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# Multiple-quantum resonant reflection of ballistic electrons from a high-frequency potential step

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It is shown that ballistic electrons of energy $E$ impinging on a high-frequency potential step of the height $V\cos\omega t$ undergo resonant reflection when $E\approx l\hbar\omega$ ($l=1, 2, \ldots$). The major contribution to the quasienergetical wave function at these resonances is made by the harmonic of nearly zero energy. For $E=\hbar\omega$, $V\ll\hbar\omega$, transmission coefficient $T=0.5$ is obtained. At $\hbar\omega<E$ the conversion of incident particles into cold ones with emission of a quantum $\hbar\omega$ is up to 40%. To observe these effects an experimental scheme with an $n^+$-$Al_xGa_{1-x}As/GaAs$ structure is suggested.

Recently the interference in transmission of ballistic electrons has been observed in a double slit experiment with $n^+$-$Al_xGa_{1-x}As/GaAs$ structures. The structures included a strip metal gate over a two-dimensional electron gas (2DEG).[1] Resonant transmission via either virtual states of a quantum well[1,2] or over-barrier resonances[1,3] has been also observed in similar structures. Therefore, it has been experimentally confirmed that for such structures the phase of the electron wave function of energy $E>E_F$ can be controlled by the stationary bias at the gate which creates one-dimensional (1D) wells or barriers across the path of ballistic electrons. In this paper we are discussing how this phase can be controlled in an analogous structure in which a stationary obstacle is *replaced* with a high-frequency (hf) potential step. Such a hf potential step can be realized by means of metallic gates separated with a narrow slit. The hf step emerges in a 2DEG beneath the slit when the gates are exposed to microwave radiation. This experimental scheme is similar to the modern application of high-frequency microwave techniques which are used for investigation of electronic transport in nanostructures with a quantum point contact[4,5] and quantum dots.[6]

Several theoretical investigations of the effect of hf fields on electron transmission through a barrier,[7] multibarrier structures,[8] and a potential well[9] suggest ideas for the solution of the problem of electron reflection from a hf step. Up to now, quantum-mechanical reflection of particles from the regions with hf potential $V(x)\cos\omega t$ and zero stationary potential has been almost unanalyzed, except for Refs. 10 and 11. This is due to the fact that in many real situations hf potential is only a small addition to nonuniform stationary potential, whereas the uniform hf field itself is known not to reflect particles because of the momentum conservation law. Nevertheless, as will be shown below, the simplest nonuniformity of a hf field in the shape of a sharp potential step becomes a semitransparent mirror for incident particles of the energy $E\approx l\hbar\omega$ ($l=1, 2, \ldots$), even without stationary field. This effect resembles multiquantum resonant reflection from a potential well in a hf field.[9]

It should be noted that some peculiarity in dynamic conductance at $E=\hbar\omega$ has been recently predicted by modeling a barrier-free interelectrode gap driven by hf voltage within perturbation theory approximation.[11,12] This approximation was successfully used earlier in Ref. 7 for electron tunneling through a high potential barrier in a hf field provided that $V\ll\hbar\omega\ll E$. However, for $\hbar\omega=E$ the electron wave function undergoes too strong perturbation by a small hf step without a barrier. Therefore, the perturbation theory cannot be applied for this case.

In contrast to Ref. 12, our analysis is based on the exact solution of the one-dimensional Schrödinger equation for quasienergetical wave function $\Psi(x,t+2\pi/\omega)=\exp[-iEt/\hbar]\,\Psi(x,t)$ in a stepwise potential by the algorithm developed in Ref. 8. This approach usually requires numerical realization,[13] so we restrict the analytical treatment to the case of a sharp hf potential step: $U(x,t)=V\,\Theta(x)\cos\omega t$, where $\Theta(x)$ is the unit step function. The wave function can be represented with a discrete set of harmonics of energies $E+n\hbar\omega$, $n=0, \pm1, \pm2, \ldots$. Thus, as we consider the resonance $E-\hbar\omega=0$, this set covers both positive and negative energies of a particle. If a particle of energy $E$ falls on the hf step from the left, the solution is given by (we put $\hbar=2m=1$ here and below)

