem we replace a two-dimensional periodic array of rods in the two-dimensional photonic band structure by impurity rods. (This system has not been previously been considered by any other means.) Again propagating modes are associated with this new impurity type. Solutions for the impurity dielectric constant as a function of impurity mode frequency and wave vector in the band gaps are given. In both cases considered, expressions of the electric fields about the impurity media are given, and results for nonlinear and frequency-dependent impurity media are presented. We will begin by first treating a system with a row of defects, and then in a second study we shall consider two-dimensional periodic arrays of defects in photonic band structures.

We consider a two-dimensional photonic band structure which in the absence of impurities is formed of infinitely long cylindrical rods of circular cross section, composed from linear dielectric material. The axes of the rods (taken to be parallel to the $x_3$ axis) are arrayed in a square lattice of lattice constant $a$ in the $x_1-x_2$ plane such that the position vectors of the rod axes are given by $\mathbf{r}_{n,m}=(n\hat{x}_1+m\hat{x}_2)a$, where $n$, $m$ are integers and the rods are surrounded by vacuum. The dielectric constant of the rods is $\epsilon=9$, their filling fraction is $f=0.4488$, and only modes propagating perpendicularly to the rod axes with electric field $\mathbf{E}$ polarized parallel to the axes of the rods are considered. This system corresponds to an experimental system recently studied by McCall *et al.*[9]

In the absence of defects or other dielectric modifications, the periodic dielectric constant of the above described system $\epsilon(\mathbf{x})$ is defined by

$$\epsilon(\mathbf{x})=\begin{cases}\epsilon, |\mathbf{x}-\mathbf{r}_{n,m}|<r, & \text{for some } n,m \text{ integers}\\ 1, & \text{otherwise}\end{cases} \tag{1}$$

where $\mathbf{x}=x_1\hat{x}_1+x_2\hat{x}_2$, $r$ is the radius of the cylinders in our system, and we take $\epsilon=9$ for the results presented below. In the presence of defects or any other modification to the dielectric properties of the system in Eq. (1), we can write the total dielectric constant $\epsilon_{\text{TOTAL}}(\mathbf{x})=\epsilon(\mathbf{x})+\delta\epsilon(\mathbf{x})$, where $\delta\epsilon(\mathbf{x})$ is the change from $\epsilon(\mathbf{x})$. Proceeding in the usual way,[5,7,8] we find that the electric field $E_3(\mathbf{x})$ of the impurity system is given by

$$E_3(\mathbf{x})=\left(\frac{\omega}{c}\right)^2\int dx'^2\; G(\mathbf{x},\mathbf{x}')\,\delta\epsilon(\mathbf{x}')E_3(\mathbf{x}'), \tag{2}$$

where

$$G(\mathbf{x},\mathbf{x}')=\sum_{\mathbf{k},j}\frac{\psi_k^{(j)}(\mathbf{x})\,\psi_k^{(j)*}(\mathbf{x}')}{\lambda_k^{(j)}-\left(\frac{\omega}{c}\right)^2}, \tag{3}$$

and $\psi_k^{(j)}(x),\lambda_k^{(j)}$ are the orthonormal eigenvectors and eigenvalues of the pure (defect-free) photonic band structure obtained from

$$[\nabla^2+\epsilon(x)\lambda_k^{(j)}]\psi_k^{(j)}(\mathbf{x})=0. \tag{4}$$

In Eq. (4), $\mathbf{k}=(k_1,k_2)$ is the wave vector in the reduced zone scheme and $j$ is a band index label.

Let us consider the addition of a row of defects to the pure photonic band structure. Specifically, we take

$$\delta\epsilon(\mathbf{x})=\Delta\sum_{l=-\infty}^{\infty} f(\mathbf{x}-l\mathbf{S}_{\mathbf{n},\mathbf{m}}), \tag{5}$$

where $\mathbf{S}_{n,m}=[n\hat{x}_1+m\hat{x}_2]a$ for $n$ and $m$ integers, and $f(\mathbf{x})$ is an arbitrary continuous function of $\mathbf{x}$ within the primitive lattice cell centered about $\mathbf{r}_{0,0}$ but zero outside the primitive lattice cell (PLC) centered about $\mathbf{r}_{00}$. Substituting Eq. (5) into Eq. (2), we find for $\mathbf{y}$ in the primitive lattice cell centered on $\mathbf{r}_{00}$,

$$E_3(\mathbf{y}+l\mathbf{S}_{n,m})=\Delta\int_{\text{PLC about }\mathbf{r}_{0,0}} dy'^2\left(\frac{\omega}{c}\right)^2\sum_h G(\mathbf{y}+l\mathbf{S}_{n,m},\mathbf{y}'+h\mathbf{S}_{n,m})f(\mathbf{y}')E_3(\mathbf{y}'+h\mathbf{S}_{n,m}), \tag{6}$$

where $l$ and $h$ range over the integers.

