Charged particles in random magnetic fields and the critical behavior in the fractional quantum Hall effect

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As a model for the transitions between plateaus in the fractional quantum Hall effect we study the critical behavior of noninteracting charged particles in a static random magnetic field with finite mean value. We argue that this model belongs to the same universality class as the integer quantum Hall effect. The universality is proved for the limiting cases of the lowest Landau level, and slowly fluctuating magnetic fields in arbitrary Landau levels. The conjecture that the universality holds in general is based on the study of the statistical properties of the corresponding random matrix model.

The integer (IQHE) and fractional quantum Hall effects (FQHE) show remarkable similarities despite the differences in their origin. While the fundamental excitation gap is due to the strong magnetic field in the IQHE,[1] strong Coulomb correlations are responsible for the gap in the FQHE.[2] However, in both effects the localization of electrons and quasiparticles, respectively, is believed to be responsible for the formation of the plateaus in the Hall conductivity.[2–4] At the transitions between successive plateaus in the IQHE scaling behavior has been observed.[5,6] This has been successfully interpreted as a disorder-induced localization-delocalization transition for noninteracting electrons.[6,7] Most remarkably, it was found experimentally that the temperature-dependent scaling behavior of the transition between the FQH plateaus at filling factors 1/3 and 2/5 is described by the same scaling exponent as the transitions between integer quantum Hall plateaus.[8] Similar agreement was obtained for the transition from filling factor 2/3 to 1.[9]

A theoretical description that makes the similarity between integer and fractional QHE explicit is the "composite fermion" (CF) theory of the FQHE.[10] It relates states of the interacting electron system at filling factor

$$\nu = \nu'/(\nu' p \pm 1) \tag{1}$$

to states of noninteracting electrons at filling factor ν' by attaching an even number p flux quanta to each electron. The magical filling factors of the interacting electron system are interpreted as filled Landau levels ($\nu' =$integer) of the CF's. Based on this approach, Jain, Kivelson, and Trivedi argued that transitions between two FQH plateaus fall into the universality class of the IQHE if these correspond to successive filled Landau levels of the CF's.[11] The transitions for which scaling behavior was observed correspond to the transitions from $\nu' = 1$ to $\nu' = 2$ for $p = 2$ and both signs in Eq. (1).[10]

Formally, the attachment of flux quanta can be achieved by the introduction of a Chern-Simons vector potential.[12,13] In a mean-field approximation this theory describes noninteracting CF's in a uniform magnetic field corresponding to the filling factor ν' of the CF's. While on the mean-field level the universality of integer and fractional QH transitions is thus manifest, the Chern-Simons field is a dynamical gauge field and one has to worry about the effects of its fluctuations. While not much is known about the influence of the dynamics of the gauge field on the localization properties, the effects of static fluctuations in the magnetic field have recently attracted a lot of attention, in particular in the context of the FQH system at filling factor 1/2. Static fluctuations in the Chern-Simons field are due to static fluctuations in the electron density that are induced by a residual disorder potential. Most discussions in the literature focused on noninteracting charged particles in a fluctuating magnetic field with vanishing mean value relevant to the filling factor 1/2. The results are rather controversial. Some authors[14–17] claim to present evidence for a localization-delocalization transition in contrast to the scaling theory[18] according to which states in two-dimensional systems are localized in the absence of a strong magnetic field. However, other authors, while observing a strong enhancement of the localization length, find no true transition.[19–21] We will consider the situation relevant to the transitions in the FQHE. Since the FQH plateaus only form if the CF Landau levels are well separated we will assume that the average magnetic field is strong compared to the fluctuations. In this limit we will show that the critical behavior is the same as that in the IQHE.

