History-dependent phenomena in spin-glass mean-field models

Zhi-de Chen
Department of Physics, Guangzhou Teachers' College, Guangzhou 510400, China
and Department of Physics, Wuhan University, Wuhan 430072, China
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The microscopic mechanism of history-dependent phenomena in spin-glass mean-field models is studied by analyzing how the applied field influences the spin-glass systems. It is found that, due to the competition between the reaction field and the applied dc field, the spin-glass system is seriously disturbed no matter how weak the dc field is. A field-dependent phase-transition mode spectrum is proposed to explain the essential spin-glass feature observed in dc measurements, including the ''plateau'' of field-cooled susceptibility, the fully field-dependent response in low-field regions, and characteristic history-dependent phenomena. The agreement between theoretical prediction and experimental observation are amazing. A comparison between the present theory and known equilibrium mean-field theory has also been made.

I. INTRODUCTION

The spin-glass (SG) phase is usually described as effectively nonergodic[1–3] because of the history-dependent phenomena and extremely long-time scales involved in its dynamics.[4,5] It is well known that effective nonergodicity is a natural result of the complex ground-state structure of SG mean-field models,[1,3] even though the microscopic mechanism is still unknown. The main difficulty is to find out an appropriate order parameter which can lead to decomposition of phase components.[3] Up to now, research on the microscopic mechanism of effective nonergodicity is rare though much work has been done on SG dynamics in recent years.[1,4–6] On the other hand, only a few works have been done based on the Thouless-Anderson-Palmer (TAP) Hamiltonian,[7] which is believed to contain more physics than the replica method. The present paper is an attempt to study the effective nonergodic behaviors of the mean-field model by the use of the TAP equation.[7] Our main interest is the history-dependent phenomena observed in dc susceptibility measurements.[8]

The present study is stimulated by one of the earliest observations that SG systems are extremely sensitive to applied dc fields even in very low field.[9] Experimental study on the effect of applied fields is rather systematic,[8] but the theoretical study is quite less,[1,10] even a satisfied answer to why SG systems are so sensitive to applied fields[11] has not yet been found. In our opinion, the history-dependent phenomena just mean that the low-temperature phase of SG systems crucially depends on the presence of dc fields in its history. The understanding about how the applied field affects SG systems is the precondition of understanding history-dependent phenomena. The present study shows that without the help of nonequilibrium statistics or detailed knowledge of the complex ground state, history-dependent phenomena appear as a natural consequence of the competition between the reaction field and applied dc field. Our work is an attempt to provide a detailed explanation for the dynamical SG transition under applied dc fields. This paper is organized as follows: Section II is devoted to analysis on the competition between reaction field and applied dc field; in Sec. III, we proposed a field-dependent phase-transition mode spectrum to understand the history-dependent phenomena; in Sec. IV, we present a detailed calculation of dc susceptibility which explains history-dependent response and irreversibility. Conclusions and discussion will be given in Sec. V.

II. THE COMPETITION BETWEEN REACTION FIELD AND dc FIELD

Our starting point is the Thouless-Anderson-Palmer (TAP) equation:[7]

$$H_i=\sum_j J_{ij}\langle S_j\rangle-\beta\langle S_i\rangle\sum_j J_{ij}^2(1-\langle S_j\rangle^2), \qquad (2.1)$$

here H_i is the effective field acting on site i. We will discuss Sherrington-Kirkpatrick (SK) model,[12] i.e., $S_i=\pm 1$, and J_{ij} will be chosen to satisfy the following distribution with $N\to\infty$:

$$P(J_{ij})=(2\pi\tilde{J}^2/N)^{-1/2}\exp(-NJ_{ij}^2/2\tilde{J}^2). \qquad (2.2)$$

It is well known that the first term on the right side of Eq. (2.1) is a cavity field, and the second term is the reaction field H_R:

$$H_R=-\beta\langle S_i\rangle\sum_j J_{ij}^2(1-\langle S_j\rangle^2), \qquad (2.3)$$

which arises from the polarization of neighboring spins $\langle S_j\rangle$ by the spin at site i.[13] It has been shown that in SG systems the reaction field has equal importance as the cavity field.[7,14] Now we present a qualitative analysis to display the role of the reaction field in a SG phase.

Notice that $\beta\Sigma_j J_{ij}^2(1-\langle S_j\rangle^2)\geq 0$, H_R always has an opposite sign to $\langle S_i\rangle$ in spite of the oscillation of J_{ij}. Suppose that the contribution of the cavity field is positive, and it will lead to a pointing-up $\langle S_i\rangle$, then the contribution of the reaction field due to the polarization of the neighboring spins by the pointing-up $\langle S_i\rangle$ (Ref. 13) will be negative, H_R will force $\langle S_i\rangle$ to point down. Likewise, suppose that the contribution of the cavity field is negative, then the reaction field will be

positive. Since the reaction field is as strong as the cavity field, $\langle S_i\rangle$ will be really frustrated when it tries to find a fixed pointing direction (up or down). In other words, the reaction field will prevent $\langle S_i\rangle$ from pointing at a fixed direction. It is in favor with a disordered phase with $\langle H_i\rangle_d=0$ (here $\langle\ \rangle_d$ is disordered average). This conclusion agrees with the fact that the nonzero convergent solution of the following equation:

$$\langle S_i\rangle=\tanh\beta H_i \tag{2.4}$$

has not been found when reaction field is included.[15]

When a dc field H_e is applied to the system, the effective field will be

$$H_i=H_e+\sum_j J_{ij}\langle S_j\rangle+H_R. \tag{2.5}$$

The net contribution of the cavity field and the dc field will be positive, and a positive $\langle S_i\rangle$ will be induced. The reaction field due to this positive $\langle S_i\rangle$ will be negative. This means that the reaction field will prevent $\langle S_i\rangle$ from pointing along the direction of the applied dc field. As a result, whether a spontaneous magnetization will appear depends on the competition between dc field and reaction field.