$$\Psi=\begin{cases} e^{ik_0x-iEt}+\sum r_n e^{-ik_nx-i(E+n\omega)t}, & x<0\\ e^{-i\frac{V}{\omega}\sin\omega t}\sum t_n e^{ik_nx-i(E+n\omega)t}, & x>0\end{cases} \tag{1}$$

where $k_n=\sqrt{E+n\omega}$. When the wave numbers $k_n$ are real, the amplitudes $r_n$ and $t_n$ determine the reflection and transmission with absorption ($n>0$) or emission ($n<0$) of $n$ quanta of alternating field. The harmonics with imaginary $k_n$ describe solutions vanishing exponentially from the point $x=0$ and thus not contributing to reflected and transmitted flows. Nevertheless, these harmonics are not to be rejected because of their importance for matching conditions,

$$\Psi\big|_{+0}=\Psi\big|_{-0},\qquad \Psi'\big|_{+0}=\Psi'\big|_{-0}. \tag{2}$$

Substituting the expansion $\exp[-i(V/\omega)\sin\omega t]$ =

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$\sum J_m(V/\omega)\exp[-im\omega t]$, where $J_m$ is the Bessel function, into (1) and equating the terms with identical time-dependent parts, $\exp[-im\omega t]$, in Eq. (2), we obtain the infinite set of linear equations. Within $N$-quantum approximation $(-N \le n \le N)$ the set is reduced to the system of $2(2N+1)$ equations with the same number of unknown amplitudes $r_n$ and $t_n$. Let us analyze this system for $E \approx \omega$ assuming that only three harmonics strongly interact: the elastic channel of energy of incident particles $E$, the harmonic $n=-1$ of energy close to the bottom of the continuum, and the harmonic $n=-2$ of negative energy. As opposed to two-channel approximation $(n = 0, -1)$, taking account of these three harmonics gives the correct estimate of the dip in the transmission coefficient. Numerical tests show that such a choice of the essential harmonics is justified, since additional channels with $n < -2$ do not affect the results for $V/\omega \ll 1$, and transitions with absorption of one, two, etc., quanta become unlikely (about 1% for $V/\omega = 0.2$) so all the channels with $n > 0$ may be rejected.

The system of six equations resulting from (2) in the three-channel approach is given by

$$t_n = J_n + \sum_{m=0}^{2} r_{-m} J_{n+m} \quad (n = 0, -1, -2),$$

$$k_n t_n = k_0 J_n - \sum_{m=0}^{2} k_{-m} r_{-m} J_{n+m}, \tag{3}$$

and is easily solved by excluding $t_n$. As the result, for the amplitudes $r_0$, $r_{-1}$, $r_{-2}$ and $t_0$, $t_{-1}$, $t_{-2}$ near the resonance $E = \omega$, in the case of low ratio $J_1/J_0 \ll 1$, we obtain

$$r_0 = \frac{1}{1+i+\alpha}, \quad r_{-1} = -\frac{2J_0}{J_1} r_0, \quad r_{-2} = -r_0,$$
$$t_0 \simeq 1 - r_0, \quad t_{-1} \simeq r_{-1}, \quad t_{-2} \simeq r_0, \tag{4}$$

where $\alpha = (2J_0/J_1)^2 (k_{-1}/k_0)$. In the resonance $k_{-1} \to 0$, and $|r_0|^2 \to 0.5$, i.e., one-half of the incident particles are reflected from the high-frequency step (Fig. 1).

