Low-frequency noise in a phonon system of disordered insulators

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The influence of slowly relaxing structural defects on some nonelectric properties of solids is considered. Fluctuations of phonon occupation numbers caused by energy exchange between the phonon system and the slowly relaxing excitations are analyzed. Fluctuations of the structure parameters caused by structural relaxations are studied. A model for the low-frequency flicker noise of Brillouin light-scattering spectrum is suggested, which relates it to spatial and temporal fluctuations of the elasto-optic coefficient. The results are compared with existing experimental data.

I. INTRODUCTION

It is well known that there is the low-frequency noise with a spectrum close to $1/f$ nearly in all physical systems.[1,2] The microscopic picture of this so-called flicker noise differs from one physical system to another, so that it is always important to understand what the origin of the flicker noise in a given system is. Until now the most intensive studies and, correspondingly, the most fascinating achievements were related to the electric noise in conducting solids. It was established that while the specific picture of the flicker noise differs from one physical system to another, the general feature is that the noise is caused by the slow dynamics of some statistically independent, affecting the carrier transport, objects with internal degrees of freedom and exponentially broad distribution of relaxation times. In the more specified forms this was first formulated as a hypothesis by Bernamount[3] and McWhorter. This hypothesis was recently proved by experiments on very small systems where a decomposition of the flicker noise into a sum of contributions each related to the given independent defect (so-called ''fluctuator'') has been observed.[4–6] In semiconductors these defects were identified with deep traps.[4,7] However, observations of the single fluctuators in metallic point contacts,[6] characterized by charge neutrality, evidenced the presence of fluctuators of more general nature: some structural-disorder-induced defects with an internal degree of freedom. The exponentially broad relaxation time distribution is a result[8] and an inherent property of any disorder.

Two-level systems (TLS's) typical for the amorphous structures[9] and responsible for many of their low-temperature properties are rather attractive candidates for the role of fluctuators. Contribution of TLS's to $1/f$ noise was first considered in Refs. 10–12. However, the TLS concept is valid only for a low-temperature region. For the broad temperature range the more general model of structural-disorder-induced fluctuators has to be used. Such a model was developed in Refs. 13,14 within a framework of the soft-potential approach.[15] Fluctuators were there identified with double-well interatomic potentials with moderately strong barriers. Application of these approaches to noise in metallic nanostructures was performed in Refs. 16,17.

The models mentioned above dealt with ''nanoscale'' fluctuators, which involve a relatively small number of atoms. However, more complex and extended defects like dislocation lines pinned by point defects may be important. Their possible role for $1/f$ noise was analyzed, e.g., in Ref. 18.

It is important that while discussing structural-disorder-induced noise one deals with the source of noise which is external with respect to the electronic system itself. It is rather ''noise of the structure'' or ''medium-induced noise,'' which causes ''medium-mediated correlation'' in the electron system[19] introduced by the scatterers. In particular, relatively long relaxation times inherent for the structure are imposed on the electron system.

Still, theoretical studies of disorder-induced flicker noise were mainly concentrated on the electric noise. Meanwhile, the structural fluctuations influences nearly *all* properties of solids because nearly all properties depend on the structural parameters. In particular, ''structural-disorder-induced'' noise manifests itself in the properties of the phonon system. One example is the flicker-type noise of Brillouin light-scattering spectrum (BLSS) intensity, which was recently observed by Musha *et al.*[20] It is also important to mention the slow-relaxation phenomena of nonelectric nature, which are generically close to the noise phenomena. Some of them were earlier shown to be evidently connected with atomic motion in *double-well* vibrational states. In dielectric systems the best-known evidences of slow-relaxation processes are the nonexponential heat release,[21–23] the internal friction,[24] and nonexponential dielectric relaxation.[12,25]

The purpose of the present paper is to study some consequences of structural-disorder-induced (or medium-induced) noise for the phonon system in insulating solids. In particular, we offer a model for the BLSS intensity low-frequency fluctuations.

In Sec. II the general expression for the noise spectrum of some fluctuating physical quantity, influenced by the fluctuations of the fluctuator occupation numbers, is obtained. Two

channels of this influence are turned out: the fluctuations of phonon occupation numbers directly caused by the fluctuators, which act as energy traps, and the fluctuations of structure parameters in the areas where fluctuators are positioned. In both cases specific estimates within a framework of the soft potential model are given. A brief discussion of this model is held in the Appendix. In Sec. III a model for the BLSS flicker noise is offered and an expression for the BLSS noise power spectral density is obtained. In Sec. IV some concluding remarks are given.

