Electromagnetic properties of composites containing elongated conducting inclusions

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(Received 3 May 1995; revised manuscript received 25 September 1995)

We present a detailed theoretical study of the dielectric and magnetic response of composites containing elongated conducting inclusions—sticks. These composites are widely used as engineering materials. They can be also considered as a model to describe many processes occurring in nature, e.g., dielectric enhancement in grain-saturated porous rocks. An approach is proposed that is based on the idea of a scale-dependent local dielectric constant. We develop an effective-medium approximation and derive an equation to calculate an effective dielectric constant of the composites in the quasistatic case and for the high frequency when there is a strong skin effect in the conducting sticks. Our theory predicts very large values of the effective dielectric constant in a wide range of the stick concentration. We find that the dielectric constant can exhibit various dispersive behaviors. It can have relaxation behavior, power-law scaling behavior, or resonance dependence on the frequency. The resonance dependence occurs when the skin effect is strong and wavelength is comparable to the stick length. Then the real part of the dielectric constant has negative values in some frequency ranges. The possibility of a wave localization is discussed in that case. We consider effective magnetic properties of the conducting stick composites. We propose that the composites with nonmagnetic components will have a giant paramagnetic response as a result of a collective interaction of the sticks with an external magnetic field.

I. INTRODUCTION AND MOTIVATION

We consider composites that contain very elongated conducting inclusions, ''sticks'' embedded in a dielectric host (see Fig. 1). The sticks are supposed to be randomly distributed and oriented. The problem to be considered here is the calculation of the macroscopic dielectric and magnetic response of the conducting stick composites. The interest in metal-dielectric composites, where conducting inclusions have a very elongated shape, is because these systems describe physical structures occurring both in nature and technology.

There are a lot of porous rocks in nature such as sandstones or similar geological formations that have channel-like or sheetlike porous structure. In spite of its great importance, there is no universal concept for rock conductivity, permeability, and dielectric susceptibility (see discussion in Refs. 1, 2). Considering the sticks as a model for the rock pores, one can reproduce the structure of porous rocks by fitting the shape and concentration of the sticks.[3] Then it is possible to calculate the rock conductivity, permeability, and dielectric susceptibility by the methods that we develop in this paper. The conducting stick composites are also important for industrial applications. Ceramic and plastic materials reinforced by carbon or metallic fibers are becoming increasingly attractive as engineering materials. The physical-chemical and mechanical properties of such materials are the subject of great interest (see Refs. 4, 5 and references therein). The dielectric properties of the materials look like an important tool for their characterization and diagnostic.[6] The results obtained in this paper may be also useful for characterization of semicontinuous metal films. The semicontinuous films, used to fabricate selective surfaces for solar photothermal energy conversion, are usually prepared by thermal evaporation or spattering of the metal on an insulating substrate. In Ref. 7 its structure has been studied by an image analysis. The authors have obtained that the elementary conducting cell has an elongated shape. This circumstance is almost evident from the electron micrographs of the films (see, e.g., Refs. 8–10). It is therefore of interest to have a picture of the variation of the film dielectric properties with the shape of conducting inclusions. After all, we will show in this paper that dielectric and magnetic properties of the conducting stick composites are unusual and very interesting in their own right. We proposed that these composites can be used to manufacture various artificial dielectrics and magnetics.

The geometric properties of composites with penetrable conducting sticks have been studied by Monte Carlo simulation[3,11–13] and analytically.[14,15] It was found that the percolation threshold p_c is inversely proportional to the stick aspect ratio $p_c \propto b/a$, where b is the radius of a stick and $2a$ is its length. The same estimation $p_c \propto b/a$ was obtained in Ref. 16 by an excluded volume explanation of Archie's law for the porous rock permeability. This important result may be explained as follows: A conducting stick intersects on average with N other sticks. When $N \ll 1$, the sticks are separated from each other and the probability to percolate through the conducting sticks is equal to zero. When $N \gg 1$, the sticks form some carpet as is shown in Fig. 1 and the probability to percolate is equal to one. There is a critical number of intersections, N_c, that correspond to the percolation threshold. An important finding of Refs. 3, 11–15 is that the critical number N_c is about one and it does not depend on the stick aspect ratio a/b in the limit $a \gg b$. The averaged number of intersections, N, can be calculated using elementary theory of probability. It appears to be proportional to the aspect ratio and to the volume concentration of the conducting sticks: $N=(4/\pi)p(a/b)$ for $d=2$ and $N=2p(a/b)$ for $d=3$ (see, e.g., Ref. 3). Therefore we have the percolation threshold $p_c = N_{c2}\pi b/(4a) \propto b/a$ for $d=2$ and $p_c = N_{c3}b/(2a) \propto b/a$ for $d=3$, where N_{c2} and N_{c3} are critical numbers

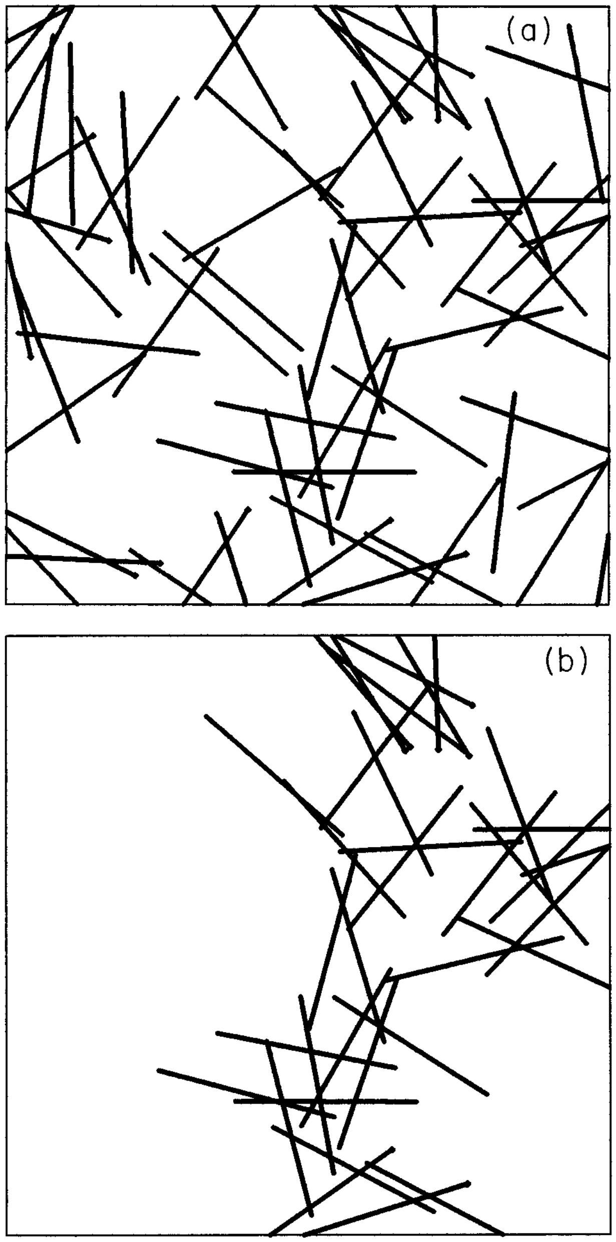

FIG. 1. Conducting stick composite. (b) Backbone of the "infinite cluster" that spans from top to bottom.

of the stick intersections for two- and three-dimensional composites, respectively. It follows from this result that the percolation threshold p_c may be very small for composites with elongated conducting inclusions. Moreover, p_c tends to zero when the aspect ratio a/b goes to infinity. Small values of the percolation threshold is one of the distinguishing features of the conducting stick composites. Another one is the anomalous dielectric response of such composites.

The dielectric response of the metal-dielectric composites has attracted the attention of many researchers for a long time. The problem to be considered is the calculation of the macroscopic, effective dielectric constant ϵ_e and magnetic permeability μ_e of the composites in terms of the dielectric and magnetic responses of its constituents. Although the results obtained so far for the composites with spherical conducting inclusions cover a wide range of methods and approximations,[6,9,17–21] extensions to nonspherical inclusions have been restricted, almost entirely, to mean-field approximations. There are two different mean-field approximations considered in the literature: Bruggeman effective-medium theory (EMT) (Refs. 6, 22, 23) and the symmetrized Maxwell-Garnet approximation introduced by Sheng (MGS) (Refs. 9, 24, 25). The EMT and MGS theories reproduce, at least qualitatively, the behavior of the effective parameters of the composites in the entire range of the conducting component concentration p. It was found that the last one gives better agreement with experimental data for the optical properties of the composites.[9] There are many approximations in the literature designed to obtain the effective parameters of the composites in the limit of small concentrations p or for the case when the properties of the constituents are only slightly different. A comprehensive list of such approximations can be found in Refs. 19–21. In Ref. 20 the authors have suggested an extrapolation procedure that being applied to a formula valid for $p \ll p_c$ gives an extension of this formula to all concentrations p. The thus obtained equations for the effective parameters give the percolation threshold p_c either equal to one or depending on the conductivity of the conducting component. In Ref. 21 the same authors suggest another symmetrization procedure that can be applied to an approximate formula to extrapolate it to all concentrations p and to all values of the parameters of the composite constituents. The procedure proposed in Ref. 21 gives, in all cases when it actually changes an original formula, the equation for the effective conductivity σ_e of the form $\sigma_e^{1/3} = (1-p)\sigma_d^{1/3} + p\sigma_m^{1/3}$, where σ_m and σ_d are the conductivities of the conducting and dielectric components, respectively. This equation implies that even for $\sigma_d = 0$ effective conductivity remains finite for all concentrations p of the conducting component. That is, the percolation threshold p_c is exactly equal to zero. Since the percolation threshold p_c for the conducting stick composites is neither equal to zero nor equal to one and the values of p_c do not depend on the stick conductivity, it is difficult to use the equations suggested in Refs. 20, 21 to find the effective parameters of the composites. For this reason we restrict further consideration to EMT and MGS theories, which give nontrivial values of p_c that are independent of the conductivities of the composite components. Nevertheless, we would like to point out that our approach is close to some extent to that proposed in Ref. 20.

The EMT and MGS theories have been developed originally to describe the properties of composites containing spherical conducting grains. Different generalizations of EMT have been suggested for composites with randomly oriented prolate conducting inclusions.[26–28] It is easy to show[29] that all these approaches give a percolation threshold that is proportional to the depolarization factor of an inclusion in the direction of its major axis $p_c \propto g_\parallel$. For very elongated inclusions the depolarization factor takes form $g_\parallel \cong (b/a)^2 \ln(a/b)$, where $2a$ and b are the stick length and radius, respectively (see, e.g., Ref. 30, Sec. 4). Therefore EMT gives the percolation threshold $p_c \propto (a/b)^2$, which is in obvious disagreement with the results of Refs. 3, 11–15. The percolation threshold given by MGS theory for the randomly oriented sticks depends on the typical shape of the dielectric regions that is assumed in the theory.[29] Since the sticks are randomly oriented, it is quite natural to suppose that the dielectric regions have on average a spherical shape.[27,28] Then the percolation threshold given by MGS theory, $p_c=0.46$, is independent of the stick shape.[29] An alternative approach corresponds to dielectric regions of the same shape as sticks.[6,9,26] Then it follows from the results of Ref. 29 that MGS theory gives $p_c \propto \sqrt{b/a}$. Therefore the results of this theory also disagree with results for the percolation threshold, $p_c \propto b/a$, obtained in Refs. 3, 11–15.

The percolation threshold is an important property since it determines the concentration of inclusions for which dramatic changes in the dielectric properties occur. A discrepancy between the values of the percolation threshold directly leads to a discrepancy between the dielectric data. Consider, for example, the composites with the stick aspect ratio $a/b>10^2$ that have been investigated in an experiment.[31] The observed values of the percolation threshold $p_c \propto b/a$ are many tens times larger than $p_c \propto (b/a)^2$ predicted by EMT and many times smaller than $p_c=0.46$ or $p_c \propto \sqrt{b/a}$ given by MGS theory. Therefore an application of the existing theories to the conducting stick composites is in question.

The percolation threshold is not a single problem with the conducting stick composites. The effective dielectric constant ϵ_e of the composite with aligned conducting spheroids has been detailed considered in Ref. 32. Suppose that the prolate spheroids with semiaxes a and b $(a>b)$ are aligned with z axis. The simple scale transformation $x=(b/a)^{1/3}x^*$, $y=(b/a)^{1/3}y^*$, $z=(a/b)^{2/3}z^*$ reduces the composite to the system of anisotropic conducting spheres distributed in an anisotropic host.[33] Therefore the percolation threshold for the composites with aligned sticks coincides with that of the spherical particles. Nevertheless, it appears that the dielectric response of aligned stick composites is quite different from that of composites with spherical inclusions even for the small concentrations considered in Ref. 32. This difference is due to long-range correlations in the interaction of the sticks. It is shown in Ref. 32 that long-range correlations are a distinct feature of sticks, while for spherical particles they are negligible (see also Ref. 34). Another important result of Ref. 32 is the possibility of the excitation of internal manifold modes in the system. This observation corresponds to the results of the present work. The high-frequency dielectric properties of the aligned stick composites have been considered in Refs. 35, 36 beyond the usual quasistatic approximation.

