Mixed-symmetry superconductivity in two-dimensional Fermi liquids

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We consider a two-dimensional (2D) isotropic Fermi liquid with attraction in both s and d channels and examine the possibility of a superconducting state with mixed s and d symmetry of the gap function. We show that both in the weak-coupling limit and at strong coupling, a mixed $s+id$ symmetry state is realized in a certain range of interaction. Phase transitions between the mixed and the pure symmetry states are second order. We also show that there is no stable mixed $s+d$ symmetry state at any coupling.

I. INTRODUCTION

The question of the order parameter symmetry is one of the central issues of high-temperature superconductivity. There is a general consensus that the superconducting gap is highly anisotropic, but whether the gap has a particular symmetry under rotations is still a matter of debate. A number of experiments on $YBa_2Cu_3O_{6+x}$ (YBCO) are roughly consistent with the d-wave symmetry[1] for which the most natural source is the exchange of magnetic fluctuations,[2] but some experiments, e.g., photoemission studies on $Bi_2Sr_2CaCu_2O_{8+x}$ (BSCCO-2212),[3,4] as well as c-axis Josephson tunneling experiments on YBCO,[5] are inconsistent with the pure d wave but more consistent with an $s+d$ state. In principle, the presence of orthorhombic distortion in, e.g., BSCCO-2212 (Ref. 6) in itself guarantees that an otherwise d-wave superconducting gap will have an admixture of the s-wave.[7] However, the superconducting state may be a mixture of s and d components even in the absence of an orthorhombic distortion.

A superconducting state with a mixed $s+d$ symmetry of the gap was first discussed in Ref. 8 and the $s+id$ state in Ref. 9. The mixed symmetry state at intermediate doping levels was also found in variational Monte Carlo studies of the t-J model.[10] An alternative possibility of symmetry mixing caused by interplane coupling was proposed in Ref. 11. More exotic mixed symmetry states have also been suggested.[12,13]

Very recent work has shown[14] that the extent of s-wave admixture is a strong function of t', the second-neighbor hopping, which varies a good deal from one high-T_c material to another. This strongly suggests that the question of s-d mixing should be looked at as a function of hole doping, and that this must be done in each high-T_c material separately.

The variational Monte Carlo calculations indicate that at the doping levels, which favor mixed symmetry states, the ground-state energy is roughly independent of the relative phase θ of s- and d-wave components. This relative phase is of great importance, since only $\theta=0$ and π states can have gap nodes. The experiments on combinations of Josephson junctions on YBCO (Ref. 15) appear to rule out a relative phase of $\pi/2$ if the s-wave amplitude is more than about 10% of the d wave. Recent photoemission work as a function of hole doping,[3,16] indicates that $\theta=0$ and that the relative amplitude of s wave and d wave depends on temperature and the hole doping level.

In view of both the experimental situation, which appears to indicate that s-d mixing is possible in some systems, and the theoretical situation where the relative phase is not well determined, additional understanding of the physics of this phase is needed. In this paper, we address the issue of whether one can obtain the $s+d$ mixed state in calculations on a simple but fairly general model. The answer we obtain is negative—we found that for any coupling, the only possible mixed symmetry in this model is $s+id$.

We consider a model of an isotropic two-dimensional Fermi liquid with attractive interaction in both s and d channels. We assume that both interactions are frequency-independent in a frequency range bounded by the cutoff frequency ω_c, and are zero for $|\omega|>\omega_c$. Obviously, when only one of the two interaction channels is present, the ground state is described by the corresponding pure symmetry gap function. When both interaction channels are present, their competition will lead to either one of the two pure symmetry superconducting states, or a mixed state, where the gap function contains both the s and the d harmonics.

In the next section we will consider the weak-coupling case, where one can use the BCS formalism. We will show that the transition between s and d symmetries occurs via an intermediate phase with mixed $s+id$ symmetry. The two phase transitions between the pure and the mixed states are second order. In Sec. III we consider the case of arbitrary coupling in the framework of the Eliashberg theory. We will show that there always exists a range of relative strengths of the s and d interactions where an $s+id$ solution exists. The analysis of the $s+d$ mixed state is more complicated. However, we can show that at least in both the weak and the strong coupling limits the $s+d$ mixed state does not occur. Our conclusions are summarized in Sec. IV. As an aside, in the Appendix we also present few simple results for the thermodynamics of a d-wave superconductor in the weak-coupling limit, which, to the best of our knowledge, have not been published anywhere else. The main feature is that the ratio of the superconducting gap to the transition temperature, $2\Delta/T_c$, for the d wave is 4.28, larger than 3.53 for the s wave.

