Instability of a tilted vortex line in magnetically coupled layered superconductors

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We calculate the line tension of a tilted stack of magnetically coupled two-dimensional pancake vortices held at the ends and show that the line tension becomes negative at an angle of 52°. We further calculate the tilt modulus at arbitrary angle θ and show that it becomes negative starting at the same angle 52° for long wavelengths. A structure for the vortex line is given, which is shown to have a lower energy.

I. INTRODUCTION

One of the important characteristics of the high-temperature oxide superconductors (HTS) is their layered structure. As a result, a number of their properties, such as the electrical resistivity, magnetic penetration depth, and critical current density, exhibit strong anisotropy as function of the angle of the applied field relative to the c axis.[1–13] For not too large anisotropy [e.g., $YBa_2Cu_3O_7$, where the anisotropy factor $\gamma=5-7$ (Refs. 14–19)], the anisotropic Ginzburg-Landau or London theories are applicable.[20–26] On the other hand, for high anisotropy [e.g., Bi-2212, where $\gamma=50-200$ (Refs. 9,27–33)] the discreteness of the structure becomes relevant. Such systems become quasi-two-dimensional, as the layers are weakly coupled by a Josephson term that expresses the tunneling current of Cooper pairs.

As a consequence of the layered structure of the HTS, the magnetic vortex line is no longer a usual Abrikosov vortex.[34] It consists of disklike or pancake-shaped current patterns with superconducting cores[35,36] of radius ξ_{ab} in the CuO planes, where ξ_{ab} is the coherence length of the order parameter in the ab plane. The two-dimensional (2D) pancake vortices in different conducting planes are connected to each other by Josephson strings.[37] This structure is best described by the Lawrence-Doniach model,[38–43] where a series of superconducting layers, governed by two-dimensional Ginsburg-Landau equations, interact with each other via a Josephson coupling term. The strength of the Josephson term depends on the anisotropy factor of the material. The higher the anisotropy, the weaker the Josephson interaction is.

When the applied magnetic field is tilted away from the direction of the crystal c axis, unusual features develop in the structure of the vortex lattice. It has been argued in Ref. 40 that for high anisotropy a combined vortex lattice, one lattice parallel to the c axis and the other parallel to the ab planes, is more favorable than a tilted vortex lattice. Some recent works have shown such a possibility (see, e.g., Refs. 44–48). Such behavior becomes even more likely when the Josephson coupling between the layers vanishes.

Given the apparent relevance of the pancake-vortex model[35] for highly anisotropic materials, such as Bi and Tl compounds, where the Josephson coupling is very weak, we here explore, within the framework of this model, the structure of the vortex line as a function of the tilting angle at low fields.

This paper is organized as follows. In Sec. II we briefly review the model of magnetically coupled 2D pancake vortices in layered superconductors.[35] In Sec. III we calculate the line tension of a vortex line tilted from the c axis and show that the line tension becomes negative at an angle of 52°, indicating that the tilted vortex line becomes unstable. In Sec. IV we calculate the tilt modulus $C_{44}(k)$ for small distortions and show that an instability of the vortex line occurs at the same angle of 52°. In Sec. V we propose another kinked structure for a vortex line that has a lower energy, and in Sec. VI we present a brief summary of our results.

II. MAGNETICALLY COUPLED 2D PANCAKE VORTICES

For a stack of superconducting layers (separated by a distance s; see Fig. 1), with one 2D pancake vortex in the central layer and no pancake vortices in the other layers, the vector potential $\mathbf{a}=\phi a_\phi(\rho,z)$ is given by[35]

$$a_\phi(\rho,z)=\int_0^\infty dq\, A(q)J_1(q\rho)Z(q,z), \tag{1}$$

where $Z(q,z_n)=\exp(-Q|z_n|)$, $n=0,\pm 1,\pm 2,\ldots$, $J_1(q\rho)$ is a Bessel function of order 1, and $\rho=\sqrt{x^2+y^2}$. The boundary condition at each plane is given by the sheet current, $\mathbf{K}=-(c/2\pi\Lambda)[\mathbf{a}+(\phi_0/2\pi)\nabla\gamma]$, where $\Lambda=2\lambda_{ab}^2/s$, ϕ_0 is the flux quantum, λ_{ab} is the penetration depth, and γ is the phase of the order parameter. For small q $(q\ll s^{-1})$,

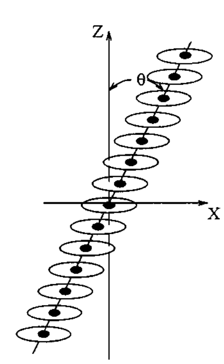

FIG. 1. A vortex line tilted by an angle θ represented as a tilted stack of 2D pancake vortices.

