Theory of the bipolar spin switch

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We extend the Valet-Fert model of the perpendicular magnetoresistance in magnetic multilayers to describe the spin accumulation and relaxation effects in the spin switch structure introduced by Johnson and calculate the output voltage of the device. In contrast to the usual treatment by Johnson, we take into account the spin relaxation in the ferromagnetic layers, and also the influence of interface resistances and interface spin flips. We show that, for thin nonmagnetic layers, the output voltage is limited by the spin relaxation in the ferromagnetic layers. We find that the interpretation of the experimental data requires surprisingly long spin diffusion lengths in both the magnetic and nonmagnetic layers, much longer than those derived from perpendicular magnetoresistance in multilayers of similar materials.

I. INTRODUCTION

The subject of this paper is the theory of the ''bipolar spin switch'' introduced by Mark Johnson.^{1,2} This device is based on the spin accumulation effects generated by injecting a spin polarized current from a ferromagnetic metal into a nonmagnetic metal.¹⁻⁵ The geometry of Johnson's spin switch^{1,2} is shown in Fig. 1. A current is driven from a ferromagnetic metal F_1 into a nonmagnetic metal N. The spin polarization of this current creates a *spin accumulation*, i.e., a *nonequilibrium magnetization* that can be described as a splitting between the chemical potentials of the spin up and spin down conduction electrons. A second ferromagnetic material $F₂$ is used to probe these chemical potentials. Its voltage depends on the relative orientation of the magnetizations in F_1 and F_2 (Refs. 1, 2):

 $V_P - V_{AP} = \Delta R_s I$ (1)

or

$$
V_{P(\text{AP})} = +(-)\Delta R_s I/2\tag{2}
$$

if the reference voltage is that of a nonmagnetic metal probe. Here *P* and AP refer respectively to the parallel and antiparallel arrangements of the magnetizations in F_1 and F_2 , and *I* is the current through the F_1/N interface.

The same spin accumulation effects play an essential role in the current-perpendicular-to-plane magnetoresistance of magnetic multilayers, or CPP-MR. $^{6-10}$ For typical experimental results in which the balance between spin accumulation and spin relaxation appears clearly, readers can refer, for example, to Yang *et al.*⁷ These experiments have been accounted for in the model worked out by Valet and Fert (VF model) to describe the spin accumulation in periodic multilayered structures.^{9,10} Here we extend the VF model to Johnson's spin switch structure. The main difference from Johnson's calculation^{1,2} is that we take into account the *spin relaxation not only in the nonmagnetic metal but also in the ferromagnetic materials*. The classical result of Johnson for ΔR_s is written as

$$
R_s = \Delta R_s / 2 = \eta_1 \eta_2 \frac{\rho \delta_s^2}{A d},\tag{3}
$$

where, in the notation of Ref. 2, ρ is the resistivity of the nonmagnetic metal, δ _s is the spin diffusion length in the nonmagnetic metal, i.e., $\delta_s = (v_F^2 \tau T_1^N/3)^{1/2}$, where τ and T_1^N are the momentum and spin relaxation times, respectively, in the nonmagnetic metal (δ_s is called l_{sf}^N in the VF model), *A* is the area of the *F*/*N* interface, *d* is the thickness of the nonmagnetic layer and is supposed to be much smaller than δ_{s} , and η_1 , η_2 are phenomenological coefficients expressing the incomplete polarization of the current in the ferromagnets. It can be pointed out that Eq. (3) depends only on the spin relaxation time T_1^N in the nonmagnetic layer $(R_s \sim T_1^N/d)$, whereas the spin accumulation is normally expected to extend by diffusion in F_1 and F_2 , and thus should be also affected by the spin relaxation in the ferromagnetic layers. Johnson's implicit assumption, as illustrated for example by Fig. 1 in Refs. 1 and 2, is that F_1 and F_2 are half-metallic ferromagnets with a zero density of states at the Fermi level for one of the spin directions (the majority spin direction or spin \uparrow). With no empty states at the Fermi level in the spin \uparrow

FIG. 1. Schematic drawing of Johnson's spin switch device. The voltage measured as shown depends on the relative orientation of the magnetizations in F_1 and F_2 .

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direction, there is no spin flip and no relaxation of the out of equilibrium magnetization within the ferromagnetic metal. $¹¹$ </sup>

In ferromagnetic metals such as Co, Ni, or Fe, it is clear that there is a nonzero density of states at the Fermi level in both spin directions. The spin relaxation time and spin diffusion length are not infinite, so that the spin accumulation and ΔR_s should be affected by the spin relaxation rates in F_1 and $F₂$. Our extension of the VF model to the spin switch problem takes into account the spin relaxation effects in both the nonmagnetic and ferromagnetic materials and, as we will show, ΔR_s depends not only on the spin relaxation time in the nonmagnetic material but also (and essentially in usual experimental conditions) on the spin relaxation and diffusion parameters in the ferromagnetic layers. Our notation for the density of states at the Fermi level will be $N(E_F)$ for the nonmagnetic metal, and $N_{\uparrow}(E_F), N_{\downarrow}(E_F)$ for the ferromagnetic material (for simplicity, we assume that F_1 and F_2 are made of the same material). In contrast to the VF model where $N(E_F)$, $N_\uparrow(E_F)$, and $N_\uparrow(E_F)$ were supposed to be equal, we have introduced three free parameters, so that, by decreasing $N_{\uparrow}(E_F)$ to zero, we can go to the half-metallic limit and compare our results with those of Johnson $Eq.$ (3)].

The second significant difference between our approach and that of Johnson is that we introduce spin dependent interface resistances in the calculation. The importance of the interface resistances in the perpendicular transport has been clearly shown by the CPP-MR experiments in multilayers and their interpretations.⁶⁻⁹ As we will show, the interface resistances can play an important role as they tend to confine the spin accumulation in the nonmagnetic layer and to increase ΔR_s . However, when some probability of spin flip scattering by the interface is also introduced, this contributes to the spin relaxation and leads to the opposite effect of a reduction of ΔR_s .

This paper is organized as follows. In Sec. II, we extend the expressions describing the spin accumulation effects in the VF model to the case with different values of the density of states at the Fermi level, $N(E_F)$, $N_\uparrow(E_F)$, and $N_\uparrow(E_F)$, and we describe how they can be applied to the spin switch geometry. In Sec. III we present the calculation of ΔR_s in the simplest case where there is no interface resistance. We introduce interface resistances and interface spin flips in Sec. IV and, in Sec. V, we relate our result for ΔR_s in the spin switch structure to the resistance change in a CPP-MR experiment. Section VI is devoted to a discussion of the results of our calculation and Sec. VII to a comparison with experiments on spin switch structures and CPP-MR experiments. Our conclusions are summarized in Sec. VIII.

A calculation based on a similar approach has been recently worked out by Hershfield and Zhao.¹² They also take into account the spin relaxation in the ferromagnetic layers and interface resistances but do not introduce the effect of spin flip at interfaces. Most of their results are similar to ours. However, our conclusion on the influence of interfaces is definitely different.

II. BASIC EQUATIONS AND OUTLINE OF THE CALCULATION

We adopt the notation of the VF model:⁹ $\uparrow(\downarrow)$ refers to the majority (minority) spin direction, $+(-)$ refers to the absolute spin direction (i.e., $\sigma = \pm 1/2$), $J_{+}(z)[J_{-}(z)]$ and $\mu_+(z)[\mu_-(z)]$ are respectively the current density in the direction *z* perpendicular to the layer planes and the electrochemical potential of the spin $+$ (spin $-)$ electrons. With the uniform distribution of the *z* component of the current assumed in the calculation, we have $J_{+(-)} = I_{+(-)} / A$. Finally we write

$$
\rho_{\uparrow(\downarrow)} = 2\rho_F^*(1 \mp \beta) \tag{4}
$$

for the resistivity of the spin \uparrow (spin \downarrow) channel in the ferromagnetic metal and, in the same way, for the nonmagnetic metal,

$$
\rho_{\uparrow(\downarrow)} = 2\rho_N^* \tag{5}
$$

 $(\rho_N^*$ is the resistivity of the metal). In Sec. IV, we shall also introduce interfacial nonspin-flip and spin-flip resistances.

