Hall effect and magnetoresistance in copper oxide metals

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(Received 27 June 1995)

We present the solution of the Boltzmann equation with an applied electric and magnetic field including skew scattering in the collision operator. The observed unusual temperature dependences of the Hall resistance and the magnetoresistance as well as the frequency dependence of the Hall resistance in the normal state of copper oxide metals is shown to arise if the skew scattering rate diverges as $T \rightarrow 0$. The implication of this phenomenological result for the microscopic theory of copper oxide metals is that the chiral response function (proportional to the magnetic field) has a singularity in the limit of zero energy and temperature.

I. INTRODUCTION

All transport properties of copper oxide high-temperature superconductors in the normal state have anomalous temperature dependences.[1] The anomalies are particularly simple for concentrations close to those for the highest T_c.[2] In this regime, the Hall "coefficient" for $YBa_2Cu_3O_{7-\delta}$, ($\delta \approx 0$) is[3] $\sim 1/T$ to a good approximation. In conventional metals, the Hall coefficient is, of course, a constant. In $La_{2-x}Sr_xCuO_4$, ($x \approx 0.15$) the Hall coefficient $R_H(T)$ does have a constant term plus the anomalous $1/T$ dependence.[4] Most of the other anomalous transport properties (resistivity, optical conductivity, Raman intensity, tunneling conductance, thermal conductivity) could be understood if one assumed that the copper oxide metals have a marginal-Fermi-liquid self-energy:[5]

$$\Sigma(\omega,q) \sim \omega \ \ln \frac{x}{\omega_c} + ix \ \mathrm{sgn}\omega, \tag{1.1}$$

where $x \approx \max(|\omega|,T)$. This assumption is of no help in understanding the anomalous $R_H(T)$; the scattering rates deduced from (1.1) again cancel in the kinetic theory expression, $R_H(T) \equiv \sigma_{xy}/\sigma_{xx}^2$ giving the customary temperature-independent behavior. More sophisticated treatments only give logarithmic corrections.

Recently, the magnetoresistance $\Delta\rho(H)/\rho$ of $YBa_2Cu_3O_7$ has been carefully measured. Given that $\rho(T) \sim T$ (to a good approximation), the conventional behavior (Kohler's rule) expected is $\Delta\rho(H)/\rho H^2 \sim \rho^{-2} \sim T^{-2}$, whereas an approximate T^{-4} dependence has been observed.[6]

The frequency-dependent Hall "coefficient" $R_H(\omega,T)$ has also recently been measured and found to be unusual.[7]

In this paper we present a solution of the Boltzmann equation with applied electric and magnetic fields including skew scattering in the collision operator. Skew scattering is present in general if a magnetic field is present and gives a scattering on the Fermi surface which is different in the right- and left-handed directions with respect to the applied magnetic field.

We do not know the physical origin of the skew scattering but suspect it is due to scattering of current carriers by some unusual chiral fluctuations. Such inelastic scattering is parametrized here by a temperature-dependent scattering rate τ_s^{-1}, just as ordinary inelastic (and elastic) scattering may be parametrized in the Boltzmann equation with a rate τ_{tr}^{-1}. The skew scattering is introduced in the simplest possible fashion to mimic a symmetry of the collision operator different from that given by ordinary scattering.

We find the remarkable result that all three of the above-mentioned magneto-transport anomalies in copper oxide metals follow if the skew scattering rate diverges as $1/T$. This phenomenology may serve as a pointer in the search of a microscopic theory.

The Hall effect and the magnetoresistance anomalies have been previously rationalized by the assumption that in copper oxide metals, the response to the Lorentz force has a different characteristic rate with a different temperature dependence than the scattering rate τ_{tr}^{-1} observed in the conductivity in zero magnetic field.[3,6,8,9] We shall comment on the feasibility of such a proposal within the framework of the theory of transport provided by the Boltzmann equation as well as its relationship to our proposal of skew scattering. Harris *et al.*[6] have already observed that the Hall effect and magnetoresistance cannot both be understood by postulating different scattering rates at different parts of the Fermi surface.[7]