In the following, we present a quantitative analysis on the competition between reaction field and applied field based on the above conclusions. It will be shown that the competition between reaction field and dc fields is crucially important for understanding history-dependent response and irreversibility.

The mathematical treatments in the present paper will follow the so-called mean-random-field theory[16] (MRFT) developed by Klein. Although MRFT is not a good mean-field theory (MFT),[12] we still choose it because of its simple mathematical treatment and transparent physical ideas. Following the MRFT, the first task is to calculate a field distribution $P(H)$ including the reaction field. According to the above analysis, the main effect of the reaction field is to keep $P(H)$ centering at $H_0=0$ (i.e., $\langle H_i\rangle_d=0$). Under this constraint, $P(H)$ can be approximately calculated by the following procedure.

First, the $\langle S_i\rangle$ on the right side of (2.1) should be positive definite, so

$$H_i^+=\sum_j J_{ij}\langle S_j\rangle-\beta m\sum_j J_{ij}^2(1-\langle S_j\rangle^2), \tag{2.6}$$

where[17]

$$m=\langle|\langle S_i\rangle|\rangle_d=\int|\tanh(\beta H)|P(H)dH, \tag{2.7}$$

which has the same meaning as the Edwards-Anderson order parameter.[11] Then a field distribution of H_i^+ can be found by[16]

$$P(H^+)=\int\prod_j P(J_{ij})dJ_{ij}\ \sigma(H^+-H_i^+). \tag{2.8}$$

Secondly, $\langle S_i\rangle$ will be negative definite, i.e.,

$$H_i^-=\sum_j J_{ij}\langle S_j\rangle+\beta m\sum_j J_{ij}^2(1-\langle S_j\rangle^2). \tag{2.9}$$

Likewise, one can find another distribution by

$$P(H^-)=\int\prod_j P(J_{ij})dJ_{ij}\ \sigma(H^--H_i^-). \tag{2.10}$$

Finally, the whole field distribution $P(H)$ including the reaction field will be

$$P(H)=[P(H^+)+P(H^-)]/2. \tag{2.11}$$

$P(H^+)$ and $P(H^-)$ can be easily found by the mathematical treatment of MRFT,[16] and the result is found to be

$$P(H)=(2\pi\tilde{J}^2q)^{-1/2}\{\exp[-(H+R)^2/2\tilde{J}^2q]+\exp[-(H-R)^2/2\tilde{J}^2q]\}/2, \tag{2.12}$$

where

$$m=(2\pi)^{-1/2}\int|\tanh Y|e^{-x^2/2}\,dx, \tag{2.13}$$

$$q=\int\tanh^2(\beta H)P(H)\,dH=(2\pi)^{-1/2}\int\tanh^2Ye^{-x^2/2}\,dx, \tag{2.14}$$

$$Y=\beta\tilde{J}q^{1/2}x-\beta R, \tag{2.15}$$

$$R=\beta\tilde{J}^2m(1-q), \tag{2.16}$$

here R effectively represents the strength of the reaction field. Magnetic susceptibility and specific heat can be found in the same way in MRFT,[16] i.e.,

$$\chi=\beta\int\operatorname{sech}^2(\beta H)P(H)dH=\beta(1-q), \tag{2.17}$$

$$C_m=k_B\beta\int H^2\operatorname{sech}^2(\beta H)P(H)dH=\sqrt{2\pi}k_B\int Y^2\operatorname{sech}^2Ye^{-x^2/2}\,dx, \tag{2.18}$$

where Y is given by (2.15). The temperature dependence of R, χ, and C_m can be obtained by a numerical solution combining Eqs. (2.13) and (2.14). Temperature dependence of R is shown in Fig. 1, where one can find that R is a monotonic decrease function of temperature. The temperature dependence of χ and C_m are found to be qualitatively the same as that of MRFT.[16]

The critical point can be found by using Eq. (2.13) or (2.14) with the approximation: $q\approx m^2$ as $T\to T_c$, the result is

$$T_c=1.272\tilde{J}/k_B. \tag{2.19}$$

This value is higher than the common one $T_{c0}=\tilde{J}/k_B$.[1] The higher critical point is probably due to the approximations used by us. Further discussions about this discrepancy will be provided later.