$$A(q)=(\phi_0/2\pi\Lambda)(q^2+\lambda_{ab}^2)^{-1/2}, \tag{2}$$

which gives, for the current in the same plane as the pancake vortex,

$$K_\phi(\rho,0)=(c\phi_0/4\pi^2\Lambda\rho)[1-(\lambda_{ab}/\Lambda)(1-e^{-\rho/\lambda_{ab}})], \tag{3}$$

while the current generated by this vortex in the other layers is

$$K_\phi(\rho,z_n)=-(c\phi_0\lambda_{ab}/4\pi^2\Lambda^2\rho)(e^{-|z_n|/\Lambda_{ab}}-e^{-r_n/\Lambda_{ab}}), \tag{4}$$

where $z_n=ns$ and $r_n=(\rho^2+z_n^2)^{1/2}$. The force acting between different vortices is given by $F_\rho=K_\phi\phi_0/c$. It can be readily seen that two vortices in the same plane repel each other, whereas they attract each other if they are in different planes.

Knowing the force acting between the vortices, we can calculate the energy of arbitrarily positioned vortices in the layers. For instance, as shown in Ref. 35 the line energy of a tilted vortex line, i.e., a straight stack of pancake vortices tilted at an angle θ from the c axis, is

$$\epsilon(\theta)=(\phi_0/4\pi\lambda_{ab})^2\ \ln[(\lambda_{ab}/\xi_{ab})(1+\cos\theta)/2\ \cos\theta]\cos\theta. \tag{5}$$

III. LINE TENSION

The line tension of a vortex $P(\theta)$ is related to its line energy $\epsilon(\theta)$ (Refs. 49,50) via the relation

$$P(\theta)=\epsilon(\theta)+\frac{\partial^2\epsilon(\theta)}{\partial\theta^2}, \tag{6}$$

where the second term on the right-hand side arises from the anisotropy of the system. In the case of magnetically coupled vortices, the line tension calculated from Eq. (5) is

$$P(\theta)=(\phi_0/4\pi\lambda_{ab})^2\frac{\cos\theta-\sin^2\theta}{\cos\theta(1+\cos\theta)}. \tag{7}$$

The line tension thus becomes negative at the angle $\theta_0=\cos^{-1}[(-1+\sqrt{5})/2]$, which is $\theta_0=51.8°$ (see Fig. 2). Notice that this angle does not depend on the characteristics of the material (λ_{ab} or s). Similarly, we expect an instability of the vortex line to occur at the same angle for a long wavelength tilting wave. This is investigated in the next section.

IV. INSTABILITY OF THE VORTEX LINE UNDER A TILTING WAVE

We calculate the elastic energy of the vortex lattice tilted at an angle θ under small fluctuations occurring along the vortex line in the low field regime in the harmonic approximation.[50,51]

$$E_{\rm el}=\frac{1}{2}\sum_{m\neq n}\sum_{\mu}\sum_{i,j}\frac{1}{2}(u^i_{\mu n}-u^i_{\mu m})\times(u^j_{\mu n}-u^j_{\mu m})\nabla_i\nabla_jV(\rho,z)|_{\rm equilib}, \tag{8}$$

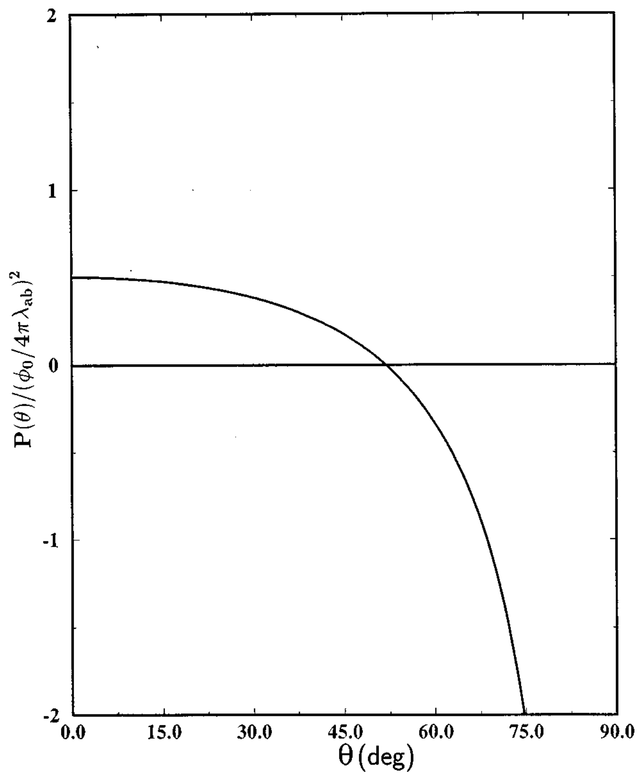

FIG. 2. The line tension $P(\theta)$, calculated from Eq. (7).

where m and n denote the planes, μ labels the pancake vortex in the plane m or n, and $i,j=x,y$. The potential V is obtained from Eq. (4).

