Gap shift and bistability in two-dimensional nonlinear optical superlattices

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We numerically investigate electromagnetic wave propagation in a two-dimensional optical superlattice with Kerr-type nonlinearity. We show that, with proper modulation depths of the refractive index, a stop gap in the spectrum is possible for a wave propagating at the Bragg angle of the structure. The location of the stop gap depends critically on the incident wave power. We demonstrate that this gap-shift effect can induce an intensity-dependent transmission. This property has important applications in optical bistable switching.

The dispersion relation provides the key to understanding electromagnetic (e.m.) wave propagation through periodic dielectric structure.[1] The solution to the dispersion relation may contain stop gaps within which the superlattice becomes perfectly reflecting. The gaps separate bands within which propagating wave solutions are allowed. In the search for photonic band-gap (PBG) materials,[2] two-dimensional (2D) periodic structures have received theoretical[3–5] and experimental[4–8] attention because superlattices of this type are relatively easy to fabricate. In this paper, we propose a 2D PBG structure, of which its refractive index is continuously modulated in two directions. We show that with the inclusion of nonlinearity of the medium the incident wave power leads to a shift in the location of the stop gap. This nonlinear mechanism can be used to construct a different class of bistable optical devices.

We consider a lossless 2D optical superlattice (OSL) with a simple Kerr-type nonlinearity shown in Fig. 1(a). We assume that the linear refractive index of a 2D nonlinear OSL is weakly, sinusoidally modulated in the $\hat{x}$ and $\hat{y}$ directions. Thus the refractive index of a 2D nonlinear OSL is given by

$$n=n_0+n_x\cos(H_x x)+n_y\cos(H_y y)+n_\alpha|E|^2/4\pi, \quad (1)$$

where n_0 is the average effective index, n_x and n_y are the index modulation depths and are taken to be rather weak, H_x and H_y are the reciprocal lattice vectors, n_α is the nonlinear coefficient of the medium, and E is the electric field of the optical wave. In what follows we will call the ratio $m=n_x/n_y$ the modulation ratio of a 2D nonlinear OSL. The 2D periodic structures described by Eq. (1) can be fabricated by using a holographic recording technique.[9] Another such structure would be a doubly periodic planar waveguide where Eq. (1) describes the distribution of an effective mode index.[10] The existence of solitary waves in the 2D nonlinear periodic medium with strong modulation depths has been demonstrated by John and Aközbek by using a variational method.[11]

As is known, the phenomenon of dynamical diffraction of an e.m. wave in a spatially periodic medium occurs when the wave vectors of incoming $\vec{k}$ and diffracted $\vec{k}+\vec{H}_\sigma$ waves fulfill the Bragg condition $|\vec{k}+\vec{H}_\sigma|\approx|\vec{k}|$, where $\vec{H}_\sigma$ is the reciprocal lattice vector. For a 2D OSL, it is possible for the incoming wave vector $\vec{k}$ to satisfy simultaneously two exact equalities for $\vec{H}_\sigma=-\vec{H}_x$ and $\vec{H}_\sigma=\vec{H}_y$. We define this situation to be the Bragg case of a 2D OSL,

$$2k_B\sin\theta_B=H_x, \quad 2k_B\cos\theta_B=H_y, \quad (2)$$

where θ_B is the Bragg angle and $k_B=n_0\omega_B/c$ the Bragg wave number with ω_B being the Bragg frequency. As it will be shown, in a 2D OSL, whether or not a frequency stop gap for a wave propagating in the vicinity of the Bragg angle occurs depends on the modulation ratio m in contrast to the properties of two-wave Bragg diffraction in one-dimensional linear periodic media.[1]

In the four-wave approximation, the Bloch waves in the periodic structure can be decomposed into

$$E(\vec{r})=\sum_\sigma E_\sigma\exp(i\vec{K}_\sigma\cdot\vec{r}), \quad (3)$$

where E_σ is the amplitude of the partial mode with wave vector $\vec{K}_\sigma$ and σ denotes o, h, $-o$, and $-h$, which are the nodes of the reciprocal lattice [Fig. 1(b)]. A useful representation of the dispersion effect caused by multiwave interaction is to introduce a new set of axes (ξ_o,ξ_h), which are defined through $\xi_o=(K_o-k_B)/k_B$, $\xi_h=(K_h-k_B)/k_B$. Then straightforward manipulations similar to the ones used in the linear multiwave diffraction dynamics lead to the nonlinear matrix equation for E_σ,