II. GENERAL EXPRESSIONS FOR THE FLUCTUATING QUANTITIES

Phenomenologically, any two-state fluctuator can be described by its interlevel splitting E and relaxation time τ. Distribution of E and τ for an ensemble of fluctuators in a disordered insulator can be found basing on some more specific model for these objects. For definiteness, our calculations will be mainly based on the soft atomic potential model, which was shown to be a successful tool to explain different phenomena both in completely amorphous (for a review see, e.g., Refs. 13,26,27) systems and in crystalline[13,16,17] systems with some degree of structural disorder. A brief review of this model is given in the Appendix. On the other hand, for some specific systems other models for the disorder-induced structural defects can be relevant.

Besides the relaxation time and the energy splitting, each fluctuator is characterized by the value of local lattice distortion. This distortion affects the ''local'' values of any parameter of solid, which, to say, depends on lattice constants. Among those parameters are the sound velocity, the phonon density of states, the elasto-optic coefficient, etc. In what follows we will be specifically interested in the elasto-optic coefficient.

In our model we assume that the presence of a fluctuator i results, in particular, in a change of the local value of the parameter, in general case denoted as $A(\mathbf{r},t)$, in some ''effective volume of a fluctuator'' Ω_i. This volume depends on a nature of the parameter A. In particular, if $A(\mathbf{r},t)$ is affected by the electric field, one should take into account the long-range electric field caused by the dipole moment of the fluctuator-induced lattice distortion. This makes the ''effective volume'' much larger than the lattice constant a:

$$\Omega_i \approx C_i[a\ln(d/a)]^3$$

even for the fluctuators of an atomic nature. Here C_i is a constant of the order of unity, and d is the mean distance between the fluctuators.

We will be interested in the temporal fluctuations of some physical quantity $\Phi(\mathbf{r},t)$, which depends on the parameters of the phonon system and on the structural parameter A. We will assume this quantity constant for a perfect homogeneous sample. Constantness is altered by the presence of disorder. In particular, dynamical processes in disorder-induced fluctuators influence $\Phi(\mathbf{r},t)$, causing its temporal fluctuations. One can find two major channels of this influence.

The first channel is due to fluctuations of the phonon temperature $T_{\rm ph}(t)$.

The second channel is due to fluctuator-induced spatial and temporal fluctuations of the parameter $A(\mathbf{r},t)$.

For any application it is necessary to calculate value of $\Phi(\mathbf{r},t)$ *averaged* over some ''active'' volume $\mathscr{V}_a$ (see Sec. III for an example):

$$\Phi_{\rm av}(t)=\int_{\mathscr{V}_a}\Phi(\mathbf{r},t)d\mathbf{r}. \tag{1}$$

Considering the fluctuations of $A(\mathbf{r},t)$ and $T_{\rm ph}(t)$ as small and linearizing Eq. (1) in the vicinity of the mean values,[31]

$$\begin{aligned} A(\mathbf{r},t)&=A^{(0)}+\delta A(\mathbf{r},t),\\ T_{\rm ph}(t)&=T^{(0)}+\delta T_{\rm ph}(t),\\ \Phi(\mathbf{r},t)&=\Phi^{(0)}+\delta\Phi(\mathbf{r},t),\\ \Phi_{\rm av}(t)&=\Phi_{\rm av}^{(0)}+\delta\Phi_{\rm av}(t), \end{aligned} \tag{2}$$

we obtain the following expression for the fluctuating part of the averaged quantity:

$$\frac{\delta\Phi_{\rm av}(t)}{\Phi_{\rm av}^{(0)}}=\frac{1}{\mathscr{V}_a}\int_{\mathscr{V}_a}\left[\frac{\partial\Phi/\partial A}{\Phi^{(0)}}\right]\Bigg|_{A^{(0)},T^{(0)}}\delta A(\mathbf{r},t)d\mathbf{r}+\left[\frac{\partial\Phi/\partial T_{\rm ph}}{\Phi^{(0)}}\right]\Bigg|_{A^{(0)},T^{(0)}}\delta T_{\rm ph}(t). \tag{3}$$