Using the expressions for $\mathbf{u}_{\mu n}$ and $V(\rho,z_n)$

$$\mathbf{u}_{\mu n}=\int\frac{d\mathbf{k}}{(2\pi)^3}e^{i\mathbf{k}_\parallel\cdot(\mathbf{R}_\mu+\mathbf{r}_{\parallel,n})}\tilde{\mathbf{u}}(\mathbf{k}), \tag{9}$$

$$V(\rho,z_n)=\int\frac{d^2q}{(2\pi)^2}(e^{i\mathbf{q}\cdot\mathbf{r}_{\parallel,n}}-1)\tilde{V}(q,z_n), \tag{10}$$

where $Q^2=q^2+\lambda_{ab}^{-2}$, $z_n=ns$, $|\mathbf{r}_{\parallel,n}|=\rho=z_n\tan\theta$, $\mathbf{k}_\parallel=(k_x,k_y)$, $r_n=z_n/\cos\theta$, and

$$\tilde{V}(q,z_n)=-\frac{\phi_0^2}{2\pi\Lambda^2}\frac{1}{q^2Q}e^{-Q|z_n|}, \tag{11}$$

which is obtained from Eq. (4), we can write the elastic energy in Fourier space as

$$E_{\rm el}=\frac{\phi_0^2n_\phi}{4\pi\Lambda^2 s}\sum_{i,j}\int\frac{d\mathbf{k}}{(2\pi)^3}\sum_{n\neq0}\int\frac{d^2q}{(2\pi)^2}\frac{q_iq_j}{q^2Q}e^{-Q|z_n|}e^{i\mathbf{q}\cdot\mathbf{r}_{\parallel,n}}\times[1-e^{i(k_zz_n+\mathbf{k}_\parallel\cdot\mathbf{r}_{\parallel,n})}]\tilde{u}_i(\mathbf{k})\tilde{u}_j(-\mathbf{k}), \tag{12}$$

where $n_\phi=B\cos\theta/\phi_0$. We may write the elastic energy $E_{\rm el}$ in the form

$$E_{\rm el}=\frac{1}{2}\sum_{i,j}\int\frac{d\mathbf{k}}{(2\pi)^3}\Phi^{ij}(\mathbf{k})\tilde{u}_i(\mathbf{k})\tilde{u}_j(-\mathbf{k}), \tag{13}$$

where $\Phi^{ij}(\mathbf{k})$ is the elastic matrix, whose diagonal elements are given by