When a dc field is involved, without considering the competition between H_e and H_R, $P(H)$ is expected to be

$$P(H+H_e)=[P(H^++H_e)+P(H^-+H_e)]/2. \tag{2.20}$$

The result is that, $P(H)$ will move its distribution center from $H_0=0$ to $H_0=H_e$ and a net spontaneous magnetization appears that will suppress the phase transition. However, in a

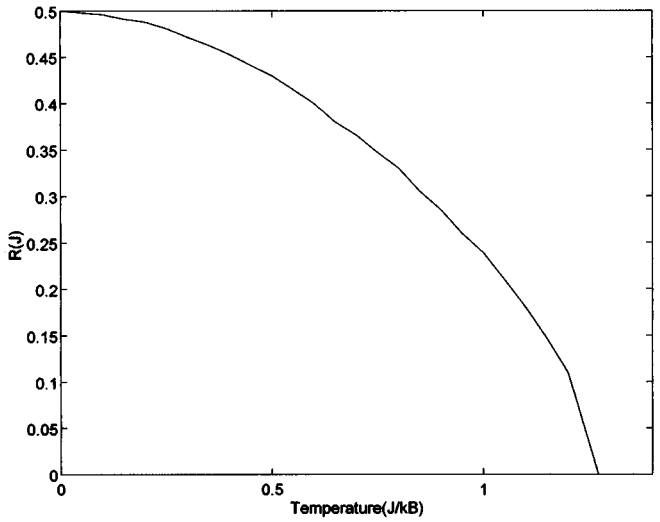

FIG. 1. Temperature dependence of R, a quantity that effectively represents the strength of reaction field. R is a monotonic decrease function of T.

SG system, $P(H+H_e)$ cannot be simply calculated by (2.20). As a mean-field approximation, it is suggested that the movement of the distribution center will happen only when the applied field is stronger than reaction field. In this paper, the main interest will be the low-field condition, i.e., $H_e \leqslant H_R$, let

$$H_e=(1-K)R, \quad 0\leqslant K\leqslant 1. \tag{2.21}$$

In this case, the field distribution $P(H+H_e)$ will be calculated under two constraints: (1) $P(H+H_e)$ still centers at $H_0=0$, and (2) the strength of the reaction field will be effectively reduced by H_e. That is

$$P(H+H_e)=\left\{ \int \prod_j P(J_{ij})dJ_{ij}\ \sigma(H-H_1^+) + \int \prod_j P(J_{ij})dJ_{ij}\ \sigma(H-H_1^-)\right\} \Big/ 2, \tag{2.22}$$

where

$$H_1^+=H_i^+ +(1-K)R, \tag{2.23}$$

$$H_1^-=H_i^- -(1-K)R, \tag{2.24}$$

provided that H_i^+ and H_i^- are given by Eqs. (2.6) and (2.9), respectively.

In the same way one can find that

$$P(H+H_e)=P_K(H)=(8\pi\tilde{J}^2q_K)^{-1/2}(\exp\{-[H+K\beta\tilde{J}^2m_K(1-q_K)]^2/2\tilde{J}^2q_K\}+\exp\{-[H-K\beta\tilde{J}^2m_K(1-q_K)]^2/2\tilde{J}^2q_K\}), \tag{2.25}$$

where

$$m_K=(2\pi)^{-1/2}\int |\tanh y_K| e^{-x^2/2}\,dx, \tag{2.26}$$

$$q_K=(2\pi)^{-1/2}\int \tanh^2 y_K e^{-x^2/2}\,dx, \tag{2.27}$$

$$y_K=\beta\tilde{J}q_K^{1/2}x-K\beta^2\tilde{J}^2m_K(1-q_K), \tag{2.28}$$

and also

$$\chi_K=\beta(1-q_K), \tag{2.29}$$

$$C_m^K=(2\pi)^{-1/2}k_B\int y_K^2\,\mathrm{sech}^2 y_K e^{-x^2/2}\,dx. \tag{2.30}$$

When a dc field H_e is involved, the staying state is characterized by the introduced parameter K:

$$K=1-H_e/R. \tag{2.21a}$$

K is just a parameter which measures the degree that the reaction field is reduced by the applied field. In the zero-field condition, where $K=1$, then distribution (2.25) reduces to distribution (2.12). In the case of $K=0$, distribution (2.25) will reduce to that found by MRFT,[16] where the reaction field was not included. Equation (2.21a) can be considered as the balance condition for two competing forces: dc fields and reaction field. When Eq. (2.21a) is satisfied, it means that the applied dc field is "neutralized" by the reaction field, or the applied field becomes effectively null.

Since K is H_e dependent, both the staying state and the differential response to applied field χ_K are fully H_e dependent. Notice that the strength of the reaction field is rather weak ($R_{\max}=0.5\tilde{J}$, c.f. Fig. 1), this fully H_e-dependent behavior holds even in the low-field regions (i.e., even when $\mu_B H_e/k_B T_c \ll 1$ is satisfied). Therefore, the observed response is fully H_e dependent and nonlinear even when the applied dc field is weak.

III. PHASE-TRANSITION MODE SPECTRUM

Since K can vary from 0 to 1 continuously, the above analysis implies that there exists a continuum of possible staying states in the low-temperature regions. Also, by using the approximation $q_K \approx m_K^2$ at $T=T_c$, one can find a formulaic critical temperature from Eq. (2.26) or (2.27):

$$T_{cK}=[(\sqrt{4K^2+1}+1)/2]^{1/2}\tilde{J}/k_B. \tag{3.1}$$

This means that there is a critical point for a given K. Although T_{cK} is just a formulaic critical point, both χ_K and C_m^K show typical critical behaviors at T_{cK}. Some representative χ_K-T curves are shown in Fig. 2(a), where one can find that sharp cusps appear at T_{cK}.