The Fourier transform of the correlation function $\langle\delta\Phi_{\rm av}(0)\,\delta\Phi_{\rm av}(t)\rangle_f\equiv\langle(\delta\Phi_{\rm av})^2\rangle_f$ has form

$$\frac{\langle(\delta\Phi_{\rm av})^2\rangle_f}{(\Phi_{\rm av}^{(0)})^2}=\left[\frac{\partial\Phi/\partial A}{\Phi^{(0)}}\right]^2\Bigg|_{A^{(0)},T^{(0)}}\left\langle\left(\frac{1}{\mathscr{V}_a}\int_{\mathscr{V}_a}\delta A(\mathbf{r})d\mathbf{r}\right)^2\right\rangle_f+\left[\frac{\partial\Phi/\partial T_{\rm ph}}{\Phi^{(0)}}\right]^2\Bigg|_{A^{(0)},T^{(0)}}\langle(\delta T_{\rm ph})^2\rangle_f. \tag{4}$$

Two channels of fluctuator influence on Φ are seen here explicitly. Now we will consider them separately in detail.

A. Phonon temperature fluctuations

Let us consider a dielectric sample with the total volume $\mathscr{V}$. Its total energy $\mathscr{E}$ consists of two parts: the energy of the phonon subsystem, $\mathscr{E}_{\rm ph}$, and the energy stored in the localized states, $\mathscr{E}_{\rm LS}$. If the whole system is closed,[32] the total energy $\mathscr{E}$ is conserved:

$$\mathscr{E}=\mathscr{E}_{\rm ph}+\mathscr{E}_{\rm LS}=\text{const}. \tag{5}$$

This relation does not allow us to consider the fluctuations of $\mathscr{E}_{\rm ph}$ and $\mathscr{E}_{\rm LS}$ independently, and gives

$$\delta\mathscr{E}_{\rm ph}(t) = -\delta\mathscr{E}_{\rm LS}(t). \tag{6}$$

The fluctuations of $\mathscr{E}_{\rm LS}$ originate from transitions in localized states. The spectrum of these fluctuations is expected to be wide due to the exponentially broad distribution of double-well fluctuator relaxation times. Equation (6) gives us the fluctuations of $\mathscr{E}_{\rm ph}$ with the *same* wide spectrum.

An expression for $\mathscr{E}_{\rm ph}$ can be written as

$$\mathscr{E}_{\rm ph} = \int d\omega g_{\rm ph}(\omega)\hbar\omega\left[N_0(\omega, T_{\rm ph}) + \frac{1}{2}\right]. \tag{7}$$

Here $g_{\rm ph}(\omega)$ is the phonon density of states. If there are some fluctuations of $\mathscr{E}_{\rm ph}$ on time scales which are longer than the phonon thermalization time, we can express these fluctuations of $\mathscr{E}_{\rm ph}$ in terms of phonon temperature fluctuations:

$$\delta\mathscr{E}_{\rm ph}(t) = \int d\omega g_{\rm ph}(\omega)\hbar\omega \left.\frac{\partial N_0(\omega, T_{\rm ph})}{\partial T_{\rm ph}}\right|_{T^{(0)}} \delta T_{\rm ph}(t)$$
$$= \frac{2\pi^2}{15}\mathscr{V}\left(\frac{k_B T^{(0)}}{\hbar s}\right)^3 k_B \delta T_{\rm ph}(t). \tag{8}$$

On the other hand, the energy stored in the localized states, $\mathscr{E}_{\rm LS}$, is

$$\mathscr{E}_{\rm LS} = \sum_i E_i n^{(i)}. \tag{9}$$

Here E_i is the energy of the ith fluctuator, $n^{(i)}$ the occupation number of its upper level, and the summation is over all fluctuators which belong to the volume $\mathscr{V}$. Fluctuations of $\mathscr{E}_{\rm LS}$ occur due to the occupation number fluctuations:

$$\delta\mathscr{E}_{\rm LS}(t) = \sum_i E_i \delta n^{(i)}(t). \tag{10}$$

Combining Eqs. (6), (8), and (10) we obtain the expression for the phonon temperature fluctuations:

$$\delta T_{\rm ph}(t) = \left[\frac{2\pi^2}{15 k_B}\mathscr{V}\left(\frac{k_B T^{(0)}}{\hbar s}\right)^3\right]^{-1} \sum_i E_i \delta n^{(i)}(t). \tag{11}$$