Consider the contribution of the channel $n = -1$ into total transmission and reflection. As $E > \omega$, this channel is opened and the reflection coefficient $R_{-1}$ is given by

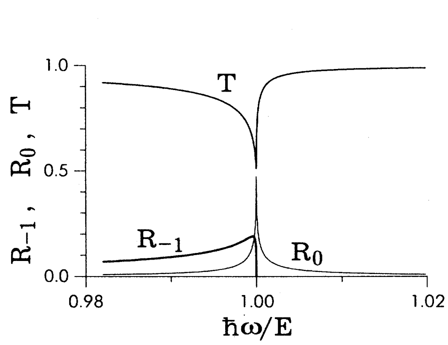


FIG. 1. The coefficients of total transmission ($T$), elastic ($R_0$), and inelastic ($R_{-1}$) reflection of particles of energy $E$ from hf step $V\Theta(x)\cos\omega t$, for $V/E = 0.5$.

$$R_{-1} = \frac{k_{-1}}{k_0}|r_{-1}|^2 = \alpha |r_0|^2 = \frac{\alpha}{1+(1+\alpha)^2}. \tag{5}$$

When $\alpha = \sqrt{2}$, the value $R_{-1}$ takes its maximum $R_{-1\,\max} = \sqrt{2}|r_0|^2 = (\sqrt{2}-1)/2 \approx 0.207$. This means that 40% of particles (20% in reflection and the same part in transmission) are scattered into inelastic channel $n = -1$, at the certain frequency $\omega_{\max}$ of the hf field, with emission of a quantum $\hbar\omega$ and reducing the energy by a factor of $(2J_0/J_1)^4/2 \approx 128\,(\omega/V)^4$. Thus, near the resonance with the bottom of the continuum the effective conversion of rapid particles to almost still ones takes place. At $\omega > E$ the channel $n = -1$ is closed. Therefore, the transport becomes elastic and is determined by the coefficients $R_0$ and $T_0$. The profile of the total reflection coefficient (as well as of the total transmission coefficient, Fig. 1) is asymmetrical with respect to the line $\omega = E$ as a result of the addition of $R_{-1}(\omega) = \alpha|r_0|^2$ to the symmetric profile $|r_0(\omega)|^2$ for $\omega < E$.

Consider the wave function in transmission $(x > 0)$ and reflection $(x < 0)$ regions. As $\omega \le E$, it is given by

$$\Psi(x \le 0, t) = \left[ e^{ik_0 x} + r_0 \left( e^{-ik_0 x} - \frac{2J_0}{J_1} e^{-ik_{-1}x + i\omega t} - e^{\kappa_{-2}x + 2i\omega t} \right) \right] e^{-iEt},$$

$$Z\Psi(x > 0, t) = \left[ e^{ik_0 x} - r_0 \left( e^{ik_0 x} + \frac{2J_0}{J_1} e^{ik_{-1}x + i\omega t} - e^{-\kappa_{-2}x + 2i\omega t} \right) \right] e^{-iEt}. \tag{6}$$

Here $Z = \exp[(iV/\omega)\sin\omega t]$ and $\kappa_{-2} = \sqrt{2\omega - E}$. As seen from (6), for $J_0/J_1 \gg 1$ the contribution of the channel $n = -1$ dominates in the wave function and the probability density approaches large values. This results from the continuity of probability density flow through the high-frequency step, since there appear many particles with the speed reduced almost to zero. As $x$ is small, channels $E$ and $E - 2\omega$ periodically compensate each other with the frequency $2\omega$. Probability density oscillates both in space and time with respect to a large constant value. When $|x| > 1/\kappa_{-2} \approx 1/\sqrt{\omega}$, the value $|\Psi|^2$ could be written as

$$|\Psi(x,t)|^2 \approx R_0 \frac{4J_0^2}{J_1^2} - \frac{4J_0}{J_1}|r_0| \times (\cos[k_0 x - \omega t - \delta_{r_0}] \pm |r_0| \cos[k_0 x \pm \omega t]),$$
$$\tan\delta_{r_0} = -\frac{1}{1+\alpha},$$

where the upper sign is taken for $x < 0$, and the lower one is for $x > 0$. A similar solution, but with the evanescent tails of the wave function, emerges for imaginary $k_{-1}$ (Fig. 2).