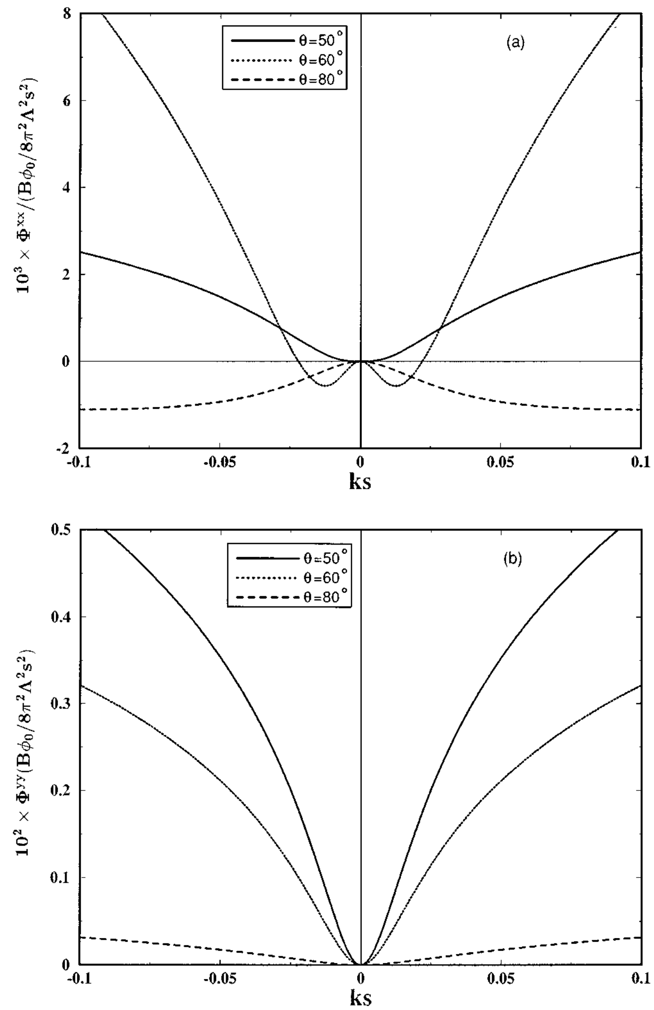

FIG. 3. (a) $\Phi^{xx}(k_z)$ as a function of $k_z s$ for $\theta = 50°$, $60°$, and $80°$. (b) $\Phi^{yy}(k_z)$ as a function of $k_z s$ for $\theta = 50°$, $60°$, and $80°$.

$$\Phi^{xx}(\mathbf{k}) = \frac{B\phi_0 \cos\theta}{16\pi^2}\sum_{n>0}\left[\frac{s\ \cos\theta}{n\lambda_{ab}}e^{-ns/\cos\theta\lambda_{ab}} - \frac{1}{n^2\ \tan^2\theta}[e^{-ns/\lambda_{ab}} - e^{-ns/\lambda_{ab}\ \cos\theta}][1-e^{i\mathbf{k}\cdot\mathbf{r}_n}]\right] \tag{14}$$

and

$$\Phi^{yy}(\mathbf{k}) = \frac{B\phi_0 \cos\theta}{16\pi^2 s^2\lambda_{ab}^2}\sum_{n>0}\left[\frac{1}{n^2\ \tan^2\theta}[e^{-ns/\lambda_{ab}} - e^{-ns/\lambda_{ab}\ \cos\theta}] \times[1-e^{-i\mathbf{k}\cdot\mathbf{r}_n}]\right]. \tag{15}$$

The off-diagonal terms are zero. These are the two transverse eingenmodes of the vortex line. $\Phi^{yy}(\mathbf{k})$ is always positive [see Fig. 3(b)]. On the other hand, $\Phi^{xx}(\mathbf{k})$ becomes negative at angles greater than 52° for small wave vectors k [see Fig. 3(a)], as expected. To show this, we note that for small wave vectors k (i.e., $ks \ll 1$), the sum over n can be done analytically,

$$\Phi^{xx}(k_\parallel, k_z) = \frac{B\phi_0}{16\pi^2\lambda_{ab}^2}\frac{\cos\theta - \sin^2\theta}{\cos\theta(1+\cos\theta)}k_z^2. \tag{16}$$

In the long wavelength limit, the tilt modulus $C_{44}(\mathbf{k})$ is given by

$$\Phi^{xx}(\mathbf{k})/k_z^2, \tag{17}$$

and the line tension $P(\theta)$ is related to the tilt modulus through the relation[50]

$$P(\theta) = \frac{\phi_0}{B}C_{44}(\mathbf{k}), \tag{18}$$

which gives back the expression given in Eq. (7).

Since the vortex line becomes unstable beyond the tilting angle 52°, this suggests that a vortex line held at its ends takes another structure, which we will investigate in the next section.

V. AN ALTERNATIVE STRUCTURE FOR THE TILTED VORTEX LINE

Since the line tension and the tilt modulus become negative at the angle of 52°, the vortex line becomes unstable. The 2D pancake vortices making up the vortex line therefore have to arrange themselves in another form. We suggest below another structure (see Fig. 4) that we show to have a lower free energy.

Using Eqs. (10) and (11) we can express the pair interaction

$$V(\rho_n, z_n) = \left(\frac{\phi_0}{2\pi\Lambda}\right)^2\int_0^\infty \frac{dq}{qQ^2}[1-J_0(q\rho)]e^{-Q|z_n|}. \tag{19}$$

This is the building block needed to calculate either the energy of a kinked structure or the energy of a tilted stack of 2D pancakes tilted by an angle θ, starting from a straight stack of 2D pancake aligned along the z axis.