The power spectral density of these fluctuations is

$$\langle(\delta T_{\rm ph})^2\rangle_f = \left[\frac{2\pi^2}{15 k_B}\mathscr{V}\left(\frac{k_B T^{(0)}}{\hbar s}\right)^3\right]^{-2} \sum_i E_i^2 \langle(\delta n^{(i)})^2\rangle_f. \tag{12}$$

Thus the quantity that characterizes the fluctuations of phonon temperature is the correlation function of the fluctuator level populations $\langle(\delta n^{(i)})^2\rangle_f$. If the defects are in equilibrium with the thermal phonons, this can be easily expressed in terms of equilibrium occupation numbers $n_0^{(i)} = [\exp(E_i/k_B T^{(0)}) + 1]^{-1}$ and the relaxation times $\tau^{(i)}$ of the defects:

$$\langle(\delta n^{(i)})^2\rangle_f = \frac{2}{\tau^{(i)}} \frac{n_0^{(i)}(1 - n_0^{(i)})}{f^2 + (1/\tau^{(i)})^2}. \tag{13}$$

Replacing the summation in Eq. (12) by the integration over the fluctuator distribution function $\mathscr{P}(E, \tau)$,

$$\sum_i \cdots \to \int dE \int d\tau \mathscr{V}\mathscr{P}(E, \tau) \cdots,$$

we obtain

$$\frac{\langle(\delta T_{\rm ph})^2\rangle_f}{(T^{(0)})^2} = \frac{225}{2\pi^4 \mathscr{V}}\left(\frac{\hbar s}{k_B T^{(0)}}\right)^6 \int dE \int d\tau \mathscr{P}(E, \tau) E^2 n_0(E)[1 - n_0(E)] \frac{\tau}{(f\tau)^2 + 1}. \tag{14}$$

An order of magnitude estimate is

$$\frac{\langle(\delta T_{\rm ph})^2\rangle_f}{(T^{(0)})^2} \approx \mathscr{P}(E \sim T^{(0)}, \tau \sim f^{-1}) T^{(0)} (q_T^6 \mathscr{V})^{-1} \propto [T^{(0)}]^{-5}. \tag{15}$$

Here q_T is a characteristic wave vector of a phonon with $\hbar\omega \sim T^{(0)}$. The relative magnitude of the fluctuations increases drastically with the temperature decrease. Therefore, these fluctuation are expected especially pronounced in small devices operating at low temperatures.

Within the soft potential model (see the Appendix) and assuming the fluctuator density of states of the order of typical for glasses $\sim 10^{33}$ erg^{-1} cm^{-3} one obtains for $T \sim 1$ K and $\mathscr{V} \sim 10^{-9}$ cm the temperature fluctuations $(\delta T/T^{(0)}) \approx 10^{-5}$.

B. Fluctuations of structure parameters

Let us find the value of the parameter $A(\mathbf{r}, t)$ averaged over some characteristic volume $\mathscr{V}_a$. Assuming that the contributions of different fluctuators are additive, one has

$$\frac{1}{\mathscr{V}_a}\int_{\mathscr{V}_a} \frac{\delta A(\mathbf{r}, t)}{A^{(0)}} d\mathbf{r} \approx \frac{1}{\mathscr{V}_a}\sum_i \Omega_i[\Delta A_1^{(i)}(1 - n^{(i)}) + \Delta A_2^{(i)} n^{(i)}]. \tag{16}$$

Here $\Delta A_1^{(i)}$ and $\Delta A_2^{(i)}$ are the corrections to A for the lower and the upper fluctuator states. Summation in the Eq. (16) is over the fluctuators belonging to the active region $\mathscr{V}_a$. Fluctuations of the left-hand side (LHS) of Eq. (16) occur due to the fluctuations of the occupation numbers $n^{(i)}$. For the mean quadratic fluctuation of A one has

$$\frac{\langle(\delta A)^2\rangle_f}{(A^{(0)})^2}\approx\frac{1}{\mathscr{V}_a^2}\sum_i \Omega_i^2\frac{(\Delta A_1^{(i)}-\Delta A_2^{(i)})^2}{(A^{(0)})^2}\langle(\delta n^{(i)})^2\rangle_f. \tag{17}$$