Let us consider the interaction of randomly oriented sticks excited by an external field E_0. The dipole moment $\mathbf{D}$ of a conducting stick may be estimated as $\mathbf{D} \propto a^3 E_0$ (Ref. 30, Sec. 3). Then the effective dielectric constant is estimated as $\epsilon_e \propto a^3 n \propto a^3 p/(ab^2)$, where n is the number of the sticks in a unit volume. We can rewrite the last expression for the dielectric constant in terms of p_c: $\epsilon_e \propto a^3 n \propto (a/b)(p/p_c)$. In this paper we are interested in composites with very elongated inclusions $(a/b \gg 1)$. Then the dielectric constant ϵ_e is large even for concentrations p, which are far below the percolation threshold p_c. We can estimate the number $n_{\rm int}$ of sticks inside the range a of the dipole interaction as $n_{\rm int} \propto a^3 n \propto \epsilon_e$. Therefore the stick dipoles start to strongly interact as soon as $\epsilon_e \gg 1$, which happens for sufficiently small stick concentrations $p<p_c \ll 1$.

The effective dielectric constant may be introduced for the composite samples whose size is much larger than the stick length a. But to calculate the effective parameters it is necessary to start with scales smaller than the stick radius $b \ll a$. The aspect ratio $a/b \gg 1$ may be considered as a minimal dimensionless correlation length of the problem. When the concentration p is increased, the correlation length is further increased. As we have pointed out above, the sticks usually strongly interact. Therefore the conducting stick composite is a system with long-ranged strong interactions. As a result, well-developed methods of the percolation theory[6,9,17,18] like renormalization group in real space or mean-field approximations cannot be directly applied to the system. Computer simulation of the dielectric properties of the conducting stick composites is also difficult. To calculate the effective dielectric constant, we have to know the electromagnetic field distribution over the system. Indeed, the field distribution is obtained usually by solution of finite-difference Laplas or Maxwell equations. In a very crude approximation the stick radius b can be taken as an elementary length. Then equations have to be solved on three-dimensional (3D) lattices whose number of sites is much larger than $(a/b)^3$. Suppose, for example, that the stick aspect ratio $a/b \approx 10^2$ then the number of the sites is of the order of 10^9. The solution of a finite difference equation on a such lattice is a difficult problem even for the most powerful modern computers.

On the experimental side results of the first attempt[31] to investigate the dielectric properties of conducting stick composites are in evident disagreement with the percolation theory.[6,17,18] The percolation theory predicts that the dielectric constant exhibits a power-law dispersive behavior $\epsilon_e \propto (\omega/\sigma_m)^{s/(t+s)} \propto (\omega/\sigma_m)^{0.3}$ in a small critical region around p_c defined by $|\Delta p|<(\omega/\sigma_m)^{1/(t+s)}$, where the critical exponents are equal to $t \cong 2.0$, $s=0.8$ for $d=3$; σ_m is the conductivity of the conducting inclusions. Out of this critical region, there should be no dispersion at all. In contrast to this prediction, the dispersive behavior of the dielectric constant ϵ_e has been observed in the microwave range 10^9–10^{10} Hz for all investigated concentrations of conducting sticks.[31] The dispersive behavior for sticks with conductivity $\sigma_m \approx 10^{14}$ sec^{-1} is similar to the Debye relaxation: $\epsilon_e(\omega) \propto 1/(1-i\omega\tau_R)$,[37] where the relaxation time τ_R depends on the stick length and conductivity. The frequency dependence of the dielectric constant $\epsilon_e(\omega)$ is resonance in form for the composites with higher stick conductivity $\sigma_m \approx 10^{17}$ sec^{-1}. The real part of the dielectric constant $\epsilon_e(\omega)$ drops down to zero at some resonance frequency ω_1 and it acquires negative values for frequencies $\omega>\omega_1$. The power-law dispersive behavior predicted by the percolation theory had not been observed in this experiment at all.

In this paper we present a comprehensive theoretical study of the dielectric and magnetic response of conducting stick composites. We study in all details the interaction of a conducting stick with an external electric field. Then we develop an effective medium theory for the conducting stick composites. Our approach is based on the well-known Bruggeman EMT. It also incorporates the idea of MGS theory, that the local environment of an inclusion may be different for different inclusions. We show that the dielectric constant is a nonlocal quantity for the considered composites. It depends on the spatial scale for scales less than the stick length. We consider a single stick in such an environment and derive an equation to find the dielectric response of the composites. Preliminary results of this work have been reported earlier.[38]

Our theory gives the percolation threshold $p_c \propto b/a$ in agreement with the results of Refs. 3, 11–15. It also reproduces the dispersive behavior obtained in Ref. 31. For the quasistatic case, when the skin effect in the sticks is negligible, we obtain the relaxation behavior of the effective dielectric constant $\epsilon_e(\omega) \propto 1/(1 - i\omega\tau_R)$, for a wide concentration range below and also above the percolation threshold. We actually determine the relaxation time τ_R in this paper. It turns out that τ_R depends on the stick shape and conductivity. For frequencies $\omega \ll \tau_R^{-1}$, the power-law dispersive behavior predicted by the percolation theory can be observed in some vicinity of the percolation threshold.

We extend our theory to the nonquasistatic case when it is a strong skin effect in the conducting sticks. Then the relaxation behavior of the effective dielectric constant $\epsilon_e(\omega)$ changes to the resonance dependence of the frequency. The real part of $\epsilon_e(\omega)$ can accept negative values when the skin effect is strong. We discuss the possibility of a wave localization in this case. The dependence $\epsilon_e(p - p_c, \omega)$ in the critical region near the percolation threshold is also considered for the strong skin effect. It seems that this dependence has a nonuniversal form in this case.

We consider the magnetic response of conducting stick composites. Randomly oriented sticks form different contours in this kind of composite. The typical area of such a contour is about a^2. An external alternative magnetic field is excited in the contour electric current $I \propto a^2$. The magnetic moment of a such current is about $M \propto a^3$. The effective magnetic permeability of the composites can be estimated as $\mu_e \propto Mp/(ab^2) \propto (p/p_c)(a/b)$. Therefore conducting stick composites can have a large magnetic response even for small concentrations of the sticks $p < p_c \ll 1$. It is necessary to stress that the magnetic response can be observed for composites consisting of nonmagnetic materials.

In this paper we calculate the magnetic moment M of the simplest contours consisting of two conducting sticks. Then we apply the approach[39] to calculate the effective magnetic permeability μ_e. The frequency dependence of the thus obtained permeability $\mu_e(\omega)$ is resonance in form. The giant paramagnetic response can be observe in some frequency range for the conducting stick composites.

The rest of the paper is organized as follows: In Sec. II we develop our mean-field approach and derive the equation for the effective dielectric constant. Then we consider the low-frequency dielectric properties of the stick composites. It is shown that the dispersive behavior of the composites strongly depends on the stick's length and conductivity. In Sec. III we deal with the high-frequency properties of the stick composites. At high frequency the interaction of a stick with an electromagnetic wave has resonance character. As a result, the composite dielectric constant is resonance in form. We also discuss the possibility of internal modes to be excited in the composites. In Sec. IV we consider the effective magnetic properties of the composites consisting of nonmagnetic materials. It is shown that the magnetic permeability of the composite can be much larger than unity in some frequency range. Section V is devoted to conclusions.

II. LOW-FREQUENCY PROPERTIES OF CONDUCTING STICK COMPOSITES

Let us consider a system of randomly oriented conducting prolate spheroids with semiaxes $a \gg b = c$. This shape is the amenable geometry to analysis and is a good affiliation for the sticks. The prolate spheroid sticks are randomly embedded in a dielectric matrix characterized by a dielectric constant ϵ_d. We neglect the direct "hard-core" interaction and assume that the sticks are penetrable. Our objective is the calculation of the effective complex dielectric constant $\epsilon_e = \epsilon_1 + i\epsilon_2$ or the effective complex conductivity $\sigma_e = -i\omega\epsilon_e/4\pi$ of the stick composite. To find the effective parameters, one has to know the distribution of the electric field $\mathbf{E}(\mathbf{r})$ and current density $\mathbf{j}(\mathbf{r})$ in the system when an external field $\mathbf{E}_0$ is applied. The effective complex conductivity σ_e is determined by the definition

$$\langle \mathbf{j}(\mathbf{r}) \rangle = \sigma_e \langle \mathbf{E}(\mathbf{r}) \rangle, \tag{1}$$

where $\langle \cdots \rangle$ denotes an average over the system volume. In the real composite, both the current $\mathbf{j}(\mathbf{r})$ and field $\mathbf{E}(\mathbf{r})$ will be highly inhomogeneous and statistically random, and it will be very difficult to calculate them precisely.

The typical correlation length of the field and current fluctuations is of the order of the stick length $2a$. Therefore the effective conductivity σ_e can be defined only for the scale l that is larger than the stick length $2a$. On the other hand, the field and current *inside* a stick are determined on a scale corresponding to the stick radius $b \ll a$. Then the conductivity σ_e and other effective parameters are determined by the field distribution in a volume larger than a^3, while the field fluctuations with volume $b^3 \ll a^3$ are important. As a result, the effective parameters of the composites essentially depend on the aspect ratio a/b, that is, on the shape of the conducting inclusions. In this situation standard methods of percolation theory cannot be applied to find the effective parameters and one has to develop new approaches. Let us illustrate the last statement by an example of effective-medium theory.

The method widely used to calculate the effective properties of a composite is a self-consistent approach known as effective-medium theory (EMT).[6,22,23] EMT has the virtue of relative mathematical and conceptual simplicity, and it is a method that provides quick insight into the effective properties of metal-dielectric composites. In EMT one makes two approximations: (a) the metal grains as well as dielectric are embedded in the *same* homogeneous effective medium that will be determined self-consistently and (b) the metal grains as well as dielectric grains are taken to have the *same* shape.

For conducting stick composites, these approximations mean that the metal and dielectric grains are assumed to have the same shape as prolate spheroids. The internal field inside a spheroid embedded in the effective medium with conductivity σ_e can be easily calculated (Ref. 30, Sec. 8). The internal fields in the conducting and dielectric spheroids averaged over all orientations are equal, respectively,[38]

$$\mathbf{E}_{\text{in m}}=\tfrac{1}{3}\mathbf{E}_{\text{in m}\parallel}+\tfrac{2}{3}\mathbf{E}_{\text{in m}\perp},$$

$$\mathbf{E}_{\text{in m}\parallel}=\frac{1}{1+g_{\parallel}(\sigma_m-\sigma_e)/\sigma_e}\,\mathbf{E}_0,$$

$$\mathbf{E}_{\text{in m}\perp}=\frac{1}{1+g_{\perp}(\sigma_m-\sigma_e)/\sigma_e}\,\mathbf{E}_0, \tag{2a}$$

$$\mathbf{E}_{\text{in d}}=\tfrac{1}{3}\mathbf{E}_{\text{in d}\parallel}+\tfrac{2}{3}\mathbf{E}_{\text{in d}\perp},$$

$$\mathbf{E}_{\text{in d}\parallel}=\frac{1}{1+g_{\parallel}(\sigma_d-\sigma_e)/\sigma_e}\,\mathbf{E}_0,$$

$$\mathbf{E}_{\text{in d}\perp}=\frac{1}{1+g_{\perp}(\sigma_d-\sigma_e)/\sigma_e}\,\mathbf{E}_0, \tag{2b}$$

where σ_m is the conductivity of the sticks, $\sigma_d=-i\omega\epsilon_d/4\pi$ is the dielectric host conductivity, and $g_{\parallel}$ and $g_{\perp}$ are the spheroid depolarization factors in the direction of the major axis and in the transverse direction, respectively. For very elongated inclusions, $g_{\parallel}\ll 1$ and $g_{\perp}\cong 1/2$ (Ref. 30, Sec. 4). The currents in the conducting and dielectric spheroids averaged over all orientations are equal,

$$\mathbf{j}_{\text{in m}}=\sigma_m\mathbf{E}_{\text{in m}},\quad \mathbf{j}_{\text{in d}}=\sigma_d\mathbf{E}_{\text{in d}}. \tag{3}$$

To find the effective conductivity σ_e, we substitute Eqs. (2) and (3) in Eq. (1) and take into account that the conducting spheroids occupy the volume fraction p of the system. Thus we obtain the following equation to determine the effective conductivity:

$$\frac{p}{3}\left[\frac{\sigma_m-\sigma_e}{\sigma_e+g_{\parallel}(\sigma_m-\sigma_e)}+\frac{2(\sigma_m-\sigma_e)}{\sigma_e+g_{\perp}(\sigma_m-\sigma_e)}\right]+\frac{1-p}{3}\left[\frac{\sigma_m-\sigma_e}{\sigma_e+g_{\parallel}(\sigma_m-\sigma_e)}+\frac{2(\sigma_m-\sigma_e)}{\sigma_e+g_{\perp}(\sigma_m-\sigma_e)}\right]=0. \tag{4}$$

This equation is equivalent to the condition that the space average of polarization of the dielectric and conducting particles embedded in the "effective medium" shall vanish. It has a transparent physical meaning. Namely, we choose the effective conductivity to cause the averaged scattered field to vanish.[40] For spherical particles ($g_{\parallel}=g_{\perp}=1/3$), Eq. (4) coincides with the usual EMT equation (see, e.g., Ref. 6) and gives the usual EMT result that the percolation threshold p_c is equal to $p_c=1/3$.

In the general case of elongated conducting particles, EMT, Eq. (4), gives the percolation threshold

$$p_c=(5-3g_{\parallel})g_{\parallel}/(1+9g_{\parallel}). \tag{5}$$

We consider here composites where the aspect ratio of the conducting particles is large $a/b\gg 1$. Then the depolarization factor is small, $g_{\parallel}\approx(b/a)^2\ln(a/b)\ll 1$ (see the Appendix) and the percolation threshold given by Eq. (5) is approximately equal to $p_c=5(b/a)^2\ln(a/b)\propto(b/a)^2$. This result is in contradiction to $p_c\propto b/a$ obtained for the stick composites in Refs. 3, 11–15 and in the experiment of Ref. 31. Therefore EMT, Eq. (4), cannot be used for the actual calculation of the effective parameters of the composites.