II. WEAK COUPLING

In this section we will consider the case when the coupling is weak in both interaction channels. In this case the

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BCS theory is valid, and the gap equation assumes the following form:[17]

$$\Delta(\vec{k}) = -\sum_{\vec{k'}} V_{\vec{k}\vec{k'}} \frac{\Delta(\vec{k'})}{2E_{\vec{k'}}}, \tag{1}$$

where

$$E_{\vec{k}} = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + |\Delta(\vec{k})|^2} = \sqrt{\varepsilon_k^2 + |\Delta(\vec{k})|^2}, \tag{2}$$

and

$$V_{\vec{k}\vec{k'}} = V_s(\vec{k},\vec{k}') + V_d(\vec{k},\vec{k}')\cos[2(\phi-\phi')] \tag{3}$$

is the interaction, which contains both s and d harmonics. The angle ϕ is defined in our two-dimensional model as $\phi = \tan^{-1}(k_y/k_x)$. Consider now a trial mixed state with arbitrary phase difference between the s-wave and the d-wave components of the order parameter:

$$\Delta(\phi) = \Delta_s + e^{i\theta}\Delta_d \cos 2\phi, \tag{4}$$

Separating the real and imaginary parts of this equation, and also the s and d components, we obtain a set of three independent equations

$$\Delta_s = -\sum_{\vec{k'}} \frac{V_s(\Delta_s + \cos\theta\Delta_d \cos 2\phi')}{2\sqrt{\epsilon_{\vec{k'}}^2 + |\Delta(\vec{k'})|^2}}, \tag{5}$$

$$\Delta_d \cos\theta = -\sum_{\vec{k'}} \frac{V_d \cos 2\phi'(\Delta_s + \cos\theta\Delta_d \cos 2\phi')}{2\sqrt{\epsilon_{\vec{k'}}^2 + |\Delta(\vec{k'})|^2}}, \tag{6}$$

$$\cos 2\phi \sin\theta = -\sum_{\vec{k'}} \frac{\sin\theta \cos 2\phi'(V_s + V_d \cos 2\phi \cos 2\phi')}{2\sqrt{\epsilon_{\vec{k'}}^2 + |\Delta(\vec{k'})|^2}}, \tag{7}$$

where $V_s = V_s(k_F, k_F)$ and $V_d = V_d(k_F, k_F)$. It is straightforward to see that if both Δ_s and Δ_d are finite, the set (5–7) can be simultaneously satisfied in only two cases, $\theta = 0$ or $\theta = \pi/2$, leading to $s+d$ or $s+id$, respectively. Thus the weak-coupling theory gives the same restricted set of possibilities for the internal phase angle that Ginzburg-Landau theory offers.[16] Below we consider these two cases separately.

A. $s+d$ state

In this case $\theta = 0$, and Eqs. (5) and (6) become

$$\Delta_s = -\int_0^{2\pi} \frac{d\phi'}{2\pi} \int_{-\omega_c}^{\omega_c} d\epsilon N(0) V_s \frac{\Delta_s + \Delta_d \cos 2\phi'}{4\sqrt{\epsilon^2 + \Delta_s^2 + 2\Delta_s\Delta_d \cos 2\phi + \Delta_d^2 \cos^2 2\phi}}, \tag{8}$$

$$\Delta_d = -\int_0^{2\phi} \frac{d\phi'}{2\pi} \int_{-\omega_c}^{\omega_c} d\epsilon N(0) V_d \frac{\cos 2\phi'(\Delta_s + \Delta_d \cos 2\phi')}{4\sqrt{\epsilon^2 + \Delta_s^2 + 2\Delta_s\Delta_d \cos 2\phi + \Delta_d^2 \cos^2 2\phi}}. \tag{9}$$