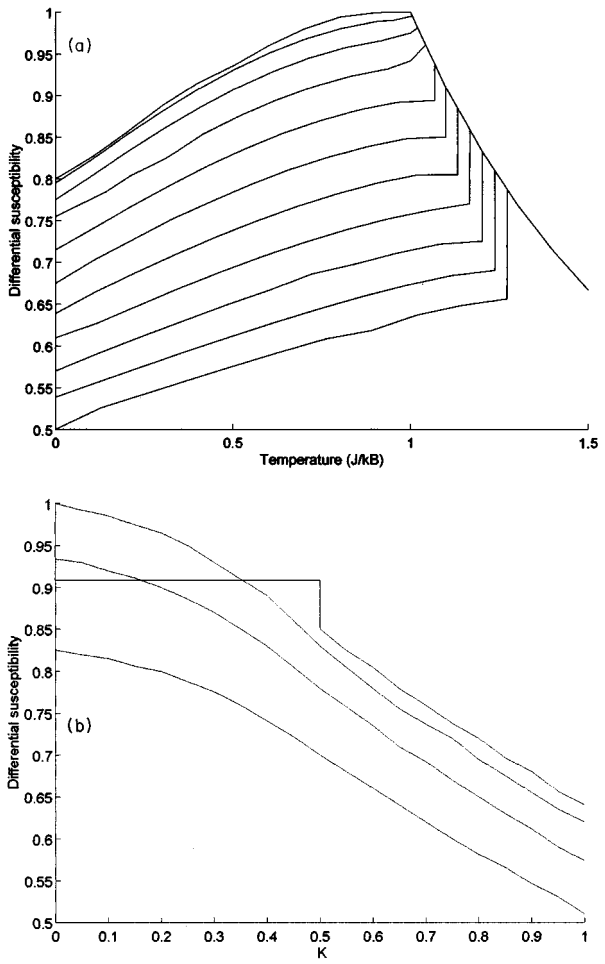

FIG. 2. (a) Some representative $\chi_K \sim T$ curves (from top to bottom: $K=0$, 0.1, 0.2, ... , 0.9, and 1). Sharp cusps of unsymmetric structures are found at the critical points of phase-transition modes (the χ axis has been displaced for a clear view of critical behaviors). These cusps are just a reproduction of susceptibility cusps observed in ac and neutron-scattering measurement. (b) Distributions $\chi(K) \sim K$ at some representative temperatures (from top to bottom: $T=1.1$, 0.9, 0.5, and 0.1 $\tilde{J}/k_B$). It is suggested that $\chi_K = 1/T$ at $T > T_{cK}$.

Based on the above result, we can consider that the staying state characterized by K is a quasi-phase-transition mode. This quasi-phase-transition mode has a quasicritical point T_{cK}, and its thermodynamical properties are described by Eqs. (2.26–2.30). Consequently, there is a continuum of quasi-phase-transition modes that will be called the phase-transition mode spectrum. The critical points of all quasi-phase-transition modes lie over some temperature region (i.e., from $T_{c0} = \tilde{J}/k_B$ to $T_{c1} = 1.272\tilde{J}/k_B$) called the critical zone. As a matter of fact, both order parameter and thermodynamical quantities have continuous distribution in the phase-transition mode spectrum. Distribution of $\chi(K)$ at some representative temperatures is shown in Fig. 2(b).

The concept of phase-transition mode and phase-transition mode spectrum are not original ideas.[18] Since the staying state characterized by K is a thermodynamical state, all the quasi-phase-transition modes in the present picture are extended, but not localized as proposed by Hertz or Jacobs.[18] There will be just one soft mode in the cooling process.

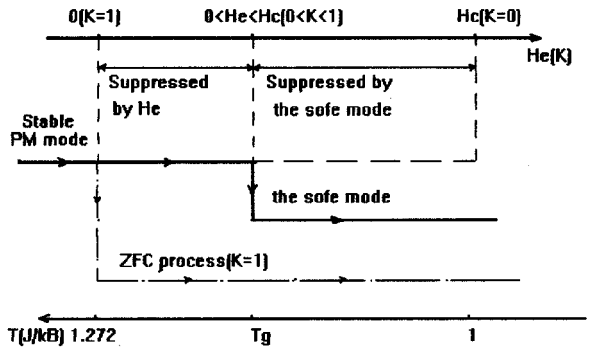

FIG. 3. Depiction of bifurcation in a SG transition over the critical zone. Phase-transition modes with higher critical points than the soft mode are suppressed by dc field, yet those with lower critical points are suppressed by the soft mode. Bifurcation due to Ising symmetry has not been included in this picture.

Now we will show how the proposed phase-transition mode spectrum works in understanding history-dependent response and irreversibility. The mode-choosing mechanism in two representative processes, the field-cooled (FC) process and the zero-field-cooled (ZFC) process, will be discussed, separately.

In a FC process, the system is cooling down with dc field H_e applied. When the system is cooled through the critical zone, the applied field will try to construct a spontaneous magnetization that will suppress the instability of quasi-phase-transition modes. On the other hand, the reaction field will act against H_e and is in favor with quasi-phase-transition modes. In the beginning, the strength of the reaction field is rather weak, thus the suppression due to the applied field will be predominant. As the temperature further lowers, the strength of the reaction field increases. At some temperatures T_g inside the critical zone, the reaction field will balance the applied field, that is,

$$H_e = R|_{T=T_g}, \quad T_{c0} \leqslant T_g \leqslant T_{c1}. \tag{3.2}$$

Consequently, the quasi-phase-transition mode with its critical point just at T_g will survive. This survival mode is the soft mode that will suppress all the other modes with critical points lower than T_g in further cooling process. This mode-choosing mechanism is schematically shown in Fig. 3. For a given H_e, the chosen soft mode can be known from Fig. 1, Eqs. (3.1), and (3.2). For examples, taking $H_e = 0.111$, 0.16, 0.2, and $0.239\tilde{J}$, the soft modes will be $K=0.8$, 0.6, 0.4, and 0 with $T_g = 1.20$, 1.13, 1.07, and $1.0\tilde{J}/k_B$, respectively. Both the chosen soft mode and T_g are H_e dependent. Dependence of T_g on H_e is shown in Fig. 4, where one can find that T_g decreases as H_e increases. When H_e exceeds some critical value,

$$H_c = R|_{T=T_{c0}} = 0.239\tilde{J}, \tag{3.3}$$

all quasi-phase-transition modes will be suppressed and no phase transition will occur.