In this paper we treat the model of noninteracting charged particles in a random magnetic field with a nonzero average magnetic field. If the average magnetic field is strong compared to the fluctuations this model describes the transitions between FQH plateaus if the charged particles are thought of as noninteracting composite fermions. We will assume that the average magnetic field is strong enough to neglect the coupling between different Landau levels. Then there are two limits in which the fluctuating magnetic field is strictly equivalent to a random electrostatic potential: first, if only states of the lowest Landau level are occupied for arbitrary correlation length of the fluctuating magnetic field; second, if the correlation length of the fluctuations is large compared to the average cyclotron radius for arbitrary Landau level index. The latter situation corresponds to the semiclassical limit studied previously.[22] In general, a fluctuating magnetic field is not equivalent to an electrostatic potential. However, in the limit of well-separated Landau levels the statistics of matrix elements of the random magnetic field Hamiltonian and a random electrostatic potential is quite similar. While the differences will be reflected in nonuniversal quantities like the density of states, we conjecture that they do not lead to different critical properties. This conjecture stands on the same footing as the universality in the IQHE that has only been demonstrated numerically for short correlation lengths in the two lowest Landau levels and in the semiclassical limit of large correlation length.[6]

Our conclusions are based on the properties of the random matrix model generated by projecting the Hamiltonian onto the Landau levels of the average magnetic field B_0. The Hamiltonian H containing the Chern-Simons vector potential $\mathbf{a}$ can be expressed as a sum of the Hamiltonian H_0 of the system with constant magnetic field $B_0\mathbf{e}_z=\nabla\times\mathbf{A}$ and a part H' due to the fluctuating Chern-Simons field,

$$H=\frac{1}{2m^*}(\mathbf{p}-e\mathbf{A}-e\mathbf{a})^2, \tag{2}$$

$$H=H_0+H', \tag{3}$$

$$H_0=\frac{1}{2m^*}(\mathbf{p}-e\mathbf{A})^2, \tag{4}$$

$$H'=\frac{1}{2m^*}\{-e[(\mathbf{p}-e\mathbf{A})\mathbf{a}+\mathbf{a}(\mathbf{p}-e\mathbf{A})]+e^2\mathbf{a}^2\}. \tag{5}$$

The matrix elements $\langle Nk|H'|N'k'\rangle$ of H' with the eigenstates $|Nk\rangle$ of H_0 form a random matrix. Its statistical properties can be compared to those of the random Landau matrix $\langle Nk|V|N'k'\rangle$ where $V(\mathbf{r})$ is a random electrostatic potential. The latter model has been extensively studied and describes the transitions between integer QH plateaus.[6] We will show that the matrix elements of H' have similar statistics to those of $V(\mathbf{r})$ in the limit of strong average magnetic field B_0, despite the rather different nature of the operators H' and $V(\mathbf{r})$. More precisely, we consider the limit in which the fluctuations of the random magnetic field $b(\mathbf{r})\mathbf{e}_z=\nabla\times\mathbf{a}(\mathbf{r})$ are small compared to the average field B_0 [we choose the average of $b(\mathbf{r})$ to vanish]. In this limit the term quadratic in $\mathbf{a}$ can be neglected in H' and the coupling between different Landau levels of H_0 becomes negligible. The intra-Landau-level matrix elements of H' are then given by

$$\langle Nk|H'|Nk'\rangle=\frac{\hbar e}{m^*}\left(\frac{1}{2}\langle Nk|b(\mathbf{r})|Nk'\rangle+\sum_{n=0}^{N-1}\langle nk|b(\mathbf{r})|nk'\rangle\right). \tag{6}$$