The basic equations are the macroscopic equations demonstrated from a microscopic Boltzmann model in Ref. 9. These equations are equivalent to those used by Johnson and co-workers, $1-4$ and van Son *et al.*⁵ The first equation is a pseudo-Ohm's law,

$$
J_{\sigma} = \frac{1}{|e|\rho_{\sigma}} \frac{d\mu_{\sigma}}{dz}.
$$
 (6)

Two other equations express the conservation of charge and spin respectively,

$$
J_+(z) + J_-(z) = \text{const} \tag{7}
$$

or

$$
\frac{dJ_+}{dz} + \frac{dJ_-}{dz} = 0\tag{8}
$$

and

$$
\frac{dJ_+}{dz} - \frac{dJ_-}{dz} = \frac{\Delta M|e|}{\mu_B T_1},\tag{9}
$$

where ΔM is the out of equilibrium magnetization and T_1 its relaxation time (unlike in Ref. 9, we come back to the conventional notation for the magnetization relaxation time, T_1 replacing $\tau_{\text{sf}}=2T_1$). T_1 has values T_1^N and T_1^F in the nonmagnetic and ferromagnetic metals, respectively. The electrochemical potentials can be written as

$$
\mu_{\pm}(z) = \mu(z) + \Delta \mu_{\pm}(z). \tag{10}
$$

 $\mu(z)$ is spin independent, while $\Delta \mu_+(z)$ and $\Delta \mu_-(z)$ are spin dependent and related to the out of equilibrium magnetization (or spin accumulation) ΔM by

$$
\Delta \mu_{\pm} = \pm \frac{\Delta M}{2 \mu_B N_{\pm}(E_F)}.
$$
\n(11)

 $N_{+}(E_F)$ and $N_{-}(E_F)$ are the density of states in the two channels and, depending on the orientation of the magnetizations, $N_+(E_F) = N_\uparrow(E_F)$ and $N_-(E_F) = N_\uparrow(E_F)$ or vice versa.

By combining Eqs. (6) – (11) we find that ΔM , $\mu_+(z)$, $\mu(z)$, $\Delta\mu_+(z)$, and $J_+(z)$ obey the same differential equation as that found in Ref. 9,

$$
\frac{d^2f(z)}{dz^2} = \frac{f(z)}{l_{\text{sf}}^{N(F)^2}}.\tag{12}
$$

The spin diffusion length in the nonmagnetic metal, l_{sf}^{N} has the same expression as in Ref. 9

$$
l_{\rm sf}^N = \left(\frac{T_1^N}{2e^2\rho_N^*N(E_F)}\right)^{1/2} \tag{13}
$$

or, if ρ_N^* is expressed in a free electron model,

 \overline{I}

$$
N_{\rm sf} = \left(\frac{\lambda \lambda_{\rm sf}}{6}\right)^{1/2} \tag{14}
$$

with $\lambda = \tau v_F$, $\lambda_{\text{sf}} = \tau_{\text{sf}} v_F = 2 T_{1}^{N} v_F$.

The new expression for the spin diffusion length in the ferromagnetic metal, l_{sf}^F , is

$$
l_{\rm sf}^F = \left\{ \frac{T_1^F}{4e^2 \rho_F^*} \left[\frac{1}{N_1(E_F)} + \frac{1}{N_1(E_F)} \right] \right\}^{1/2},\tag{15}
$$

which reduces to the expression of Ref. 9 when ρ_F^* is expressed in a free electron model and $N_{\uparrow}(E_F) = N_{\uparrow}(E_F)$. The functions $\mu_{\pm}(z)$, $J_{\pm}(z)$, etc. are therefore linear combinations of constant, linear, and exponential terms $\left[\exp(\pm z/l_{\rm sf}^{N(F)})\right]$.

As illustrated by Fig. $2(a)$, the difficulty of Johnson's geometry for a theoretical treatment is that the current lines curve down progressively. The current arrives along the horizontal direction of the figure and goes out vertically. Some current lines, illustrated by line 1, cross N , penetrate F_2 and then curve down to go out along *N*. Some others turn down within *N* between the F_1/N and F_2/N interfaces; see line 2. Some others, see line 3, have practically turned to the vertical direction before penetrating F_1/N interfaces. An exact solution of the problem would depend inevitably on the geometrical parameters of the structure, so that we have adopted the following approximate solution.

We consider the current distribution obtained by adding up the current distributions of Figs. $2(b)$ and $2(c)$. In Fig. 2(b), a uniform current *I*/2 flows horizontally from $z = -\infty$ to $z=+\infty$ (as in a conventional CPP-MR experiment). In Fig. 2(c), a current *I*/2 flows from $z=-\infty$ to $z=0$, while an equal current *I*/2 flows in the opposite direction from $z = +\infty$ to $z=0$. The plane $z=0$ is a drain collecting the two currents, so that there is a total current *I* downwards. Adding up the two distributions of Figs. $2(b)$ and $2(c)$ leads to the current distribution of Fig. 2(d), where a current *I* flows from $z = -\infty$ to $z=0$, and at $z=0$, curves down abruptly to the vertical direction. In the real distribution of Fig. $2(a)$, some of the current lines curve down after $z=0$, some before. However, since the scaling length of the spin accumulation effects is very long, the solution with an abrupt turn at $z=0$ should provide us with an approximate solution to the problem.

What we have to calculate now is the electrochemical potential $\mu_+(z)$ for both the current distributions of Figs. $2(b)$ and $2(c)$ and successively for antiparallel AP) and parallel (P) arrangements of the magnetizations in F_1 and F_2 . This will be done first in Sec. III without interfacial resistance and interfacial spin relaxation, consequently not only the charge current but also the spin current and the electro-

FIG. 2. (a) Current distribution in the spin switch device. (b) Current distribution for a conventional current-perpendicular-toplane geometry with current $I/2$. (c) Current distribution with two uniform currents *I*/2 flowing from $z = \pm \infty$ to a drain in the $z=0$ plane. A total current *I* flows downwards in the $z=0$ plane. (d) Current distribution obtained by adding up the current distributions $~$ (b) and $~$ (c), as an approximation of $~$ (a).

chemical potentials are continuous at the F_1/N and N/F_2 interfaces in this case. In Sec. IV we take into account the interfacial resistance and interfacial spin relaxation. This leads to a discontinuity of the electrochemical potentials related to the interface resistance and a discontinuity of the spin currents related to the spin flip interface resistance.

III. CALCULATION WITHOUT INTERFACE RESISTANCE

We first calculate the electrochemical potential $\mu_+(z)$ for the situation of Fig. $2(b)$ (horizontal current density $J/2$ from $z=-\infty$ to $z=+\infty$) and we will consider successively AP and P configurations of F_1 and F_2 . We choose the value of μ ₊ at $z=0$ as reference and write

$$
\mu_{\pm}(z) = -e\rho_F^*(1-\beta^2) \frac{J}{2} (z+a) + e\rho_N^* \frac{J}{2} a + \delta \mu^{F_1}(z) + \Delta \mu_{\pm}^{F_1}(z) \quad \text{in } F_1,
$$
 (16)

$$
\mu_{\pm}(z) = -e\rho_N^* \frac{J}{2} z + \Delta \mu_{\pm}^N(z) \quad \text{in } N,\tag{17}
$$

$$
\mu_{\pm}(z) = -e\rho_F^*(1-\beta^2) \frac{J}{2} (z-a) - e\rho_N^* \frac{J}{2} a + \delta \mu^{F_2}(z) \n+ \Delta \mu_{\pm}^{F_2}(z) \quad \text{in } F_2,
$$
\n(18)

where the terms $\delta \mu^{F_1}(z)$, $\Delta \mu_{\pm}^{F_1}(z)$, $\Delta \mu_{\pm}^{N}(z)$, $\delta \mu^{F_2}(z)$, $\Delta \mu_{\pm}^{F_2}(z)$ are generated by the spin accumulation effects and, according to Eq. (12) , are linear combinations of constant, linear, and exponential terms $\left[\exp(\pm z/l_{\text{sf}}^F)\right]$ in F_1 and F_2 , $\exp(\pm z/l_{\rm sf}^N)$ in *N*].