II. BOLTZMANN EQUATION FOR TRANSPORT INCLUDING SKEW SCATTERING

The Boltzmann equation for the distribution function $f_k(\mathbf{r},t)$,

$$f(\mathbf{k},\mathbf{r},t) \equiv f_{\mathbf{k}}^0 + g_{\mathbf{k}}(t), \tag{2.1}$$

for a spatially uniform electric field $\mathbf{E}(t)$ and magnetic field $\mathbf{B}(t)$ is

$$\frac{\partial g_{\mathbf{k}}}{\partial t} + e\mathbf{E}\cdot\mathbf{v}_{\mathbf{k}} \frac{\partial f_{\mathbf{k}}^0}{\partial \varepsilon_{\mathbf{k}}} + \frac{e}{\hbar c}(\mathbf{v}_{\mathbf{k}} \times \mathbf{B}) \cdot \frac{\partial g_{\mathbf{k}}}{\partial \mathbf{k}} = C_{\mathbf{k}}, \tag{2.2}$$

where $C_{\mathbf{k}}$ is the collision operator, f_k^0 is the equilibrium (Fermi-Dirac) distribution, and $\mathbf{v_k}=\hbar^{-1}\nabla_{\mathbf{k}}\varepsilon_{\mathbf{k}}$.

In the linearized approximation the collision operator may be written

$$C_{\mathbf{k}}=\int d\mathbf{k}'[C(\mathbf{k},\mathbf{k}')g(\mathbf{k}')-C(\mathbf{k}',\mathbf{k})g(\mathbf{k})] \quad (2.3a)$$

$$=-g(\mathbf{k})/\tau(\mathbf{k})+\int d\mathbf{k}'C(\mathbf{k},\mathbf{k}')g(\mathbf{k}'). \quad (2.3b)$$

In (2.3b) the first term, $-g(\mathbf{k})/\tau(\mathbf{k})$, represents the "scattering out" of particles from $\mathbf{k}$, while the other term represents the "scattering in" to $\mathbf{k}$.

For a time-independent perturbation, (2.2) is formally solved as

$$g=eA^{-1}\left[\mathbf{E}\cdot\mathbf{v}\left(\frac{-\partial f_0}{d\varepsilon}\right)\right], \quad (2.4)$$

where

$$\langle\mathbf{k}|A|\mathbf{k}'\rangle=\left(1/\tau(\mathbf{k})+\frac{e}{\hbar c}\,\mathbf{v_k}\times\mathbf{B}\cdot\nabla_{\mathbf{k}}\right)\delta(\mathbf{k}-\mathbf{k}')-C(\mathbf{k},\mathbf{k}'). \quad (2.5)$$

Quite generally C can be written as a sum of two parts:

$$C(\mathbf{k},\mathbf{k}')=C_n(\mathbf{k},\mathbf{k}')+C_s(\mathbf{k},\mathbf{k}'), \quad (2.6)$$

where C_n is the "normal part" which is symmetric in interchange of $\mathbf{k}$ and $\mathbf{k}'$

$$C_n(\mathbf{k},\mathbf{k}')=C_n(k',\mathbf{k}), \quad (2.7)$$

and C_s $(\mathbf{k},\mathbf{k}')$ is the "skew" or antisymmetric part

$$C_s(\mathbf{k},\mathbf{k}')=-C_s(\mathbf{k}',\mathbf{k}), \quad (2.8)$$

which satisfies

$$\int d\mathbf{k}'C_s(\mathbf{k},\mathbf{k}')=0. \quad (2.9)$$

Such a term is present only if time-reversal invariance is broken, for instance if a magnetic field $\mathbf{B}$ is present. A specific representation of $C_s(\mathbf{k},\mathbf{k}')$, for example, is $(\mathbf{k}\times\mathbf{k}')\cdot B$. It follows from (2.9) that $\tau(\mathbf{k})$ does not depend on C_s.