If we write

$$E_3(\mathbf{y}+l\mathbf{S}_{n,m})=e^{i\mathbf{q}\cdot l\mathbf{S}_{n,m}}E_{30}(\mathbf{y}), \tag{7}$$

where $\mathbf{q}=q[n\hat{x}_1+m\hat{x}_2]/\sqrt{n^2+m^2}$ for

$$-\frac{\pi}{\sqrt{(n^2+m^2)}a}\leqslant q\leqslant\frac{\pi}{\sqrt{(n^2+m^2)}a},$$

then

$$E_{30}(\mathbf{y})=\Delta\int_{\text{PLC about }\mathbf{r}_{0,0}} dy'^2\left(\frac{\omega}{c}\right)^2\left[G_0(\mathbf{y},\mathbf{y}')+2\sum_{l=1}^{\infty}G_l(\mathbf{y},\mathbf{y}')\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]\right]f(\mathbf{y}')E_{30}(\mathbf{y}'), \tag{8}$$

where

$$G_l(\mathbf{y},\mathbf{y}')=G(\mathbf{y}+l\mathbf{S}_{n,m},\mathbf{y}'). \tag{9}$$

(Note: The equation for a single-site impurity centered about $\mathbf{r}_{0,0}$ is just given by

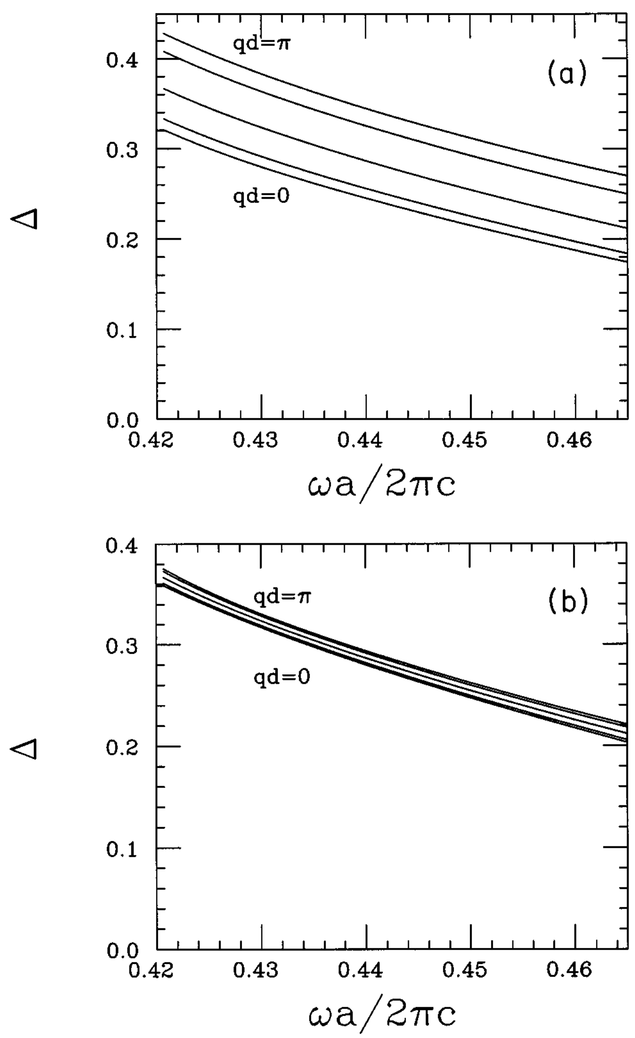


FIG. 1. Plot of Δ versus $\omega/c$ for (a) $\mathbf{S}_{1,0}$ and (b) $\mathbf{S}_{1,1}$. Curves are labeled from bottom to top as $qd=0$, $\pi/4$, $\pi/2$, $3\pi/4$, and $\pi$. All lengths are measured in units of $a$.