The boundary conditions when there is no interface resistance and no interfacial spin flips are the following: $\mu_+(z)$ and $\mu_-(z)$ are continuous at $z = \pm a$ ($a = t_N/2$, t_N is the thickness of *N*); the charge current $[J_+(z)+J_-(z)]$ is equal to *J*/2 everywhere; and the spin current proportional to $[J_{+}(z)-J_{-}(z)]$ is continuous at $z=\pm a$, where the current densities J_{\pm} are related to μ_{\pm} by Eq. (6).

In the case of an AP configuration (majority spins with a positive orientation in F_1 and negative in F_2), the solution can be written as

$$
\delta \mu^{F_1}(z) = \delta \mu_F - \frac{\frac{1}{\rho_\uparrow N_\uparrow(E_F)} - \frac{1}{\rho_\downarrow N_\downarrow(E_F)}}{\frac{1}{\rho_\uparrow} + \frac{1}{\rho_\downarrow}} \mu_F \exp(z/l_{\rm sf}^F),\tag{19}
$$

$$
\Delta \mu_{\pm}^{F_1}(z) = \pm \frac{1}{N_{\uparrow(1)}(E_F)} \ \mu_F \exp(z/l_{\rm sf}^F), \tag{20}
$$

$$
\Delta \mu_{\pm}^{N}(z) = \pm \frac{2}{N(E_F)} \mu_N \cosh(z/l_{\text{sf}}^{N}), \tag{21}
$$

$$
\delta \mu^{F_2}(z) = -\delta \mu_F + \frac{\frac{1}{\rho_\uparrow N_\uparrow(E_F)} - \frac{1}{\rho_\downarrow N_\downarrow(E_F)}}{\frac{1}{\rho_\uparrow} + \frac{1}{\rho_\downarrow}} \mu_F \exp(-z/l_{\rm sf}^F),\tag{22}
$$

$$
\Delta \mu_{\pm}^{F_2}(z) = \pm \frac{1}{N_{\downarrow(\uparrow)}(E_F)} \ \mu_F \exp(-z/l_{\rm sf}^F), \tag{23}
$$

with

$$
\mu_N = \frac{e\beta J N(E_F)}{4\left[\frac{1}{\rho_N^* l_{\rm sf}^N} \sinh\left(\frac{a}{l_{\rm sf}^N}\right) + \frac{1}{\rho_F^* l_{\rm sf}^N} \cosh\left(\frac{a}{l_{\rm sf}^N}\right)\right]},\tag{24}
$$

$$
\mu_F = \frac{4\,\mu_N \text{cosh}\left(\frac{a}{l_{\text{sf}}^N}\right) \text{exp}\left(\frac{a}{l_{\text{sf}}^N}\right)}{N(E_F)\left(\frac{1}{N_\uparrow(E_F)} + \frac{1}{N_\downarrow(E_F)}\right)},\tag{25}
$$
\n
$$
2\,\beta\,\mu_N \text{cosh}\left(\frac{a}{l_N^N}\right)
$$

$$
\delta \mu_F = \frac{P \mu_N \cos \omega \left(\frac{l_N^N}{l_{\rm sf}^N}\right)}{N(E_F)}.
$$
 (26)

For the spin switch problem, we are interested in the voltage difference between $z = +\infty$ and $z = 0$. We ignore the contribution from linear terms which will be cancelled by terms with opposite signs of the current distribution of Fig. $2(c)$. We keep

$$
V_{\rm AP} = \frac{-\delta\mu_F}{e} = \frac{-\beta^2 J \cosh\left(\frac{a}{l_{\rm sf}^N}\right)}{2\left[\frac{1}{\rho_N^* l_{\rm sf}^N} \sinh\left(\frac{a}{l_{\rm sf}^N}\right) + \frac{1}{\rho_F^* l_{\rm sf}^F} \cosh\left(\frac{a}{l_{\rm sf}^N}\right)\right]}.
$$
\n(27)

In the case of a P configuration (majority spin in the positive orientation in both F_1 and F_2) the solution is written as

$$
\delta \mu^{F_1}(z) = \delta \mu'_F - \frac{\frac{1}{\rho_\uparrow N_\uparrow(E_F)} - \frac{1}{\rho_\downarrow N_\downarrow(E_F)}}{\frac{1}{\rho_\uparrow} + \frac{1}{\rho_\downarrow}} \mu'_F \exp(z/l_{\text{sf}}^F),\tag{28}
$$

$$
\Delta \mu_{\pm}^{F_1}(z) = \pm \frac{1}{N_{\uparrow(\downarrow)}(E_F)} \ \mu_F' \exp(z/l_{\text{sf}}^F), \tag{29}
$$

$$
\Delta \mu_{\pm}^{N}(z) = \pm \frac{2}{N(E_F)} \mu_N \sinh(z/l_{\rm sf}^{N}), \tag{30}
$$

$$
\delta \mu^{F_2}(z) = -\delta \mu'_F + \frac{\frac{1}{\rho_\uparrow N_\uparrow(E_F)} - \frac{1}{\rho_\downarrow N_\downarrow(E_F)}}{\frac{1}{\rho_\uparrow} + \frac{1}{\rho_\downarrow}} \mu'_F \exp(-z/l_{\text{sf}}^F),\tag{31}
$$

$$
\Delta \mu_{\pm}^{F_2}(z) = \mp \frac{1}{N_{\uparrow(\downarrow)}(E_F)} \mu_F' \exp(-z/l_{\text{sf}}^F). \tag{32}
$$

The expression of μ_N is still given by Eq. (24), while μ'_F and $\delta \mu_F'$ can be obtained by replacing cosh by sinh in Eqs. (25) and (26). The voltage difference between $z = +\infty$ and $z = 0$ (contribution from linear terms excluded) is written as

$$
V_P = \frac{-\delta\mu'_F}{e} = \frac{-\beta^2 J \sinh\left(\frac{a}{l_{\rm sf}^N}\right)}{2\left[\frac{1}{\rho_N^* l_{\rm sf}^N} \cosh\left(\frac{a}{l_{\rm sf}^N}\right) + \frac{1}{\rho_F^* l_{\rm sf}^F} \sinh\left(\frac{a}{l_{\rm sf}^N}\right)\right]}.
$$
(33)

For the solution of Fig. $2(c)$, the variation of the electrochemical potential can be calculated in a similar way. We find that the voltage differences between $z = +\infty$ and $z = 0$, V'_{AP} and V'_{P} for the AP and *P* configurations, respectively, are related to V_{AP} and V_P calculated above by

$$
V'_{AP} = -V_P,
$$

$$
V_P' = -V_{\rm AP},\tag{34}
$$

as it can also be predicte from time reversal symmetry arguments.