Since the applied field is effectively null at $T \leqslant T_g$, the system will keep staying at the soft mode in both cooling and warming sweep at $T \leqslant T_g$. In other words, the chosen mode

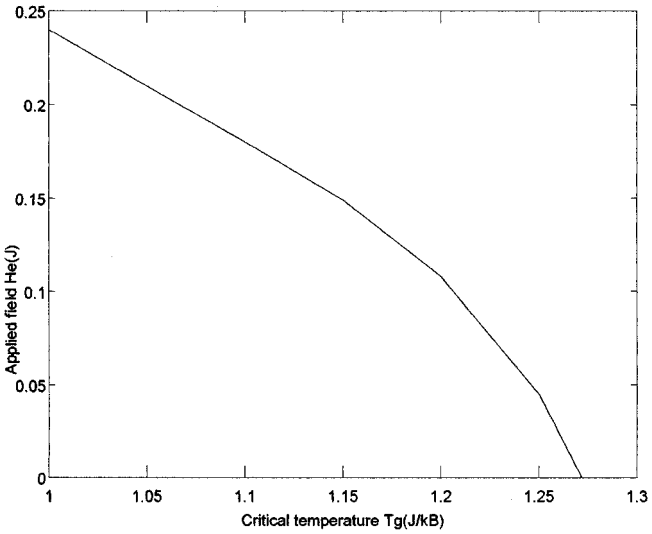

FIG. 4. Dependence of T_g on the applied field. T_g is a monotonic decrease function of H_e and when $H_e > 0.239\tilde{J}$, there is no phase transition.

in a FC process is T independent at $T \leqslant T_g$, and thus a FC process is reversible for a fixed applied field.

In a ZFC process, the system is cooled down with null H_e, the soft mode will be $K=1$ mode when cooling through the critical zone. The system will keep staying at the $K=1$ mode at $T < T_{c1}$ in zero-field condition. When H_e is applied at some measuring temperature T_0, the $K=1$ mode will become unstable since the balance between H_e and reaction field breaks. The system will be driven to another stable mode K_S, in which the applied field and reaction field balance again. K_S is given according to the balance condition (2.21a), i.e.,

$$K_S = 1 - H_e/R|_{T=T_0}. \tag{3.4}$$

The application of H_e leads to an evolution of staying state from the $K=1$ to K_S mode. Experimental evidences for this evolution of staying state have been found in both dc (Ref. 8) and ac measurement.[19]

Suppose that the system is warmed up to T_1 by a step change ΔT, the balance between H_R and H_e will be broken since the strength of H_R decreases as T increases. Consequently, the K_S mode becomes unstable at T_1, and the staying state will evolve to another stable mode K_{S1}: $K_{S1} = 1 - R/H_e|_{T=T_1}$. As a result, the chosen mode in the warming sweep is T dependent. When the system is warmed up above the critical zone and then cooled down, this cooling down process is exactly a FC process, thus the system will be locked in the soft mode chosen by the applied field in the cooling sweep. Depiction of this history-dependent mode-choosing mechanism is shown in Fig. 5.

Now we turn to discuss how irreversible behaviors appear during the warming sweep in a ZFC process.[3,8] For simplicity, the experimental process[8] will be schematically simulated in Fig. 6 by using $H_e = 0.239\tilde{J}$. Starting from 0, the system is zero-field cooled to $A(T_A = 0.1\tilde{J}/k_B)$, then H_e is applied and the system is warmed to $B(T_B = 0.4\tilde{J}/k_B)$. The chosen modes at A and B are $K_S = 0.518$ and 0.467, respectively, after H_e is applied. Next an interrupting cooling down

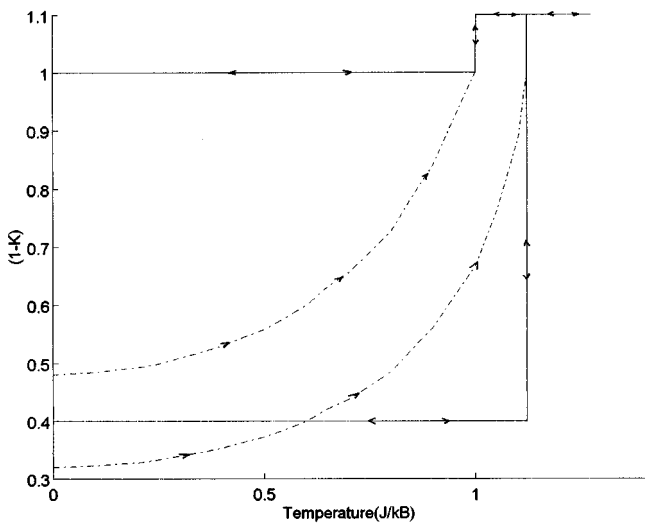

FIG. 5. History-dependent phenomena in dc measurements: mode-choosing results in both FC and ZFC process for two dc fields of $H_e = 0.239\tilde{J}$ (top) and $0.16\tilde{J}$. Reversible FC processes are shown in full line with reversible marks, irreversible ZFC processes are shown in dash-point line, and broken line means paramagnetic (PM) phase.

