Resonant macroscopic quantum tunneling in small Josephson junctions: Effect of temperature

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Within the well-established quantum picture of Josephson junctions, we study the possibility of resonant macroscopic quantum tunneling. Expressions for the lifetime of metastable states are obtained and the effects of temperature on the main features of the phenomenon are discussed. A "saturation temperature" for the process, below which any temperature effect is negligible, is identified. Quantitative considerations are also given on the actual experimental accessibility to these measurements.

The quantum behavior of the Josephson phase results in a host of interesting phenomena including macroscopic quantum tunneling (MQT),[1] energy-level quantization (ELQ),[2] macroscopic quantum coherence of two states,[1,3] and many states (Bloch oscillations),[4] and the Coulomb blockade of Cooper pair tunneling.[5] Macroscopic quantum tunneling has been the phenomenon which has attracted the attention of the investigators, both from a theoretical and experimental point of view. Experiments on current-biased Josephson tunnel junctions with microwave irradiation support the idea of the existence of both MQT (Refs. 6 and 7) and ELQ,[7] and the data are in good agreement with the theory.[1,2,8] Manifestations of new Coulomb and microscopic effects in tunnel junctions of low capacitance have also attracted considerable interest, both for the physics involved and in view of applications.[4,5,9] In this paper we wish to discuss, within the well established quantum picture of the junction, the effect of the temperature on the resonant macroscopic quantum tunneling (RMQT) between levels with the same energy in neighboring wells of the potential shape describing the junction. This process produces sharp voltage peaks in the current-voltage (I-V) characteristics at given current values, which can be measured in junctions with suitable parameters, and characteristic peaks in the switching current distribution. The phenomenon has been already analyzed by different authors.[10–13] In Ref. 10, in particular, a detailed analysis of the resonant tunnel dynamics is presented for the $T=0$ case, with a complete description of the experimental conditions for a check of the phenomenon. From this $T=0$ analysis, it is clear that the lifetime of the voltage peaks has a relevant role for the observability of such steps. In fact the lifetime decreases as the voltage amplitude increases; therefore the junction parameters must be chosen in order to "balance between measuring an extremely small signal of lengthy duration and a larger signal of extremely short duration."[10] The lifetime is also related to the escape rate out to the free running state producing characteristic peaks in the switching current distribution. This appears, at the moment, the most readily observable effect arising from RMQT. In Ref. 12, general expressions for the main features of the phenomenon are given, but no systematic study of the effect of temperature is presented.

The present work extends the $T=0$ analysis of Schmidt *et al.* studying the lifetime and the escape rate including the effect of temperature. This is obtained by solving the master equation for the level occupancy probability, proposed by Larkin and Ovchinnikov.[2] In the following we briefly discuss the theoretical approach and the main results. In particular, we find a sort of "crossover temperature" for the process, below which any temperature effect is negligible. This temperature is related to the quantum temperature of the junction and it can provide an indication of the temperature range of experimental interest in choosing the actual working conditions.

The macroscopic quantum variable which describes the dynamics of a Josephson junction is the phase difference 2ϕ of the order parameters of the two superconductors forming the Josephson junction. The dissipation of the system is typically described in terms of an effective resistance R of the junction (resistively shunted junction model).[14]

Within this model the junction dynamics can be described in terms of a mechanical analog, namely a particle performing its motion in a washboard potential $U(\phi)=-E_0(2\alpha\phi+\cos 2\phi)$ with a viscosity coefficient $\eta=1/RC$. Here $E_0=\hbar I_c/2e$ is the Josephson coupling energy, α is the bias current I normalized to the critical one I_c, $\alpha=I/I_c$. The system is initially trapped in a relative minimum of the washboard potential $U(\phi)$ shown in Fig. 1, but it can escape from the well via either MQT or thermal activation. As is well known, two stable solutions are possible. In one solution the particle is trapped in a potential well with exponentially small voltage ($V=0$ state), in the other it is running down the potential with a certain nonzero mean velocity ($V\neq 0$). In a quantum picture we must consider the presence of energy levels in a potential well of the washboard potential associated with a Josephson tunnel junction.[2,7] In the weak friction limit, there are, in fact, sharp and well separated energy levels inside each potential well, and we must consider the probabilities $w_{j,k}$ for transitions from the kth into jth level and the probability γ_j of tunneling through the barrier, probability which strongly depends on the energy-level position E_j. For values of the normalized current α smaller than $\alpha_0\approx 0.2$ the ground state in the first well will lie on an energy smaller than the upper level of the second well (see Fig. 1).