This is our main result. It contains only matrix elements of the gauge-invariant local magnetic field $b(\mathbf{r})$. The quantity $\hbar\omega_c(\mathbf{r})=\hbar eb(\mathbf{r})/m^*$ is the deviation of the local cyclotron energy from the average value $\hbar\Omega=\hbar eB_0/m^*$. It follows from Eq. (6) that in the lowest Landau level $N=0$ the random magnetic field is indistinguishable from a random electrostatic potential $V(\mathbf{r})=\hbar\omega_c(\mathbf{r})/2$, irrespective of the statistical properties of the random magnetic field $b(\mathbf{r})$, as has already been noted.[15] When the magnetic field varies sufficiently slowly on the scale of the cyclotron orbit radius $R_c=(2N+1)^{1/2}l_0$, $l_0^2=\hbar/(eB_0)$, its matrix elements become independent of the Landau level index and the random magnetic field is strictly equivalent to the random electrostatic potential $V(\mathbf{r})=(N+1/2)\hbar\omega_c(\mathbf{r})$. In both limits the random magnetic field manifests itself only in the fluctuating cyclotron energy in the Nth Landau level. We can thus apply all the known results for electrostatic disorder to the present system. In particular, the critical behavior is identical and the localization length diverges in the center of the Landau levels with the same exponent as in the IQHE. It further readily follows that the density of states for a white noise distribution of the magnetic field in the lowest Landau level is given by Wegner's result.[23] This is in contrast to the situation in high Landau levels where the density of states differs considerably from the electrostatic disorder case.[24] As the charge e entering the Hamiltonian (2) is the charge of the electrons the critical conductivity of the CF's is the same as that of the electrons as has recently been seen experimentally.[25]

In Ref. 15 it was argued that, in general, the random magnetic field is equivalent to the potential $V(\mathbf{r})=(N+1/2)\hbar\omega_c(\mathbf{r})$ plus gradient corrections. We can discuss this statement if we express Eq. (6) in terms of the Fourier coefficients of $\omega_c(\mathbf{r})$, $\omega_c(\mathbf{r})=\Sigma_{\mathbf{G}}\omega_{\mathbf{G}}\exp(i\mathbf{G}\cdot\mathbf{r})$,

$$\langle Nm|H'|Nm'\rangle=\hbar\sum_{\mathbf{G}}\omega_{\mathbf{G}}e^{-G^2l_0^2/2}G_{m,m'}\left(\frac{Gl_0}{\sqrt{2}}\right)\times\left[\frac{1}{2}L_N\left(\frac{G^2l_0^2}{2}\right)+\sum_{n=0}^{N-1}L_n\left(\frac{G^2l_0^2}{2}\right)\right], \tag{7}$$

where m,m' are angular momentum quantum numbers in the symmetric gauge $\mathbf{A}(\mathbf{r})=B_0(-y\mathbf{e}_x+x\mathbf{e}_y)/2$ and $G_{m,m'}(x)=(m'!/m!)^{1/2}x^{m-m'}L_{m'}^{m-m'}(x^2)$. An effective electrostatic potential $V(\mathbf{r})$, $V(\mathbf{r})=\Sigma_{\mathbf{G}}V_{\mathbf{G}}\exp(i\mathbf{G}\cdot\mathbf{r})$, has the same matrix elements, if

$$V_{\mathbf{G}}L_N\left(\frac{G^2l_0^2}{2}\right)=\hbar\omega_{\mathbf{G}}\left[\frac{1}{2}L_N\left(\frac{G^2l_0^2}{2}\right)+\sum_{n=0}^{N-1}L_n\left(\frac{G^2l_0^2}{2}\right)\right]. \tag{8}$$

We see that the effective electrostatic potential only exists if $\omega_{\mathbf{G}}=0$ for $G^2l_0^2/2$ equal to the zeros of $L_N(x)$. For arbitrary random magnetic fields this is only fulfilled in the lowest Landau level. In particular, there is no effective potential for a white-noise distribution of the magnetic field in higher Landau levels. If the magnetic field is sufficiently smooth, such that $\omega_{\mathbf{G}}=0$ for $G^2l_0^2/2\geqslant x_N^{(1)}$, where $x_N^{(1)}$ is the first zero of $L_N(x)$, then the inverse of $L_N(G^2l_0^2/2)$ can be expanded into a power series in $G^2l_0^2/2$ and the effective potential exists and can be written as a power series in $-l_0^2\nabla^2$ acting on $\hbar\omega_c(\mathbf{r})$, as claimed in Ref. 15.