Finally, if we define ΔR , by Eq. (1), we find

$$
\Delta R_s = \frac{V_P - V_{AP} + V_P' - V_{AP}'}{AJ} \tag{35}
$$

or

$$
\Delta R_s = \frac{\beta^2}{A} \left(\frac{\cosh\left(\frac{t_N}{2I_{\rm sf}^N}\right)}{\frac{1}{\rho_N^* l_{\rm sf}^N} \sinh\left(\frac{t_N}{2I_{\rm sf}^N}\right) + \frac{1}{\rho_F^* l_{\rm sf}^F} \cosh\left(\frac{t_N}{2I_{\rm sf}^N}\right)} - \frac{\sinh\left(\frac{t_N}{2I_{\rm sf}^N}\right)}{\frac{1}{\rho_N^* l_{\rm sf}^N} \cosh\left(\frac{t_N}{2I_{\rm sf}^N}\right) + \frac{1}{\rho_F^* l_{\rm sf}^F} \sinh\left(\frac{t_N}{2I_{\rm sf}^N}\right)} \right). \tag{36}
$$

After some algebraic calculation, this leads to

$$
\Delta R_s = \frac{2\beta^2 \rho_N^* l_{\text{sf}}^N / A}{\left[1 + \left(\frac{\rho_N^* l_{\text{sf}}^N}{\rho_F^* l_{\text{sf}}^F}\right)^2\right] \sinh\left(\frac{t_N}{l_{\text{sf}}^N}\right) + 2\frac{\rho_N^* l_{\text{sf}}^N}{\rho_F^* l_{\text{sf}}^F} \cosh\left(\frac{t_N}{l_{\text{sf}}^N}\right)}\,. \tag{37}
$$

Johnson's expression in Ref. 2, i.e., Eq. (3) in the present article, can be found by taking the half-metallic limit for the ferromagnetic metal [i.e. $(P_F^* l_{\text{sf}}^F)^{-1} = 0$] and the limit where t_N is much smaller than l_{sf}^N . In our notation, this gives

$$
\Delta R_s = \frac{2\beta^2 \rho_N^* l_{\rm sf}^{N^2}}{A t_N}.
$$
\n(38)

IV. CALCULATION WITH INTERFACE RESISTANCES

A. Without interface spin flips

The importance of interface electron scattering had been shown by a large number of giant magnetoresistance measurements.13 We will first consider the effects associated with scattering without spin flips. In the geometry with the current perpendicular to the layer plane, this scattering is expressed by introducing interface resistances, $6-10$ and extensive data on the value and spin dependence of these interface resistances have already been derived from experiments. $6-8$ The notation in the VF model for the interface resistances is 9

$$
r_{\uparrow(\downarrow)} = 2r_b^*(1 \mp \gamma). \tag{39}
$$

These interface resistances introduce discontinuity in the variation of the electrochemical potentials at the interfaces, for example,

$$
\frac{1}{e} \left[\mu_{\pm}(z = z_0^+) - \mu_{\pm}(z = z_0^-) \right] = r_{\pm} J_{\pm}(z = z_0) \tag{40}
$$

for an interface at $z = z_0$ $[z = z_0^{+(-)}]$ means just before (after) the interface]. Typical experimental values of r_b^* in systems such as Co/Cu, Co/Ag, NiFe/Cu, or NiFe/Ag are in the range of $10^{-15} \Omega \text{ m}^2$ (f $\Omega \text{ m}^2$), with spin asymmetry coefficients in the range $0.7-0.8$ (to be compared with bulk spin asymmetry coefficients $\beta \approx 0.5$ in the same systems).

To take into account the effect of interface resistance, we replace the continuity condition for the electrochemical potentials in the preceding section by Eq. (40) . This has two effects. (1) If γ is different from β , this changes the polarization of the injected current. (2) The spin accumulation is more confined in the nonmagnetic layer when there is an interface resistance.

The first effect is not essential. It is clearly expected that when γ is larger (smaller) than β , the current polarization and ΔR_s will be increased (decreased). The second effect, related to spin confinement by interface resistances, is more interesting. To focus on the confinement effect, we will put the general expressions for γ different from β in Appendix A and, in this section, we assume that γ and β are equal. By introducing Eq. (40) in the calculation of the preceding section and $\gamma = \beta$, we get the following expression for ΔR_s instead of Eq. (37) :

$$
\Delta R_s = \frac{2\beta^2 \rho_N^* l_{\rm sf}^N A}{\left[1 + \left(\frac{\rho_N^* l_{\rm sf}^N}{\rho_F^* l_{\rm sf}^r + r_b^*}\right)^2\right] \sinh\left(\frac{t_N}{l_{\rm sf}^N}\right) + 2\frac{\rho_N^* l_{\rm sf}^N}{\rho_F^* l_{\rm sf}^r + r_b^*} \cosh\left(\frac{t_N}{l_{\rm sf}^N}\right)}.
$$
\n(41)

It can be checked easily that Eq. (41) reduces to Eq. (37) in the absence of interface resistance, i.e., when r_b^* equals zero.

B. With interface spin flips

The interface resistances $r₁$ and $r₁$, Eq. (39), introduced in theories of CPP-MR are supposed to be associated with scattering without spin flip (with contributions from diffuse scatterings by interface roughness and Landauer type contributions from specular scattering by interface potential steps).¹⁰ Rigorously, and whatever the mechanism (roughness or Landauer) involved is, some proportion of scattering with spin flip is expected if one takes into account the spin orbit coupling in the scattering potentials or in the wave functions. For 3*d* or noble metals, the order of magnitude of this proportion is only 10^{-2} so that the spin flip scattering by interfaces has been neglected in the VF model of the CPP-MR. However, as we show below, this small proportion of spin flip interface scattering can be important for the spin switch problem and is taken into account by introducing spin flip interface resistance in our calculation. The effect of the interface spin flip is to contribute to the relaxation of the spin accumulation and to produce a discontinuity in the spin current as expressed by the following equation:

$$
\frac{1}{e} \left(\overline{\mu_+ - \mu_-} \right)_{z_0} = r_b^{\text{sf}} \left[(J_+ - J_-)_{z_0^-} - (J_+ - J_-)_{z_0^+} \right], \tag{42}
$$

where $(\mu_+ - \mu_-)_{e_0}$ is the mean value of $[\mu_+(z) - \mu_-(z)]$ in the interface region at $z=z_0$. As the thickness of the interfaces (a few \AA) is much smaller than the spin diffusion length, the variation of $\mu_+(z)$ and $\mu_-(z)$ can be treated as linear within this thickness, so that Eq. (42) can be written as

$$
\frac{1}{2e} \left[\mu_+(z_0^-) + \mu_+(z_0^+) - \mu_-(z_0^-) - \mu_-(z_0^+) \right]
$$

$$
= r_b^{\text{sf}} \left[(J_+ - J_-)_{z_0^-} - (J_+ - J_-)_{z_0^+} \right]. \tag{43}
$$

Equation (43) replaces the continuity equation of the spin current used in Sec. III. Without interface spin flip, r_b^{sf} is infinite and the continuity of the spin current is recovered. In addition, Eq. (40) must be written for the mean value of the currents $J_{\pm}(z=z_0) = 1/2[J_{\pm}(z_0^+) + J_{\pm}(z_0^-)].$

We have performed again the calculation of Sec. III by taking into account not only r_b^* but also r_b^{sf} . We find for ΔR_s

$$
\Delta R_{s} = \frac{2\beta^{2} \rho_{N}^{*} l_{sf}^{N} A}{\left[1 + \left(\frac{\rho_{N}^{*} l_{sf}^{N} (\rho_{F}^{*} l_{sf}^{F} + r_{b}^{s f})}{r_{b}^{s f} (\rho_{F}^{*} l_{sf}^{F} + r_{b}^{s f})}\right)^{2}\right] \sinh\left(\frac{t_{N}}{l_{sf}^{N}}\right)} + 2 \frac{\rho_{N}^{*} l_{sf}^{N} (\rho_{F}^{*} l_{sf}^{F} + r_{b}^{s f})}{r_{b}^{s f} (\rho_{F}^{*} l_{sf}^{F} + r_{b}^{*} + \frac{r_{b}^{*} \rho_{F}^{*} l_{sf}^{F}}{r_{b}^{s f}})} \cosh\left(\frac{t_{N}}{l_{sf}^{N}}\right)}.
$$
\n(44)

It can be checked that Eq. (44) reduces to Eq. (41) when r_b^{sf} is infinite and to Eq. (37) when, in addition, r_b^* is zero.