Replacing the summation by the integration over the distribution function $\mathscr{P}(E,\tau)$, and taking into account the explicit expression, Eq. (13), for $\langle(\delta n^{(i)})^2\rangle_f$ one obtains

$$\frac{\langle(\delta A)^2\rangle_f}{(A^{(0)})^2}\approx\frac{2}{\mathscr{V}_a}\frac{\langle(\Delta A_1-\Delta A_2)^2\rangle}{(A^{(0)})^2}\langle\Omega\rangle^2\int dE\int d\tau\mathscr{P}(E,\tau)n_0(E)[1-n_0(E)]\frac{\tau}{(f\tau)^2+1}$$
$$\approx\frac{\langle(\Delta A_1-\Delta A_2)^2\rangle}{(A^{(0)})^2}\mathscr{P}(E\sim T,\tau\sim f^{-1})T\langle\Omega\rangle\frac{\langle\Omega\rangle}{\mathscr{V}_a}. \tag{18}$$

C. Comparison of two obtained noise contributions

Let us compare two obtained contributions to the fluctuations of $\Phi_{\rm av}$. If Φ is not too strong (nonexponential) a function of T and A, the ratio of the first, Eq. (14), and the second, Eq. (18), noise contributions is

$$R(T^{(0)})\approx\frac{\mathscr{V}_a}{\mathscr{V}}\frac{1}{\langle\Omega\rangle^2 q_T^6}\frac{(\phi^{(0)})^2}{\langle(\Delta\phi_1-\Delta\phi_2)^2\rangle}. \tag{19}$$

One sees that R as compared with unity may be both small and large, depending on the parameters of the system and on the temperature. Thus two noise contributions under discussion can compete. In the low-temperature region $[R(T^{(0)})>1]$ the contribution due to the phonon temperature fluctuations dominates, while in the region of high temperatures $[R(T^{(0)})<1]$ fluctuations of $\Phi_{\rm av}$ are mainly due to fluctuations of the structure parameter A.

The crossover temperature $T_{\rm cr}$ between these two regimes depends, in particular, on the ratio of the characteristic volume of the active region of the sample and its total volume. For most experiments this ratio is much smaller than unity, so that $T_{\rm cr}$ is rather small and nearly in all temperature ranges the contribution to the noise due to fluctuations of structure parameters dominates. However, in small samples this ratio is of the order of unity, and the phonon temperature fluctuations may become observable.

III. BLSS INTENSITY NOISE POWER SPECTRAL DENSITY

Now let us consider a special case of BLSS noise. It is well known that light is, in the course of its propagation through a dielectric media, scattered inelastically due to fluctuations of the dielectric susceptibility. The general expression for the differential cross-section of this scattering in the spatially homogeneous system is[33]

$$\frac{d^2\sigma}{d\Omega\,d\omega_2}=\left(\frac{\omega_2}{c}\right)^4\frac{\omega_2}{\omega_1}\mathscr{V}\mathscr{V}_a\sum_{i,f}\Phi(\mathscr{E}_i)\delta\left(\frac{\mathscr{E}_i-\mathscr{E}_f}{\hbar}-(\omega_2-\omega_1)\right)$$
$$\times|\mathbf{e}_2\cdot\hat{\delta}\chi\cdot\mathbf{e}_1|^2. \tag{20}$$

Here $\omega_1,\mathbf{e}_1$ and $\omega_2,\mathbf{e}_2$ are the frequencies and the polarization vectors of the incident and the scattered photons, respectively, c is the light velocity, $\Phi(\mathscr{E}_i)$ is the probability for the system to be in a given state $|i\rangle$ with energy $\mathscr{E}_i$ and $\mathscr{V}$ the total volume of the sample. $\mathscr{V}_a$ is the volume of that active part of the sample which contributes to the detected scattered light. This volume is determined by the spatial resolution of the light detector.