Performing the integration over the frequency and doing standard manipulations, we obtain

$$\left(1 - \frac{2g_s}{g_d}\right)\alpha = g_s f(\alpha), \tag{10}$$

where $g_s = -V_s N(0)/4$, $g_d = -V_d N(0)/4$, $\alpha \equiv \Delta_s/\Delta_d$, and the function $f(\alpha)$ is given by

$$f(\alpha) = \int_0^{\pi} \frac{dx}{2\pi}(2\alpha\cos x - 1)(\cos x + \alpha)\ln(\alpha + \cos x)^2. \tag{11}$$

The graphical solution of Eq. (10) is shown in Fig. 1. It is easy to see that in the limit of $\alpha \to \infty$, $f(\alpha) \sim \alpha$, while in the limit of $\alpha \to 0$, $f(\alpha) \sim -\alpha/2$. If we start out with $g_s = 0$, then, naturally, the only solution is $\alpha = 0$, i.e., pure d wave. As g_s increases, the slope of the straight line on Fig. 1 decreases, and at

$$g_s^{(1)} = \frac{g_d}{2+g_d} \tag{12}$$

the lines first cross at $\alpha = \infty$. However, the pure d-wave solution does not become unstable at this point. If we increase g_s even further, we find three solutions: at $\alpha = 0$ (pure d wave), $\alpha = \infty$ (pure s wave), and at some finite α_m, which corresponds to a mixed state. As g_s increases further, α_m decreases and becomes zero at

$$g_s^{(2)} = \frac{g_d}{2 - g_d/2}. \tag{13}$$

For larger g_s, there exists only one solution: $\alpha = \infty$, which corresponds to a pure s-wave state.

We see therefore that the d-wave solution exists at $0 < g_s < g_s^{(2)}$, while the s-wave solution exists at $g_s > g_s^{(1)}$. The key point is that $g_s^{(2)} > g_s^{(1)}$, such that there is an intermediate region $g_s^{(1)} < g_s < g_s^{(2)}$, where both pure solutions exist together with the $s+d$ solution (see Fig. 2). To verify which solutions are stable, we computed the second derivatives of the energy and found that the two pure solutions are stable in the intermediate region, while the $s+d$ state actually corresponds to a maximum rather than a minimum of energy. Clearly then, the $s+d$ state is unstable; if it was the only mixed state allowed, then the system would simply undergo a first-order transition between the two pure states with a region where hysteresis is possible between $g_s^{(1)}$ and $g_s^{(2)}$.

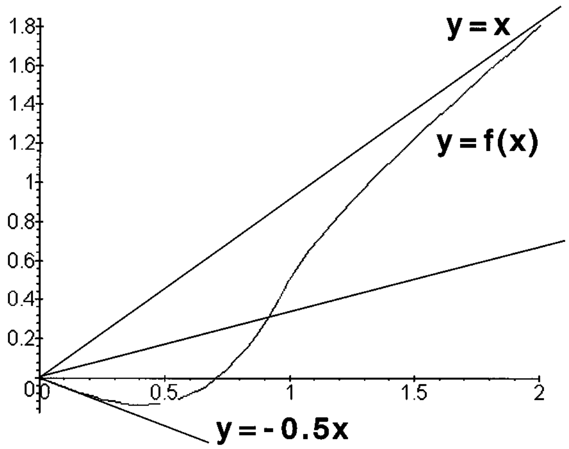

FIG. 1. Graphical solution of Eq. (10) is given by the intersection of the curve $f(\alpha)$ and the straight line.

B. $s+id$ state

In the case of $\theta=\pi/2$, we follow the same procedure. Now the coupled gap equations have the following form:

$$\Delta_d=\int_0^{2\pi}\frac{d\phi'}{2\pi}\int_{-\omega_c}^{\omega_c}d\epsilon g_d\frac{\Delta_d\cos^2 2\phi'}{\sqrt{\epsilon^2+\Delta_s^2+\Delta_d^2\cos^2 2\phi}},\qquad(14)$$

$$\Delta_s=\int_0^{2\pi}\frac{d\phi'}{2\pi}\int_{-\omega_c}^{\omega_c}d\epsilon g_s\frac{\Delta_s}{\sqrt{\epsilon^2+\Delta_s^2+\Delta_d^2\cos^2 2\phi}}.\qquad(15)$$