V. CONNECTION WITH CPP-MR EXPERIMENTS

All the results presented in Sec. III and IV are for the resistance ΔR_s defined for the Johnson geometry by Eq. (1) and related to a voltage difference between F_2 and *N*. But our calculation can also be applied to the interpretation of CPP-MR experiments. In the CPP-MR geometry of Fig. 2(b), the extra voltage between $z = +L$ and $z = -L$ with *L* much larger than l_{sf}^F is $-2\delta\mu_F/e$ for an AF configuration (or $-2\delta\mu_F^{\prime}/e$ for a *F* configuration), that is twice the voltage V_{AP} (or V_P) measured in the Johnson geometry and given by Eq. (27) [or Eq. (33)]. The resistance change in this CPP-MR experiment is therefore written as

$$
\Delta R^{\text{CPP}} = \frac{2(V_P - V_{AP})}{AJ/2} = 2\Delta R_s \tag{45}
$$

and can be calculated from the expressions of ΔR_s given in Sec. III and IV.

Equation (45) can also be applied to the CPP-MR of multilayers when the thickness of the magnetic layers is much larger than l_{sf}^F . In this case, Eq. (45) expresses the resistance change per period of the multilayer. In the case without interface spin flip, one obtains equivalent expressions to those of the VF model⁹ in the limit $t_F \ge l_{\text{sf}}^F$.

VI. DISCUSSION

We begin by discussing our results without interface resistance. We have used Eq. (37) with $\rho_N^* = 10^{-8} \Omega$ m, ρ_F^* $=10^{-7}$ Ω m, β =0.5, l_{sf}^N = 1.5 μ m to calculate ΔR_s as a function of t_N for several value of l_{sf}^F (the values of ρ_N^* , ρ_F^* , β are in the typical range derived from CPP measurements on Co/Cu in several groups,⁶⁻⁸ the value of l_{sf}^{N} is that derived by Johnson¹ for Au, and is about 10 times larger than that found from CPP-MR experiments on Co/Cu in Ref. 8). In Figs. $3(a)$ and 3(b) we show plots of ΔR_s and $t_N \Delta R_s$ versus t_N for different values of $l_{\text{sf}}^{\bar{F}}$.

For $t_N \ge l_{\text{sf}}^N$, the asymptotic variation of ΔR_s is always an exponential decrease as $\exp(-t_N/l_{\rm sf}^N)$, and the prefactor of the exponential decreases when l_{sf}^F decreases. In the opposite limit, $t_N \ll l_{\text{sf}}^N$, the influence of l_{sf}^F is even more pronounced. First, for zero spin relaxation in the ferromagnetic material (i.e., for $\rho_F^* l_{\text{sf}}^F = \infty$ or "half metallic limit") and for $t_N \ll l_{\text{sf}}^N$, Eq. (37) reduces to Johnson's result,² that is, in our notation

$$
\Delta R_s = \frac{2\beta^2 \rho_N^* l_{\rm sf}^{N^2}}{A t_N} \sim \frac{T_1^N}{A t_N}.
$$
\n(46)

 At_N/T_1^N is simply the spin relaxation rate in the volume At_N of the layer *N*, so that Eq. (46) means that ΔR_s is entirely governed by the balance between spin injection and spin relaxation in the volume At_N . Consequently, $\Delta R_s(t_N\Delta R_s)$ diverges (tends to a constant) as t_N tends to zero; see curves 1 in Figs. 3(a) and 3(b). Also ΔR_s tends to infinity at any value of t_N when the spin relaxation time in N tends to infinity.

When the spin relaxation time in ferromagnetic material is finite, the second term in the denominator of Eq. (37) has a nonzero value, so that ΔR_s does not diverge as t_N tends to zero. This can be seen in Fig. 3. More precisely, as t_N tends to zero $(t_N \ll l_{\text{sf}}^N)$, Eq. (37) becomes

$$
\Delta R \approx \frac{2\beta^2 \rho_N^* l_N^{N^2}}{A[t_N + 2l_{\text{sf}}^F (l_{\text{sf}}^N / l_{\text{sf}}^F)^2 (\rho_N^* / \rho_F^*)]},\tag{47}
$$

 ΔR_s tends to a finite value that depends only on the parameters of the ferromagnetic metal:

$$
(\Delta R_s)_{t_N \to 0} = \beta^2 \rho_F^* l_{\text{sf}}^F / A. \tag{48}
$$

By expressing l_{sf}^F from Eq. (15), we obtain

$$
(\Delta R_s)_{t_N \to 0} = \frac{\beta^2}{A} \left[\frac{\rho_F^* T_1^F}{4e^2} \left(\frac{1}{N_\uparrow(E_F)} + \frac{1}{N_\downarrow(E_F)} \right) \right]^{1/2}.
$$
 (49)

We check that, when one of the densities of states in *F* tends to zero ("half-metallic limit," without spin relaxation in F), $(\Delta R_s)_{t_N\to 0}$ tends to infinity, which is consistent with the divergence as t_N^{-1} we have found in the limit of infinite l_{sf}^F , Eq. $(46).$

The difference between the ''half-metallic limit'' and the case of metals such as Co or Fe is easy to understand. In the ''half-metallic limit,'' spin relaxation occurs only in *N* and the number of spin flips per unit time is proportional to t_N and to the splitting between the spin \uparrow and spin \downarrow electrochemical potentials. These spin flips balance the spin current injected in *N*. When t_N tends to zero, the balance can be conserved only by increasing the potential splitting to infinity. In contrast, in the general case, spin relaxation processes exist also in the ferromagnetic layers. More precisely, as spin accumulation and relaxation extend over about l_{sf}^F in F_1 and *F*₂, *t_N* is replaced by $[t_N + 2l_{\text{sf}}^F(l_{\text{sf}}^N / l_{\text{sf}}^F)^2(\rho_N^*/\rho_F^*)]$ where the additional thickness with relaxation, $2l_{sf}^F$, is weighted by a factor taking into account the difference in the relaxation and diffusion parameters in *F* and *N*.

We point out that, in Eq. (49), $(\Delta R_s)_{t_N\to 0}$ increases when at least one of the densities of states at E_F became small. Thus a strong ferromagnetic metal (Co, Ni) in which one of the densities of states is relatively small (*s* band only) is intermediate between the half-metallic case $[N_{\uparrow}(E_F)=0,$ $(\Delta R_s)_{t_N=0}$ = ∞] and the case of a weak ferromagnetic metal with a large density of states in both spin directions (Fe).