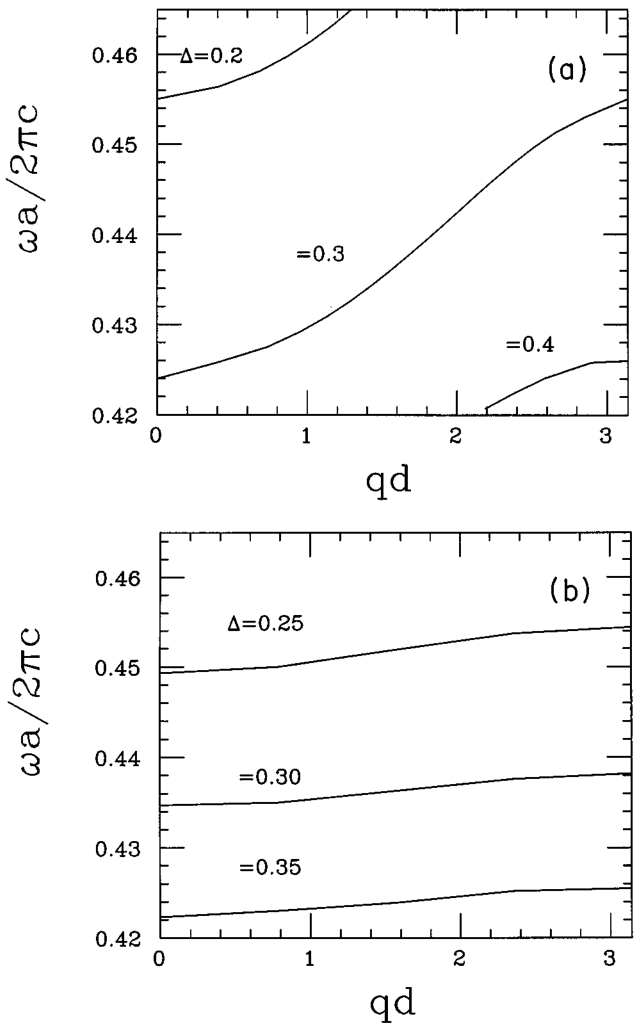


FIG. 2. Plot of $\omega/c$ versus $qd$ for (a) $\mathbf{S}_{1,0}$ and (b) $\mathbf{S}_{1,1}$. Curves are labeled as states of constant Δ. All lengths are measured in units of $a$.

$$E_3(\mathbf{y})=\Delta\int_{\text{PLC about }\mathbf{r}_{0,0}} dy'^2\left(\frac{\omega}{c}\right)^2 G_0(\mathbf{y},\mathbf{y}')f(\mathbf{y}')E_3(\mathbf{y}'), \tag{10}$$

and its solution yields localized impurity modes bound to the single impurity site.) The solution of Eq. (10) yields impurity modes of wave vector **q** traveling along the row of impurity rods.

In the case that $f(\mathbf{y})=\delta(\mathbf{y})$, Eq. (8) can be readily solved for Δ as a function of **q** and $\omega/c$ chosen in the photonic band gap. We find

$$\Delta=\left\{\left(\frac{\omega}{c}\right)^2\left[G_0(\mathbf{0},\mathbf{0})+2\sum_{l=1}^{\infty} G_l(\mathbf{0},\mathbf{0})\cos[\mathbf{q}\cdot l\mathbf{S}_{\mathbf{n},\mathbf{m}}]\right]\right\}^{-1}, \tag{11}$$

where $\mathbf{0}=0(\hat{\mathbf{x}}_1+\hat{\mathbf{x}}_2)$ gives the value of Δ needed to observe a defect mode of wave vector **q** at a frequency $\omega/c$ chosen to be in a photonic band gap. These defect modes are then propagating modes, confined to move along the row of impurities. As we shall see below, the electric-field intensity of the defect modes decays quickly with increasing distance perpendicularly from the plane of defects.

In Fig. 1 we plot results for Δ versus $\omega/c$ in the second band gap ($0.414\leqslant\omega a/2\pi c\leqslant 0.468$) of our photonic band structure for states of impurity mode wave vector **q**. In Figs. 1(a) and 1(b) results are shown, respectively, for $\mathbf{S}_{10}$ and $\mathbf{S}_{11}$ type impurities. Curves are presented in each case for a number of values of $qd$ where $d=a$ for the $\mathbf{S}_{1,0}$ case and $d=\sqrt{2}\,a$ for the $\mathbf{S}_{1,1}$ case are the lattice constants of the respective rows of impurities.