Concerning the sensitivity of investigated resonance to the parameters of the hf potential, we can note the fol-

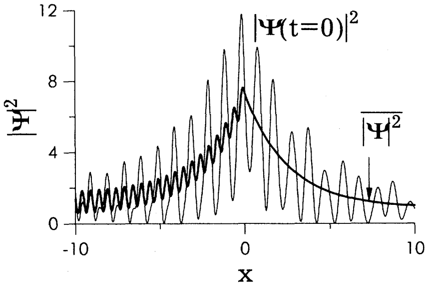


FIG. 2. Probability density at $t = 0$ (thin) and averaged in time (thick) for elastic scattering of particles of energy $E$ by hf step $V\Theta(x)\cos\omega t$, for $V/E = 0.5$ and $\hbar\omega/E = 1.001$. The amplitude of incident wave is taken to be 1. Unit of measure along the $x$ axis is the de Broglie wavelength, $h/\sqrt{2mE}$, of an incident particle.

lowing. By means of numerical simulations we proved that the profile of the hf step should not necessarily be rectangular; it can be smoothed and stretched over some interval. However, to reach reflection up to $R_0 = 0.5$, a quarter of the wavelength of incident particle, $\lambda/4$, must be greater than the width of this interval. Calculations performed with the software described in Ref. 13 demonstrate how hf step broadening affects $T(\omega)$ (Fig. 3). Particles do not reflect from a wide step. The presence of a scatterer in the form of a nonzero stationary potential (e.g., stationary potential step, $\delta$ well, or $\delta$ barrier) makes the dip in the transmission coefficient narrower and shallower. Thus, for observation of the resonance with the bottom of the continuum an abrupt enough hf step and absence of fluctuations of stationary potential are required.

As the amplitude of alternating field increases, for $V/\omega \geq 1$ the width of the resonance $E = \omega$ becomes larger and there appear simultaneously other dips in $T(\omega)$ which correspond to $E = l\omega$, $l = 2, 3, \ldots$ (Fig. 3). In this case, the time dependence of $|\Psi(x,t)|^2$ indicates that generation of the second, third, etc., harmonic possible. To describe correctly these multiquantum resonances up to the fifth one it is required to use up to 25

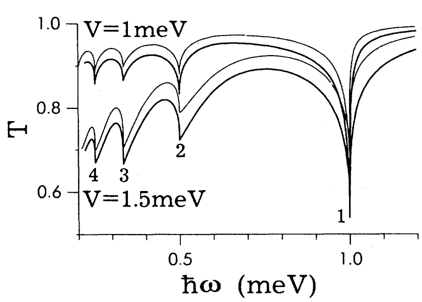


FIG. 3. Multiquantum resonances $\hbar\omega = E/l$, $l = 1, 2, 3, 4$ in total transmission coefficient for rectangular (thick curves) and smoothed (thin curves) hf steps for $E = 1$ meV, and $m = 0.067m_e$. Each pair of curves is marked with its $V$. For thin curves, the smoothing width is $\lambda/4 = 37.5$ nm.

channels in calculations.

The predicted dips in the transmission coefficient could be revealed in the hf response of $n^+$-$Al_xGa_{1-x}As/GaAs$ semiconductor split gate structures with 2DEG and ballistic electrons. In perfect structures with a $\delta$-doped $n^+$-$Al_xGa_{1-x}As$ layer and thick spacer, at the temperature near 10 mK, the electron mean-free path exceeds 10 $\mu$m, which is two orders of magnitude higher than the distance between charged impurities in the $\delta$-doped layer. Thus, fluctuations of potential due to nonuniform charge distribution in the $\delta$ layer are suppressed in an electron gas. Numerical modeling[14] confirms this fact, since the average amplitude of fluctuations of potential is much less than 0.1 meV (for $E_F = 3.7$ meV in the structure with a 40-nm spacer). This level of fluctuations is tolerable to observe the discussed reflection from the hf step. Furthermore, the width of the hf step cannot be less than the width of the slit between gates, and the latter is greater than or about 10 nm, as attainable by present technology. Thus, we need to install a hf step with the width less than or equal to a quarter of the de Broglie wavelength, $\lambda/4$ nm $\approx 37.5/\sqrt{E}$ meV. Hence, the energy of motion of ballistic electrons in the normal direction to the slit should not exceed 1 meV. Since the total energy of ballistic electrons is greater than $E_F - E_0 \approx 3.5$ meV, $E_0$ being the bottom of the 2DEG conduction band, their falling to the hf step must be oblique. Then the dependences $T(E)$ and $R(E)$ obtained above will hold, with $E$ being the energy of motion normal to the slit.