Since, in general, the random magnetic field is not equivalent to an electrostatic potential even in the limit of strong magnetic field it is not evident that it has the same critical behavior. According to Eq. (6) the random matrix $\langle Nk|H'|Nk'\rangle$ is equivalent not to a single random Landau matrix but to a superposition of N random Landau matrices, all containing the same electrostatic potential but different Landau levels. This leads to differences in physical properties like the density of states. By studying the statistical properties of the matrix elements of H' and comparing them to those of an electrostatic random potential we can argue that these differences are irrelevant for the critical behavior of the system. To this end, we briefly review the construction of the random Landau matrix for electrostatic potentials. For Gaussian correlations of a scalar potential $V(\mathbf{r})$,

$$\overline{V(\mathbf{r})V(\mathbf{r}')}=\frac{V_0^2}{2\pi\sigma^2}\exp\left(-\frac{|\mathbf{r}-\mathbf{r}'|^2}{2\sigma^2}\right), \tag{9}$$

matrix elements $\langle Nk|V|Nk'\rangle$ in Landau gauge are given in terms of uncorrelated random numbers $u(x,k)$, $\overline{u(x,k)u(x',k')}=\delta(x-x')\delta_{k,-k'}$, by[6]

$$\langle Nk_1|V|Nk_2\rangle=\frac{V_0\beta l_0}{\sqrt{2L_y}\pi\sigma}\exp\left(-\frac{\kappa^2 l_0^2\beta^2}{4}\right)\times\int d\xi u(\beta\xi+Kl_0,\kappa l_0)e^{-\xi^2}F_N^V(\xi,\kappa l_0;\sigma), \tag{10}$$

where $K=(k_1+k_2)/2$, $\kappa=k_1-k_2$,

$$F_N^V(\xi,x;\sigma)=(2^N N!)^{-1}\int d\eta\,\exp\left(-\frac{l_0^2+\sigma^2}{\sigma^2}\eta\right)\times H_N\left(\eta+\frac{\xi}{\beta}-\frac{x}{2}\right)H_N\left(\eta+\frac{\xi}{\beta}+\frac{x}{2}\right), \tag{11}$$

L_y is the width of the system, and $\beta^2=(\sigma^2+l_0^2)/l_0^2$. Using this result for Gaussian correlations of the magnetic field,

$$\overline{b(\mathbf{r})b(\mathbf{r}')}=\frac{b_0^2 l_0^2}{\sigma^2}\exp\left(-\frac{|\mathbf{r}-\mathbf{r}'|^2}{2\sigma^2}\right), \tag{12}$$

the matrix elements of H' are given by

$$\langle Nk_1|H'|Nk_2\rangle=\frac{V_0\beta l_0}{\sqrt{2L_y}\pi\sigma}\exp\left(-\frac{\kappa^2 l_0^2\beta^2}{4}\right)\times\int d\xi u(\beta\xi+Kl_0,\kappa l_0)e^{-\xi^2}F_N^B(\xi,\kappa l_0;\sigma), \tag{13}$$

where $V_0^2=2\pi\hbar e b_0^2/(m^{*2}B_0)$, and

$$F_N^B(\xi,x;\sigma)=\frac{1}{2}F_N^V(\xi,x;\sigma)+\sum_{n=0}^{N-1}F_n^V(\xi,x;\sigma). \tag{14}$$

Equations (10) and (13) differ only in the weight functions $F_N^{V,B}$. In both cases these are polynomials of degree $2N$ in ξ and x. The critical behavior in the IQHE is universal if it is the same for all polynomials $F_N^V(\xi,x;\sigma)$. This has been numerically checked for the parameter combinations $(N,\sigma)=(0,0)$, $(0,l_0)$, and $(1,l_0)$.[26] For all other values of N and σ universality in the IQHE is a conjecture. The important features of Eqs. (10) and (13) seem to be the Gaussian factors of ξ and κ that reflect the Landau quantization, while the particular form of the polynomial weight function seems to be rather irrelevant. We therefore conjecture that random matrices of the form (10) and (13) have the same critical behavior for any weight function $F_N(\xi,x;\sigma)$ that is a polynomial in ξ and κ. This implies in particular that the model under consideration belongs to the same universality class as the IQHE.