The current is given by

$$\mathbf{j}=e\int d\mathbf{k}\;\mathbf{v_k}g(\mathbf{k}), \quad (2.10)$$

so that using (2.4) the conductivity tensor is

$$\sigma^{\mu\nu}=e^2\int d\mathbf{k}\;d\mathbf{k}'\mathbf{v}_{\mathbf{k}}^{\mu}A^{-1}_{\mathbf{k},\mathbf{k}'}\mathbf{v}^{\nu}_{\mathbf{k}'}\left(-\frac{\partial f_0}{\partial\varepsilon_{\mathbf{k}'}}\right). \quad (2.11)$$

Defining

$$\tau_{\mathrm{tr}}^{-1}(\mathbf{k},\mathbf{k}')=(\tau^{-1}-C_n)_{\mathbf{k},\mathbf{k}'}, \quad (2.12)$$

we can expand A^{-1} in powers of $\mathbf{B}$ (assuming that C_s varies linearly with $\mathbf{B}$), to get

$$A^{-1}=\tau_{\mathrm{tr}}-\tau_{\mathrm{tr}}(\mathbf{v}\times\mathbf{B}\cdot\nabla-C_s)\tau_{\mathrm{tr}}+\tau_{\mathrm{tr}}(\mathbf{v}\times\mathbf{B}\cdot\nabla-C_s)\tau_{\mathrm{tr}}(\mathbf{v}\times\mathbf{B}\cdot\nabla-C_s)\tau_{\mathrm{tr}}+O(B^3). \quad (2.13)$$

Using (2.13) in (2.11) yields the conductivity in zero field σ_{xx} from the first term of (2.13), the Hall conductivity σ_{xy} from the second term of (2.13) which is linear in $\mathbf{B}$, and the magnetoresistance $\Delta\sigma_{xx}=\sigma_{xx}(B)-\sigma_{xx}$ from the third term which is quadratic in $\mathbf{B}$.

Some general features of all the transport coefficients are now apparent from Eqs. (2.11) and (2.13). First, the skew scattering term C_s always occurs in combination with the Lorentz force term $\mathbf{v}\times\mathbf{B}\cdot\nabla$. Indeed, it should be regarded as a temperature-dependent renormalization of the Lorentz force. Second the Lorentz force in (2.13) always appears multiplied by τ_{tr}^{-2} where τ_{tr}^{-1} is precisely the same transport rate which appears in the conductivity σ_{xx} at $B=0$. The suggestion by Anderson[3,6,8] that the Lorentz force term be multiplied by $\tau_{\mathrm{tr}}^{-1}\tau_H^{-1}$, where τ_H^{-1} is a rate with some different physical origin and temperature dependence cannot directly be implemented within the general phenomenological theory of transport coefficients given by the Boltzmann equation. New physics may however be sought within the Boltzmann framework from the skew scattering C_s.

III. EVALUATION OF TRANSPORT COEFFICIENTS

In order to express our principal physical point clearly, we now make some simplifications. We consider the normal scattering matrix $C_n(\mathbf{k},\mathbf{k}')$ to be independent of $\mathbf{k}$ and $\mathbf{k}'$ for $\mathbf{k},\mathbf{k}'$ close to the Fermi surface such that ε_k, $\varepsilon_{k'}$ are $O(kT)$ of the Fermi energy. Then τ_{tr}^{-1} depends only on the temperature. With the transport properties of copper oxides in mind we consider the case of two dimensions only.

For $T\ll E_F$, the integrals in (2.11) can be expressed as surface integrals on the Fermi surface by using $df/d\varepsilon_{\mathbf{k}}=-\delta(\varepsilon_{\mathbf{k}}-\varepsilon_F)$ and choosing coordinates $\mathbf{k}\to(\varepsilon,s)$ where $\varepsilon=\varepsilon_k$ and s is normal to the constant energy contour $\varepsilon_{\mathbf{k}}$. The transport coefficients obtained from Eqs. (2.11)–(2.13) are evaluated in terms of

$$l(s)\equiv\tau v(s) \quad (3.1)$$

and

$$\beta(s,s')=\frac{\hbar c}{eB}\int\frac{d\varepsilon}{v(s)}\,C_s(\varepsilon,s,\varepsilon_F,s')\,\frac{1}{v(s')}. \quad (3.2)$$