To understand the reason for this discrepancy let us examine the basic approximations of EMT. It is obvious that suggestion (b), "the metal grains as well as dielectric ones are taken to have the same shape," is not correct for stick composites. Indeed, the dielectric in such composites cannot be considered as an aggregation of individual grains. It fills all the space between randomly oriented conducting sticks, and the averaged shape of the dielectric regions is more spherical than prolate spheroid. Then the field in the dielectric regions is given by[6]

$$\mathbf{E}_{\text{in d}}=\frac{3\sigma_e}{2\sigma_e+\sigma_d}\,\mathbf{E}_0, \tag{6}$$

where we suppose that the dielectric regions are surrounded by the "effective medium" with conductivity σ_e.

Since we assume that the dielectric regions have a spherical shape that is different from the shape of the conducting sticks, there is no reason to assume that local environments of dielectric regions and conducting sticks are the same. It means that the first approximation (a) of the standard EMT, "the metal grains as well dielectric are embedded in the *same* homogeneous effective medium," should be also revised. At this point we follow the ideas of the symmetrized Maxwell-Garnett approximation.[24]

Let us consider a single-well conducting stick placed in an external field E_0 directed along it. The field is close to zero at the stick surface, and it recovers the value E_0 at a distance $l>a$ only. Clearly, the "effective medium" in the distance l, $b<l<a$, is most important in the formation of the internal field $\mathbf{E}_{\text{in m}}$ in the stick. On the other hand, we can prescribe bulk effective properties to the domain of the composite, that the size l is much larger than the stick length $2a$. For the smaller scale l, the sticks will be cut off and effective conductivity is a function of the scale l: $\sigma_e=\sigma(l)$. When $l\ll a$ the conductivity $\sigma(l)$ will be equal to $\sigma(l)\cong\sigma_d$ for the stick concentration $p\ll 1$ considered here. The scale-dependent conductivity $\sigma(l)$ will recover its bulk value $\sigma(l)=\sigma_e$ when $l\gg a$. We accept the simplest assumption: A conducting stick is surrounded in the composite by the medium with conductivity

$$\sigma(l)=\sigma_d+(\sigma_e-\sigma_d)\,\frac{l}{a},\quad l<a,$$

$$\sigma(l)=\sigma_e,\quad l>a. \tag{7}$$

To find the internal field $\mathbf{E}_{\text{in m}}$ in a conducting stick embedded in a such "effective medium," note that Eq. (2a) for the field is obtained under very general conditions (Ref. 30, Sec. 8) and it should be true for the scale-dependent stick environment. We calculate the internal field for this case in the Appendix and show that $\mathbf{E}_{\text{in}\,\mathbf{m}}$ is still given by Eqs. (2a) where the depolarization factors $g_{\parallel}, g_{\perp}$ now take the form

$$g_{\parallel}=\frac{b^2\sigma_e}{a^2\sigma_d}\ln\left(1+\frac{a\sigma_d}{b\sigma_e}\right)\equiv\frac{b^2\epsilon_e}{a^2\epsilon_d}\ln\left(1+\frac{a\epsilon_d}{b\epsilon_e}\right),\quad g_{\perp}=\frac{1}{2},\tag{8}$$

in the limit $a\gg b$. The depolarization factor $g_{\parallel}$ is reduced to the usual expression $g_{\parallel}=(b/a)^2\ln(a/b)$ in the dilute case when $\sigma_e\cong\sigma_d$.

Now we would like to summarize the main assumptions of our effective-medium theory for conducting stick composites (EMTSC). To obtain the EMTSC equation for the effective complex conductivity, we make two approximations.

(a*) Each conducting stick is embedded in the effective medium with conductivity $\sigma(l)$ that depends on the scale l by means of Eq. (7). The conductivity $\sigma(l)$ equals the effective conductivity σ_e for $l>a$. The value of σ_e will be determined self-consistently.

(b*) The dielectric regions are taken to have a spherical shape and embedded in the effective medium with conductivity σ_e.

To find the effective conductivity σ_e, we substitute Eqs. (2a), (6), and (3) into Eq. (1). Thus we obtain the following EMTSC equation to determine the effective conductivity:

$$\frac{p}{3}\left[\frac{\sigma_m-\sigma_e}{\sigma_e+g_{\parallel}(\sigma_m-\sigma_e)}+\frac{2(\sigma_m-\sigma_e)}{\sigma_e+g_{\perp}(\sigma_m-\sigma_e)}\right]+(1-p)\,\frac{3(\sigma_d-\sigma_e)}{2\sigma_e+\sigma_d}=0,\tag{9}$$

where the polarization factors $g_{\parallel}$ and $g_{\perp}$ are given by Eq. (8).

Equation (9) is much simplified in the case of the high-conducting elongated inclusions ($\sigma_m\gg\omega$, $a\gg b$) considered in the paper.

(1) We are interesting in the effective conductivity for the stick concentration p which is below and in a vicinity of the percolation threshold $p_c\propto b/a\ll 1$. Therefore we neglect the concentration p in comparison to the one in the second term of Eq. (9).

(2) Since the concentration $p\ll 1$, the effective conductivity σ_e is much smaller than the stick conductivity $\sigma_e\ll\sigma_m$. Therefore we neglect σ_e in comparison with σ_m in all terms of Eq. (9).

(3) We can neglect the stick polarizability in a direction perpendicular to an external field since it is much smaller than the polarizability in the direction of the field. That is, we can neglect the second term in the square brackets in Eq. (9).

After these simplifications Eq. (9) takes the form

$$\frac{1}{3}\,p\,\frac{\sigma_m}{\sigma_e}\,\frac{1}{1+(b^2\sigma_m/a^2\sigma_d)\ln(1+a\sigma_d/b\sigma_e)}+3\,\frac{\sigma_d-\sigma_e}{2\sigma_e+\sigma_d}=0.\tag{10}$$

This is the basic EMTSC equation for the effective parameters of the composite containing elongated conducting inclusions. Let us consider the behavior of σ_e obtained from Eq. (10) in some extreme cases.

In the static limit $\omega\to 0$, $\sigma_d\to 0$, the composite conductivity σ_e decreases with decreasing the stick concentration p and it vanishes at the percolation threshold, which is equal to

$$p_c=\frac{9}{2}\,\frac{b}{a}.\tag{11}$$

This result for p_c is in best agreement with the result $p_c\propto b/a$ obtained in Refs. 3, 11–15 and in the experiment of Ref. 31. The static conductivity vanishes near the percolation threshold as

$$\sigma_e(\tau)=\frac{b}{a}\,\sigma_m\tau^t,\quad \tau\geqslant 0,\tag{12}$$

where $\tau=(p-p_c)/p_c$ is the reduced concentration of the conducting sticks, $t=1$. It follows from Eq. (12) that $\sigma_e\ll\sigma_m$ for concentrations $\tau<1$, in agreement with the assumption (1) made for Eq. (10). To understand this result, let us consider sticks that belong to the backbone of an infinite cluster. Each stick that belongs to the backbone cuts a finite number of other backbone sticks [see Fig. 1(b)]. Therefore the backbone consists of segments whose length is proportional to the stick length $2a$. Since the conductance of such a segment is proportional to b/a, we immediately obtain this small factor in Eq. (12).

The EMTSC developed in this paper has disadvantages characteristic of mean-field theories. For example, it gives the "conductivity" critical exponent $t=1$ in Eq. (12) instead of $t\cong 2.0$ given by percolation theory.[6] Nevertheless, we believe that EMTSC qualitatively reproduces the distinguishing features of the composites. Thus the very small factor b/a should also appear in a "true" scaling equation for the static conductivity $\sigma_e(\tau)$ since the conductance of an elementary bond in the backbone is proportional to this factor. We suggest that the scaling equation for $\sigma_e(\tau)$ has the form

$$\sigma_e(\tau)=\frac{b}{a}\,\sigma_m\tau^t,\tag{13}$$

where the critical exponent t is equal to $t\cong 2.0$.

The static dielectric constant $\epsilon_e(\tau)$ is predicted by EMTSC to diverge when p_c is approached from either side,

$$\epsilon_e(\tau)=\frac{1}{2}\,\epsilon_d\,\frac{a}{b}\,|\tau|^{-s},\tag{14}$$

where the critical exponent s is equal to $s=1$. Note that a large factor $a/b\gg 1$ appears in Eq. (14). We guess that this factor is due to the large polarizability of a conducting stick discussed above. As a result, the dielectric constant $\epsilon_e(\tau)$ achieves very large values in the critical region $|\tau|\ll 1$ near the percolation threshold p_c as shown in Fig. 2 (the dashed line in Fig. 2 indicates the position of p_c). The "dielectric" critical exponent $s=1$ in Eq. (14) is not far away from the known value $s=0.8$.[6] Therefore we believe that Eq. (14) may be used for a quantitative estimation of $\epsilon_e(\tau)$ in the vicinity of p_c.

The dielectric constant ϵ_e does not diverge at the percolation threshold for finite frequency ω, but the real part ϵ_1 of the complex dielectric constant $\epsilon_e=\epsilon_1+i\epsilon_2=4i\pi\sigma_e/\omega$ still has a maximum at the percolation threshold as it follows from the solution of Eq. (10). Let us consider the scaling

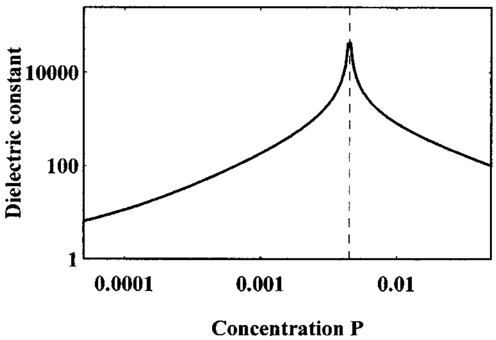

FIG. 2. Static dielectric constant of the conducting stick composite as a function of the stick concentration p; the stick aspect ratio and dielectric constant of the host are chosen to be $a/b=1000$ and $\epsilon_d=2$, respectively. The dashed line signifies the position of the percolation threshold p_c.

properties of the effective dielectric constant $\epsilon_e(\tau,\omega)$ in the critical region $|\tau|\ll 1$, $\sigma_m/\omega \gg 1$ near the percolation threshold. The critical behavior of $\epsilon_e(\tau,\omega)$ for ordinary metal dielectric composites is believed[6,41,42] to be described by the scaling expression

$$\epsilon_e(\tau,\omega)=\epsilon_d|\tau|^{-s}\phi\left[|\tau|\left(\frac{4\pi i\sigma_m}{\epsilon_d\omega}\right)^{1/(t+s)}\right], \qquad (15a)$$

where the scaling function $\phi(z)$ has the following asymptotic behavior:

$$\phi(z)\cong\begin{cases}1, & |z|\gg 1, \quad \tau<0,\\ z^s, & |z|\ll 1,\\ z^{t+s}+1, & |z|\gg 1, \quad \tau>0.\end{cases} \qquad (15b)$$

This scaling behavior has been obtained in some experiments and numerical simulations (see Refs. 6, 43, 44, and references therein) that have been done for the composites with approximately round conducting grains. For $\omega\to 0$, Eqs. (15a), (15b) give the asymptotic behavior of the effective dielectric constant $\epsilon_e\cong\epsilon_d|\tau|^{-s}$ and effective conductivity $\sigma_e\cong\sigma_m\tau^t$ for $\tau>0$ that is somewhat different from the asymptotic behavior of these quantities for the conducting stick composites given by Eqs. (13) and (14). To incorporate Eqs. (13) and (14) in the scaling approach, we suggest rewriting the scaling Eq. (15a) in the form

$$\epsilon_e(\tau,\omega)=\frac{a}{b}\,\epsilon_d|\tau|^{-s}\phi\left[|\tau|\left(\frac{i}{\omega^*}\right)^{1/(t+s)}\right], \qquad (15c)$$

where the scaling function $\phi(z)$ is still given by Eq. (15b), while the dimensionless frequency $\omega^*=(a/b)^2(\epsilon_d\omega/4\pi\sigma_m)$ to the square of the stick aspect ratio a/b. Equation (15c) can be considered as a generalization of Eq. (15a) to the conducting stick composites. This equation has a scaling form and gives the asymptotics (13) and (14).

From Eqs. (15b), (15c), it follows that when both $|\tau|$ and ω/σ_m are small in the critical region around p_c defined by

$$|\tau|<\left(\frac{a}{b}\right)^{2/(t+s)}\left(\frac{\epsilon_d\omega}{4\pi\sigma_m}\right)^{1/(t+s)}, \qquad (16)$$

the complex dielectric constant exhibits a power-law dispersive behavior

$$\epsilon_1(\omega)\propto\epsilon_2(\omega)\propto\left(\frac{a}{b}\right)^{(t-s)/(t+s)}\left(\frac{\omega}{\sigma_m}\right)^{s/(t+s)}. \qquad (17)$$

Note that the concentration range where the dielectric constant has the scaling dispersive behavior (17) increases by a factor $(a/b)^{2/(t+s)}$ for the composites containing elongated conducting inclusions with the aspect ratio $a/b\gg 1$. Consider, for example, a hypothetical composite prepared from industrial carbon fibers with a thickness of about 1 μm that are cut into sticks with length 1 mm. The typical conductivity of such carbon fibers is about $\sigma_m\cong 10^{14}$ sec^{-1}. Therefore the dimensionless frequency in Eq. (15c) is of the order 10^9 sec$^{-1}/\omega$. Suppose that we measure the dielectric constant in the vicinity $|\tau|<0.1$ of the percolation threshold. Then the power-law dispersive behavior given by Eq. (17) should be observed in the frequency band 100 kHz$<\nu<$100 MHz, where $\nu\equiv\omega/2\pi$. One can compare this frequency band with the band for the scaling dispersion, 10^{11} Hz$<\nu<10^{14}$ Hz, given by the usual scaling expression (15a). We believe that the conducting stick composites can be used as a convenient reference system to verify *quantitatively* the predictions of the percolation theory, e.g., Eqs. (15) as well as some modern approaches (see, e.g., Ref. 45).