Integrating over frequency and doing standard manipulations, we obtain that the mixed state exists if

$$1-\frac{2g_s}{g_d}=g_s\left(\frac{1}{2}+\alpha^2-\alpha\sqrt{1+\alpha^2}\right).\qquad(16)$$

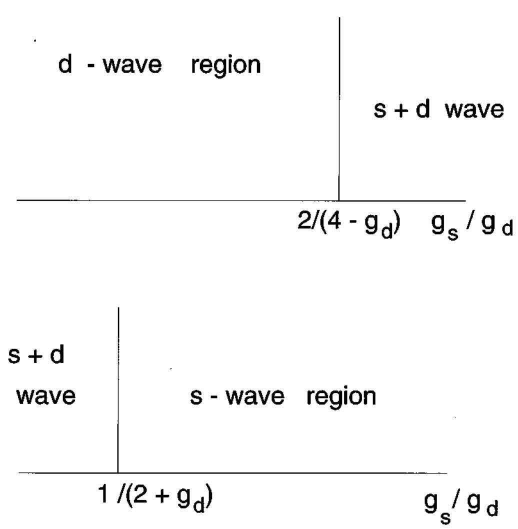

FIG. 2. The location of the phase boundaries for the trial $s+d$ state. The critical point of the transition from $s+d$ to s occurs inside the d phase and vice versa, meaning that there is no region of $s+d$ mixed phase.

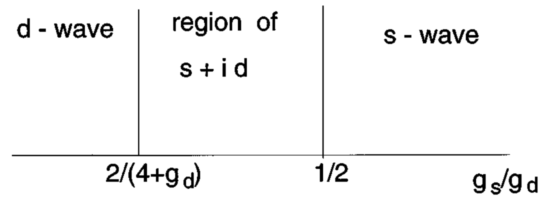

FIG. 3. The phase diagram of the superconductor at zero temperature depending on the ratio of coupling strengths in the two channels.

As before, $\alpha=\Delta_s/\Delta_d$.

As we start out with $g_s=0$, Eq. (16) has no solution, and the gap has pure d symmetry ($\alpha\equiv 0$). However, contrary to the previous case, here a solution of (16) first appears at

$$g_s^{(1i)}=\frac{g_d}{2+g_d/2},\qquad(17)$$

for the same $\alpha=0$. As we increase g_s from $g_s^{(1i)}$, α and, therefore, Δ_s increases continuously, satisfying

$$\alpha=\frac{g_s/2+2g_s/g_d-1}{\sqrt{2g_s(1-2g_s/g_d)}},\qquad(18)$$

and becomes infinite at $g_s^{(2i)}=g_d/2$. Clearly, in this situation, we have a second-order phase transition from pure d to a mixed $s+id$ symmetry state at $g_s=g_s^{(1i)}$, and a second-order transition from a mixed state to pure s state at $g_s=g_s^{(2i)}>g_s^{(1i)}$. In other words, the two pure states, which are stable with respect to $s+d$ mixture, are in fact unstable (for corresponding g_s) with respect to $s+id$ mixture, and in between $g_s^{(1i)}$ and $g_s^{(2i)}$ the $s+id$ state is the equilibrium state of a system (see Fig. 3).

III. STRONG COUPLING

In order to be certain that our results are not an artifact of the weak-coupling approximation, we perform the calculations in the strong-coupling regime. We follow the Eliashberg formalism[18,19] at zero temperature. We assume that the frequency cutoff $\omega_c\ll\epsilon_F$, so that vertex corrections can be neglected according to Migdal theorem.[20]

In the Eliashberg approach, one preserves the frequency dependence of the gap and substitutes the full quasiparticle Green's function in the gap equation. In explicit form, the equations are

$$\tilde\Delta(\vec{k},\omega)$$
$$=-\int\frac{d\omega'}{2\pi}\sum_{\vec{k'}}V_{\vec{k}\vec{k'}}\frac{\tilde\Delta(\vec{k'},\omega')}{\Omega^2(\vec{k'},\omega')+\tilde\Delta^2(\vec{k'},\omega')+\xi^2(\vec{k'})},$$

$$\Omega(\vec{k},\omega)$$

$$=\omega-\int\frac{d\omega'}{2\pi}\sum_{\vec{k'}}V_{\vec{k}\vec{k'}}\frac{\Omega(\vec{k'},\omega')}{\Omega^2(\vec{k'},\omega')+\tilde{\Delta}^2(\vec{k'},\omega')+\xi^2(\vec{k'})},\tag{19}$$