We now proceed to the case with nonzero interface resistance and $\gamma = \beta$ but without interface spin flip ($r'_b \neq 0$, $r_b^{\text{sf}} = \infty$). In Figs. 4(a) and 4(b), we have plotted the variation with t_N for ΔR_s and $t_N \Delta R_s$ calculated from Eq. (41) with the same values of ρ_N^* , ρ_F^* , β , l_{sf}^N as in Fig. 3, l_{sf}^F = 0.5 μ m and for different values of r_b^* (the curves 1 for $r_b^* = 0$ corre-

FIG. 4. Influence of the interface resistance. ΔR_s in (a) and $t_N\Delta R_s$ in (b) are plotted versus t_N for $\gamma = \beta$ with several values of r_b^* indicated in the figures. The other parameters are those of curve 3 in Fig. 3. One sees that the introduction of interface resistance (without interface spin flip) enhances ΔR_s . An infinite value of r_b^* (curve 4) restores the divergences of ΔR_s in t_N^{-1} in the half metallic limit.

sponds to the curves 3 in Fig. 3). It turns out that ΔR_s increases when r_b^* increases and an infinite value of r_b^* restores the divergence of ΔR_s as t_N^{-1} that we had found in the "half-metallic limit" (curves 1 in Fig. 3). For a finite value of r_b^* , the limit of ΔR_s when t_N tends to zero $(t_N \ll l_{\text{sf}}^N)$ is now

$$
(\Delta R_s)_{t_N \to 0} = \beta^2 [\rho_F^* l_{\text{sf}}^F + r_b^*] / A, \tag{50}
$$

which is larger than the value $\beta^2 \rho_F^* l_{sf}^F / A$ for $r_b^* = 0$, Eq. (48). This is because the spin accumulation is now more confined into *N* by the interface resistance. When r_b^* tends to infinity, the spin accumulation is totally confined and the divergence as $T_1^N/A t_N$ is restored.

To give rise to a non-negligible effect on $(\Delta R_s)_{t_N\to 0}$ and more generally on ΔR_s , the interface resistance r_b^* must reach values of the order of $\rho_F^* l_{sf}^F$, i.e., $5 \times 10^{-14} \Omega \text{ m}^2$ with the typical values we have supposed for ρ_F^* and l_{sf}^F . This can be inferred from Eq. (50) and also seen directly in Fig. 4. In contrast, the experimental values of r_b^* in systems such as Co/Cu, Co/Ag, or NiFe/Ag are always around 5×10^{-16} Ω m², that is much too small to produce an effect on ΔR_s . Even if we take the much smaller value $l_{\text{sf}}^{N}=0.044 \mu m$ derived from CPP experiments on Co/Cu by Piraux *et al.*, ⁸ values of r_b^* around 5×10^{-16} Ω m² are still much too small to

FIG. 5. Influence of the interface spin flips. ΔR_s in (a) and $t_N \Delta R_s$ in (b) are plotted versus t_N for several values of r_b^{sf} indicated in the figures. The other parameters are those of curve 2 in Fig. 4, which is identical to curve 1 for $r_b^{\text{sf}} = \infty$ (no spin flip).

produce a significant confinement of the spin accumulation in the normal metal and a definite increase of ΔR_s . Nevertheless, this does not rule out that significant effects could be obtained by increasing r_b^* above the typical values found up to now in metallic structures. This could be done, for example, by introducing insulating materials and tunnel resistances at the interfaces. The first condition, however, is the spin dependence of these interface resistances. We recall that, in Sec. IV and the discussion above, we have assumed that γ equals β . In contrast, as shown in Appendix A, the enhancement of ΔR_s by the interface resistance vanishes when γ tends to zero. In addition, at this point, we must also consider that any interface resistance should also come with spin flip interface resistance, which we will discuss now.

In Figs. 5(a) and 5(b) we show the variation of ΔR_s and $t_N \Delta R_s$ with t_{N_s} calculated from Eq. (44) with the values of $\rho_N^*, \rho_F^*, \beta, l_{\text{sf}}^N, l_{\text{sf}}^F$ already used for Fig. 4, $r_b^* = 5 \times 10^{-14}$ Ω m² (value required to produce a significant increase of ΔR_s , see curves 2 in Fig. 4) and for several values of r_b^{sf} . As expected, ΔR_s is reduced when some spin flip is introduced and r_b^{sf} progressively decreased from infinity (we recall that $r_b^{\text{sf}} = \infty$ means that there is no interfacial spin flip).

In Appendix B we estimate the value of r_b^{sf} expected from spin-orbit scatterings at interfaces and develop a more quantitative discussion. It turns out that, if the proportion of spin flip scattering is similar in the scattering processes at interfaces and inside the ferromagnetic layers (for similar spinorbit interactions), the enhancement of ΔR_s by the interface resistances is, in first approximation, balanced by its reduction by interface spin flip.

In conclusion, we find that, in metallic systems, ΔR_s cannot be significantly enhanced by the interface resistances. The usual experimental values of r_b^* are generally much too small and, moreover, any enhancement by exceptionally large interface resistances, should be strongly reduced by the effects of interface spin flips. However, as mentioned above, this does not rule out the possibility of interesting effects by other types of nonmetallic interfaces (tunnel junctions, etc.).

VII. COMPARISON WITH EXPERIMENTAL RESULTS

The first important result of our calculation is that $\Delta R_s(t_N)$ does not diverge as t_N tends to zero but saturates at a constant value for $t_N \ll l_{\text{sf}}^N$. As pointed out in Sec. VI, with typical values of r_b^* measured in metallic multilayers (around 5×10^{-16} Ω m²), the influence of r_b^* can be neglected and ΔR_s is expected to saturate at the value $\beta^2 \rho_F^* l_{sf}^F / A$. The corresponding result for CPP-MR experiments is obtained from Eq. (45) :

$$
\Delta R_s^{\text{CPP}}(t_N \ll l_{\text{sf}}^N, t_F \gg l_{\text{sf}}^F) = 2\Delta R_s(t_N \ll l_{\text{sf}}^N) = 2\beta^2 \rho_F^* l_{\text{sf}}^F / A,\tag{51}
$$

where ΔR_s^{CPP} is the resistance change for one period.

Before discussing spin switch data, we begin by showing that the above result, Eq. (51) , has been clearly observed in CPP-MR measurements. If one considers that, for $t_F \ge l_{\text{sf}}^F$, t_N , the resistance R_{AF}^{CPP} is practically that of a thickness t_F with resistivity $\rho_F^*/(1-\beta^2)$,

$$
R_{\rm AF}^{\rm CPP} \approx t_F \rho_F^* (1 - \beta^2) / A,\tag{52}
$$

which leads to a MR ratio independent of t_N and inversely proportional to t_F :

$$
\left(\frac{\Delta R}{R_{\rm AF}}\right)_{t_N \ll l_{\rm sf}^N}^{\rm CPP} \approx \frac{2\beta^2}{1-\beta^2} \frac{l_{\rm sf}^F}{t_F}.
$$
\n(53)

This behavior has been clearly observed in Co/Cu multilayered nanowires with thin Cu layers and thick Co layers $(t_F$ in the range $0.06-1$ μ m); see Fig. 4 in Ref. 8. The complete analysis of the CPP-MR measurements in Ref. 8 has given values of β , γ (0.36 and 0.85, respectively) close to those already derived for Co/Cu by the Michigan State group⁶ and has also obtained $l_{\text{sf}}^{\text{Cu}}(77 \text{ K}) = 140 \text{ nm}$. The value of l_{sf}^F derived from the fit of the linear variation of $\Delta R/R$ with t_F^{-1} is 44 nm. We will compare these values of l_{sf}^N and l_{sf}^F derived from CPP-MR with those required to account for spin switch data in the same thickness range.

We proceed to the discussion of spin switch data obtained by Johnson^{1,2} on NiFe/Au/NiFe trilayers (NiFe=permalloy) and already discussed by this author on the basis of Eq. (3) , which is equivalent to our Eq. (46) derived in the limit of zero spin relaxation in the ferromagnetic layers. As pointed out by Johnson, the fit of the experimental data with Eq. (3) , even when $\eta_1 \eta_2 \ (\equiv \beta^2)$ is assumed to be equal to 1, leads to surprisingly long spin diffusion lengths in the nonmagnetic layers, either $l_{\text{sf}}^N = 1.5 \ \mu \text{m}$ to account for the variation of ΔR_s with t_N , or a three times larger value to account for the amplitude of ΔR_s .