Consider now the behavior of the dielectric constant ϵ_e for concentrations p below the percolation threshold out of the critical region. We will still suppose that $|\epsilon_e|/\epsilon_d\gg 1$ (see Fig. 2) and neglect ϵ_d in comparison with ϵ_e in the second term of Eq. (10). Then Eq. (10) takes the form

$$\epsilon_e(p,\omega)=\epsilon_d\,\frac{2}{9}\,p\,\frac{a^2}{b^2\ln(a/b)}\times\frac{1}{\ln[a\epsilon_d/b\epsilon_e(p,\omega)]/\ln(a/b)-i\omega^*}, \qquad (18)$$

where we introduce the dimensionless frequency

$$\omega^*=\left(\frac{a}{b}\right)^2\frac{|\epsilon_d/\epsilon_m|}{\ln(a/b)}=\left(\frac{a}{b}\right)^2\frac{\epsilon_d\omega}{4\pi\sigma_m\ln(a/b)}. \qquad (19)$$

It is interesting to note that the thus obtained dimensionless frequency ω^* coincides up to a factor $1/\ln(a/b)$ with dimensionless frequency which emerges in the scaling Eq. (15c), which has been obtained in the different concentration ranges. The dimensionless frequency ω^* is an important parameter that determines, as we show below, the dependence $\epsilon_e(p,\omega)$ for the entire range of concentration. One can rewrite Eq. (19) for the dimensionless frequency as $\omega^*=\omega\tau_R$, where τ_R is the stick relaxation time. Electric charges somehow distributed over a stick will relax to their stationary distribution in time τ_R. For spherical particles the relaxation time coincides with Maxwell's time $1/(4\pi\sigma_m)$ and is negligibly small usually. The relaxation time $\tau_R\propto(a/b)^2/(4\pi\sigma_m)$ for the conducting sticks may be many orders of magnitude larger than Maxwell's time. This is the origin of the relaxation behavior of the effective dielectric constant obtained in the experiment of Ref. 31 and discussed below.

For the stick concentration $p<p_c$ out of the critical range, Eq. (18) yields that $|\epsilon_e(p)|<(a/b)\epsilon_d$. To make an estimation of ϵ_e, we use a crude approximation neglecting $\ln(\epsilon_d/\epsilon_e)$ in comparison with $\ln(a/b)$ in Eq. (18). Thus we obtain an explicit form of the effective dielectric constant:

$$\epsilon_e(p,\omega)\cong\epsilon_d\,\frac{2}{9}\,p\,\frac{a^2}{b^2\,\ln a/b}\,\frac{1}{1-i\omega^*}. \tag{20}$$

It follows from Eq. (20) that the dielectric constant ϵ_e takes large values even for the concentration $p\ll p_c$ due to the large aspect ratio $a/b\gg 1$. Since the dielectric constant ϵ_e achieves sufficiently large value far below the percolation threshold (see Fig. 2), this effect is not connected to the formation of large, critical conducting clusters considered in percolation theory.[6,17,18] Therefore it is easy to reproduce the large values ϵ_e in a real experiment.[31] This is one of the interesting features of the stick composites that may be important for engineering applications. We propose that these composites can be used to prepare an artificial dielectric with a high dielectric constant.

Another interesting feature of Eqs. (18) and (20) is that they give dispersive behavior of the effective dielectric constant $\epsilon_e(p,\omega^*)$ that is close to the relaxation behavior. This relaxation behavior cannot be obtain from the usual percolation theory[6] and is closer to that given by the Maxwell-Garnet approximation.[9] The fact that the behavior of composites with elongated conducting inclusions is better described by the Maxwell-Garnet approximation is discussed in the literature (see, e.g., Ref. 9), but until now has no proper explanation. Note that the relaxation behavior give by Eqs. (18)–(20) should be observed for the concentration range $(b/a)^2<p<b/a$, that is, in a wide concentration range below the percolation threshold. This prediction of the theory is in the best *qualitative* agreement with experiment.[31] A detailed comparison with experiment will be present elsewhere.

To obtain the behavior of the effective dielectric constant in entire range of the concentrations, we solve the EMTSC equation (10) numerically. Thus the calculated real $\epsilon_1(\tau)$ and imaginary $\epsilon_2(\tau)$ parts of the dielectric constant we present in Fig. 3 for different values of the dimensionless frequency ω^*. The dependence $\epsilon_1(\tau)$ has "percolationlike" behavior with a sharp peak at p_c when $\omega^*\ll 1$. The peak in $\epsilon_1(\tau)$ becomes wider and wider, and it shifts to the larger concentration far away from the percolation threshold when the frequency ω^* increases. Since the dielectric constant has no anomaly at the percolation threshold when $\omega^*\geq 1$, we believe that the EMTSC equation (10) gives now quantitatively correct results for all stick concentrations including the percolation threshold. For $\omega^*\geq 1$ the effective dielectric constant has a relaxation dispersion $\epsilon_e(\omega)\propto 1/(1-i\omega\tau_R)$ in the entire range of concentrations p while the relaxation time τ_R somewhat decreases with increasing p above the percolation threshold p_c. We obtain from this result and scaling, Eqs. (15b)–(17), that in the critical concentration region $|\tau|\ll 1$ near p_c the effective dielectric constant $\epsilon_e(\omega^*)$ changes with the frequency as follows: $\epsilon_e\propto\epsilon_d(a/b)|\tau|^{-s}$, $\tau<0$, and $\epsilon_e\propto\epsilon_d(a/b)(|\tau|^{-s}+(i/\omega^*)\tau^t)$, $\tau>0$ for $\omega^*<|\tau|^{t+s}$, $\epsilon_e\propto\epsilon_d(a/b)\omega^{*s|(t+s)}$ for $|\tau|^{t+s}<\omega^*<1$: finally we have $\epsilon_e\propto\epsilon_d(a/b)/(1-i\omega^*)$ for $\omega^*>1$. Note that the dispersive behavior $\epsilon_e(\omega)$ can be described in any particular range of ω^* and τ by one of the equations suggested in Ref. 19. But none of these equations can reproduce $\epsilon_e(\omega)$ as a whole. Our approach has another advantage over Ref. 19 because it allows one to express explicitly the parameters in the dispersive equations in terms of the universal critical exponents t and s, dimensionless concentration τ, etc. Out of the critical concentration region, the effective dielectric constant either accepts their static values for $\omega^*<1$ or it has a relaxation dispersion when $\omega^*>1$ as we have mentioned above.

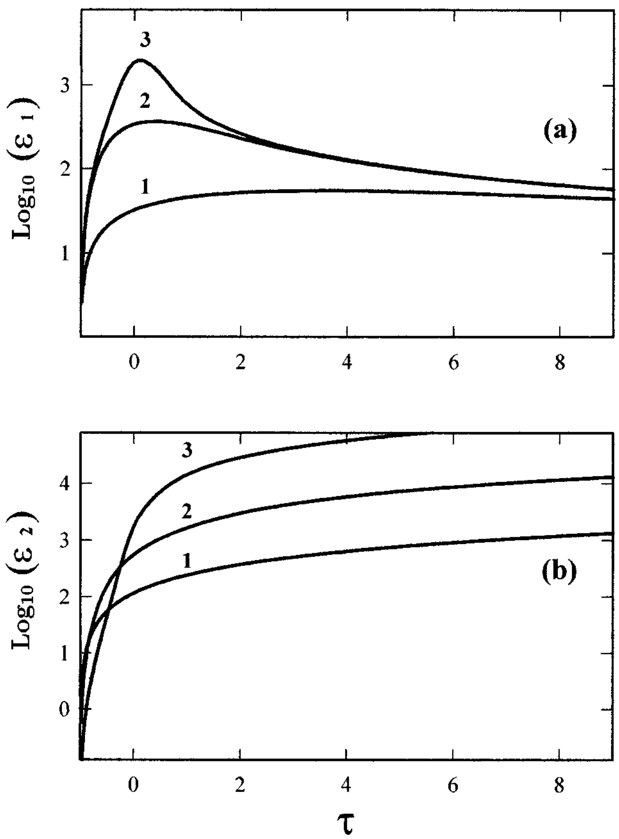

FIG. 3. Dependence of real $\epsilon_1(\tau)$ (a) and imaginary $\epsilon_2(\tau)$ (b) parts of the effective dielectric constant ϵ_e on the dimensionless stick concentration $\tau=(p-p_c)/p_c$ for different calues of the dimensionless frequency $\omega^*=(a/b)^2[\epsilon_d\omega/4\pi\sigma_m\ln(a/b)]$: $1-\omega^*=1.0$, $2-\omega^*=0.1$, $3-\omega^*=0.01$. The aspect ratio is equal to $a/b=1000$, $\epsilon_d=2$.

It follows from the above discussion that the dispersive behavior of stick composites can be very different from that of ordinary metal-dielectric composites with spherical particles. We believe that stick composites may be considered as a special class of percolating systems.

III. HIGH-FREQUENCY DIELECTRIC RESPONSE OF THE CONDUCTING STICK COMPOSITES

Composite materials containing conducting sticks dispersed in a dielectric matrix have new and unusual properties at high frequency. When the frequency ω increases, the wavelength $\lambda=2\pi c/\omega$ of an external electromagnetic field starts to be of the order of the stick length $2a$. At first glance the sticks behave now like microantennas and an external wave should be scattered in all directions. We propose in this paper that composite materials still have well-defined dielectric and magnetic properties for the high-frequency case in contrast to the "antenna" picture.

The reason for this "effective-medium" behavior is that a very thin conducting stick interacts with an external field like a dipole. Therefore the conducting stick composite can be considered as a system of dipoles regardless of the wavelength and be described in terms of effective parameters. Namely, we can still use the effective dielectric constant ϵ_e or effective conductivity $\sigma_e = -i\omega\epsilon_e/4\pi$ to describe the interaction of the composites with an external electromagnetic wave. The formation of large conducting clusters near the percolation threshold may initiate some scattering only.

Since conducting stick composites are supposed to have effective parameters for all concentrations p out of the percolation threshold, we can use the theory developed in Sec. II to calculate the effective conductivity σ_e. Certainly, this theory has to be modified to take into account nonquasistatic effects. The problem of the effective parameters of composites had been considered for the nonquasistatic case in Refs. 36, 46, 47. It has been shown there how the mean-field approach can be extended to find the composite dielectric constant and magnetic permeability at high frequency. We can summarize the results of Refs. 36, 46, 47 in the following way: One has to solve Maxwell's equations to find the polarizability for a particle in the composite illuminated by an electromagnetic wave. The particle is supposed to be embedded in the "effective medium" with conductivity σ_e. Then the effective conductivity σ_e is determined by the condition that the averaged polarizability of all particles shall vanish. The polarizability of the dielectric regions that are assumed to have spherical shape is known (see, e.g., Refs. 6, 30). The problem is reduced to the calculation of the polarizability of an elongated conducting inclusion for the nonquasistatic case.

The diffraction of electromagnetic waves on a conducting stick is a classical problem of classical electrodynamics. A rather complex theory of this process is presented in Refs. 48, 49. But until now it has not been realized that the problem can be solved analytically in the case of very elongated sticks when the aspect ratio a/b is so large that $\ln(a/b) \gg 1$.

Consider a conducting stick of length $2a$ and radius b illuminated by an electromagnetic wave. For simplicity we will suppose that the electric field in the wave is directed along the stick. The stick is supposed to be embedded in a medium with effective conductivity σ_e. The external electric field will excite the electric current $I(z)$ in the stick, where z is the coordinate along the stick, measured from its midpoint. The dependence $I(z)$ will be nontrivial when the wavelength λ is of the order or smaller than the stick length. It will be also a nontrivial charge distribution $q(z)$ along the stick in this case. The charge distribution $q(z)$ determines the polarizability of the stick. To find $I(z)$ and $q(z)$, it is convenient to introduce the potential $U(z)$ of the charges $q(z)$ distributed over the stick surface. From the equation for the electric induction (see, e.g., Ref. 30), $\nabla \cdot \mathbf{D} = 4\pi p_{\text{ext}}$, we obtain the equation

$$-\frac{\epsilon_m}{\sigma_m}\frac{dI(z)}{dz} = 4\pi q(z), \tag{21}$$

which connects the values of charge $q(z)$ and current $I(z)$ in the stick. The stick charge per unit length $q(z)$ is connected to the potential $U(z)$ via the specific capacitance C given by Eq. (A4): $q(z) = CU(z)$. Substituting this relation into Eq. (21), we have the equation

$$-\frac{dI(z)}{dz} = 4\pi\frac{\sigma_m}{\epsilon_m}CU = 4\pi\frac{\sigma_d}{2\ln(1+a\sigma_d/b\sigma_e)}U, \tag{22}$$

which connects the current $I(z)$ and the surface potential $U(z)$. Otherwise, the electric current $I(z)$ and potential $U(z)$ are connected by the usual Ohm's law

$$-\frac{dU(z)}{dz} = \left(R - i\frac{\omega L}{c^2}\right)I(z) - E_0, \tag{23}$$

where E_0 is an amplitude of the external field and R and L are the impedance and inductance per unit length of the stick, respectively. To obtain a closed equation for the current $I(z)$, we differentiate Eq. (22) with respect to z and substitute the result into Eq. (23) for $dU(z)/dz$. Thus we have at the second-order differentiate equation for $I(z)$

$$\frac{d^2I(z)}{dz^2} = \frac{2\pi\sigma_d}{\ln(1+a\sigma_d/b\sigma_e)}\left[\left(R - i\frac{\omega L}{c^2}\right)I(z) - E_0\right], \tag{24}$$

with boundary conditions corresponding to vanishing the current at the ends of the stick,

$$I(-a) = 0, \quad I(a) = 0. \tag{25}$$

Solution of this equation gives the current distribution $I(z)$ in a conducting stick that is illuminated by an electromagnetic wave. Then we can calculate the charge distribution and, therefore, the polarizability of the stick.