where $\xi(\vec{q})$ is the renormalized single-particle energy, $\xi(q)=v_F(q-p_F),\tilde{\Delta}(\vec{q},\omega)$ is the anomalous part of the self-energy, and $\Omega(\vec{q},\omega)$ is the antisymmetric in ω part of the normal self-energy. As $\omega_c\ll\epsilon_F$, the integration over $|\vec{k}'|$ is confined to a region near the Fermi surface and can be substituted by the integration over ξ, as in BCS theory. This integration is straightforward and yields

$$\tilde{\Delta}(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{(g_s+g_d\cos2\phi)\tilde{\Delta}(\omega',\phi)}{\sqrt{|\Omega(\omega',\phi)|^2+|\tilde{\Delta}(\omega',\phi)|^2}}$$

$$\Omega(\omega,\phi)=\omega+\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\times\frac{(g_s+g_d\cos2\phi)\Omega(\omega',\phi)}{\sqrt{|\Omega(\omega',\phi)|^2+|\tilde{\Delta}(\omega',\phi)|^2}}.\tag{20}$$

We now again consider the two mixed states separately.

A. $s+id$ state

Consider first a mixed $s+id$ state. For this symmetry, the angular decomposition of the self-energy functions yields

$$\tilde{\Delta}(\omega,\phi)=\tilde{\Delta}_s(\omega)+i\tilde{\Delta}_d(\omega)\cos(2\phi),\tag{21}$$

$$\Omega(\omega,\phi)=\Omega_s(\omega)+i\Omega_d(\omega)\cos(2\phi).\tag{22}$$

Accordingly, Eqs. (20) can be broken down into four equations

$$\tilde{\Delta}_d(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_d\tilde{\Delta}_d(\omega')\cos^2 2\phi}{\sqrt{\Omega_s^2(\omega')+\Omega_d^2(\omega')+\tilde{\Delta}_s^2(\omega')+\tilde{\Delta}_d^2(\omega')\cos^2 2\phi}},\tag{23}$$

$$\tilde{\Delta}_s(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_s\tilde{\Delta}^s(\omega')}{\sqrt{\Omega_s^2(\omega')+\Omega_d^2(\omega')+\tilde{\Delta}_s^2(\omega')+\tilde{\Delta}_d^2(\omega')\cos^2 2\phi}},\tag{24}$$

$$\Omega_d(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_d\Omega_d(\omega')\cos^2 2\phi}{\sqrt{\Omega_s^2(\omega')+\Omega_d^2(\omega')+\tilde{\Delta}_s^2(\omega')+\tilde{\Delta}_d^2(\omega')\cos^2 2\phi}},\tag{25}$$

$$\Omega_s(\omega)=\omega+\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_s\Omega_s(\omega')}{\sqrt{\Omega_s^2(\omega')+\Omega_d^2(\omega')+\tilde{\Delta}_s^2(\omega')+\tilde{\Delta}_d^2(\omega')\cos^2 2\phi}}.\tag{26}$$

Equation (25) is homogeneous in Ω_d. For weak coupling its only solution was $\Omega_d\equiv0$. In principle, at strong coupling, there is a chance that above some threshold there exists a solution with a nonzero Ω_d. We will not consider this rather exotic possibility and will instead assume that the stable solution of Eq. (25) corresponds to $\Omega_d=0$ for all couplings.

We now follow the same approach as at weak coupling and look for the transition points between pure s and mixed $s+id$, and pure d and mixed $s+id$ states. In the former case, we linearize the above set of equations around $\tilde{\Delta}_d=0$ and obtain

$$\tilde{\Delta}_d(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_d\tilde{\Delta}_d(\omega')\cos^2 2\phi}{\sqrt{\Omega_s^2(\omega')+\tilde{\Delta}_s^2(\omega')}},$$

$$\tilde{\Delta}_s(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_s\tilde{\Delta}_s(\omega')}{\sqrt{\Omega_s^2(\omega')+\tilde{\Delta}_s^2(\omega')}}.\tag{27}$$

Equations (27) are obviously satisfied when $g_s^{(2i)}=g_d/2$, the same as for weak coupling.