In the following we restrict our analysis to the case of the ground state of one well lying on energy close to the first excited level in the subsequent well. In this case the particle

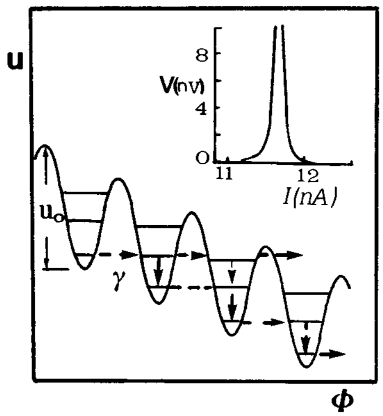

FIG. 1. Potential $U(\phi)$ describing the junction dynamics. A sharp voltage peak in the I-V characteristics appears when the ground state lies on an energy level very close to one of the first excited states in the next well; this is shown in the inset for the following junction parameters: $E_0/\hbar\omega_j=2.5$, $T=50$ mK, $I_c=200$ nA, and $R/R_Q=100$, where $R_Q=\pi\hbar/2e^2=6.45$ kΩ.

coming via MQT from the ground state will go into the first level and after can either decay or perform a successive tunneling (see Fig. 1). These competitive processes lead to different results. For each current value we must, in fact, define two possible junction states which produce different observable solutions: one is the "free running state" where the system after a series of successive tunneling is freely running down the potential slope with energy E larger than barrier energy $U_0(E>U_0)$. This state is characterized by junction voltage corresponding to the quasiparticle branch of the I-V curve. In the other state, which we call "resonant state," the system performs a sequence of tunneling and decay processes, being trapped in one of the wells at a certain energy level $(E=E_j<U_0)$. The resonant state is characterized by junction voltage corresponding to the phase mobility due to the tunneling across the barrier.[10–13] Assuming that tunneling from the ground state to the first excited state is the most relevant process, the average voltage will be $eV\approx\pi\hbar\gamma_0$. The tunnel probability γ_0 has the resonant behavior for current values for which the ground state corresponds to one of the excited states of the next well.[10–13] This produces a fast increase of the voltage at certain current values and voltage steps on the I-V curve. The probability ρ of finding the system in the resonant state is obviously $\rho=\Sigma_j\rho_j$, where ρ_j is the occupancy probability of the jth level. The junction dynamics is well described by the following kinetic equation:

$$\partial\rho_j/\partial t=\Sigma_k(w_{j,k}\rho_k-w_{k,j}\rho_j)-\gamma_j\rho_j+\gamma_{j-1}\rho_{j-1}, \quad (1)$$

where $k=0,\ldots,N$ ($N+1$ is the number of levels inside the well, referring 0 to the ground state), and γ_j is the tunnel rate probability from the jth level. Only transitions $w_{j,j\pm1}$ between close levels are relevant. The last term in Eq. (1) indicates that the particle tunneling from the $(j-1)$th level goes into the jth level in the next well. Following a rather standard procedure for quasistationary processes, we can now obtain a general expression for the lifetime of the resonant state. Assuming initially the junction in the resonant state, $\rho(t=0)=1$, $\rho(t)$ tends to reduce with time due to an escape rate Γ to the free running state. The lifetime τ of the resonant state can be assumed as the reverse of Γ, $\tau=\Gamma^{-1}$. As usual, in quasistationary condition Γ is defined as

$$\Gamma=-(1/\rho)\partial\rho/\partial t. \quad (2)$$

Summing the system of Eq. (1) over all j, the following expression for $\partial\rho/\partial t$ is obtained:

$$\partial\rho/\partial t=-(w_\infty+\gamma_N)\rho_N, \quad (3)$$

where w_∞ is the transition probability to the continuum.