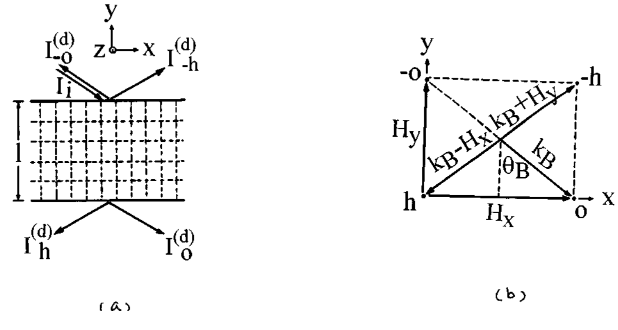

FIG. 1. (a) Schematic of a two-dimensional optical superlattice and (b) exact Bragg conditions in a two-dimensional optical superlattice.

$$\begin{vmatrix} 4(\delta-\xi_o)+\Delta M_0 & M_x+\Delta M_x & \Delta M_{x-y} & M_y+\Delta M_y^* \\ M_x+\Delta M_x^* & 4(\delta-\xi_h)+\Delta M_0 & M_y+\Delta M_y^* & \Delta M_{x+y}^* \\ \Delta M_{x-y}^* & M_y+\Delta M_y & 4(\delta+\xi_o)+\Delta M_0 & M_x+\Delta M_x^* \\ M_y+\Delta M_y & \Delta M_{x+y} & M_x+\Delta M_x & 4(\delta+\xi_h)+\Delta M_0 \end{vmatrix} \begin{vmatrix} E_o \\ E_h \\ E_{-o} \\ E_{-h} \end{vmatrix} = 0, \tag{4}$$

where $M_x = 2n_x/n_0$ and $M_y = 2n_y/n_0$ are the linear index modulation strengths and $\delta = (\omega - \omega_B)/\omega_B$ is a frequency dephasing parameter of the operating frequency ω from the Bragg frequency ω_B and is assumed to be small. The field-dependent index modulation strengths in Eq. (4) are expressed as

$$\Delta M_x = M_\alpha(E_o E_h^* + E_{-h} E_{-o}^*),$$

$$\Delta M_y = M_\alpha(E_{-o} E_h^* + E_{-h} E_o^*),$$

$$\Delta M_{x+y} = M_\alpha E_{-h} E_h^*,$$

$$\Delta M_{x-y} = M_\alpha E_o E_{-o}^*,$$

$$\Delta M_0 = M_\alpha \sum_\sigma |E_\sigma|^2,$$

where $M_\alpha = n_\alpha/n_0$ and σ takes o, h, $-o$, and $-h$. To obtain Eq. (4), the approximations $1-K_o^2/k^2 \approx 2(\delta-\xi_o)$, $1-K_h^2/k^2 \approx 2(\delta-\xi_h)$, $1-K_{-o}^2/k^2 \approx 2(\delta+\xi_o)$, and $1-K_{-h}^2/k^2 \approx 2(\delta+\xi_h)$ were used, which follow from the inequalities $n_x \ll n_0$, $n_y \ll n_0$, and $\delta \ll 1$. The relation between ξ_o and ξ_h is, in the geometry of Fig. 1, $\xi_o = \xi_h - \eta \sin 2\theta_B$, where η is the angular deviation from the Bragg angle. Though a 2D nonlinear OSL can be studied by using the formalism proposed above for any arbitrary angular deviation, we shall treat only the simple case of Bragg incidence.

In the limit of low power, the field-dependent terms are negligible. In this case, setting the determinant of coefficients to zero, we obtained the dispersion equation,

$$\xi_o^4 + B\xi_o^2 + C = 0, \tag{5}$$

where

$$B = \left(\frac{M_y}{2}\right)^2 \left[-\delta^2 + \frac{(1-m^2)}{2}\right],$$

$$C = \left(\frac{M_y}{2}\right)^4 \left[\frac{1}{16}(1-m^2)^2 + \delta^4 - \frac{\delta^2}{2}(m^2+1)\right].$$

It is readily obtained that Eq. (5) yields real values for ξ_o for all δ when $m>1$. For a 2D OSL with $m<1$, however, the regime where $|\delta| < (1-m)M_y/4$ corresponds to all ξ_o's having an imaginary part and thus to evanescent Bloch waves. Outside the regime, ξ_o has real-value solutions indicating propagating Bloch waves. So a frequency stop gap centered at the Bragg frequency appears if $m<1$. Under the Bragg incidence, the width of the stop gap is given by $\Delta\omega = (1-m)M_y\omega_B/4$. For convenience, we adopt two variables $\Omega = 2(\omega-\omega_B)/\Delta\omega$ and $L = k_B l/\cos\theta_B$ in the following discussion, where l is the thickness of the 2D nonlinear OSL. As an example, we choose a 2D nonlinear OSL with struc-