Now consider the transition from the d-wave state into the mixed state. Linearizing Eqs. (23)–(26) with respect to $\tilde{\Delta}_s$, we get

$$\tilde{\Delta}_d(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_d\tilde{\Delta}_d(\omega')}{2\sqrt{\Omega_s^2(\omega')+\tilde{\Delta}_d^2(\omega')\cos^2 2\phi}}+\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_d\tilde{\Delta}_d(\omega')\cos4\phi}{2\sqrt{\Omega_s^2(\omega')+\tilde{\Delta}_d^2(\omega')\cos^2 2\phi}},\tag{28}$$

$$\tilde{\Delta}_s(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_s\tilde{\Delta}_s(\omega')}{\sqrt{\Omega_s^2(\omega')+\tilde{\Delta}_d^2(\omega')\cos^2 2\phi}}.\tag{29}$$

We first observe that if the second term in Eq. (28) were absent, the d-wave solution would become unstable at $g_s^{(1i)}=g_d/2=g_s^{(2i)}$. We now show that this second term yields a negative correction to the first term, independent of what $\tilde{\Delta}_d(\omega')$ and $\Omega_s(\omega')$ are. Indeed, let us perform angular integration first. For the first term in (28), the integrand [apart from $g_d\,\tilde{\Delta}_d(\omega')$] is positive, and the integration yields a positive result. For the second term, we have to evaluate

$$I=\int_0^{\pi}\frac{\cos\phi d\phi}{\sqrt{a+b\cos\phi}}, \tag{30}$$

where $a>b>0$. Doing simple manipulations, we obtain

$$I=\int_0^{\pi/2}d\phi\left(\frac{\cos\phi}{\sqrt{a+b\cos\phi}}-\frac{\cos\phi}{\sqrt{a-b\cos\phi}}\right). \tag{31}$$

Since $\cos\phi$ is positive when $0<\phi<\pi/2$, then I is negative. This simple argument shows that the second term in (28) effectively reduces the value of g_d to $g_d^{\mathrm{eff}}<g_d$. Clearly then, the critical value $g_s^{(1i)}=g_d^{\mathrm{eff}}/2$ for the transition between the pure d state and the $s+id$ mixed state will be *smaller* than $g_d/2\equiv g^{(2i)}$.

The above consideration shows that for arbitrary interaction strength there exists a range of g_s/g_d, where neither of the pure states is stable. We further expanded near each of the transition points up to cubic terms in either Δ_s or Δ_d and indeed found a nonzero solution for this intermediate range of couplings, which implies that in the intermediate region the gap has an $s+id$ symmetry. It turns out therefore that the phase diagram at weak and strong couplings is essentially the same. It is nevertheless interesting that the strong-coupling corrections tend make the mixing of the s wave into a predominantly d wave more favorable.

B. $s+d$ state

Now we study whether it is possible to obtain the $s+d$ mixed symmetry state in equilibrium. This case is significantly more complicated because, unlike the $s+id$ case, the square of the gap function now contains a term that is linear in both s and d components.

However, we will show that at least in the limit of very strong coupling there is no region of $s+d$ symmetry. Indeed, consider the transition, at $g_s=g_s^{(1)}$, between the pure s and the mixed $s+d$ states. Eliashberg equations linearized in $\tilde{\Delta}_d$ and Ω_d read

$$\tilde{\Delta}_s(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_d\tilde{\Delta}_s(\omega')}{\sqrt{\Omega_s^2(\omega')+\tilde{\Delta}_d^2(\omega')}}, \tag{32}$$

$$\Omega_s(\omega)=\omega+\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_s\Omega_s(\omega')}{\sqrt{\Omega_s^2(\omega')+\tilde{\Delta}_d^2(\omega')}}, \tag{33}$$

$$\tilde{\Delta}_d(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_d\tilde{\Delta}_d(\omega')\cos^2 2\phi}{\sqrt{\Omega_s^2(\omega')+\tilde{\Delta}_d^2(\omega')\cos^2 2\phi}}-\frac{g_d\tilde{\Delta}_d(\omega')[\tilde{\Delta}_d(\omega')\tilde{\Delta}_s(\omega')+\Omega_d(\omega')\Omega_s(\omega')]\cos^2 2\phi}{2[\Omega_s^2(\omega')+\tilde{\Delta}_s^2(\omega')]^{3/2}}, \tag{34}$$