In quasistationary conditions ρ_N can be related to ρ by finding the steady-state solution of Eq. (1). By simple calculations we obtain

$$\rho=(1+\Sigma_jA_j)\rho_N, \quad j=0,..,N-1 \quad (4)$$

where

$$A_j=\Pi_k[w_{k,k+1}/(w_{k+1,k}+\gamma_k)], \quad k=j,\ldots,N-1. \quad (5)$$

By definition [Eq. (2)] we obtain

$$\Gamma=(w_\infty+\gamma_N)/(1+\Sigma_jA_j), \quad j=0,..,N-1. \quad (6)$$

In this way we have a general expression for Γ as a function of $w_{j,k}$ and γ_k, which strongly depends on temperature T and junction parameters. At low temperature $[KT\ll\hbar\omega_j$, where $\omega_j^2=(1-\alpha^2)^{1/2}2eI_c/\hbar C]$, we can neglect the transition probability $w_{k+1,k}$ toward higher levels, being $w_{k,k+1}/w_{k+1,k}\approx\exp[(E_{k+1}-E_k)/KT]\gg1$, and we also expect $w_{k,k+1}\gg\gamma_k$ (except very close to the resonant conditions). In this case Eq. (6) reduces to $\Gamma\approx\gamma_N/\Pi_k(w_{k,k+1}/\gamma_k)$ and therefore Γ strongly decreases with increasing the ratio $w_{k,k+1}/\gamma_k$. This corresponds to the trivial physical consideration that Γ is small when the competing decay probability is much larger than tunnel probability, the alternance of tunneling to the first level and decay down to the ground state being the most likely process.

In order to plot Γ at various temperatures and junction parameters, we now calculate the positions of the levels E_j, the transitions between levels $w_{j,k}$ and the tunnel probabilities γ_j with the aid of the expressions reported in Ref. 2. The cubic approximation for the Josephson potential has been used.

For γ_0 close to the resonance we actually use the formula obtained by Larkin *et al.*[15] calculated for $KT\ll\hbar\omega_j$:

$$\gamma_0=A\omega_j\exp(2S+1)\{\omega_j/\varepsilon[1+\coth(\varepsilon/2KT)] + B/\omega_j(\omega_j/\varepsilon)^2\}. \quad (7)$$

Here $A=(3)^{1/2}\hbar^2/8Re^2$, $B=e^2/2\pi^2C$ (R and C are the resistance and the capacitance of the junction), S is an action in the classically forbidden region, and ε is the energy difference referring to resonant energy levels in different wells. Equation (7) is valid only for large enough values of ε, namely when $\varepsilon>\hbar^2\omega_j/(Re^2)$.[16] As it is easy to see, the transition probability is an asymmetric function of the energy difference ε. The first term in Eq. (7) is connected with the emission (or absorption) of phonon at small energy and, for low-temperature values, is strongly asymmetric. The second term is connected with the emission of excitation with energy equal to the energy difference between neighboring lev-

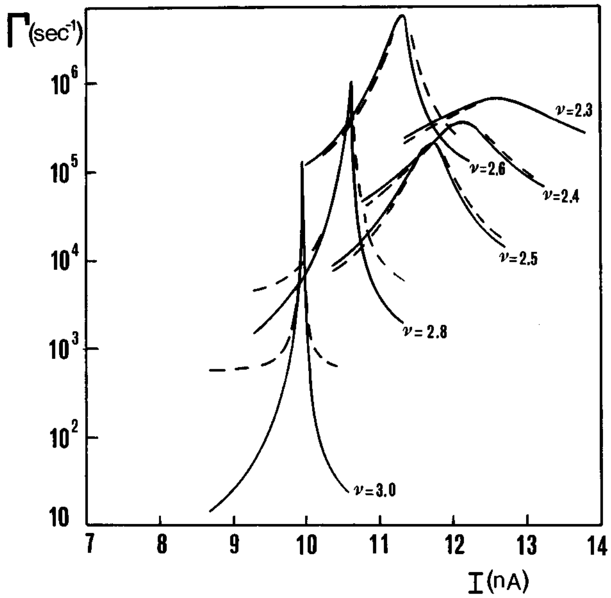

FIG. 2. Escape rate as a function of the current in the region of the resonance for different values of ν at $T=50$ mK (solid lines) and $T=200$ mK (dashed lines).

els in one potential well.[15] The results of the calculations at different temperatures are reported in Figs. 2–5.