In order to obtain a better understanding of the dispersion relations of the gap impurity modes, we use the results in Fig. 1 to present in Fig. 2 plots of the impurity mode frequencies as a function of $qd$ for fixed Δ. Again Figs. 2(a) and 2(b) are for $\mathbf{S}_{1,0}$ and $\mathbf{S}_{1,1}$ impurities, respectively. It is seen that for fixed $\Delta=0.2$, 0.3, and 0.4 in Fig. 2(a) and for fixed $\Delta=0.25$, 0.30, 0.35 in Fig. 2(b), there are gaps in the frequency spectrum of propagating impurity modes. For fixed $\Delta=0.2$ and 0.4 in Fig. 2(a) there are gaps in $qd$ for the propagation of impurity modes.

An expression for the electric field of the defect mode in the case that $f(\mathbf{y})=\delta(\mathbf{y})$ is given from

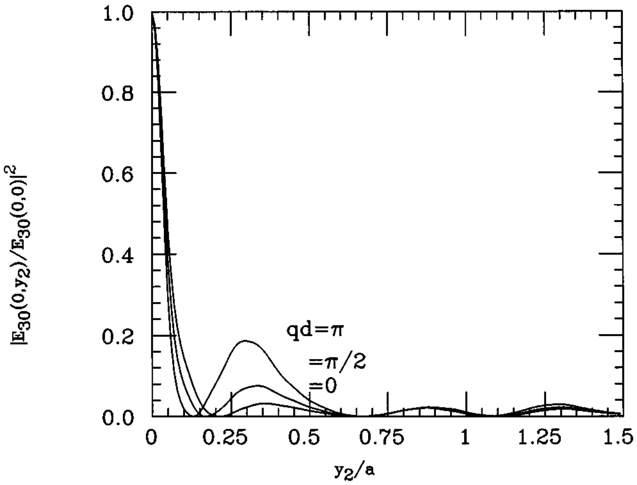


FIG. 3. Plot of $|E_{30}(0,y_2)/E_{30}(0,0)|^2$ versus $y_2$. The curves are labeled as states of $qd$ and $\omega a/2\pi c = 0.450$.

$$E_{30}(\mathbf{y})/E_{30}(\mathbf{0}) = \Delta\left(\frac{\omega}{c}\right)^2\left[G_0(\mathbf{y},\mathbf{0}) + 2\sum_{l=1}^{\infty} G_l(\mathbf{y},\mathbf{0})\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]\right], \quad (12)$$

where $\Delta$ is obtained from Eq. (11). Specifically, the modulus squared of the electric field of the defect mode divided by $|E_{30}(0)|^2$ is given by

$$\left|\frac{E_{30}(\mathbf{y})}{E_{30}(\mathbf{0})}\right|^2 = \left|\frac{G_0(\mathbf{y},\mathbf{0}) + 2\sum_{l=1}^{\infty} G_l(\mathbf{y},\mathbf{0})\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]}{G_0(\mathbf{0},\mathbf{0}) + 2\sum_{l=1}^{\infty} G_l(\mathbf{0},\mathbf{0})\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]}\right|^2. \quad (13)$$

In Fig. 3 we present results for $|E_{30}(\mathbf{y})/E_{30}(\mathbf{0})|^2$ versus $y_2$ for $\mathbf{S}_{1,0}$ for $y_1 = 0$. The curves are again labeled **qd**.

In the case in which we take

$$f(\mathbf{y}) = \begin{cases} \frac{1}{4t^2}, & |y_1|,|y_2| \leq t \\ 0, & \text{otherwise,} \end{cases} \quad (14)$$

Eq. (8) can be discretized into the form of a matrix eigenvalue problem. We have done this for $t = 0.01a$ and $0.10a$ using a 25-pt. Gaussian quadrature in $\mathbf{y}'$. For a given $qd$ and impurity mode frequency, $\omega/c$, in the band gap there are now a number of eigenvalue solutions for $\Delta$ which support an impurity mode of frequency $\omega/c$ in the gap. Only in the case that $f(\mathbf{y}) = \delta(\mathbf{y})$ do we find a single solution for $\Delta$ corresponding to an $\omega/c$ in the gap. In Fig. 4 we plot $\Delta$ versus impurity mode frequency $\omega/c$ in the $0.414 \leq \omega a/2\pi c \leq 0.468$ band gap, presenting curves of various $qd$. We do not show all of the eigenvalues for $\Delta$ but present only the eigenvalues which are closest to the results for $f(\mathbf{y}) = \delta(\mathbf{y})$ shown in Fig. 1.