The explicit form of the scattering tensor $\hat{\delta}\chi=(\delta\chi)_{\mu\nu}$ depends on what type of light scattering dominates. For the Brillouin scattering by acoustic phonons one has[33]

$$(\delta\chi)_{\mu\nu}=\phi_{\mu\nu\lambda\gamma}u_{\lambda\gamma}, \tag{21}$$

where $\phi_{\mu\nu\lambda\gamma}$ is the elasto-optic coefficient tensor, and

$$u_{\lambda\gamma}=iq_\gamma\epsilon_\lambda\sqrt{\frac{\hbar}{2\rho\mathscr{V}\omega_{\mathbf{q}}}}\times\begin{cases}\sqrt{N_0(\omega_{\mathbf{q}},T_{\rm ph})+1},\\ \text{photon emission (Stokes)},\\ \sqrt{N_0(\omega_{\mathbf{q}},T_{\rm ph})},\\ \text{photon absorption (anti-Stokes)}\end{cases} \tag{22}$$

is the phonon-induced strain. Here ϵ_λ are the phonon polarization vector components, ρ is the bulk density and $\mathbf{q}$ and $\omega_{\mathbf{q}}$ are the phonon wave vector and frequency, respectively. $N_0(\omega,T_{\rm ph})$ is the Plank function with the phonon temperature $T_{\rm ph}$.

Both the momentum

$$\mathbf{k}_1=\mathbf{k}_2\pm\mathbf{q} \tag{23}$$

and the energy

$$\omega_1=\omega_2\pm\omega_{\mathbf{q}} \tag{24}$$

conservation laws must be satisfied in course of light scattering, where $\mathbf{k}_{1(2)}$ is the wave vector of the incident (scattered) photons; a double sign refers to Stokes and anti-Stokes processes. This restriction leads to a relation for the frequency shift of light scattered at some angle θ:

$$|\omega_2-\omega_1|=\omega_{\mathbf{q}}=2\omega_1\left(\frac{s}{c}\right)\nu\sin(\theta/2)\ll\omega_1=k_1c, \tag{25}$$

where s is the sound velocity, and ν the refractive index of the specimen. This shift is very small, as compared with ω_1, for all possible angles θ, and so we will not distinguish between the frequencies of incident and scattered photons. Furthermore, in the experimentally actual temperature range we can consider this shift to be much smaller than the phonon temperature $k_BT_{\rm ph}$.

Using the high-temperature limit for the Plank function $N_0(\omega, T_{\rm ph}) \sim k_B T_{\rm ph}/\hbar\omega_{\mathbf{q}}$ and for the sake of brevity omitting the dependence of the cross section, Eq. (20), on the photon polarization and symmetry of the medium, one obtains[33] the total cross section of light scattering in the form

$$\sigma = \left(\frac{\omega_2}{c}\right)^4 \mathscr{V}_a \frac{k_B T_{\rm ph}}{2\rho s^2} \phi^2. \tag{26}$$

Here and below we do not distinguish between Stokes and anti-Stokes scattering components, neglecting unity in comparison with the large Plank factor $k_B T_{\rm ph}/\hbar\omega_{\mathbf{q}} \gg 1$.

The total intensity of scattered light $I^{\rm sc}_{\rm tot}$ can be expressed as:

$$I^{\rm sc}_{\rm tot} = I^{\rm in}\sigma, \tag{27}$$

where $I^{\rm in}$ is the intensity of the incident light flux.

Up to now we have considered our sample to be spatially homogeneous and the scattering process to be stationary. This simple picture is altered by the presence of disorder. In particular, dynamical processes in disorder-induced fluctuators can influence the total intensity of the BLSS, causing its temporal fluctuations. In order to avoid confusion, we would like to emphasize here that we discuss 1/f fluctuations of the total *intensity* of the BLSS, that is, of quantity *integrated over* the light frequency shifts. In principle, we could discuss fluctuations of the form of the BLSS *line* characterized by linewidth and line height. In this case we would show that it is line height that fluctuates with 1/f spectrum, while the linewidth remains constant, at least in the leading approximation.

Turning back to the fluctuations of the total BLSS intensity, we would like to determine, as in Sec. II, two major channels via which the fluctuations in question originate.

The first channel is due to fluctuations of the phonon temperature $T_{\rm ph}(t)$.

The second channel is due to fluctuator-induced spatial and temporal fluctuations of the elasto-optic coefficient $\phi(\mathbf{r},t)$.