is taken: from B to A. Since the strength of reaction field increases as T lowers and H_e is just "neutralized" at B, H_e will become effectively null at $T \leqslant T_A$, as in a FC process. Consequently, the system will be locked at $K_S = 0.467$ in both cooling ($B \rightarrow A$) and warming ($A \rightarrow B$) processes, that is, irreversibility appears. When the system is warmed up above B, the balance between H_e and H_R will lose again, and the the warming process comes back to the original ZFC process. Similar irreversible behaviors can be found by interrupting cooling downs starting at C, D, and E as shown in

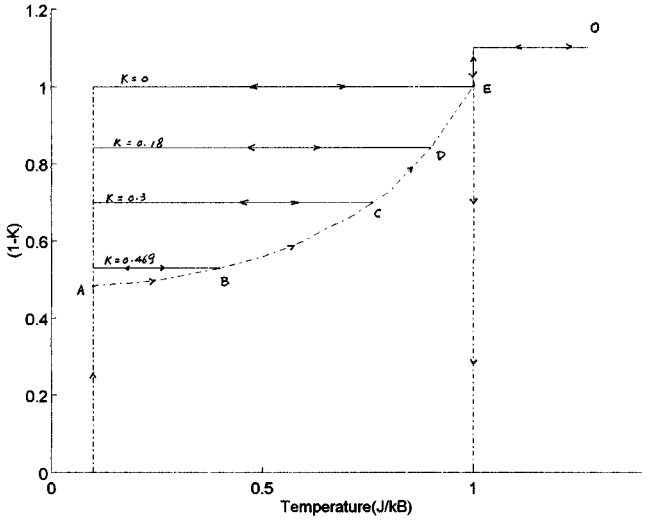

FIG. 6. A schematic demonstration for irreversibility in a ZFC process of $H_e = 0.239\tilde{J}$, locking modes in interrupting cooling down are marked above the lines. When H_e has been applied, a cooling down followed a warming up is reversible (say, $B \rightarrow A \rightarrow B$), yet a warming up followed a cooling down (say, $A \rightarrow B \rightarrow A$) is irreversible. For a given set of (H_e, T) (say, $H_e = 0.239\tilde{J}$, $T = 0.1\ \tilde{J}/k_B$), the chosen mode is fully history dependent.

Fig. 6. The mode-choosing mechanism shown in Fig. 6 explains the experimental observations that[3,8] a cooling down following a warming up is reversible, yet a warming-up following a cooling down is irreversible.

IV. HISTORY-DEPENDENT RESPONSE

In this section, the corresponding dc susceptibility will be calculated by using the mode-choosing mechanism provided in the last section. In a dc measurement, dc susceptibility $\chi_{\rm dc}$ is found by

$$\chi_{\rm dc}=M/H_e=\frac{1}{H_e}\int_0^{H_e}\frac{dM}{dH}\,dH, \tag{4.1}$$

here M is the induced magnetization by applied field H_e. The former analysis implies that $\chi_{\rm dc}\neq dM/dH$ even when $\mu_B H_e/k_B T_c \ll 1$. In the present picture, $\chi_{\rm dc}$ can be expressed as

$$\chi_{\rm dc}=\frac{1}{H_e}\int_1^{K_0}\chi_K(dK/dH_e)^{-1}\,dK, \tag{4.2}$$

where K_0 is the chosen mode, χ_K is the differential susceptibility of the Kth mode, and dK/dH_e means how the chosen mode varies with H_e. It has been shown that the mode-choosing mechanism (i.e., dK/dH_e) is history dependent, consequently, $\chi_{\rm dc}$ will be history dependent.

In a ZFC process, the variation of chosen mode with H_e in the warming sweep obeys Eq. (3.4), this implies that

$$(dK/dH_e)|_{\rm ZFC}=-1/R, \tag{4.3}$$

substituting this result to (4.2) and using (3.4), ZFC susceptibility is found to be

$$\chi_{\rm ZFC}=\frac{1}{1-K_0}\int_{K_0}^1\chi_K\,dK=\bar{\chi}_{K_0}. \tag{4.4}$$

$\chi_{\rm ZFC}$ is the average susceptibility over modes from K_0 to 1, that can be found by using the results provided in Fig. 2.

In a FC process, the chosen mode varies with H_e above T_g, but will be T independent below T_g. This means that dK/dH_e, and thus dM/dH, will be H_e dependent above T_g, but becomes T independent below T_g. Consequently, $\chi_{\rm FC}$ will be T independent at $T<T_g$, that is

$$\chi_{\rm FC}(T)=\chi_{\rm FC}(T_g),\quad T<T_g, \tag{4.5}$$

the well-known plateau of FC susceptibility.[8]

In the temperature regions above T_g, FC susceptibility will be calculated by

$$\chi_{\rm FC}=(M_1+M_2)/H_e. \tag{4.6}$$

M_1 and M_2 are given by

$$M_1=\int_0^R\frac{dM}{dH}\,dH=R\int_0^1\chi_K\,dK=R\bar{\chi}_0, \tag{4.7}$$

$$M_2=\int_R^{H_e}\frac{dM}{dH}\,dH=M_{\rm SK}(H_e)-M_{\rm SK}(R), \tag{4.8}$$

where $M_{\rm SK}(H)$ is given by the following coupled equations as in MRFT (Ref. 16) or SK theory:[12]

$$M_{\rm SK}(H)=\sqrt{2\pi}\int\tanh(\beta\tilde{J}q^{1/2}x+\beta H)e^{-x^2/2}\,dx, \tag{4.9}$$

$$q=\sqrt{2\pi}\int\tanh^2(\beta\tilde{J}q^{1/2}x+\beta H)e^{-x^2/2}\,dx. \tag{4.10}$$