The results in Eq. (11) for the case of $\delta$-function defects can be readily extended to consider the effects of nonlinearity on the defect gap modes predicted by Eq. (11). Specifically, if we take the Kerr form,

$$\delta\epsilon(\mathbf{x}) = \Delta_0(1 + \chi|E_3(\mathbf{x})|^2)\sum_{l=-\infty}^{\infty} \delta(\mathbf{x} - l\mathbf{S}_{n,m}), \quad (15)$$

then substituting into Eq. (2) we find solutions for $\Delta_0$ as a function of impurity (in the gap) mode frequency $\omega$ given by

$$\Delta_0 = \left\{\left(\frac{\omega}{c}\right)^2 (1 + \chi|E_0|^2)\left[G_0(\mathbf{0},\mathbf{0}) + 2\sum_{l=1}^{\infty} G_l(\mathbf{0},\mathbf{0})\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]\right]\right\}^{-1}, \quad (16)$$

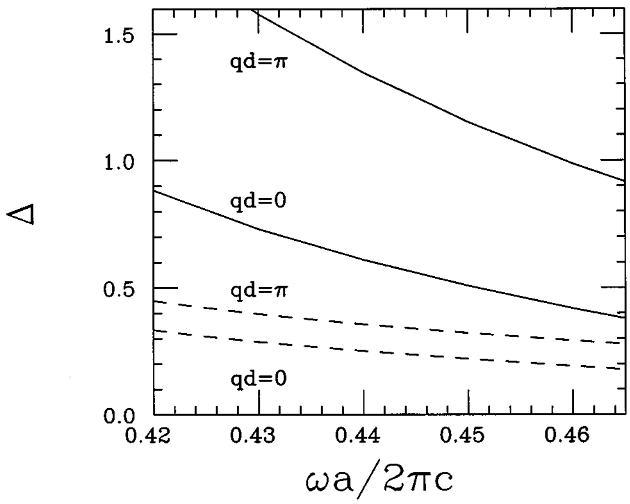


FIG. 4. Plot of $\Delta$ versus $\omega/c$ for $t = 0.01a$ (dashed) and $t = 0.1a$ (solid). Curves of $qd = 0$ and $qd = \pi$ are labeled. For each $t$, the curves for $0 < qd < \pi$ fall between the $qd = 0$ and $qd = \pi$ limits. All lengths are measured in units of $a$.

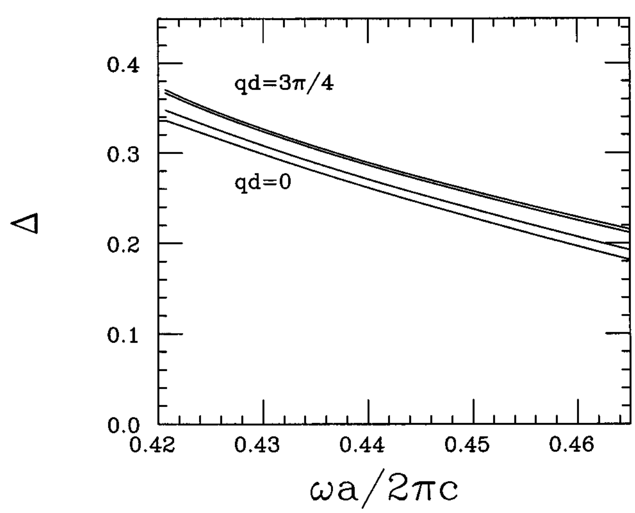


FIG. 5. Plot of $\Delta$ versus $\omega/c$ for the two-dimensional impurity array described in the text. The curves are labeled bottom to top as states of $qd = 0$, $\pi/4$, $\pi/2, \pi$, and $3\pi/4$. (Note: The curves for $qd = \pi/2$ and $\pi$ coincide.) All lengths are measured in units of $a$.