When we introduce spin relaxation in F_1 and F_2 , and if we neglect the influence of interface resistance (since typical

FIG. 6. The variation of $t_N \Delta R_s$ versus t_N is compared with experimental data on NiFe/Au/NiFe structures (Refs. 1, 2). The curves have been calculated without interface resistance and without interface spin flips, in (a) with $l_{\text{sf}}^N = 1.5 \mu m$ and for several values of l_{sf}^F , in (b) with l_{sf}^F = 30 μ m and for several values of l_{sf}^N . The other parameters are $\rho_N^* = 0.48 \mu \Omega$ cm (from Ref. 1 for Au), $\rho_F^* = 16 \mu\Omega$ cm and $\beta = 0.56$ for permalloy [thus corresponds to $\rho_F = \rho_F^*(1-\beta^2) = 12 \mu \Omega \text{ cm}$ from results on permalloy in Ref. 6. In (a) the curves for $l_{\text{sf}}^F \ge 10 \ \mu \text{m}$ reproduce approximately the variation with t_N but the calculated values of ΔR_s are smaller by about a factor of 10. In (b) it turns out that considerably large values of the spin diffusion lengths (l_{sf}^F =30 μ m, l_{sf}^N =7–8 μ m) are required to account for the absolute value of ΔR_s at small t_N .

values of r_b^* are too small by two orders of magnitude to produce a significant effect; see Sec. VI), ΔR_s is given by Eq. (37) . The curves shown and compared with experimental data in Fig. 6 have been calculated with $\rho_N^* = 0.48 \mu \Omega$ cm (from Ref. 1 for Au), $\rho_F^* = 16 \mu \Omega$ cm and $\beta = 0.56$ for permalloy (this corresponds to a resistivity of 12 $\mu\Omega$ cm for NiFe) from results on permalloy in Ref. 6. In Fig. $6(a)$, we show curves calculated with $l_{\text{sf}}^{N}=1.5 \mu$ m and several values of l_{sf}^F , and in Fig. 6(b), with $l_{\text{sf}}^F = 30 \ \mu \text{m}$ and several values of l_{sf}^N . In Fig. 6(a), a reasonable fit of the variation with t_N can be obtained if the values of l_{sf}^F are above 10 μ m, but the calculated values of ΔR_s are too small by about a factor of 10. To account for the amplitude of ΔR_s , it is necessary to increase the spin diffusion lengths in both materials. As shown in Fig. 6(b), we can account for the values of ΔR_s at small values of t_N for $l_M^N \approx 7-8$ μ m, $l_{sf}^F \approx 30$ μ m, but then, the decrease of $t_{N}\Delta R_s$ for $t_N>1$ μ m is not reproduced. Hershfield and $Zhao¹²$ have fitted their model with the same experimental results and also conclude that similar long spin diffusion lengths are required.

The really puzzling result of the above discussion is that the interpretation of the spin switch data of Johnson requires much longer spin diffusion lengths than those derived from CPP-MR experiments $[l_{\text{sf}}^N=0.15 \mu m$ and $l_{\text{sf}}^F=0.044 \mu m$ from data on Co/Cu (Ref. 8)]. One can argue that the materials are different, NiFe/Au for the spin switch measurements and Co/Cu for the CPP-MR. But this goes in the wrong direction as Au has a stronger spin-orbit interaction than Cu and therefore a faster spin relaxation rate. On the other hand, NiFe and Co have similar spin-orbit constants but the chemically disordered and more resistive NiFe should have a shorter spin diffusion length. It has also been pointed out by Yang *et al.*⁷ that the spin diffusion lengths derived from CPP-MR are in agreement with those estimated from ESR experiments, 14 so that finally the puzzling question is why the interpretation of the spin switch data requires so long spin diffusion lengths.

An alternative possibility to interpret the spin switch data with realistic spin diffusion lengths is to consider the possible influence of the interface resistances, r_b^* in our notation. In Sec. VI we have seen that typical values of r_b^* for metallic systems around $10^{-15} - 10^{-16} \Omega$ m² are much too small to enhance ΔR_s . To enhance significantly ΔR_s and account for the experimental data with more realistic spin diffusion lengths, values of r_b^* of the order of 5×10^{-13} Ω m² are required; see, for example, curves 3 in Fig. 4. In addition, by considering only interface resistances without spin flip, one overestimates the enhancement of ΔR_s . It turns out from the discussion in Appendix B that, with similar spin-orbit interactions and thus similar proportions of spin flip scattering at the interfaces and in the ferromagnetic layers, the enhancement of ΔR_s should be strongly reduced. Therefore it seems unlikely that the interface resistance can play a significant role. Finally, it remains the puzzling result that the interpretation of Johnson's data requires much longer spin diffusion lengths than those derived from CPP-MR (Refs. 7, 8) or ESR (Ref. 14) measurements.

VIII. CONCLUSIONS

We have extended the VF model $9,10$ of the CPP-MR in multilayers to calculate the output voltage of Johnson's spin switch.^{1,2} This output voltage, $V_P - V_{AP} = \Delta R_s I$, is governed by the balance between the injection of spins by the current and the relaxation of the resulting spin accumulation. In the approach of Johnson,^{1,2} only the spin relaxation within the nonmagnetic layers is taken into account, which leads to a divergence of ΔR_s as t_N^{-1} when the nonmagnetic thickness tends to zero. In our calculation, we have taken into account that spin relaxation also takes place in the part of the magnetic layers into which the spin accumulation extends, and, also, we have introduced the effect of interface resistances and interface spin flips. The main results can be summarized as follows.

(a) ΔR_s depends on the spin relaxation times in both the ferromagnetic and nonmagnetic layers. For example, as the nonmagnetic thickness t_N tends to zero, ΔR_s does not diverges as t_N^{-1} but saturates at a constant value determined by the parameters (spin relaxation time, resistivity) of the ferromagnetic material $(\Delta R_s = \beta^2 \rho_F^* l_{\text{sf}}^F / A)$.

FIG. 3. Influence of the spin diffusion length in the ferromagnetic layers, l_{sf}^F : ΔR_s in (a) and $t_N \Delta R_s$ in (b) are plotted versus t_N for several values of l_{sf}^F indicated in the figures. The other parameters are $l_{\text{sf}}^N = 1.5 \mu m$ [derived for Au by Johnson (Ref. 1)] and typical values found in CPP-MR experiments on Co/Cu multilayers: $\rho_N^* = 10^{-8} \Omega$ m, $\rho_F^* = 10^{-7} \Omega$ m, $\beta = 0.5$. Curve 1, for an infinite l_{sf}^F , exhibits the divergence of ΔR_s in t_N^{-1} in the half metallic limit $(i.e., without spin relaxation in *F*) (Refs. 1, 2).$

(b) Although interface resistances can confine the spin accumulation into the nonmagnetic layer and thus enhance ΔR_s , a significant enhancement is unlikely for two reasons: first, ΔR_s is not enhanced when one introduces a realistic proportion of spin-orbit scattering at the interfaces; secondly, even when one ignores the spin flips by spin-orbit interactions at the interfaces, the typical values of interface resistances in magnetic systems are much too small to produce a significant effect. However, it cannot be ruled out that other types of interface junctions, with high resistance and small proportion of transmission with spin flip, could help to confine the spin accumulation and enhance ΔR_s .

(c) There is a close connection between ΔR_s and the resistance change in a CPP-MR experiment, ΔR^{CPP} ($t_F \gg l_{\text{sf}}^F$, 1 period)= $2\Delta R_s$, so that it is fruitful to compare spin switch and CPP-MR experimental data. It turns out that the interpretation of so far reported spin switch experiments^{1,2} requires much larger spin diffusion lengths than those derived from CPP-MR experiments.⁸

~d! There is a definite convergence between the results of our model and those obtained by Hershfield and Zhao.¹² A minor difference is that they have not introduced interface spin flips and arrive at different conclusions on the influence of interfaces.