In order to take these both into account we introduce the time- and spatial-dependent intensity of scattered light per unity of volume, $I^{\rm sc}(\mathbf{r},t)$:

$$I^{\rm sc}(\mathbf{r},t) = I^{\rm in}\left(\frac{\omega_2}{c}\right)^4 \frac{1}{16\pi^2} \frac{k_B T_{\rm ph}(t)}{2\rho s^2} \phi^2(\mathbf{r},t). \tag{28}$$

The total intensity of scattered light $I^{\rm sc}_{\rm tot}(t)$ can be written as

$$I^{\rm sc}_{\rm tot}(t) = \int_{\mathscr{V}_a} I^{\rm sc}(\mathbf{r},t)\, d\mathbf{r}. \tag{29}$$

The power spectral density (PSD) of the BLSS intensity noise is generally given by Eq. (4). The BLSS is usually measured at rather high temperatures and in rather large samples. Due to this fact the contribution to the noise due to phonon temperature fluctuations is negligible, while the contribution of elasto-optic constant fluctuations is estimated as

$$\frac{\langle(\delta I^{\rm sc}_{\rm tot})^2\rangle_f}{(I^{\rm sc(0)}_{\rm tot})^2} \approx 2\frac{\langle(\Delta\phi_1 - \Delta\phi_2)^2\rangle}{(\phi^{(0)})^2} \mathscr{P}(E \sim T^{(0)}, \tau \sim f^{-1}) \times T^{(0)} \times \langle\Omega\rangle \frac{\langle\Omega\rangle}{\mathscr{V}_a}. \tag{30}$$

Finally, within the soft potential model we obtain

$$\left[\frac{\langle(\delta I^{\rm sc}_{\rm tot})^2\rangle_f}{(I^{\rm sc(0)}_{\rm tot})^2}\right]_\phi \approx 2\pi \frac{P_0 \eta_L^{5/2}}{\mathscr{V}_a f} \frac{\langle(\Delta\phi_1 - \Delta\phi_2)^2\rangle}{(\phi^{(0)})^2} \langle\Omega\rangle^2 \times \left(\frac{k_B T^{(0)}}{W}\right)^{5/4} \left(\ln\frac{1}{f\tau_0}\right)^{-3/4}. \tag{31}$$

Let us compare the obtained results with existing experimental data on low-frequency BLSS intensity noise in dielectrics.[20] Flicker-type BLSS intensity noise was observed with PSD $\approx 10^{-7}/f$ for a relatively large value of $\mathscr{V}_a \sim 10^{-4}\,\mathrm{cm}^3$. One may see that the model of ''atomic-scale'' fluctuators is hardly able to explain the experimental results quantitatively even for large fluctuator density of states, $\sim 10^{33}\ \mathrm{erg}^{-1}\ \mathrm{cm}^{-3}$, typical for glasses. However, the very $1/f$ spectrum of the fluctuations observed at relatively small frequencies (up to 10^{-4} Hz) is unambiguous evidence[34] for the effect of structural relaxation. We believe that the large magnitude of noise observed is related to a contribution of more complex structural defects like dislocation segments. Phenomenologically their contribution is described by Eqs. (18), (30) with $\mathscr{P}$ and Ω considered as density of states and characteristic volume of ''macrodefects.'' In this case the ratio $\langle\Omega\rangle/\mathscr{V}_a$ can be large enough to support the necessary fluctuation amplitude.

IV. CONCLUSIONS

In summary, we have shown that the slow dynamics of disorder-induced localized vibrational states may produce low-frequency noise of parameters related to the phonon system of solids. The fluctuations of phonon occupation numbers related to energy exchange between the phonon systems and slowly relaxing degrees of freedom are considered and are shown to be pronounced at low temperatures and for small enough samples. The fluctuation of structure parameters affected by the structural relaxation are studied as well. A model for the low-frequency noise of Brillouin light-scattering spectrum is offered, which explains it as a result of the spatial and the temporal fluctuations of the elasto-optic coefficient, determining the strength of Brillouin light scattering. The results are compared with the existing experimental data.

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APPENDIX: SOFT POTENTIAL MODEL

In order to describe the low-frequency vibrational states responsible, in particular, for the slow-relaxation phenomena, we will use the soft potential model (SPM). This model was proposed in Ref. 15 as an extension of the well-known two-level system (TLS) model[9] for a more wide temperature range and was further developed in Refs. 28,13,29,30,26,27.