where $|E_0|$ is the amplitude of the electric field at $\mathbf{r}_{0,0}$. We see that from Eq. (16) that $\Delta_0$ now depends on the field strength $E_0$. This allows for the adjustment of the impurity mode frequencies for fixed $\Delta_0$ and $\mathbf{q}$ through changes in $E_0$. (Similarly, for fixed $\Delta_0$ and $\omega$, adjustments in $\mathbf{q}$ can be made by changing $E_0$.) Comparing Eqs. (11) and (16), we see that

$$\Delta=(1+\chi|E_0|^2)\Delta_0, \tag{17}$$

so that the results for $\Delta$ in Fig. 1 are readily taken over to solve the nonlinear problem.

An additional case of interest is the system obtained from Eqs. (1) and (2) upon taking the $2D$ periodic array of defects

$$\delta\epsilon(\mathbf{x})=\Delta\sum_{l=-\infty}^{\infty}\sum_{l'=-\infty}^{\infty}f(\mathbf{x}-l\mathbf{S}_{n,m}-l'S_{n',m'}), \tag{18}$$

where $\mathbf{S}_{n,m}=[n\mathbf{x}_1+m\hat{x}_2]a$, $\mathbf{S}_{n',m'}=[n'\hat{x}_1+m'\hat{x}_2]a$ for $n,m,n',m'$ integers are orthogonal vectors, and $f(x)$ is defined as in Eq. (5). This gives a 2D periodic system but the periodicity of $\delta\epsilon(\mathbf{x})$ may or may not be the same as that of $\epsilon(\mathbf{x})$.

Substituting Eq. (18) in to Eqs. (1) and (2) and writing

$$E_3(\mathbf{y}+l\mathbf{S}_{n,m}+l'\mathbf{S}_{n',m'})=e^{i\mathbf{q}\cdot[l\mathbf{S}_{n,m}+l'\mathbf{S}_{n',m'}]}E_{30}(\mathbf{y}), \tag{19}$$

for $\mathbf{y}$ in the primitive lattice cell of $\epsilon(\mathbf{x})$ centered about $\mathbf{r}_{0,0}$ and $\mathbf{q}$ a wave vector in the first Brillouin zone of the lattice with primitive lattice vectors $\mathbf{S}_{n,m}$ and $\mathbf{S}_{n',m'}$, we find

$$E_{30}(\mathbf{y})=\Delta\int_{\text{PLC about }\mathbf{r}_{00}}dy'^2\left(\frac{\omega}{c}\right)^2\Bigg[G_{00}(\mathbf{y},\mathbf{y}')+2\sum_{l=1}^{\infty}\{G_{l0}(\mathbf{y},\mathbf{y}')\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]+G_{0l}(\mathbf{y},\mathbf{y}')\cos[\mathbf{q}\cdot l\mathbf{S}_{n',m'}]\}$$
$$+4\sum_{l=1}^{\infty}\sum_{l'=1}^{\infty}G_{ll'}(\mathbf{y},\mathbf{y}')\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]\cos[\mathbf{q}\cdot l'\mathbf{S}_{n',m'}]\Bigg]E_{30}(\mathbf{y}'), \tag{20}$$

where

$$G_{ll'}(\mathbf{y},\mathbf{y}')=G(\mathbf{y}+l\mathbf{S}_{n,m}+l'\mathbf{S}_{n',m'},\mathbf{y}'). \tag{21}$$

The solution of Eq. (20) then yields traveling-wave impurity modes of wave vector $\mathbf{q}$.

If we now take $f(\mathbf{x})=\delta(\mathbf{x})$, we find

$$\Delta=\Bigg\{\left(\frac{\omega}{c}\right)^2\Bigg(G_{00}(\mathbf{0},\mathbf{0})+2\sum_{l=1}^{\infty}\{G_{l0}(\mathbf{0},\mathbf{0})\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]+G_{0,l}(\mathbf{0},\mathbf{0})\cos[\mathbf{q}\cdot lS_{n',m'}]\}$$
$$+4\sum_{l=1}^{\infty}\sum_{l'=1}^{\infty}G_{ll'}(\mathbf{0},\mathbf{0})\cos[\mathbf{q}\cdot l\mathbf{S}_{n,m}]\cos[\mathbf{q}\cdot l'\mathbf{S}_{n',m'}]\Bigg)\Bigg\}^{-1} \tag{22}$$