A suggested scheme of a possible device is shown in Fig. 4. We think it will allow us to observe transmission,

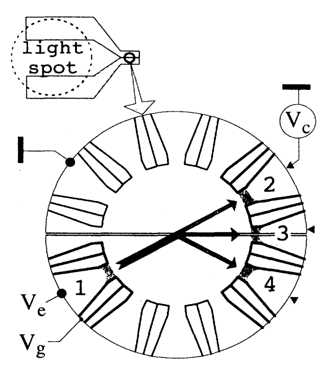


FIG. 4. Suggested scheme of experiment. Strip gates transfer the alternating voltage from the area exposed to microwave irradiation to the slit (at the top). The size of the light spot is about 1 mm while the slit is less than 40 nm wide. Beneath the slit, in the 2DEG plane, 12 point contacts (PC's) are formed by etching around a dial which is about 6 $\mu$m in diameter (magnified, at the bottom). The dark lines that outline the gates of PC's denote the grooves less than 100 nm wide etched in the $n^+$-$Al_xGa_{1-x}As/GaAs$ structure up to the spacer layer. Control and measurement contacts to different regions of 2DEG are marked with circles and triangles, respectively. Quantum PC's used as the emitter (1) and collectors (2–4) are shaded, while other PC's are kept open to apply the voltage $V_e$ between the emitter and base. The forked arrow specifies the flows of ballistic electrons.

reflection, and abrupt cooling of electrons at the same time. For this purpose, the point contacts are placed in such a way that electrons are emitted at an angle (30° in Fig. 4) to the slit. Under microwave irradiation the hf step appears beneath the slit which is acting as antenna. Similar antennas were already used to impose a hf addition to the potential of a quantum point contact[4] and of a quantum dot.[6] In our case the amplitude of the hf step is expected to be 0.1–1 meV at the frequency 1 THz. In other words, the power of the irradiation source should be sufficient to provide hf charge redistribution at the surface of the structure, in the $\delta$-doped layer and in the 2DEG (at least near the slit). This assumption has been already made in application to split gate structures irradiated with microwaves.[5,15]

In the device proposed, the emitter (1) and collector (2–4) point contacts are made by etching. Their penetrability is controlled with the voltages $V_g$ applied to narrow 2DEG regions. These regions are separated from the base, emitter, and collector by etching. To supply ballistic electrons with kinetic energy $\approx 5$ meV in the base, it is biased by 1.5 mV with respect to the emitter. The current $I \approx 10^{-7}$ A through the multimode quantum point contact (1) with resistance $R \approx 10^3\ \Omega$ determines the energy spread of ballistic electrons, $\Delta E = eIR \approx 0.1$ meV. The collector point contact (2) serves to receive ballistic electrons passed through the hf step elastically. It is placed opposite to the emitter contact. The electrons which are elastically reflected from the hf step and turned by 60° are received by the contact (4). The contact (3) serves for catching the electrons deflected by 30° due to the abrupt decrease of the normal velocity component via inelastic reflection and transmission. As in Ref. 3, the response of a collector contact to an electron hit is measured by the voltage difference between the base and a collector (2–4) whereas the emitter current is kept constant. Each contact can be calibrated by the flow of ballistic electrons from the opposite point contact in the absence of the hf field. The device also allows impingement angles of 60° and 90°.

In summary, it is shown analytically and numerically that a hf potential step acts like a semitransparent mirror for the particles of energy $E = l\hbar\omega$. An experimental scheme is suggested to observe elastic and inelastic resonant reflection of ballistic electrons from the hf step. The hf step appears at the electron path in the 2D electron gas of $n^+$-$Al_xGa_{1-x}As/GaAs$ semiconductor heterostructure below the narrow slit between strip gates when it is subjected to microwave irradiation.

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