To determine the impedance R and inductance L in Eq. (25), we take a conducting stick in the form of a prolate spheroid with semiaxes $a \gg b$. Since the section area of a spheroid at coordinate z is equal to $\pi b^2[1-(z/a)^2]$, we have the following expression for the impedance:

$$R = \frac{1}{\pi b^2[1-(z/a)^2]\sigma_m^*}, \tag{26}$$

where σ_m^* is the stick conductivity that is renormalized to take into account a skin effect in the conducting sticks. We assume that the conductivity σ_m^* changes due to the skin effect in the same way as the conductivity of a long wire (see, e.g., Ref. 30, Sec. 61) of radius b,

$$\sigma_m^* = \sigma_m f(\Delta), \quad f(\Delta) = \frac{(1-i)}{\Delta}\frac{J_1[(1+i)\Delta]}{J_0[(1+i)\Delta]}, \tag{27}$$

where J_0 and J_1 are the Bessel functions of zero and first order, respectively, and the parameter Δ is equal to the ratio of the stick radius b to the skin depth $\sigma = c/\sqrt{2\pi\sigma_m\omega}$, i.e.,

$$\Delta = b\sqrt{2\pi\sigma_m\omega}/c. \tag{28}$$

When the skin effect is weak, i.e., $\Delta \ll 1$, the function $f(\Delta) \cong 1$ and renormalized conductivity σ_m^* is equal to the stick conductivity $\sigma_m^* \cong \sigma_m$. In the opposite case of a strong skin effect ($\Delta \gg 1$), the current I flows in the thin skin layer at the surface of the stick. Then Eq. (30) gives $f(\Delta) = (1-i)/\Delta$, $\sigma_m^* = (1-i)\sigma_m/\Delta \ll \sigma_m$.

The inductance L per unit length of a stick is calculated in the Appendix [see Eq. (A9)] and given by the equation

$$L \cong 2\ln\left(\frac{a}{b}\right) + 2i\sqrt{\epsilon_d}ka, \tag{29}$$

where $k=\omega/c=2\pi/\lambda$ is a wave vector of the external field. The first term in Eq. (29) is the usual inductance of a long wire (see, e.g., Ref. 30, Sec. 34). The second term emerges for the nonquasistatic case. For the very thin sticks considered in this section, $\ln(a/b) \gg 1$ and the second term in Eq. (28) is much smaller than first one. Nevertheless, it appears to be very important when a stick is at resonance with an electromagnetic field.

For further consideration it is convenient to rewrite Eqs. (24) and (25) in terms of the dimensionless coordinate $z_1=z/a$ and dimensionless current $I_1=I/(\sigma_m f(\Delta)\pi b^2 E_0)$ and introduce the dimensionless relaxation parameter

$$i\gamma = f(\Delta)\,\frac{\sigma_m}{\sigma_d}\,g_\parallel = f(\Delta)\,\frac{\sigma_m}{\sigma_d}\left(\frac{b}{a}\right)^2 \ln\left(1+\frac{a\epsilon_d}{b\epsilon_e}\right) \tag{30}$$

and dimensionless stick frequency Ω,

$$\Omega^2 = (ak)^2 LC = \epsilon_d (ak)^2\,\frac{\ln(a/b)+i\sqrt{\epsilon_d}ka}{\ln(1+a\epsilon_d/b\epsilon_e)}, \tag{31}$$

where $k=\omega/c$. Substituting the thus determined parameters into Eqs. (24) and (25), we obtain the following equation for the dimensionless current $I_1(z_1)$:

$$\frac{d^2 I_1(z_1)}{dz_1^2} = \left[-i\,\frac{2}{\gamma(1-z_1^2)} - \Omega^2\right] I_1(z_1) + \frac{2i}{\gamma},$$

$$I_1(-1)=0, \quad I_1(1)=0. \tag{32}$$

In order to understand the physical meaning of Eq. (32), let us consider two limiting cases. When the skin depth is weak, the combination $\Omega^2\gamma \propto \Delta^2 \ll 1$. Therefore we can neglect the second term in square brackets in Eq. (32) and find the current

$$I_1(z_1) = \frac{(1-z_1^2)}{1+i\gamma}. \tag{33}$$

The electric field $E_{\text{in } \mathbf{m}\parallel}$ inside a conducting stick is uniform when the skin effect is negligible and it equals $E_{\text{in } \mathbf{m}\parallel} = E_0 - dU/dz$. From Eqs. (32) and (33), we obtain the following expression for the internal field:

$$E_{\text{in } \mathbf{m}\parallel} = E_0\,\frac{1}{1+(\sigma_m/\sigma_d)(b^2/a^2)\ln(1+a\sigma_d/b\sigma_e)}$$

$$= E_0\,\frac{1}{1+g_\parallel \sigma_m/\sigma_e}, \tag{34}$$

where $g_\parallel$ is the depolarization factor given by Eq. (8). As one can expect, the field $E_{\text{in } \mathbf{m}\parallel}$ given by Eq. (34) coincides with the quasistatic internal field in a prolate conducting spheroid $\mathbf{E}_{\text{in } \mathbf{m}\parallel}$ [see Eq. (2a)] for the case $\sigma_m \gg \sigma_e$ considered here.

In the opposite case of a strong skin effect the product $\Omega^2\gamma \propto \Delta \gg 1$. Therefore we neglect the first term in the square brackets in Eq. (32), obtaining

$$I_1(z_1) = \frac{2i}{\Omega^2\gamma}\left(\frac{\cos(\Omega z_1)}{\cos(\Omega)} - 1\right). \tag{35}$$

In the diluting case $p \ll (b/a)^2$, the effective dielectric constant $\epsilon_e \cong \epsilon_d$ and the stick frequency is equal to $\Omega = ka\sqrt{\epsilon_d} = 2\pi a\sqrt{\epsilon_d}/\lambda$. Therefore the current will have resonances when the wavelength $\lambda_n = 4a\sqrt{\epsilon_d}/(2n-1)$, $n=1,2,\ldots$. It is the well-known antenna resonances.[48,49]

In the general case of arbitrary Ω and γ, the solution of Eq. (32) cannot be expressed as a finite set of any known special functions.[50] Therefore we integrate this equation numerically. Experience with numerical integration of Eq. (32) has shown that for not very short wavelengths $(\lambda > \lambda_2)$ the solution can be approximated by a simple formula

$$I_1(z_1) = \frac{1-z_1^2}{1+i\gamma\cos\Omega}. \tag{36}$$

This equation can be considered as some interpolation from Eqs. (33)–(35).

When the current I_1 is known, we can calculate the specific polarizability P_m of a conducting stick,

$$P_m = 4\pi\,\frac{\mathbf{D}}{VE_0} = \frac{1}{E_0}\,\frac{3}{ab^2}\int_{-a}^{a} z\,\frac{q(z)}{\epsilon_e}\,dz = \frac{3}{2}\,f(\Delta)\,\frac{\sigma_m}{\sigma_e}\int_0^1 I_1(z_1)\,dz_1, \tag{37}$$

where $\mathbf{D}$ and $V=4\pi ab^2/3$ are the dipole moment and volume of the stick, respectively, and the function $f(\Delta)$ is given by Eq. (30). Substituting Eq. (36) for the current in Eq. (37), we obtain the equation

$$P_m = f(\Delta)\,\frac{\sigma_m}{\sigma_e}\,\frac{1}{1+i\gamma\cos\Omega} = f(\Delta)\,\frac{\sigma_m}{\sigma_e}\,\frac{1}{1+f(\Delta)(\sigma_m/\sigma_e)(b^2/a^2)\ln(1+a\epsilon_d/b\epsilon_e)\cos\Omega} \tag{38}$$

for the polarizability P_m of a conducting stick that is illuminated by an electromagnetic wave. The stick is supposed to be parallel to the direction of the electric field in the wave.

Up to now we considered sticks that are aligned with an electric field in the incident electromagnetic wave. The composites we are interested in contain randomly oriented conducting sticks. In this case we have to improve Eq. (38) for the stick polarizability. Consider a conducting stick that is aligned with some unit vector $\mathbf{n}$. Suppose that the stick is illuminated by an electromagnetic wave with electric field

$$\mathbf{E} = \mathbf{E}_0 \exp[i(\mathbf{k}_e \cdot \mathbf{r})], \tag{39}$$

where $\mathbf{k}_e$ is the wave vector of the wave inside the composite. The current I in a very elongated stick is excited by the component of the electric field

$$\mathbf{E}_\parallel(z)=\mathbf{n}(\mathbf{n}\cdot\mathbf{E}_0)\exp[i(\mathbf{k}_e\cdot\mathbf{n})z], \tag{40}$$

which is parallel to the stick, where z is a coordinate along the stick. The field $\mathbf{E}_\parallel$ averaged over all stick orientations is aligned with the external field $\mathbf{E}_0$. Its amplitude is equal to

$$E_0^*(z)=\frac{E_0}{(k_ez)^2}\left(\frac{\sin(k_ez)}{k_ez}-\cos(k_ez)\right). \tag{41}$$

The current flows in a stick are linear functions of the field $\mathbf{E}_\parallel$. Since the averaged field $\mathbf{E}_\parallel$ is aligned with $\mathbf{E}_0$ the current averaged over the stick orientations is also aligned with the external field $\mathbf{E}_0$. To obtain the current $\langle\langle I_1(z_1)\rangle\rangle$ averaged over the stick orientations and, therefore, the averaged stick polarizability $\langle\langle P_m\rangle\rangle$, one has to substitute the field $E_0^*(z)$ given by Eq. (41) into Eq. (24) instead of the field E_0. Then the current $\langle\langle I_1(z_1)\rangle\rangle$, polarizability $\langle\langle P_m\rangle\rangle$, and, therefore, effective dielectric constant will depend on the frequency ω but, what is more, on the value of the wave vector k_e. This means that a conducting stick composite is a medium with some spatial dispersion. This result is easy to understand when we recall that the characteristic scale of an inhomogeneity in the composites is the stick length $2a$. Moreover, one can use the bulk value of the dielectric constant ϵ_e for the scales larger than a only, as discussed in Sec. II. So it is not surprising that the interaction of an electromagnetic wave with the composites has a nonlocal character and, therefore, spatial dispersion takes place. On can expect that additional waves will be excited in the composites in the presence of a strong spatial dispersion.[51,52] These questions call for further investigation.

In this paper we consider the wavelength $\lambda>\lambda_2$; therefore, we expand $E_0^*(z)$ in a series

$$E_0^*(z)=\frac{E_0}{3}\left(1-\frac{(k_ez)^2}{10}+\cdots\right) \tag{42}$$

and restrict ourselves to the first term. Since the averaged field is equal to $E_0^*(z)=E_0/3$, the averaged current is equal to $\langle\langle I_1(z_1)\rangle\rangle=I_1(z_1)/3$ where the stick current $I_1(z_1)$ is still given by Eq. (36). As a result, the stick polarizability $\langle\langle P_m\rangle\rangle$ averaged over all orientations is equal to $P_m/3$ where P_m is given by Eq. (38).

Inasmuch as the sticks are randomly oriented, the dielectric regions of the composites are supposed to have a spherical shape [see discussion after Eq. (5)]. The specific polarizability of a dielectric region is given by the usual quasistatic equation (see, e.g., Ref. 6)

$$P_d=\frac{3(\sigma_d-\sigma_e)}{2\sigma_e+\sigma_d}. \tag{43}$$

The effective dielectric constant of the composites σ_e is determined by the self-consistent condition that the polarizability averaged over all inclusions shall vanish,[36,46,47]

$$p\langle\langle P_m\rangle\rangle+(1-p)P_d=\frac{p}{3}\,\frac{f(\Delta)\sigma_m/\sigma_e}{1+f(\Delta)(\sigma_m/\sigma_e)(b^2/a^2)\ln(1+a\epsilon_d/b\epsilon_e)\cos\Omega}+3(1-p)\,\frac{\sigma_d-\sigma_e}{2\sigma_e+\sigma_d}=0. \tag{44}$$

This equation differs from the EMTSC equation (10) obtained for the quasistatic case in that (a) the conductivity of the sticks σ_m is replaced by the renormalized conductivity $\sigma_m^*=f(\Delta)\sigma_m$ given by Eq. (27), and (b) the "resonance" factor cos Ω appears in the denominator of the first term.