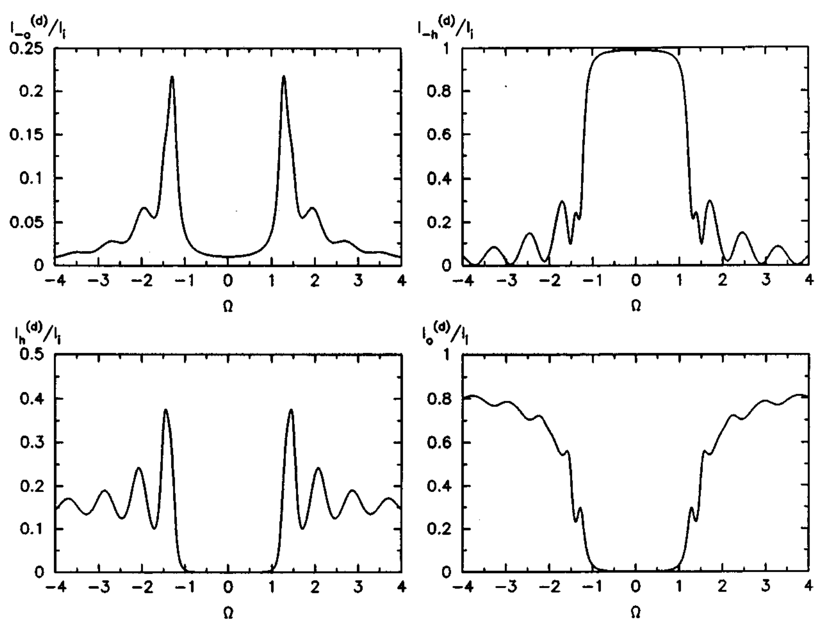

FIG. 2. Linear transmission coefficients of four diffracted waves of the 2D nonlinear OSL as a function of the frequency detuning parameter Ω.

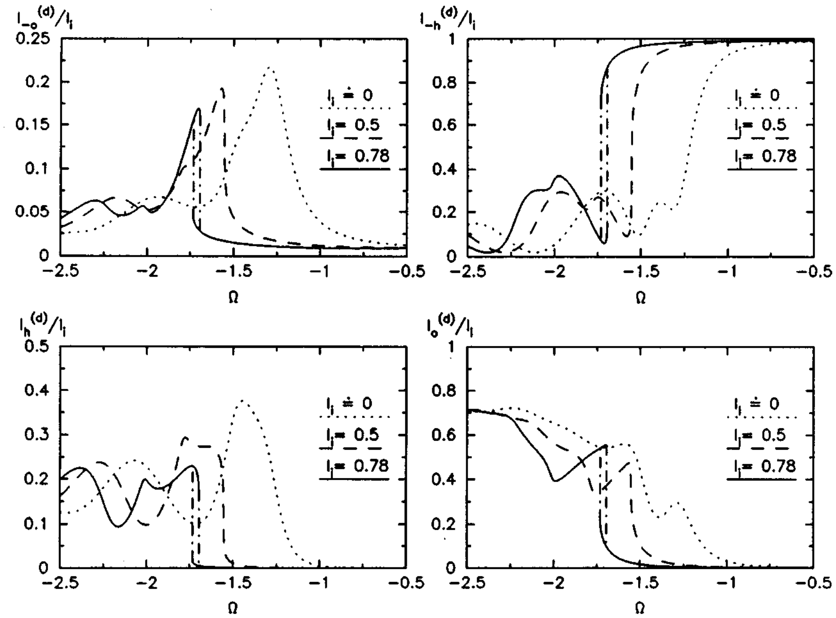

FIG. 3. Nonlinear transmission coefficients of four diffracted waves of the 2D nonlinear OSL as a function of the frequency detuning parameter Ω with different input intensities.

ture parameters $M_\alpha = 10^{-4}$, $L = 3.5\times10^4$, $M_x = 4.5\times10^{-5}$, and $M_y = 4.5\times10^{-4}$, thus with a modulation ratio $m = 0.1$. The incoming wave intensity I_i is normalized to $I_{i,c}$ with $M_\alpha I_{i,c} = 10^{-4}$.

In Fig. 2, we plot the linear transmission coefficients as functions of the frequency detuning parameter Ω of the incident wave. Clearly, total reflection (i.e., a stop gap) occurs whenever $|\Omega| < 1$.