specifies the $\Delta$ needed to obtain a mode of wave vector $\mathbf{q}$ and frequency $\omega$ in the band gap. In obtaining Eq. (22) we assume that Eq. (18) is a weak enough perturbation that the edges of the band gap are little changed from those in the unperturbed system. This will be the case for small $\Delta$ and for $|\mathbf{S}_{n,m}|,|\mathbf{S}_{n',m'}|\gg a$. In the case of the introduction of a nonlinear Kerr-type medium giving

$$\delta\epsilon(x)=\Delta_0[1+\chi|E_{30}(x)|^2]\sum_{l=-\infty}^{\infty}\sum_{l'=-\infty}^{\infty}\delta(\mathbf{x}-l\mathbf{S}_{n,m}-l'\mathbf{S}_{n',m'}), \tag{23}$$

we find that

$$\Delta_0=\Delta/(1+\chi|E_0|^2), \tag{24}$$

where $\Delta$ is from Eq. (22) and $|E_0|$ is the field amplitude at the site of the nonlinearity. Again the bands of modes in the gaps of the $\epsilon(\mathbf{x})$ system can be frequency tuned in the case $x\neq 0$ by adjusting $E_0$. The general solution of Eq. (20) for arbitrary $f(\mathbf{x})$ is again obtained by discretizing Eq. (20) into the form of a matrix eigenvalue problem for $\Delta$.

In Fig. 5 we present results from Eq. (22) evaluated for our $\epsilon$=9 system in the case that $\{\mathbf{S}_{1,1},\mathbf{S}_{1,-1},\mathbf{S}_{-1,1},\mathbf{S}_{-1,1}\}$ are the only nonzero $\mathbf{S}_{n,m}$ and $\mathbf{S}_{n',m'}$ in Eq. (18). We plot $\Delta$ versus $\omega$ in the second band gap for curves labeled $qd$ where $\mathbf{q}=q\sqrt{2}\hat{x}_1$ for $-\pi/\sqrt{2}a\leqslant q\leqslant\pi/\sqrt{2}a$ and $d=\sqrt{2}a$ is the lattice constant of the impurity array.

In this paper we present an exact Green's-function calculation for a two-dimensional photonic band structure with an infinite row of impurities or an infinite two-dimensional array of impurities. Previous Green's-function treatments considered only single impurities or finite impurity clusters. Unlike results for single impurities and finite clusters of impurities the modes found in our infinite impurity systems are propagating, not localized modes. Conditions on the dielectric constant of the impurity material are found for impurity modes of a given frequency in the gap and wave vector to exist in the impurity system. These results are used to discuss the problem of nonlinear impurity dielectric materials. In addition, the solution of the problem of frequency-dependent impurity dielectric material[8] with frequency-dependent dielectric constant $\epsilon_{\mathrm{imp}}(\omega)$ for either of the systems considered above can be obtained as a solution of the equation

$$\Delta(\omega,\mathbf{q})=A_i[\epsilon_{\mathrm{imp}}(\omega)-\epsilon], \tag{25}$$

where $A_i$ is the area in units of the lattice constant of a single impurity rod in the row of rods for rows of defects or the two-dimensional array of rods for the periodic array of defects. In Eq. (25), $\Delta(\omega,\mathbf{q})$ is one of the frequency- and wave-vector-dependent solutions for $\Delta$ given in either Figs. 1, 4, or 5, and Eq. (25) is solved self-consistently for $\omega$ as discussed in Ref. 8.

The types of impurity modes discussed above suggest themselves to various technological applications. Specifically (1) rows of imperfections have been suggested as alternatives to optic fibers in optical circuitry,[1] and (2) two-dimensional arrays of impurities can be used to create very narrow band transmission filters in the band gaps of photonic band structures. The introduction of nonlinearity into the imperfections forming these impurity systems allows for the tuning of the impurity modes in frequency and wave vector by varying the amplitude of the electric field, and the use of frequency-dependent impurity dielectric materials is found to facilitate the observation of impurity gap modes near the dielectric resonances of such materials.[8] One additionally useful feature of photonic band structures that has been demonstrated is that, as photonic band structures can be constructed of very low loss dielectric materials, the impurity modes can often exhibit states of very high $Q$.[1]

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