We consider different values of the ratio $E_0/\hbar\omega_j$, typically referred to as ν in literature.[10] This parameter can be also defined in terms of the ratio of the Josephson energy and the charging energy $E_c=e^2/C$, $\nu=(2E_0/E_c)^{1/2}$. It is connected with the number of levels inside the well and it increases with increasing the capacitance. This adimensional quantity is of the most importance to account for the effect of the junction parameters. The values considered refer to junction capacitance ranging between 7–22 fF, and critical current $I_c=200$ nA. The resistance value, which also strongly affects the results, has been fixed to $R=645$ kΩ in order to well satisfy the condition of validity for Eq. (7). Junctions

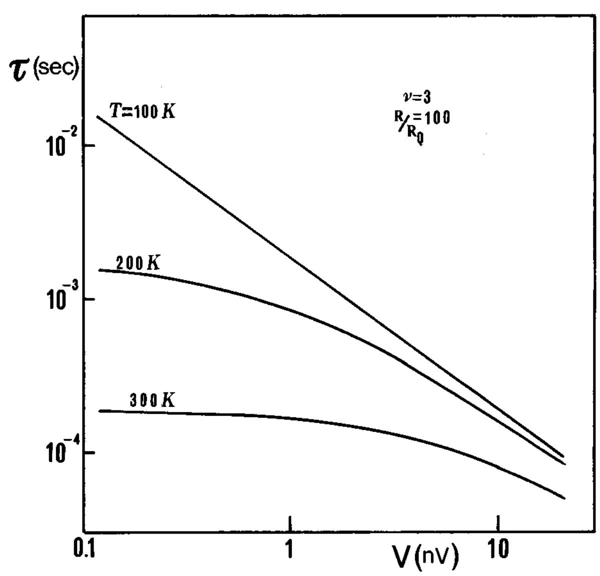

FIG. 3. Lifetime of the resonant state as a function of the voltage along the step, at different temperatures and for $\nu=3$.

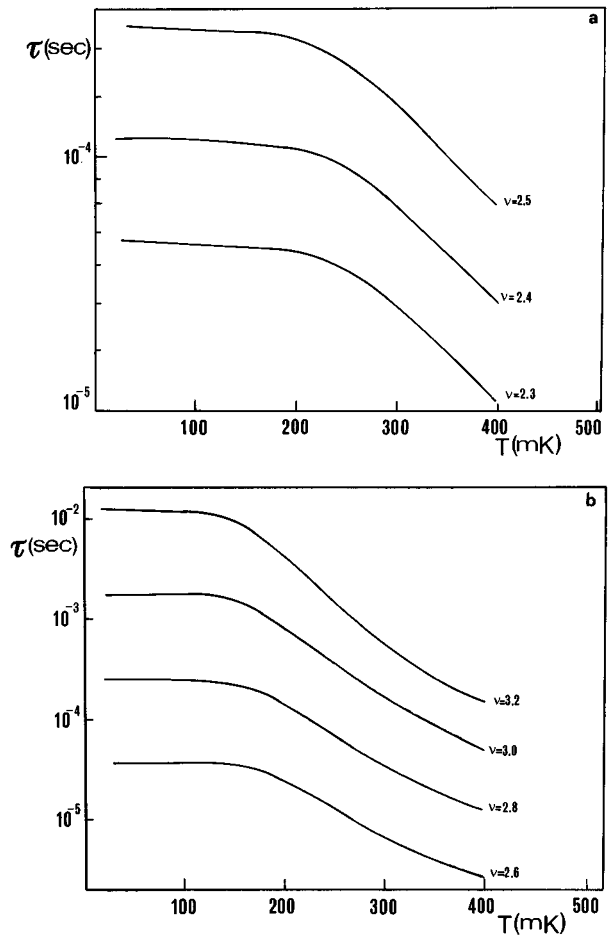

FIG. 4. Lifetime of the resonant state as a function of temperature, for different values of ν, at fixed voltage of the resonant state: (a) $V=10$ nV and two levels in the potential well; (b) $V=1$ nV and three levels in the potential well.