We will now briefly derive the main result Eq. (6). In a complex notation for vectors in the x-y plane, $z=x+iy$, we can express the Hamiltonian

$$H'=-\frac{e\sqrt{2}\hbar}{4m^* l_0}(\hat{a}_0\bar{a}+\hat{a}_0^\dagger a)+\frac{e^2}{4m^*}a\bar{a}+\text{H.c.}, \tag{15}$$

where $\bar{a}$ denotes the complex conjugate of a, in terms of inter-Landau-level operators

$$\hat{a}_0=\frac{l_0}{\sqrt{2}\hbar}(\Pi_x^0+i\Pi_y^0)\quad\text{and}\quad\hat{a}_0^\dagger=\frac{l_0}{\sqrt{2}\hbar}(\Pi_x^0-i\Pi_y^0),$$

where $\mathbf{\Pi}^0=\mathbf{p}-e\mathbf{A}$. Using a Coulomb gauge for the Chern-Simons vector potential $\mathbf{a},\nabla\cdot\mathbf{a}=0$, we have the following relations between the commutators:

$$[\hat{a}_0^\dagger,a]+[\hat{a}_0,\bar{a}]=0, \tag{16}$$

$$[\hat{a}_0^\dagger,a]-[\hat{a}_0,\bar{a}]=\sqrt{2}l_0 b(z), \tag{17}$$

so that $H'=H_1+H_2+H_3$, with[27]

$$H_1=\frac{1}{2}\hbar\frac{eb}{m^*}, \tag{18}$$

$$H_2=-\frac{e\hbar}{\sqrt{2}ml_0}(\hat{a}_0^\dagger a+\bar{a}\hat{a}_0), \tag{19}$$

$$H_3=\frac{e^2}{2m^*}a\bar{a}. \tag{20}$$

The matrix elements of H_3 are on the average smaller by a factor of b_0/B_0 than the matrix elements of H_1 and H_2 and can be neglected in the limit $b_0\ll B_0$. Applying the Landau level ladder operators $\hat{a}_0$ we get a recursion relation for the matrix elements of H_2 (for clarity we suppress the dependence on the intra-Landau-level quantum numbers k)

$$\begin{aligned}\langle N|H_2|N\rangle&=-\frac{\hbar e}{\sqrt{2}m^* l_0}\langle N|\hat{a}_0^\dagger a+\bar{a}\hat{a}_0|N\rangle\\&=\frac{\hbar e}{m^*}\langle N-1|b|N-1\rangle+\langle N-1|H_2|N-1\rangle\\&=\frac{\hbar e}{m^*}\sum_{n=0}^{N-1}\langle n|b|n\rangle,\end{aligned} \tag{21}$$

thus leading to Eq. (6).

In conclusion, we have studied charged quantum particles in a random magnetic field in the limit that the fluctuations are much weaker than the average magnetic field. This model arises in an approximate treatment of the fermionic Chern-

Simons theory of the FQHE. We have shown that the model studied is strictly equivalent to an electrostatic disorder potential in the two limits of the lowest Landau level and of slowly varying magnetic field. A comparison of the statistical properties of this model with known results for electrostatic disorder led us to conjecture that these two models have the same critical behavior. This implies that static fluctuations of the Chern-Simons vector potential do not change the critical behavior of ‘‘composite fermions’’ and that transitions between Landau levels of the CF’s belong to the same universality class as the IQHE.

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