$$\Omega_d(\omega)=\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}d\omega'\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{g_d\Omega_d(\omega')}{\sqrt{\Omega_s^2(\omega)+\tilde{\Delta}_d^2(\omega)\cos^2 2\phi}}-\frac{g_d\tilde{\Delta}_d(\omega')[\tilde{\Delta}_d(\omega')\tilde{\Delta}_s(\omega')+\Omega_d(\omega')\Omega_s(\omega')]\cos^2 2\phi}{2[\Omega_s^2(\omega')+\tilde{\Delta}_s^2(\omega')]^{3/2}}. \tag{35}$$

We now show that for $g_s\gg 1$ and $\omega\ll\omega_0$, $\tilde{\Delta}_s(\omega)\gg\Omega_s(\omega)$. Indeed, suppose that this is true. Then it is easy to see that in the region of frequencies we are interested in, $\tilde{\Delta}_s(\omega)$ is frequency independent and equal to $g_s\omega_0$, while $\Omega_s(\omega)$ is linear in ω. Letting $\Omega_s(\omega)=\lambda\omega$, we can rewrite the inequality as $g_s=g_s^{(1)}\gg\lambda$. Then, Eq. (33) becomes

$$\lambda=1+\frac{g_s^{(1)}\lambda}{\omega}\int_{\omega-\omega_0/2}^{\omega+\omega_0/2}\frac{\omega' d\omega'}{\sqrt{\lambda^2\omega'^2+\tilde{\Delta}_s^2}}. \tag{36}$$

Solving (36), we obtain for $g_s^{(1)}\gg 1$

$$\lambda=2(g_s^{(1)})^{2/3}\ll g_s^{(1)}, \tag{37}$$

thus justifying the assumption that $\tilde{\Delta}(\omega)\gg\Omega_s(\omega)$. Furthermore, in this limit, Eq. (34) reduces to

$$1=\frac{g_d}{2}\int_{-\omega_0/2}^{\omega_0/2}\frac{\lambda^2\omega^2 d\omega}{(\tilde{\Delta}_s^2+\lambda^2\omega^2)^{3/2}}. \tag{38}$$

Using (37) we finally obtain that

$$g_s^{(1)}=\left(\frac{g_d}{6}\right)^{3/5}\ll g_d. \tag{39}$$

We see, therefore, that in the strong-coupling limit the hypothetical transition between a mixed $s+d$ and a pure s state occurs at $g_s=g_s^{(1)}\ll g_d$. However, for this ratio of couplings, both the pure s wave and the mixed state clearly must be unstable with respect to the pure d-wave state. A similar analysis shows that the transition between the pure d and the mixed state occurs at $g_s=g_s^{(2)}\sim g_d$. As a result, we again have $g_s^{(2)}>g_s^{(1)}$, which, just as in the weak-coupling case, implies that there is no region of mixed $s+d$ symmetry.

IV. SUMMARY

We have studied in this paper a two-dimensional isotropic Fermi liquid with attractive interaction in both s and d channels. We considered the weak-coupling limit and also applied the Eliashberg formalism at strong coupling.

The phase diagram of the superconductor turns out to be the same at weak and strong coupling. It displays a region with a mixed $s+id$ symmetry gap when the coupling strengths in the two channels are of the same order of magnitude. The phase transitions between the mixed state and the pure s and d states are second order.

We have shown that in the weak- and strong-coupling limits a mixed $s+d$ state does not occur. Intuitively, this can be interpreted as the propensity of the system to choose the state in which the amplitude of the gap function has the largest value. This is also in agreement with the Ginsburg-Landau considerations,[11,16] which suggests that in the absence of orthorhombic distortion the $s+id$ state has lower energy than the $s+d$ state.

Indeed, the model we considered is oversimplified: the two-dimensional isotropic Fermi liquid captures some of features of the high-T_c materials; however, the isotropic system is never close to a magnetic or a metal-insulator transition. Our analysis of the $s+d$ versus $s+id$ question, therefore, does not include such effects. In this sense, what we have shown here is that if the $s+d$ state is realized in real materials, some nontrivial physical effect of the proximity to these phase transitions is likely to be the cause.