We hope that our work will incite experimentalists to study spin switch structures of Johnson's type or to devise new microstructures based on spin accumulation effects.

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APPENDIX A: CALCULATION WITH DIFFERENT SPIN ASYMMETRIES FOR THE FERROMAGNETIC LAYERS AND THE INTERFACE RESISTANCES

We have performed the same calculation as in Sec. IV A, again with only non-spin-flip interface resistance but now in the general case where the bulk and interface spin asymmetry coefficients, β and γ , are different. Instead of Eq. (41), we find

$$
\Delta R_{s} = \frac{2\rho_{N}^{*}l_{\rm sf}^{N}(\beta\rho_{F}^{*}l_{\rm sf}^{F} + \gamma r_{b}^{*})^{2}/A(\rho_{F}^{*}l_{\rm sf}^{F} + r_{b}^{*})^{2}}{1 + \left(\frac{\rho_{N}^{*}l_{\rm sf}^{N}}{\rho_{F}^{*}l_{\rm sf}^{F} + r_{b}^{*}}\right)^{2}\sinh\left(\frac{t_{N}}{l_{\rm sf}^{N}}\right) + 2\frac{\rho_{N}^{*}l_{\rm sf}^{N}}{\rho_{F}^{*}l_{\rm sf}^{F} + r_{b}^{*}}\cosh\left(\frac{t_{N}}{l_{\rm sf}^{N}}\right)}.
$$
\n(A1)

For $\beta = \gamma$, Eq. (A1) reduces to Eq. (41). It is easy to check that, compared to the value predicted by Eq. (41), ΔR_s is larger (smaller) when γ is larger (smaller) than β . For $\gamma=0$, Eq. (A1) becomes

$$
\Delta R_s = \frac{2\beta^2 \rho_N^* l_{sf}^N A (1 + r_b^* / \rho_F^* l_{sf}^F)^2}{\left[1 + \left(\frac{\rho_N^* l_{sf}^N}{\rho_F^* l_{sf}^F + r_b^*}\right)^2\right] \sinh\left(\frac{t_N}{l_{sf}^N}\right) + 2\frac{\rho_N^* l_{sf}^N}{\rho_F^* l_{sf}^F + r_b^*} \cosh\left(\frac{t_N}{l_{sf}^N}\right)}.
$$
\n(A2)

In the limit $t_N \ll l_{\text{sf}}^N$, for example, Eq. (A2) leads to

$$
(\Delta R_s)_{t_{N\to 0}} = \frac{\beta^2 (\rho_F^* l_{\text{sf}}^F)^2}{A (\rho_F^* l_{\text{sf}}^F + r_b^*)}.
$$
 (A3)

Comparing with Eq. (50) for $\beta = \gamma$, it turns out that, with $\gamma=0$, ΔR_s is smaller than in the absence of interface resistance and tends to zero when r_b^* becomes much larger than $\rho_F^* l_{\text{sf}}^F$.

APPENDIX B: INFLUENCE OF INTERFACE SPIN FLIPS

We begin by estimating the spin flip interface resistance r_b^{sf} and relating r_b^{sf} to the interface resistance r_b^* . We adopt the simple picture of an interfacial layer of thickness $2\Delta z$ in which structural and chemical disorders produce strong electron scattering. The spin-orbit part of the scattering potentials makes that a proportion p_I of the scatterings is with spin flip and contributes to the spin-lattice relaxation. For scattering in systems involving 3*d* and 4*s* metals, 10^{-2} is a typical value for p_I .¹⁴ The momentum and spin relaxation times in the interfacial layer, respectively τ_I and T_1^I , are in the ratio *pI*

$$
\frac{\tau_I}{T_1^I} \approx p_I. \tag{B1}
$$

From Eq. (9) , the discontinuity of the spin current between $z=-\Delta z$ and $z=+\Delta z$ on each side of the interface at $z=0$ is related to the nonequilibrium magnetization ΔM in the interfacial layer by

$$
\frac{1}{e} [(J_{+} - J_{-})_{-\Delta z} - (J_{+} - J_{-})_{+\Delta z}] = \frac{\Delta z \Delta M}{\mu_{B} T_{1}'}.
$$
 (B2)

For an approximate calculation, we suppose $N_+(E_F) = N_-(E_F) = N(E_F)$ and write

$$
\Delta M \approx \mu_B (\overline{\mu_+ - \mu_-}) N(E_F). \tag{B3}
$$

Introducing Eqs. $(B1)$ and $(B3)$ in Eq. $(B2)$ and identifying to Eq. (42) leads to

$$
r_b^{\text{sf}} \approx \frac{\tau_I}{p_I e^2 N(E_F) \Delta z}
$$
 (B4)

and, after relating $\Delta z/\tau_I$ to the interface resistance r_b^* in a free electron model, we obtain

$$
\tau_b^{\text{sf}} \approx \frac{r_0^2}{p_I r_b^*},\tag{B5}
$$

with

$$
r_0 = \frac{\hbar k_F}{ne^2}.
$$
 (B6)

In a free electron model and with similar approximations, we can write

$$
\rho_F^* l_{\text{sf}}^F \approx \frac{r_0}{\sqrt{p}},\tag{B7}
$$

where p is now the mean proportion of spin-orbit scattering inside the ferromagnetic layers. A typical value of *p* for a ferromagnetic metal like Co is again 10^{-2} . Equations (B5) and (B7) can be introduced in the expressions of ΔR_s , Eq. (44), to estimate the combined effect of r_b^* and r_b^{sf} . We consider, for example, the saturation value of ΔR_s when t_N tends to zero, that is, if we neglect the terms $r_b^* \rho_F^* l_{sf}^F / r_b^s$,

$$
(\Delta R_s)_{t_N \to 0} = \frac{\beta^2 (\rho_F^* l_{\text{sf}}^F + r_b^*)}{A (1 + \rho_F^* l_{\text{sf}}^F / r_b^{\text{sf}})}.
$$
 (B8)

Introducing Eqs. $(B5)$ and $(B7)$ in Eq. $(B8)$ leads to

$$
(\Delta R_s)_{t_N \to 0} = \frac{\beta^2 r_0 (r_0 + \sqrt{p} r_b^*)}{A(\sqrt{p} r_0 + p_I r_b^*)}.
$$
 (B9)

We first check that, without interface spin flip, i.e., $p_I=0$, Eq. (B9) becomes

$$
(\Delta R_s)_{t_N \to 0} = \frac{\beta^2}{A} \left(\frac{r_0}{\sqrt{p}} + r_b^* \right) = \beta^2 (\rho_F^* l_{\text{sf}}^F + r_b^*)/A
$$
\n(B10)

in agreement with Eq. (50) . Then, we suppose similar spin flip proportions in bulk and interface scattering, i.e., $p_I = p$. Equation (B9) becomes

$$
(\Delta R_s)_{t_N \to 0} = \frac{\beta^2 r_0}{A\sqrt{p}} = \beta^2 \rho_F^* l_{\text{sf}}^F / A, \tag{B11}
$$

which is the expression derived for $r_b^* = 0$, Eq. (48). This means that there is no enhancement of ΔR_s by the interface resistances when the scatterings producing these interface resistances have the same spin flip proportion as the scattering within the layers.

To summarize, the introduction of interface resistance can improve the confinement of spin accumulation in the nonmagnetic layer and enhance ΔR_s only when (a) r_b^* is larger than the typical values of interface resistance in metallic multilayers by two or three orders of magnitude and (b) the spin flip proportion in the scattering by interfaces is much smaller than in bulk scattering, which in general should not be the case for most multilayered systems investigated up to now.

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