According to this model, the localized states can be described by the soft anharmonic effective potentials

$$V(x)=V_0\left[\eta\left(\frac{x}{a}\right)^2+\xi\left(\frac{x}{a}\right)^3+\left(\frac{x}{a}\right)^4\right]. \tag{A1}$$

Here x is the generalized coordinate, a is the characteristic length of the order of interatomic spacing, and V_0 the energy of atomic scale ($\approx$10 eV). The parameters η and ξ are random. In the region of *soft* potentials ($|\eta|,|\xi|\ll 1$) the distribution function of these parameters has the form

$$P(\eta,\xi)=|\eta|P_0, \tag{A2}$$

where P_0 is a constant.

For $\xi^2/\eta_L<\eta_L/|\eta|$, negative η and $|\eta|>3\eta_L$, where

$$\eta_L=\left(\frac{\hbar^2}{2Ma^2V_0}\right)^{1/3}\approx 10^{-2}$$

is the important small parameter of the model (M is the characteristic atomic mass), the two lowest levels in the potential Eq. (A1) form a two-level system with the energy splitting E:

$$E=\sqrt{\Delta_0^2+\Delta^2}. \tag{A3}$$

Here Δ and Δ_0 are the asymmetry and the tunneling splitting, respectively:

$$\Delta\approx\frac{W}{\sqrt{2}}\frac{|\xi|}{\eta_L^{1/2}}\left(\frac{|\eta|}{\eta_L}\right)^{3/2}, \tag{A4}$$

$$\Delta_0\approx W\exp\left[-\frac{\sqrt{2}}{3}\left(\frac{|\eta|}{\eta_L}\right)^{3/2}\right]. \tag{A5}$$

The characteristic energy W is the interlevel spacing in the quartic potential, Eq. (A1) with $\eta=\xi=0$:

$$W\approx V_0\eta_L^2\approx k_BT(T\approx 10\ \text{K}). \tag{A6}$$

The barrier height V_B between two minima of the double-well fluctuator potential, Eq. (A1), for $\Delta\ll V_B$ is

$$V_B=\frac{W}{4}\left(\frac{|\eta|}{\eta_L}\right)^2. \tag{A7}$$

At low temperatures the transitions between the levels of the fluctuator occur mainly due to tunneling, while at higher temperatures the transitions due to classical activation dominate. The crossover temperature T_c from tunneling to activation was shown to depend weakly on the fluctuator barrier height and is given by [13,14]

$$k_BT_c\approx\frac{W}{\pi}\left(\frac{4V_B}{W}\right)^{1/4}=O(W). \tag{A8}$$

In our further analysis we restrict ourselves to the temperature region $k_BT\gg W$, which is more likely for flicker noise experiments (see, e.g., Ref. 20). On the other hand, it can be shown that at low temperatures $k_BT\ll W$, the physical picture of noise in question does not change qualitatively (see, e.g., Refs. 13,14). In the region $k_BT\gg W$ transitions between the wells of a fluctuator occur mainly due to activation processes with a rate given by

$$\frac{1}{\tau(T)}=\frac{1}{\tau_0}\exp\left(-\frac{V_B}{k_BT}\right), \tag{A9}$$

where $\tau_0^{-1}\approx 10^{12}-10^{13}\ \text{sec}^{-1}$ is the attempt rate, and T the lattice temperature.

The main quantity which we will need in our further analysis is the distribution function of the variables E and τ, $\mathscr{P}(E,\tau)$. It can be expressed in terms of $P(\eta,\xi)$, Eq. (A2):

$$\mathscr{P}(E,\tau)=\int d\eta\int d\xi\,\delta(E-E(\eta,\xi))\,\delta(\tau-\tau(\eta,\xi))|\eta|P_0, \tag{A10}$$

where $E(\eta,\xi)$ and $\tau(\eta,\xi)$ are given by Eqs. (A3) and Eq. (A9), respectively. Neglecting Δ_0, which is exponentially small for fluctuators with high barriers, we finally obtain

$$\mathscr{P}(E,\tau)\approx\frac{P_0\eta_L^{5/2}}{W\tau}\left(\ln\frac{\tau}{\tau_0}\right)^{-3/4}\left(\frac{k_BT}{W}\right)^{1/4}, \tag{A11}$$

which does not depend on E in a certain energy interval.

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