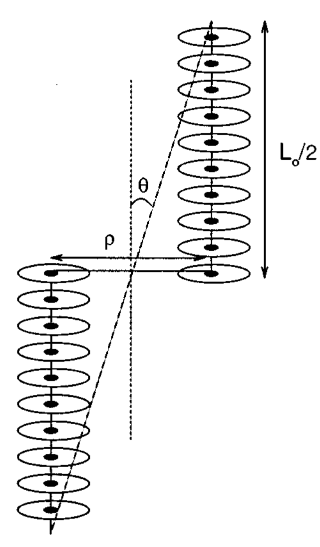

FIG. 4. The "kinked structure," with L_0 the height of the stack and $\rho = L_0\ \tan\theta$.

Although we consider a stack of 2D pancakes of finite height L_0, for the kinked structure we take advantage of the exponentially decaying force between the pancakes to sum over an infinite number of pancakes.

The energy required to deform a straight stack of pancakes into the kinked structure shown in Fig. 4 is given by

$$E_{\text{kink}}=\left(\frac{\phi_0}{2\pi\Lambda}\right)^2\sum_{m=1}^{\infty}\sum_{n=m}^{\infty}\int_0^{\infty}\frac{dq}{qQ}[1-J_0(q\rho)]e^{-Qz_n}, \tag{20}$$

where $\rho=L_0\tan\theta$.

This equation can be rewritten as

$$E_{\text{kink}}=\left(\frac{\phi_0}{2\pi\Lambda}\right)^2\sum_{n=1}^{\infty}\int_0^{\infty}\frac{dq}{qQ}[1-J_0(q\rho)]ne^{-Qz_n}. \tag{21}$$

For small s, the sum over n can be converted to an integral, and E_{kink} takes the form

$$E_{\text{kink}}=\left(\frac{\phi_0}{2\pi\lambda_{ab}}\right)^2\frac{\lambda_{ab}^3}{s^2}[(e^{-\rho/\lambda_{ab}}-1)+\ln(\rho/\lambda_{ab}) -E_i(-\rho/\lambda_{ab})+C], \tag{22}$$

where E_i is the exponential-integral function[52] and C is Euler's constant, $C=0.577\ldots$.

On the other hand, the energy required to tilt a stack of a straight stack of 2D pancakes of height L_0 by an angle θ is given by[35]

$$E_{\text{tilt}}=\left(\frac{\phi_0}{4\pi\lambda_{ab}}\right)^2\frac{\lambda_{ab}^2}{s^2}L_0\ \ln\left(\frac{1+\cos\theta}{2\ \cos\theta}\right). \tag{23}$$

It can be seen from the two expressions of energies, E_{kink} and E_{tilt}, that for large L_0, $E_{\text{kink}}\sim\ln(L_0)$ while $E_{\text{tilt}}\sim L_0$. Therefore, for moderately large angles and a large number of pancakes the kinked struture requires less energy to be formed than the tilted structure. As shown in Fig. 5, when we start tilting the straight stack of 2D pancakes away from a vertical configuration, the tilted structure is first energetically more favorable, but the kinked structure then becomes energetically more stable at small angles ($\sim 5°$).

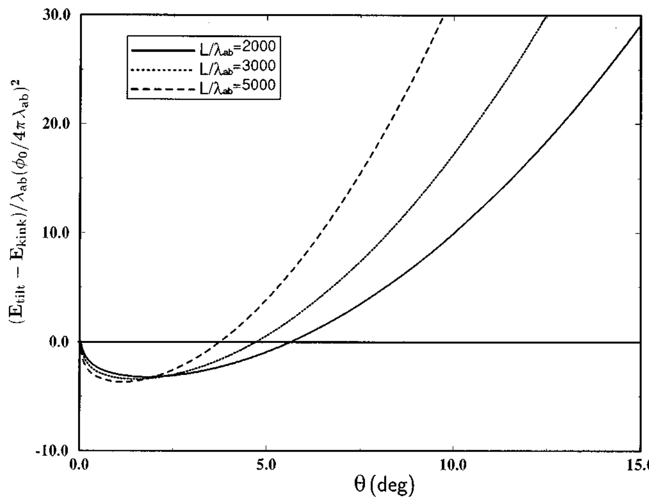

FIG. 5. Energy comparison of a kinked and a tilted stack of 2D pancakes for $L_0/\lambda_{ab}=2000$, 3000, and 5000.

VI. CONCLUSION

We have shown that a tilted vortex line, i.e., a tilted stack of 2D pancake vortices, with the tilt maintained by holding the pancake vortices at the ends of the stack, becomes unstable at a universal angle (52°). This instability, which is due to the absence of the Josephson coupling, is seen from both the expressions of the tilt modulus and the line tension. For moderately large angles a kinked structure, such as that shown in Fig. 4, has a lower energy than that of a uniformly tilted stack of pancake vortices.

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