Let us consider a solution of Eq. (44) for the conducting stick concentration p below the percolation threshold $(b/a)^2<p<b/a$. The absolute values of the effective conductivity σ_e are large as compared with $|\sigma_d|$ for such concentrations. Then, neglecting σ_d in the second term of Eq. (44), we have

$$\frac{\sigma_e}{\sigma_d}\equiv\frac{\epsilon_e}{\epsilon_d}=\frac{2}{9}\,p\,\frac{f(\Delta)\sigma_m/\sigma_d}{1+f(\Delta)(\sigma_m/\sigma_d)(b/a)^2\,\ln[1+(a/b)\epsilon_d/\epsilon_e]\cos\Omega}. \tag{45}$$

This equation is similar to Eq. (18) obtained in Sec. II for the quasistatic case. The absolute values of the effective dielectric constant ϵ_e calculated in the quasistatic approximation (see Figs. 2, 3) are less than $(a/b)\epsilon_d$ for the concentration $p<b/a$. Let us suppose that the same inequality takes place in the nonquasistatic case considered here. For a qualitative analysis of Eq. (45), we therefore neglect $\ln(\epsilon_d/\epsilon_e)$ in comparison with $\ln(a/b)$ in Eqs. (45) and (31). After these simplifications σ_e disappears from right-hand side of Eq. (45) and this equation becomes an explicit expression for the effective conductivity or dielectric constant,

$$\epsilon_e=\epsilon_d\,\frac{2}{9}\,p\,\frac{a^2}{b^2\,\ln(a/b)}\,\frac{1}{\cos\Omega+\sigma_d/f(\Delta)\sigma_m}, \tag{46}$$

where the function $f(\Delta)$ is given by Eq. (27). Substituting into Eq. (46) the definition of the stick frequency Ω given by Eq. (31) and $\sigma_d=-i\epsilon_d\omega/4\pi$, we have

$$\epsilon_e=\epsilon_d\,\frac{2}{9}\,p\,\frac{a^2}{b^2\,\ln(a/b)}\,\frac{1}{\cos((\sqrt{\epsilon_d}ak)\sqrt{1+i\sqrt{\epsilon_d}ak/\ln(a/b)})-i[\epsilon_d/\Delta^2f(\Delta)](ak)^2/2\,\ln(a/b)}, \tag{47}$$

where $k=\omega/c$ and the parameter Δ is given by Eq. (28). When the skin effect is strong, the parameter $\Delta \gg 1$ and function $f(\Delta)\propto 1/\Delta$. Therefore we can neglect the second term in the denominator of Eq. (47). Expanding the first term in the denominator of Eq. (47) in a series of the small parameter $1/\ln(a/b)$, we have

$$\epsilon_e=\epsilon_d \frac{p}{p_c} \frac{a}{b\ \ln(a/b)} \frac{1}{\cos(\sqrt{\epsilon_d}ak)-i\epsilon_d(ak)^2/2\ \ln(a/b)}. \tag{48}$$

From this equation it follows that the effective dielectric constant of the conducting stick composite has resonance behavior when the skin effect is strong. The resonance frequencies ω_n are determined by the condition $\sqrt{\epsilon_d}ak=(\pi/2)(2n-1)$, $n=1,2,...,$ and therefore they are equal to $\omega_n=(\pi/2)(2n-1)c/(a\sqrt{\epsilon_d})$. It is interesting to point out that the thus obtained effective dielectric constant is independent of the metal conductivity σ_m.

Consider the behavior of the effective dielectric constant near the lowest resonance frequency $\omega_1=(\pi/2)c/a\sqrt{\epsilon_d}$. Expanding the denominator of Eq. (48) in a power series of $\omega-\omega_1$, we have

$$\epsilon_e=\epsilon_d \frac{p}{p_c} \frac{a}{b\ \ln(a/b)} \frac{1}{(\omega_1-\omega)/\omega_1-i\pi^2/8\ \ln(a/b)}, \tag{49}$$

where imaginary part of the denominator $\pi^2/(8\ln(a/b))\ll 1$ since we suppose that $\ln(a/b)\gg 1$. This equation gives a well-developed resonance behavior of the effective dielectric constant ϵ_e. At the resonance frequency ω_1, the real part of ϵ_e changes its sign and becomes negative when $\omega>\omega_1$. The imaginary part of ϵ_e has a maximum at the resonance and its value

$$\epsilon_2(\omega_1)=\epsilon_d p \frac{16}{9\pi^2} \frac{a^2}{b^2} \tag{50}$$

does not depend on the conductivity of the sticks for the case of a strong skin effect considered here. Therefore the imaginary part of the effective dielectric constant does not vanish for composites with stick conductivity $\sigma_m\to\infty$. We believe that the presence of the effective losses is due to exciting internal modes in such composites. These modes are bounded around the sticks and cannot exit from the composite. When $\sigma_m\to\infty$ and the dielectric host has no losses, the amplitude of these modes will continuously increase in time. In all real composites there are some losses in the conducting sticks as well as in the dielectric host. Therefore the internal field should stabilize at some large value. In other words, it should be a wave localization in the conducting stick composites when $\sigma_m\to\infty$. The field distribution in the composites cannot be found in terms of the effective-medium theory developed in this paper. The field distribution and the spectrum of the internal modes are questions for further consideration.

In the general case of an arbitrary skin effect, we solve Eq. (44) numerically. The thus obtained effective dielectric constant ϵ_e depends on the stick concentration p, aspect ratio a/b and dimensionless frequency ω^* [see Eq. (19)] as in the quasistatic case. The behavior of ϵ_e also depends on the parameter Δ given by Eq. (28), i.e., on the ratio of the stick radius b to the skin depth δ. Considering Eqs. (44) and (45), we see that the parameters ϵ_d, p, a/b, ω^*, and Δ fully characterize the dielectric response of the conducting stick composites. The parameter Δ can be considered as an electrical "goodness" of a conducting stick. When the parameter $\Delta\gg 1$, the behavior of the effective dielectric constant is similar to that given by Eq. (46). We return to the quasistatic situation when $\Delta\ll 1$. In this case neither the skin effect nor resonance effects are important.

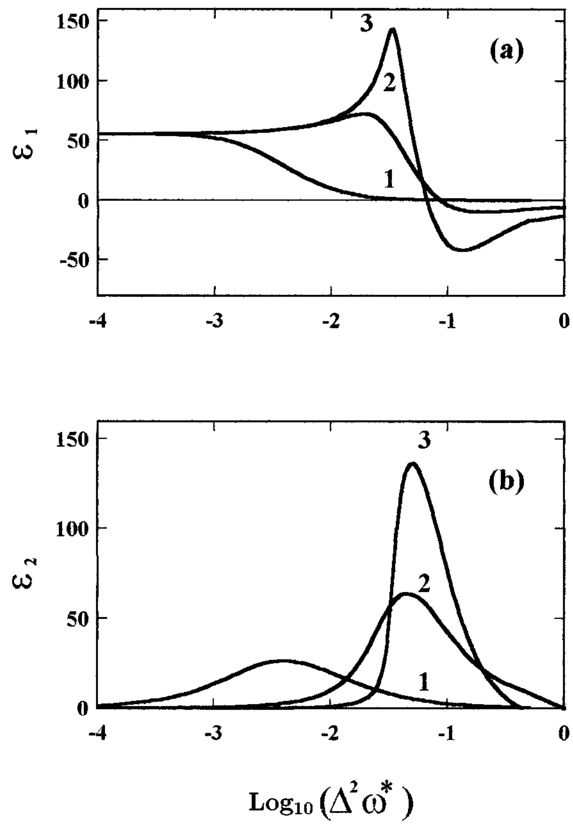

FIG. 4. Dispersion curves of real $\epsilon_1(\omega^*)$ (a) and imaginary $\epsilon_2(\omega^*)$ (b) parts of the effective dielectric constant ϵ_e for different values the parameter $\Delta=b/\delta$, the ratio of stick radius b to skin depth $\delta=c/\sqrt{2\pi\sigma_m\omega}$: $1-\Delta=0.1$, $2-\Delta=0.5$, $3-\Delta=5.0$. The stick concentration $p=0.1p_c$, aspect ratio $a/b=1000$, $\epsilon_d=2$.

In Fig. 4 we show the dependence of the effective dielectric constant ϵ_e on the frequency ω^* for different values of the parameter Δ. The stick concentration p is chosen to be sufficiently small, $p=0.1p_c$. The dispersive behavior is close to the relaxation behavior given by Eq. (26) for the weak skin effect: $\Delta\leq 0.1$. The dispersive behavior dramatically changes when the parameter Δ increases to $\Delta\geq 0.5$. Now the dependence of the effective dielectric constant $\epsilon_e(\omega^*)$ has a resonance character. The real part of $\epsilon_e(\omega^*)$ increases with frequency: it has a sharp maximum at the resonance frequency and then drops to negative values. The imaginary part of $\epsilon_e(\omega^*)$ has a well-developed maximum at the resonance frequency.

In Fig. 5 we present $\epsilon_e(\omega^*)$ for the concentration $p=0.9p_c$ close to the percolation threshold. When the skin effect is weak ($\Delta=0.1$), the real and imaginary parts of the effective dielectric constant decrease according to the power

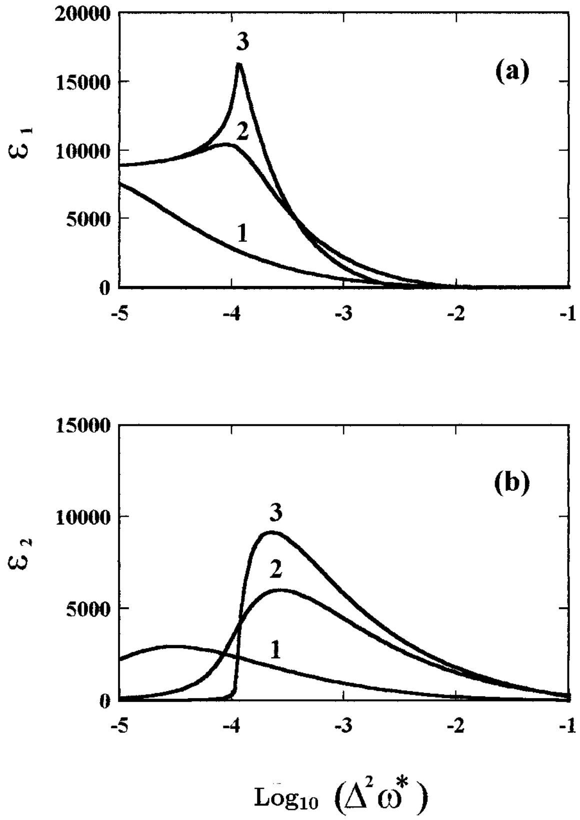

FIG. 5. Dispersion curves of real $\epsilon_1(\omega^*)$ (a) and imaginary $\epsilon_2(\omega^*)$ (b) parts of the effective dielectric constant ϵ_e for different values the parameter $\Delta=b/\delta$, the ratio of stick radius b to skin depth $\delta=c/\sqrt{2\pi\sigma_m\omega}$: $1-\Delta=0.1$, $2-\Delta=0.5$, $3-\Delta=5.0$. The stick concentration $p=0.9p_c$, aspect ratio $a/b=1000$, $\epsilon_d=2$.

law as discussed in Sec. II. This critical behavior continues in a wide range of the dimensionless frequency ω^* turning into the relaxation dispersion for $\omega^*>1$. The dispersive behavior changes when the parameter $\Delta\geqslant 0.5$ in a similar fashion as in the dilute case discussed above. In this paper we do not consider in detail the behavior of the effective dielectric constant ϵ_e in the critical region $p\cong p_c$ for the nonquasistatic case. We point out that the only real part of ϵ_e takes small negative values in a wide frequency range for ω^* larger than the resonance frequency, as shown in Fig. 5. It should be noted that the effective-medium theory developed here cannot pretend to an accurate estimation of the effective parameters in the critical region. Moreover, some "critical opalescence" can take place for the concentration p near the percolation threshold. All these questions deserve further study.

IV. HIGH-FREQUENCY MAGNETIC RESPONSE OF THE CONDUCTING STICK COMPOSITES

Consider the metal-dielectric composite containing conducting sticks dispersed in a dielectric host. The sticks are much elongated, that is, the aspect ratio $a/b\gg 1$. We suppose that the volume concentration p of the conducting sticks is less than the percolation threshold $p<p_c\propto b/a$. We also suppose that neither sticks nor dielectric host has magnetic properties. At first glance the composite has no magnetic proper-

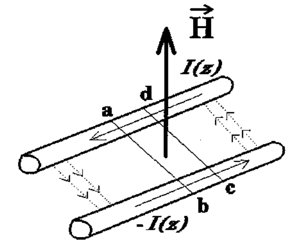

FIG. 6. Currents in the two-stick contour excited by an external magnetic field **H**. The displacement currents that short the currents in the conducting sticks are shown by dashed lines.

ties under such conditions. Indeed, the magnetic response of a single conducting stick is small even at high frequency (Ref. 30, Sec. 59). Since the concentration $p\ll 1$, the response of the entire composite should be negligible small.

In reality, as we will see below, the composites may have a giant paramagnetic response at some frequencies. The reason for such behavior of the conducting stick composites is the collective response of the sticks to the high-frequency magnetic field. The sticks form various closed circuits in the composite. The external magnetic field excites electric currents in these stick contours. Magnetic moments of the currents flowing in the stick contours are in charge for the magnetic response of the composite.