with these parameters can be built by standard fabrication technique, although to obtain an effective impedance $R \gg R_Q$ in any real experimental configuration great care in insulating the junction from the external electronics and environment is required and it may present a difficulty.[7]

In Fig. 2 we plot the escape rate as a function of the current for ν ranging between 2.3 and 3, and at different temperatures ($T=50$ and 200 mK). As a general consideration, we note that the escape rate presents, for current close to the resonance, a peak whose sharpness increases with increasing ν and with decreasing temperature. Γ ranges between $10–10^5$ sec^{-1} for $\nu=3$ and $T=50$ mK, whereas is almost flat (10^6 sec^{-1}) for $\nu=2.3$ and $T=200$ mK. The effect of temperature is larger for larger ν values, mostly because of different "quantum temperatures" $T_Q=\hbar\omega_j/K$, being $T_Q=1.51$ K for $\nu=3$ and $T_Q=1.97$ K for $\nu=2.3$. These Γ values could be quite accessible in measurements of the current switching distributions using the existing techniques.[6,7]

In order to discuss the possibility of observing the RMQT voltage peaks on the supercurrent branch of the I-V characteristics, we consider the lifetime τ of the resonant state which can be defined as the reverse of Γ, $\tau=\Gamma^{-1}$.

In Fig. 3 we plot the lifetime τ as a function of the voltage along the step for $\nu=3$ and at various temperatures. Finally considering a figure of merit $\chi=V\tau^{1/2}$, we find that χ is rang-

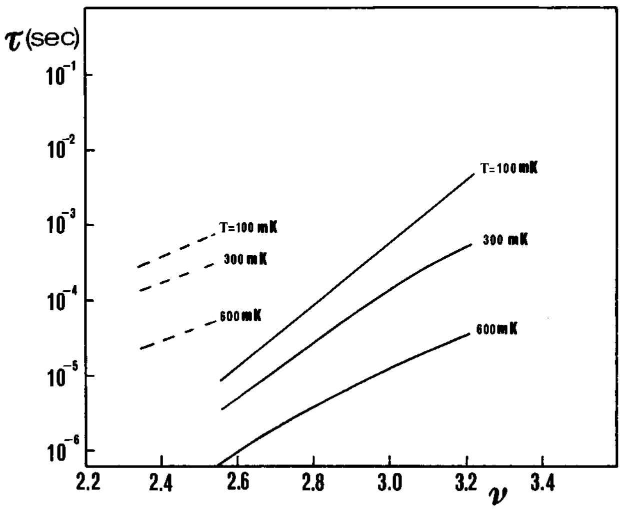

FIG. 5. Lifetime of the resonant state as a function of $\nu = E_0/\hbar\omega_j$, for different values of temperature, at fixed voltage of the resonant state, $V=3$ nV. The curves present a discontinuity at ν value for which the levels inside the well change from two (dashed lines) to three (solid lines).

ing between 10^{-11}–10^{-10} $\mathrm{VHz}^{-1/2}$, making a direct observation of the voltage peaks on the I-V curve a hard goal.

In Fig. 4, we show τ vs T, at a fixed step voltage on the I-V curve and different values of ν: $V=10$ nV for $\nu=2.3$–2.5 (two levels in the well) and $V=1$ nV for $\nu=2.6$–3.2 (three levels in the well). This temperature dependence shows an exponential behavior with saturation at low temperatures. For the whole family of curves a saturation temperature T_S can be identified, which is related to the quantum temperature, roughly as $T_S = T_Q/10$. Although this temperature cannot be defined by a precise value, due to its slight dependence on other junction parameters and voltage amplitude, it can provide an indication of the temperature range of experimental interest in choosing the actual working conditions.