Consequently, $\chi_{\rm FC}$ above T_g will be given by

$$\chi_{\rm FC}=M_{\rm SK}(H_e)/H_e-\Delta\chi, \tag{4.11}$$

$$\Delta\chi=(M_{\rm SK}(R)-R\bar{\chi}_0)/H_e. \tag{4.12}$$

In the temperature regions above the critical zone (i.e., $T>T_{c1}$), where $R=0$, and thus $\Delta\chi=0$. This implies that

$$\chi_{\rm FC}=M_{\rm SK}(H_e)/H_e,\quad T>T_{c1}. \tag{4.13}$$

As a result, Curie law will be obeyed at $T>T_{c1}$ in low-field regions. On the other hand, since $\Delta\chi>0$ at $T<T_{c1}$, $\chi_{\rm FC}$ will be lower than that of Curie law before T_g is reached as observed in experiments.[20]

Some numerical results are shown in Figs. 7(a), 7(b), and 7(c). Curves shown in Fig. 7 are typical history-dependent phenomena observed in dc measurements. We have found that, both the ''plateau temperature'' (where the plateau appears in a FC process) and ''reversibility temperature'' (where $\chi_{\rm FC}$ and $\chi_{\rm ZFC}$ join)[8] decrease as dc fields increase. Also, $\chi_{\rm ZFC}$ curves of different fields are found to be quasi-parallel and obey

$$\chi_{\rm ZFC}(H_1)>\chi_{\rm ZFC}(H_2),\quad H_1>H_2. \tag{4.14}$$

These results are in good agreement with experimental observations,[8] yet in a FC process, the following relation:

$$\chi_{\rm FC}(H_1)<\chi_{\rm FC}(H_2),\quad H_1>H_2, \tag{4.15}$$

is found to hold only above the critical point, but breaks below the critical point. The reason for this discrepancy is unclear.

V. CONCLUSION AND DISCUSSION

In previous sections, a field-dependent phase-transition mode spectrum is proposed to understand the history-dependent phenomena and irreversibility. Theoretical predictions of the present paper explains the essential SG features observed in the dc measurements. In the present picture, the separation of paramagnetic (PM) and SG phases is not just a critical point, but over a finite temperature region (i.e., the critical zone) as indicated by the experiment[4] and computer simulation.[21] Bifurcation (or tree branching) in a SG transition[3] is continuous over the critical zone, the applied field serves as a ''switchman'' that will switch on a specified bifurcation (or branching) when cooling through the critical zone.

The kernel of the present theory is the proposed phase-transition mode spectrum which suggests an infinite number of possible staying states in low-temperature regions. This picture is reminiscent of the well-known ground-state structure of SG systems. In order to provide a complete descrip-