In this work we restrict ourselves to the simplest stick circuit consisting of two sticks only. Suppose at the beginning that the sticks are parallel to each other as shown in Fig. 6. We also suppose that an external magnetic field $\mathbf{H}=\mathbf{H}_0\exp(-i\omega t)$ is applied perpendicular to the plane of the circuit. This field will excite a circular current I in the system of two parallel sticks. The circular current I flows in one stick in one direction and in another stick in the opposite direction as shown in Fig. 6. The displacement currents flowing between two sticks short the circuit. The considered two-stick circuit is nothing but the well-known two-wire transmission line excited by the external magnetic field. The current I in the two-wire line can be calculated from Telegrapher's equation (see, e.g., Refs. 49, 53)

Electrodynamics processes in line of two wires separated by the distance d are determined by the impedance per unit length,

$$Z=\frac{1}{f(\Delta)\sigma_m\pi b^2}-\frac{i\omega}{c^2}L_2, \tag{51}$$

where σ_m and b are the stick conductivity and radius, respectively; the function $f(\Delta)$ given by Eqs. (27) and (28) emerges from the skin effect, and L_2 is the self-inductance per unit length of a system of two parallel straight wires having cross section of radii b (Ref. 30, Sec. 33),

$$L_2=4\mu_e\ln\left(\frac{d}{b}\right), \tag{52}$$

where d is the distance between axes of the wires, and μ_e is the effective magnetic permeability of the composite. The value of μ_e will be determined self-consistently. Another important parameter is the mutual capacity per unit length of two wires (Ref. 30, Sec. 3),

$$C_2 = \frac{\epsilon_d}{4\ln(d/b)}. \tag{53}$$

The capacitance C_2 determines the value of the displacement currents flowing in between two wires. Since we consider two nearest sticks, we put ϵ_d in Eq. (53) for the interstick capacitance. This approximation is good when the distance d between the sticks is smaller than their length a [see Eq. (7)].

Following the procedure described in Sec. III, we introduce the current I as a current in one stick. This current depends on the coordinate z along the stick. We also introduce the potential difference $U(z)$ between two sticks. Then we get a first equation to determine $I(z)$ and $U(z)$ from the first Maxwell's equation written in integral form,

$$\oint_{(a,b,c,d)} \mathbf{E}\, d\mathbf{l} = \frac{i\omega}{c} \oiint_S H\, dS, \tag{54}$$

where $S = d\times dz$ is the area restricted by the contour (a,b,c,d) as shown in Fig. 6. From Eq. (54) it follows that

$$-\frac{dU(z)}{dz} = ZI(z) + \frac{id\omega}{c} H_0. \tag{55}$$

The current $I(z)$ depends on the coordinate z since it can go out from one stick and come into another stick. The second equation for $I(z)$ and $U(z)$ we obtain from the charge conservation law considering the currents in the sticks and displacement current between them,

$$\frac{dI(z)}{dz} = i\omega C_2 U(z). \tag{56}$$

Combination of Eqs. (55) and (56) gives the second-order differential equation for the current,

$$\frac{d^2 I(z)}{dz^2} = -g^2 I(z) + \frac{C_2 d\omega^2}{c} H_0,$$

$$-a < z < a,$$

$$I(-a) = I(a) = 0, \tag{57}$$

where the parameter g equals

$$g = k\sqrt{\epsilon_d \mu_e + i\frac{\epsilon_d}{2\Delta^2 f(\Delta)\ln(d/b)}}, \tag{58}$$

where $k = \omega/c$ is the wave vector of the external field: the parameter Δ and function $f(\Delta)$ are given by Eqs. (28) and (27), respectively.

We solve Eq. (57) for the current $I(z)$ and calculate the magnetic moment $\mathbf{m}$ of the circuit of two sticks,

$$\mathbf{m} = \frac{1}{2c}\int [\mathbf{r}\times \mathbf{j}(\mathbf{r})]d\mathbf{r}, \tag{59}$$

where $\mathbf{j}(\mathbf{r})$ is the density of the current in two conducting sticks or density of the displacement currents. Integration in Eq. (59) goes over two conducting sticks as well as over the space between them where the displacement currents are flowing. From Eqs. (57)–(59) we obtain the magnetic moment the system of two sticks,

$$m = 2H_0 a^3 C_2 (kd)^2 \frac{\tan(ga) - ga}{(ga)^3}. \tag{60}$$

Note that the thus obtained moment takes on large values when the wavelength $\lambda = 2\pi/k$ is of the order of the stick length $2a$ and the skin effect is strong. Indeed, for the case of a strong skin effect when the ratio of the stick radius b to the skin depth δ is large, $\Delta = b/\delta \gg 1$, we can neglect the imaginary part of the parameter g in Eq. (58). Then we can estimate $ga \approx ak \approx 1$, $kd \approx d/a$, and have $m \propto H_0 a d^2$. One can compare the thus estimated moment m with the moment of a single stick, $m_1 \propto H_0 a b^2$. Since the concentrations $p \ll 1$, the typical distance d between two sticks is much larger than the stick radius b. Therefore the "collective" moment m is much larger than the moment m_1 of a single stick, $m/m_1 \propto (d/b)^2 \gg 1$. For this reason the conducting stick composite may have a large magnetic response even for very small concentrations of the sticks.

Let us now estimate quantitatively the effective magnetic permeability of the conducting stick composites. We are interested in the effective properties of the composites where conducting sticks are randomly oriented. Consider a conducting stick directed along some vector $\mathbf{n}_1$ and take its center as the center of coordinates. Suppose that the nearest-neighboring stick is aligned with the vector $\mathbf{n}_2$ and has its center at the coordinate $\mathbf{r}_0$. As a first approximation, we assume that the moment of a such system is still given by Eq. (60), but we substitute in this equation the averaged distance d_{12} between the stick,

$$d_{12}^2 = \frac{1}{2a}\int_{-a}^{a} [\mathbf{n}_1 z - (\mathbf{r}_0 + \mathbf{n}_z) z]^2 dz. \tag{61}$$

Integrating Eq. (61) and averaging the result over the direction of vector $\mathbf{n}_2$ and over the direction and length of vector $\mathbf{r}_0$, we have the averaged distance $\langle d^2\rangle$ between any two nearest-neighbor sticks,

$$\langle d^2\rangle = \tfrac{8}{3}a^2 + \langle r_0^2\rangle. \tag{62}$$

The averaged square distance between centers of two nearest-neighbor sticks can be estimated as $\langle r_0^2\rangle \approx n^{-2/3} \approx a^{2/3} b^{4/3}/p^{2/3}$, where n is the number of the sticks in a unit volume. Since the averaged distance $\langle r_0^2\rangle$ is much smaller than a^2, for the concentrations $p > (b/a)^2$ we neglect $\langle r_0^2\rangle$ in comparison with $\langle a_0^2\rangle$ and set $\langle d^2\rangle = 8a^2/3$. Substituting the value of the averaged distance, i.e.,

$$\langle d\rangle = \sqrt{\langle d^2\rangle} = \sqrt{(8/3)}a, \tag{63}$$

in Eq. (60), we have the magnetic moment m of two randomly oriented sticks. The thus obtained moment can have an arbitrary orientation. The component of the magnetic moment that is parallel to $\mathbf{H}_0$ makes a contribution to the total moment $\mathbf{M}$ of the composite only. Averaging over the direction of the stick moment gives the factor 1/3. Then we obtain from Eq. (60) the magnetic moment per unit volume of the conducting stick composite,

$$\mathbf{M} = \frac{1}{3}\mathbf{H}_0 n a^3 C_2 \frac{8}{3}(ka)^2 \frac{\tan(ga) - ga}{(ga)^3}, \tag{64}$$

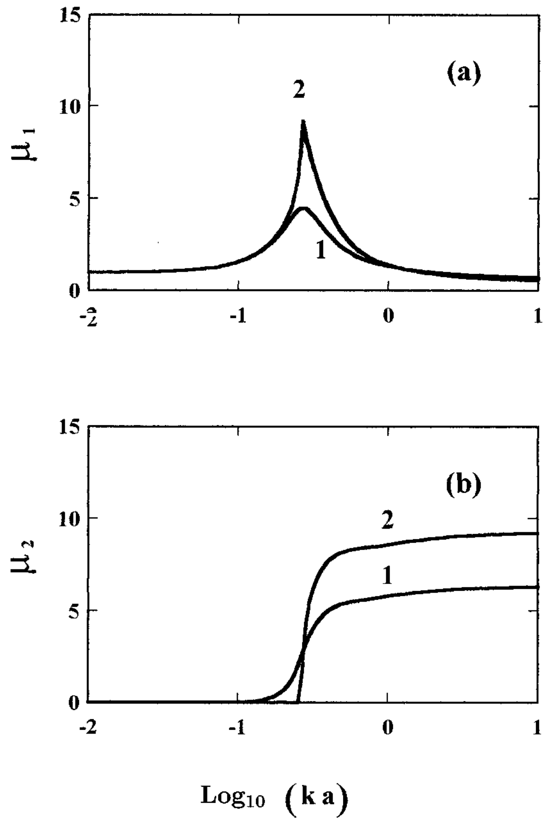

FIG. 7. Dispersion curves of real $\mu_1(ka)$ (a) and imaginary $\mu_2(ka)$ (b) parts of the effective magnetic permeability μ_e for different values the parameter $\Delta = b/\delta$, the ratio of stick radius b to skin depth $\delta = c/\sqrt{2\pi\sigma_m\omega}$: $1-\Delta=0.1$, $2-\Delta=5.0$. The stick concentration $p=0.1p_c$, aspect ratio $a/b=1000$, $\epsilon_d=2$.

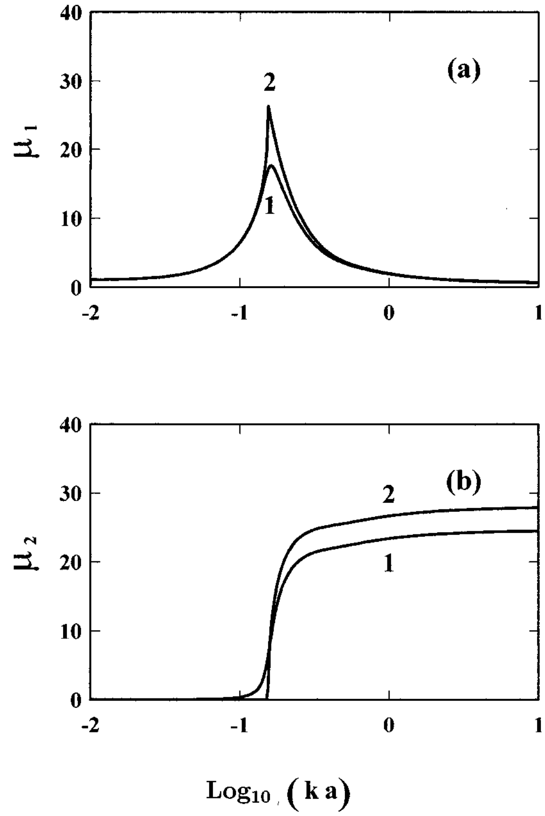

FIG. 8. Dispersion curves of real $\mu_1(ka)$ (a) and imaginary $\mu_2(ka)$ (b) parts of the effective magnetic permeability μ_e for different values the parameter $\Delta = b/\delta$, the ratio of stick radius b to skin depth $\delta = c/\sqrt{2\pi\sigma_m\omega}$: $1-\Delta=0.1$, $2-\Delta=5.0$. The stick concentration $p=0.9p_c$, aspect ratio $a/b=1000$, $\epsilon_d=2$.

where $n=p/(4/3\pi a b^2)$ is the density of the sticks, $C_2=\epsilon_d/[4\ln(\langle d\rangle/b)]\cong\epsilon_d[4\ln(a/b)]$ and the parameter g is given by Eq. (58) where we substitute the average distance $\langle d\rangle$, i.e.,

$$g=k\sqrt{\epsilon_d\mu_e+i\,\frac{\epsilon_d}{2\Delta^2 f(\Delta)\ln(\langle d\rangle/b)}}\cong k\sqrt{\epsilon_d\mu_e+i\,\frac{\epsilon_d}{2\Delta^2 f(\Delta)\ln(a/b)}}. \tag{65}$$

Taking into account the definition of the effective magnetic permeability $\mu_e\mathbf{H}_0=\mathbf{H}_0+4\pi\mathbf{M}$, we obtain from Eq. (64) the following equation for the effective magnetic permeability:

$$\mu_e=1+\frac{2}{3}\,p\,\frac{a^2}{b^2}\,\frac{\epsilon_d(ka)^2}{\ln(a/b)}\,\frac{\tan(ga)-ga}{(ga)^3}, \tag{66}$$

where the parameter g is given by Eq. (65). The thus obtained effective magnetic permeability μ_e of the conducting stick composites is shown in Figs. 7 and 8 for the stick concentrations $p=0.1p_c$ and $p=0.9p_c$, respectively. The real part of the effective permeability $\mu_1(\omega)$ as a function of the frequency has a positive maximum that increases in magnitude and shifts to lower frequencies when the concentration increases. The permeability achieves the value $\mu_1\approx 10.0$ at the resonance even for a sufficiently small concentration $p=0.1p_c$. We name this phenomenon the "giant paramagnetic response of the conducting stick composites." The imaginary part of the effective permeability $\mu_2(\omega)$ is a step function of the frequency. It almost equals zero for small frequencies: $\mu_2(\omega)$ builds up at the frequency corresponding to the maximum of $\mu_1(\omega)$, and then $\mu_2(\omega)$ decays very smoothly. It is interesting to point out that this behavior of $\mu_2(\omega)$ to some extent is opposite to that of the imaginary part of the effective dielectric constant (see Figs. 4 and 5).

The magnetic permeability of the percolating composites has attracted the attention of many researchers.[39,54–57] In all these works the authors have considered the magnetic response of percolating clusters made up from conducting inclusions that are in Ohmic contact with each other. As a result, they have obtained the diamagnetic response for the percolating composites. In the case of the conducting stick composites, the contours that are excited by an external magnetic field include C and L elements. The currents excited in the contours are shifted in phase relative to the field. As a result, the composites have a *paramagnetic* response.