Finally in Fig. 5 we report the dependence of τ as function of ν at different temperatures. Here the step voltage has been fixed at $V=3$ nV. We note an exponential dependence with a strong discontinuity while changing the levels in the well from two to three. In fact, close to the resonance, it is relatively easy to find the system in excited levels, and the escape rate is therefore dominated by the tunnel probability from the upper level, which strongly depends on its energy. The upper energy level is very close to the barrier energy when one more level can stay in the well.

In conclusion, we have investigated the phenomenon of resonant tunneling which occurs in small current-biased Josephson junctions. This produces voltage spikes on the I-V curve as well as typical peaks in the escape rate to the free running state. Our approach, allows us to include the temperature in the calculations, both for what concerns the resonant tunnel probability γ_0, and the escape rate to the free running state. Our results, extrapolated to $T=0$, are in good agreement with those obtained in Ref. 10 with a different approach, confirming the great difficulties one would find in trying to detect the voltage steps. This is due to the small lifetime ($\tau \approx 0.1$–1 msec) for voltages large enough to be measured ($V \approx 1$ nV) in the low-temperature range ($T < 200$ mK). The effect of RMQT on the switching current distributions seems at the moment the most readily observable effect in current-biased Josephson junctions. For what concerns the temperature behavior of lifetime, we found an exponential dependence with a saturation at low temperatures ($T < T_Q/10$).

Very recently we become aware of an important experimental result[17] which, though referring to a superconducting quantum interference device (SQUID) rather than a single junction structure, clearly show the occurrence of RMQT in a macroscopic system. The dynamic of the SQUID may be simpler since after each tunneling event a different observable state is reached, while in a single current-biased junction this is obtained only after a sequence of tunneling events. The experimental peaks appear wider than the extremely sharp ones predicted by the theory for a single junction. Such a large broadening could be hardly ascribed only to intrinsic dissipation. Any quantitative comparison would require a further detailed analysis specific of the SQUID system.

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[1] A. D. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); See also A. J. Leggett, in *Quantum Tunneling in Condensed Media*, edited by Yu. Kagan and A. J. Leggett (Elsevier, Amsterdam, 1992), and references therein.
[2] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **91**, 318 (1986) [Sov. Phys. JETP **64**, 185 (1986)].
[3] C. D. Tesche, Phys. Rev. Lett. **64**, 2358 (1990).
[4] K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. **59**, 347 (1985).
[5] D. V. Averin and K. K. Likharev, J. Low Temp. Phys. **62**, 345 (1986).
[6] S. Washburn *et al.*, Phys. Rev. Lett. **54**, 2712 (1985); D. B. Schwartz *et al.*, *ibid.* **55**, 1547 (1985).
[7] J. M. Martinis *et al.*, Phys. Rev. B **35**, 4682 (1987); and references therein.
[8] D. Esteve *et al.*, Phys. Rev. B **34**, 158 (1986).
[9] L. J. Geerligs *et al.*, Phys. Rev. Lett. **63**, 326 (1989).
[10] J. M. Schmidt *et al.*, Phys. Rev. B **43**, 229 (1991).
[11] N. Hatakenaka *et al.*, Phys. Rev. B **42**, 3987 (1990).
[12] A. V. Zhuravlev and A. B. Zorin, Sov. J. Low Temp. Phys. **16**, 102 (1990).
[13] Yu. N. Ovchinnikov *et al.*, J. Supercond. **5**, 481 (1992).
[14] A. Barone and G. Paternò, *Physics and Applications of Josephson Effect* (Wiley, New York, 1982), Chap. 6.
[15] A. I. Larkin *et al.*, Physica B **152**, 266 (1988).
[16] This condition is well verified up to extremely close to the resonance for R values chosen in the plots.
[17] R. Rouse *et al.*, Phys. Rev. Lett. **75**, 1614 (1995); and (unpublished).