Figure 3 shows the nonlinear transmission coefficients on the low-frequency side of the stop gap of our interest. It is seen that due to the nonlinearity of the medium two quantitative changes in the stop gap occur. First, the position of the stop gap shifts to the low-frequency side as the input power increases. This effect may be used to design an intensity-driven optical limiter as suggested by Scalora *et al.* for 1D nonlinear PBG materials.[12] Second, with increasing input wave energy the width of the stop gap also expands. In this example, the intensity for bistability is found to be $M_\alpha I_i = 5\times10^{-5}$, a value that is comparable to a nonlinear distributed feedback structure[13] where for operating frequency within the gap bistability is mediated by the excitation of gap solitons.[14–17] For an input intensity of $M_\alpha I_i = 7.8\times10^{-5}$, the coefficients exhibit clear bivalued features at the low-frequency side near the stop gap.

The relations between the relative outputs and the input are shown in Fig. 4 for two beams of different frequencies. Both frequencies are tuned in the passband below the stop gap (linear case) so that the nonlinear effect can shift the gap towards the operating frequency as the intensity is increased. The critical detuning for which the bistability just starts to

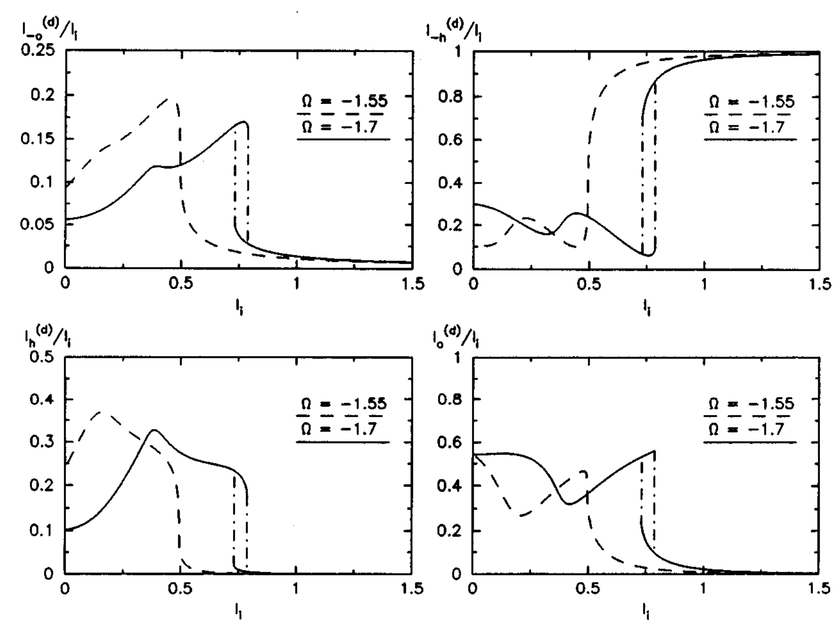

FIG. 4. Relative outputs as a function of input of the 2D nonlinear OSL for two different frequency detunings.

occur is -1.55. For beams with frequency detuning larger than 1.55 below the gap, it will initially be transmitted. As the beam intensity increases, its effect is to widen and shift the gap towards the operating frequency; it will do so to such extent that the operating frequency will find itself inside the gap. Diffracted beams $I_o^{(d)}$ and $I_h^{(d)}$ then shut off. With decreasing input, the relative outputs do not retrace their original paths and output-versus-input functions exhibit typical bistable hysteresis.

It is important to note that the bistable switching behaviors shown in Fig. 4 are different from that of the index-modulation bistability mechanism in a 2D nonlinear OSL previously proposed by us[18] in that the latter is, in effect, a switching between different transmitting states (TS's), rather than between a nontransmitting state (NTS) and a TS because the modulation ratio of the 2D nonlinear OSL discussed there is larger than unity, thus with no stop gap as specified above. The bistability revealed here, however, is based on the occurrence of the stop gap and the nonlinear effect on it. Therefore the optical response exhibits bistable switching between TS's and NTS's (at least for exiting beams $I_o^{(d)}$ and $I_h^{(d)}$) for a light beam with its frequency operating in the band below the gap. With optimizing structure parameters, we can also achieve a high contrast at the switching on and switching off for the beam $I_{-h}^{(d)}$.

The results we presented here for the case of $M_\alpha > 0$ are also sustained for a 2D OSL with a negative nonlinear coefficient. In this case we select the operating frequency in the passband but above the gap because the nonlinear effect now is to shift the stop gap to the high-frequency side of the gap.

In summary, we have shown that for a 2D OSL with proper index modulation depths, a stop gap in the linear spectra is possible for light propagating in the vicinity of the Bragg angle. The inclusion of nonlinearity in the superlattice alters substantially the position and width of the stop gap. For a light beam with its frequency in the band, an appropriate change of the input power can switch the exiting beams between TS's and NTS's. These properties may have applications in designing actual 2D nonlinear OSL bistable devices.

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