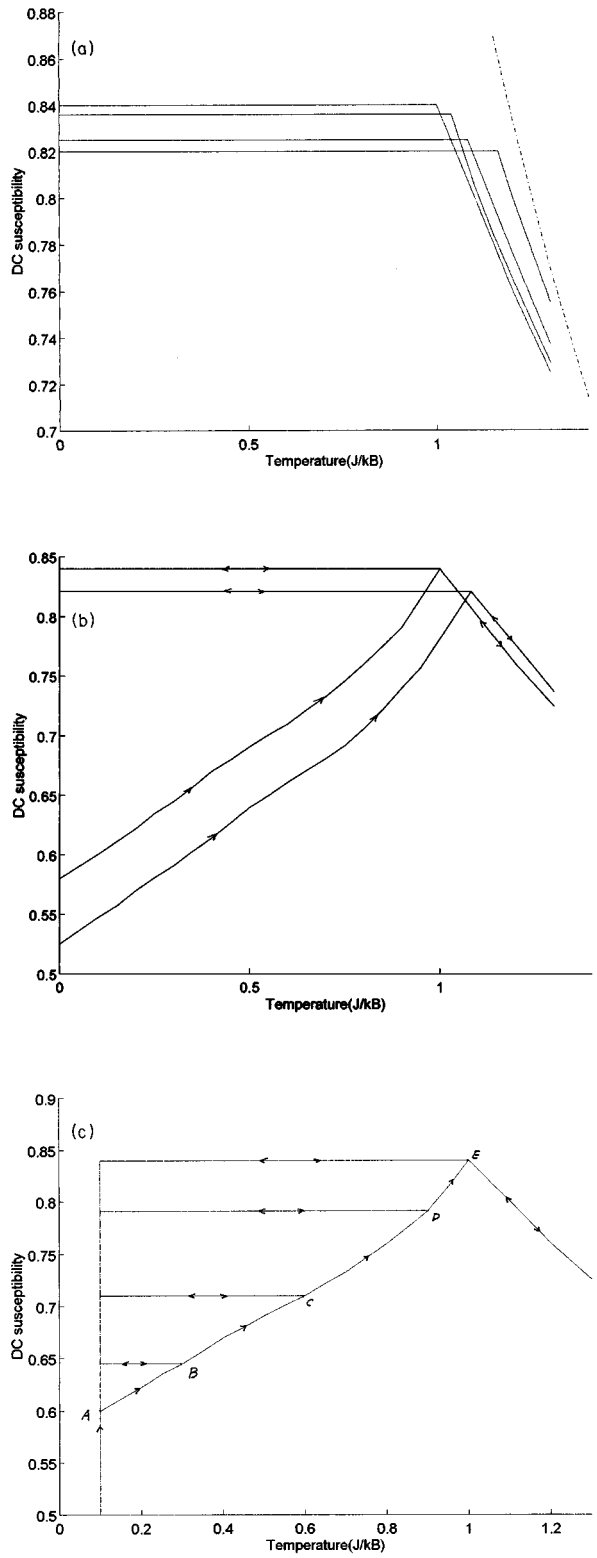

FIG. 7. (a) Plateaux in FC susceptibility: some representative $\chi_{\mathrm{FC}}\sim T$ curves of $H_e=0.239$, 0.2, 0.16, and 0.111 $\tilde{J}$ (from top to bottom). The Curie law is shown as a broken line for reference. Here the plateau temperature is just the critical temperature of the soft mode. (b) Typical history-dependent response in dc measurements: a complete set of $\chi_{\mathrm{FC}}\sim T$ *and* $\chi_{\mathrm{ZFC}}\sim T$ curves of $H_e=0.239\tilde{J}$ (top) and $0.16\tilde{J}$ (cf. Fig. 5). (c) A demonstration of irreversibility in a ZFC process of $H_e=0.239\tilde{J}$ (cf. Fig. 6). Here one can see that, for a given set of (H_e,T) (say, $H_e=0.239\tilde{J}$, $T=0.1\tilde{J}/k_B$), the measured response is fully history dependent.

tion for this complex ordered phase, a continuous function $q(x)$ with a mysterious variable $x(0\leqslant x\leqslant 1)$ is used as order parameter in the Parisi theory.[22] In the present picture, a continuous distribution function $q(K)$ $(0\leqslant K\leqslant 1)$ is employed as the order parameter. This coincidence is very striking though $q(K)$ has a different meaning from $q(x)$. It is not clear yet whether $q(K)$ can correspond to another ''gauge'' of Sompolinsky's theory.[1,23] The important point is that the proposed order parameter $q(K)$ is able to make decomposition of phase components.[3]

A quantitative comparison between the present theory and equilibrium MFT still has some problems. One is the higher critical point in the zero-field condition. It seems to us this higher critical point is probably due to the approximations used, including the proposed approximation to include the reaction field and the mean-random-field approximation.[24] Another discrepancy comes from the comparison between the so-called Almeida-Thouless (AT) line[25] and the field dependence of the critical point provided in Fig. 4. It is true that two lines have qualitatively the same field dependence, that is, T_c decreases as H_e increases. However, from a warming direction, the AT line terminates at $(H_e=0,\ T_c=T_{c0})$ while our T_c-H_e line starts at $(H_e=H_c,\ T=T_{c0})$. Obviously this discrepancy is due to the wrong critical point we find in zero-field condition.

Now we turn to discuss the stability of the quasi-phase-transition mode. Theoretical research and numerical simulation[26] shows that an equilibrium $P(H)$ should have two basic characters: (1) $P(H)$ has a minimum at $H=0$ and (2) $P(0)|_{T\to 0}=0$. However, distribution (2.25) tells that for all K's, $P_K(H)$ has a maximum at $H=0$ and $P_K(0)|_{T\to 0}\neq 0$. This clearly implies $P_K(H)$ is not an equilibrium distribution, and thus all phase-transition modes are nonequilibrium. The fact that $P_K(H)$ is not an equilibrium distribution is not a surprise result for two reasons. The first is that MRFT is not a good way to seek an equilibrium field distribution.[12] Another reason is that the reaction field always resists a fixed pointing direction of $\langle S_i\rangle$, and thus $P_K(H)$ always has a maximum at $H=0$ even when $T\to 0$. This implies that when $T\to 0$, approaching to equilibrium requires a vanishing reaction field. This conclusion is consistent with the equilibrium requirement of TAP theory,[7] since $T\to 0$, $1-q\propto T^2$ means $R=\beta\tilde{J}^2 m(1-q)|_{T\to 0}\to 0$. In the present theory, $R|_{T\to 0}\neq 0$, so $P_K(0)|_{T\to 0}\neq 0$.

The nonequilibrium nature of the phase-transition mode spectrum is consistent with recent experimental observation that SG systems are out of nonequilibrium during measurements.[4,6] Experimental evidence for the existence of quasi-phase-transition modes is of special interest. In principle, the nonequilibrium quasi-phase-transition mode is not expected to be directly observable in a static measurement like dc measurement as assured in the present paper, e.g., cusps shown in Fig. 2 are smeared out in dc measurement (cf. Fig. 7). It can be expected that the quasi-phase-transition mode can be directly observed in dynamical measurements, like ac susceptibility that exactly measures the differential susceptibility[19] and neutron scattering. In fact, susceptibility cusps shown in Fig. 2 have been observed in both ac (Ref. 27) and neutron-scattering[28] measurement. Further analysis on how SG systems respond to a varying field (like ac fields) is necessary to explain this coincidence.

Finally, I want to emphasis that the main point of the present theory is to propose a possible way for describing an effectively nonergodic SG phase in the mean-field model. This is different from finding an exact solution of the mean-field model. Further work is necessary to seek the relation between the present picture and the known equilibrium theory.

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