In this work we consider stick circuits consisting of two sticks only. Therefore the thus obtained Eq. (66) for the effective magnetic permeability μ_e of the composites may be not quantitative true in the critical region. In this region one has to consider the circuits forming by three, four, etc., sticks. The application of EMT is also in question for the vicinity of the percolation threshold. Therefore the critical behavior of the magnetic permeability is open for further consideration.

V. SUMMARY AND CONCLUSIONS

We present a comprehensive study of electrodynamics properties of the metal-dielectric composites containing elongated conducting inclusions, sticks dispersed in a dielectric host. The distinguishing feature of these composites is a scale-dependent conductivity and dielectric constant. We develop an effective-medium theory that takes into account this scale dependence. Thus we get an equation to determine the effective parameters of the composites. The effective dielectric constant determined by this means is quite different from that of the ordinary metal-dielectric composites with spherical conducting particles. The maximum of the dielectric constant may be shifted far away from the percolation threshold. The dispersive behavior of the conducting stick composites is also different from that of the ordinary composites. At low frequencies it is close to the relaxation behavior in a wide concentration range below and also above the percolation threshold. A power-law critical dispersive behavior is still predicted in the vicinity of the percolation threshold. But in contrast to the usual composites, the scaling function for the effective conductivity incorporates a large parameter, the stick aspect ratio $a/b \gg 1$. As a result, the concentration range when the critical, power-law dispersion takes place increases greatly, while the frequency range for the critical dispersion drastically shifts to lower frequencies.

Conducting stick composites have new and unusual properties at high frequency when the skin effect in a stick is significant. For example, the effective dielectric constant has a resonance at some frequencies. Its real part vanishes at the resonance and acquires negative values for a frequency larger than the resonance. The imaginary part of the effective dielectric constant has a maximum at the resonance. The dispersive behavior does not depend on the stick conductivity and takes some universal form when the stick conductivity tends to infinity. Wave localization may occur in the system in his case.

We propose that conducting stick composites consisting of nonmagnetic particles have a large magnetic response at high frequency. The effective magnetic properties are due to the collective interaction of the sticks with an external magnetic field. A giant paramagnetic response can take place in some frequency range.

One can conclude that conducting stick composites is a new class of percolating systems. In spite of the study performed in this work, many questions concerning the electrodynamics properties of the composites are still unclear. The critical behavior at high frequency is the most intriguing problem. From the results of this work, it follows that spatial dispersion may take place in such composites. The new, internal modes can be excited around the conducting sticks. Then the electrical and magnetic field distributions in the stick composites may be quite different from that of the ordinary composites. All these questions are open for further consideration.

ACKNOWLEDGMENTS

We would like to acknowledge very useful conversations with A. S. Antonov, K. N. Rozanov, and A. P. Vinogradov. This research was partially supported by Commissariat a l'energie Atomique of France under Contract No. 65243 and by the Russian Fund of Fundamental Researches under Grant No. 95-02-06443A. Part of this work was done while one of the authors (A.K.S.) was at Liege University.

APPENDIX

In Sec. III we have obtained the equation for the charge and current in a conducting stick using an intuitive approach. In this appendix we rederive Eq. (24) to make all assumptions clear. An explicit expression for the quasistatic field in the stick is obtained as a by-product.

Let us consider a conducting stick in the composite, a prolate conducting spheroid with semiaxes $a \gg b$ that is illuminated by an electromagnetic wave with frequency ω. The direction of the major axis is supposed to coincide with direction of the electric field $\mathbf{E}_0 \exp(-i\omega t)$ in the wave. Let $q(z)$ be the charge per unit length induced on the surface of the stick and z the coordinate along the major axis of the stick, measured from its midpoint. The electric potential of the charge $q(z)$ is given by the solution of Maxwell's equations (see, e.g., Ref. 49, p. 377)

$$U(z) = \oiint \frac{q(z')/2\pi\rho(z')\exp(ik_e|\mathbf{r}-\mathbf{r}'|)}{\epsilon(|\mathbf{r}-\mathbf{r}'|)|\mathbf{r}-\mathbf{r}'|}\,ds' \cong \int_{\substack{|z'|<a\\|z-z'|<b}} \frac{q(z')\exp(ik_e|z-z'|)}{\epsilon(|z-z'|)|z-z'|}\,dz', \tag{A1}$$

where $\mathbf{r}$ is a point on the surface at coordinate z, $\epsilon(|z-z'|)$ is the scale-dependent dielectric constant of the composites given by Eq. (7), $\rho(z) = b\sqrt{1-z^2/a^2}$ is the radius of the stick cross section at coordinate z, $k_e = \sqrt{\epsilon(|z-z'|)}k$, and $k = \omega/c$ is the wave vector of the external field. Integration in the first integral goes over the total surface of the stick: in going to the second expression, we neglect terms of the order of $b/a \ll 1$. The denominator of the integrals in Eq. (A1) have some singularity at $z = z'$; therefore, we put $k_e = k_e|_{z=z'} = \sqrt{\epsilon_d}k$ in the exponent. We divide the last integral in Eq. (1) into two parts putting $q(z')\exp(i\sqrt{\epsilon_d}k|z-z'|) \equiv q(z) + [q(z')\exp(i\sqrt{\epsilon_d}k|z-z'|) - q(z)]$, i.e.,

$$U(z) = q(z)\int_{\substack{|z'|<a\\|z-z'|<b}} \frac{1}{\epsilon(|z-z'|)|z-z'|}\,dz' + \int_{\substack{|z'|<a\\|z-z'|<b}} \frac{q(z')\exp(i\sqrt{\epsilon_d}k|z-z'|) - q(z)}{\epsilon(|z-z'|)|z-z'|}\,dz'. \tag{A2}$$

Since $a \gg b$, we have, for points not too near the ends of the stick,

$$q(z)\int_{\substack{|z'|<a\\|z-z'|<b}} \frac{1}{\epsilon(|z-z'|)|z-z'|}\,dz' = \frac{1}{C}\,q(z), \tag{A3}$$

where

$$C = \epsilon_d\,\frac{1}{2\ln(1 + a\epsilon_d/b\epsilon_e)} \tag{A4}$$

is the capacitance per unit length of the stick embedded in the effective medium with a scale-dependent dielectric constant $\epsilon(l)$, and ϵ_e is the effective dielectric constant of the composite, i.e., the limiting value of $\epsilon(l)$ for $l>a$ [see Eq. (7) and discussion there].

In the integral which contains the difference $q(z')\exp(ik_e|z-z'|)-q(z)$, we resolve the exponent into real and imaginary parts. Thus the second term in Eq. (A2) becomes

$$\int_{|z'|<a} \frac{q(z')\cos(\sqrt{\epsilon_d}k|z-z'|)-q(z)}{\epsilon(|z-z'|)|z-z'|}\,dz' + i\int_{|z'|<a} \frac{q(z')\sin(\sqrt{\epsilon_d}k|z-z'|)}{\epsilon(|z-z'|)|z-z'|}\,dz'. \quad \text{(A5)}$$

Since the first integral is real and has no singularity at $z=z'$, it gives some correction to the capacitance C. We will neglect this correction as it is small in comparison with the leading term given by Eqs. (A3) and (A4). The second integral is exactly equal zero for $z=0$ since $q(z)$ is an odd function—the total charge is zero. We may neglect this term for all z that are not too close to the ends of the stick. This assumption is invalid near the ends of the stick, but in calculating the dipole moment that region is unimportant. Therefore we assume that connection between the charge $q(z)$ and potential $U(z)$ is local and given by

$$U(z)=\frac{1}{C}\,q(z). \quad \text{(A6)}$$

This result is obtained with so-called *logarithmic accuracy*: its relative error is of the order $1/\ln(a/b)$, and the ratio a/b is assumed to be so large that its logarithm is large.

Consider now the vector potential $\mathbf{A}(z)$ induced by the current $I(z)$ flowing in the stick. We obtain the vector potential following the same basic pattern as above. Thus we have with the same logarithmic accuracy the following expression for the component A_z of the vector potential $\mathbf{A}(z)$:

$$A_z(z)=\frac{1}{c}\int_{\substack{|z'|<a\\|z-z'|<b}} \frac{I(z')\exp(ik_e|z-z'|)}{|z-z'|}\,dz' \cong \frac{2I(z)}{c}\ln\left(\frac{a}{b}\right)+\frac{i}{c}\int_{-a}^{a}\frac{I(z')\sin(\sqrt{\epsilon_d}k|z-z'|)}{|z-z'|}\,dz'. \quad \text{(A7)}$$

Note that we do not take into account the effective magnetic permeability μ_e of the composites introduced by Sec. IV. We neglect μ_e for the reason that the value of the integral in Eq. (A7) is determined mainly by a singularity at $z=z'$, while the μ_e is formed on much larger scales [see Eq. (63) and the following discussion]. The last term in Eq. (A7) does not vanish since the current $I(z)$ is an even function. It is a sufficiently smooth function for not very short wavelengths. We expand the exponent into a series of k and put, rather arbitrary, $\int I(z')dz'=2aI(z)$. Thus we obtain the local connection between the vector potential and current, i.e.,

$$A_z(z)\cong\frac{L}{c}\,I(z), \quad \text{(A8)}$$

where L is the inductance per unit length,

$$L=2\ln\left(\frac{a}{b}\right)+i2\sqrt{\epsilon_d}ka. \quad \text{(A9)}$$

Note that $\sin(\sqrt{\epsilon_d}k|z-z'|)/|z-z'|\to\pi\delta(z-z')$ when $k\to\infty$. Therefore Eq. (A7) also gives the local relation (A8) in the short wave limit, but the imaginary part of the inductance saturates at the value $i\pi$ and is independent of k in this limit. In any case the second term in Eq. (A9) is much smaller than $2\ln(a/b)\gg1$. Nevertheless, we preserve the imaginary part of the inductance since it has a profound impact in a stick response to external fields when the resonance conditions are fulfilled [see Eqs. (44)–(46)].

We suppose that the stick is excited by the external field $\mathbf{E}_0\exp(-i\omega t)$ that is parallel to its axis. Then electric field at some point z on the surface is equal to

$$E(z)=E_0-\frac{dU(z)}{dz}+i\,\frac{\omega}{c}\,A_z(z). \quad \text{(A10)}$$

Substituting here Eqs. (A6) and (A8), we have

$$E(z)=E_0-\frac{1}{C}\frac{dq(z)}{dz}+i\,\frac{\omega}{c^2}\,LI(z). \quad \text{(A11)}$$

Then we use the continuity condition for electricity $dI/dz=i\omega q$, obtaining

$$E(z)=E_0-\frac{1}{i\omega C}\frac{d^2I(z)}{dz^2}+i\,\frac{\omega}{c^2}\,LI(z). \quad \text{(A12)}$$

On the other hand, the field $E(z)$ is proportional to the current $E(z)=R(z)I(z)$, where R is called the complex resistance or impedance of the conductor. In the quasistatic case, $R(z)=1/(\pi r(z)^2\sigma_m)$, where σ_m is the stick conductivity and $r(z)=b\sqrt{1-(z/a)^2}$ is the stick radius at coordinate z. When the skin effect in the conducting stick is not negligible, the impedance R is given by Eq. (26). Substituting $E(z)=R(z)I(z)$ in Eq. (A12), we obtain the wanted Eq. (24) for the current in a stick illuminated by an electromagnetic wave.

Note that one can use the original Eqs. (A1) and (A7) for the potentials instead to expand them in terms of $1/lag(a/b)$. Then Eq. (24) becomes an integral equation for the current $I(z)$. This one-dimensional integral equation can be easily solved numerically. Nevertheless, we use the "local" approximation described above since we believe that the accuracy of the effective-medium approach makes no sense to further improve the current estimation.

For the quasistatic case, we neglect the inductance and substitute $R=1/(\pi r^2\sigma_m)$ in Eq. (A12), obtaining

$$\frac{d^2I(z)}{dz^2}=-\frac{i\omega C}{\sigma_m\pi b^2(1-z^2/a^2)}I(z)+i\omega CE_0,$$

$$I(-a)=0,\quad I(a)=0. \quad \text{(A13)}$$

The solution of this equation matching the boundary conditions is

$$I(z)=\pi b^2\sigma_m\,\frac{1-z^2/a^2}{1+2\pi i(\sigma_m/\omega)(b^2/a^2C)}\,E_0. \quad \text{(A14)}$$

Then Eq. (A12) gives the internal field $E_{\rm in}=E(z)$ in the stick that is uniform and equals

$$E_{\rm in}=\frac{1}{1+2\pi i(\sigma_m/\omega)(b^2/a^2C)}E_0. \qquad (A15)$$

Substituting here Eq. (A4) for the capacitance C and $\sigma_d=-i\omega\epsilon_d/4\pi$, we have

$$E_{\rm in}=\frac{1}{1+(b^2\epsilon_e/a^2\epsilon_d)\ln(1+a\epsilon_d/b\epsilon_e)\sigma_m/\sigma_e}E_0. \qquad (A16)$$

Comparing this result and field $\mathbf{E}_{{\rm in}\ m\parallel}$ in Eq. (2a) for the case $\sigma_m \gg \sigma_e$, considered here, we obtain Eq. (8) for the depolarization factor $g_\parallel$. For the scale-independent environment, we return to the equation

$$g_\parallel=\frac{b^2}{a^2}\ln\left(\frac{a}{b}\right), \qquad (A17)$$

which can be also obtained by direct expansion of the known expression for $g_\parallel$ (see, e.g., Ref. 30, Sec. 4) in a series of $b/a \ll 1$. When the external field is across the stick, the local field distribution is determined by the scale of the stick radius $b \ll a$. Therefore the depolarization factor across the stick, $g_\perp$, is still given by the usual expression for the much prolate spheroid, i.e., $g_\perp \cong 1/2$.

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