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APPENDIX

In this appendix, we present weak-coupling calculations for the thermodynamics of a pure d-wave superconductor. The gap equation at zero temperature is

$$1=g_d\int_{-\omega_0/2}^{\omega_0/2}d\epsilon\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{\cos^2 2\phi}{\sqrt{\epsilon^2+\Delta_d^2\cos^2 2\phi}}. \qquad (A1)$$

After simple manipulations we arrive at

$$\Delta_d=\frac{2\omega_0}{\sqrt{e}}\exp\left(-\frac{1}{g_d}\right). \qquad (A2)$$

At the same time, the transition temperature is given by [17]

$$T_c=\frac{\gamma\omega_0}{\pi}\exp\left(-\frac{1}{g_d}\right), \qquad (A3)$$

where $\ln\gamma=C\approx0.577$ is Euler's constant. Then

$$\frac{2\Delta_d}{T_c}=\frac{4\pi}{\sqrt{e}\gamma}\approx4.28. \qquad (A4)$$

Note, this ratio for the s wave is 3.53. Finally, starting with the gap equation at finite temperature

$$1=g_d\int_{-\omega_0/2}^{\omega_0/2}d\epsilon\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{\cos^2 2\phi}{\sqrt{\epsilon^2+\Delta_d^2}}=1, \qquad (A5)$$

and performing standard expansions at $T\ll T_c$ and $T_c-T\ll T_c$ [17] we find the following expressions for the temperature dependence of the superconducting gap:

$$\Delta_d(T)=\Delta_d(0)\left[1-0.37\left(\frac{T}{T_c}\right)^3\right], \quad T\ll T_c$$

$$\Delta_d(T)=1.65\Delta_d(0)\sqrt{1-T/T_c}, \quad T_c-T\ll T_c. \qquad (A6)$$

[1] D.J. Scalapino, Phys. Rep. **250**, 331 (1995) is a recent review of both experiment and theory of d-wave superconductivity in high-T_c materials.
[2] P. Monthoux, A.V. Balatsky, and D. Pines, Phys. Rev. Lett. **67**, 3448 (1991).
[3] J. Ma, C. Quitmann, R.J. Kelley, H. Berger, G. Margaritondo, and M. Onellion, Science **267**, 862 (1995).
[4] H. Ding, J.C. Campuzano, and G. Jennings, Phys. Rev. Lett. **74**, 2784 (1995).
[5] A.G. Sun, D.A. Gajewski, M.B. Maple, and R.C. Dynes, Phys. Rev. Lett. **72**, 2267 (1994).
[6] M.D. Kirk, J. Nogami, A.A. Baski, D.B. Mitzi, A. Kapitulnik, T.H. Geballe, and C.F. Quate, Science **242**, 1673 (1988).
[7] Q.P. Li, B.E.C. Koltenbah, and R. Joynt, Phys. Rev. B **48**, 437 (1993).
[8] A.E. Ruckenstein, P.J. Hirschfeld, and J. Apel, Phys. Rev. B **36**, 857 (1987).
[9] G. Kotliar, Phys. Rev. B **37**, 3664 (1988).
[10] G.J. Chen, R. Joynt, F.C. Zhang, and C. Gros, Phys. Rev. B **42**, 2662 (1990).
[11] K. Kuboki and P.A. Lee (unpublished).
[12] M. Sigrist, D. Bailey, and R.B. Laughlin, Phys. Rev. Lett. **74**, 3249 (1995).
[13] A.G. Abanov and P.B. Wiegmann (unpublished).
[14] B.E.C. Koltenbah and R. Joynt, Bull. Am. Phys. Soc. **40**, 307 (1995); and (unpublished).
[15] D. van Harlingen, Rev. Mod. Phys. **67**, 515 (1995) reviews this evidence.
[16] J. Betouras and R. Joynt Europhys. Lett. **31**, 119 (1995).
[17] See, e.g., E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics, Part II* (Pergamon, New York, 1991).
[18] G.M. Eliashberg, Sov. Phys. JETP **11**, 696 (1960).
[19] D.J. Scalapino, J.R. Schrieffer, and J.W. Wilkins, Phys. Rev. **148**, 263 (1966).
[20] A.B. Migdal, Sov. Phys. JETP **7**, 41 (1958).