We find with electric field along the x direction,

$$\sigma_{xx}=e^2\int ds\;l(s)\cos^2\phi(s), \quad (3.3)$$

$$\sigma_{xy}=e^2\omega_c\left[-\int ds\;l(s)\cos\phi(s)\,\frac{d}{ds}\,[l(s)\sin\,\phi(s)]+\int ds\;ds'\,\beta(s,s')l(s)\sin\phi(s)l(s')\cos\phi(s')\right], \quad (3.4)$$

$$\Delta\sigma_{xx}=e^2\omega_c^2\left\{\int ds\ l(s)\left[\frac{d}{ds}[l(s)\cos\phi(s)]\right]^2-2\int ds\ ds'\,l(s)\cos\phi(s)\beta(s,s')l(s')\ \frac{d}{ds'}[l(s')\cos\phi(s')]\right.$$
$$\left.-\int ds\ ds'd''l(s')\beta(s,s')\beta(s',s'')l(s)\cos\phi(s)l(s'')\cos\phi(s'')\right\}. \tag{3.5}$$

In (3.3)–(3.5) ϕ is the angle between $V_{\mathbf{k}_F}(s)$ and the electric field. In (3.4), the first term—the Lorentz force contribution can be written as proportional to

$$\int ds\boldsymbol{l}(s)\times\frac{d}{ds}\boldsymbol{l}(s).$$

At particle-hole symmetry (for energies less than T near the Fermi surface) this term is zero—a well-known result for the Hall coefficient. However, the skew scattering contribution—the second term in (3.4) is, in general, not zero even when the Fermi surface is particle-hole symmetric (but the full Hamiltonian is not).

These equations can be put in a particularly attractive form due originally to Ong[6,10] and co-workers. Defining

$$\sum(s)=\frac{l(s)\cos^2\phi(s)}{\sigma_{xx}}, \tag{3.6}$$

and

$$\Theta_H(s)=\frac{1}{\cos\phi(s)}\left[\frac{d}{ds}\,l(s)\sin\phi(s)\right.$$
$$\left.+\int ds'\beta(s,s')l(s')\sin\phi(s')\right], \tag{3.7}$$

(and assuming the Fermi surface has fourfold symmetry), it is possible to show that the Hall angle $\langle\Theta_H\rangle=-\sigma_{xy}/\sigma_{xx}$ for small fields is given by

$$\langle\Theta_H\rangle=\int ds\sum(s)\Theta_H(s). \tag{3.8}$$

Similarly

$$\frac{\Delta\sigma_{xx}}{\sigma_{xx}}=\int ds\sum(s)\Theta_H^2(s)\equiv\langle\Theta_H^2\rangle. \tag{3.9}$$

The magnetoresistance obtained by inverting the conductivity tensor is then

$$\frac{\Delta\rho}{\rho}=\frac{gD\sigma_{xx}}{\sigma_{xx}}-\left(\frac{\sigma_{xy}}{\sigma_{xx}}\right)^2=[(\Theta_H-\langle\Theta_H\rangle)^2]. \tag{3.10}$$

IV. EXPERIMENTAL RESULTS IN COPPER OXIDES

Extensive results for electrical transport experiments in a magnetic field are available for the compounds $YBa_2Cu_3O_{7-\delta}$ (123) and $La_{x-x}Sr_xCuO_4$ (214) at the "ideal" compositions $\delta\approx0.1$ and $x\approx0.15$, respectively. The zero-field resistivity in both is $\rho(T)=\rho_0+\rho_1T$ to a high accuracy. The temperature-dependent part implies a scattering rate $\tau^{-1}\sim\tau_0^{-1}+\lambda kT$, according to Eq. (3.3).

In these materials, the thermal conductivity in the normal state is constant to a good approximation, i.e., the Wiedemann-Franz law is observed. This implies that no distinction should be made in the temperature dependence of the momentum relaxation rate and the energy or quasiparticle linewidth. This is often true if the self-energy is nearly momentum independent in the situation where the Fermi surface is not isotropic, so that conservation of momentum does not imply conservation of current. This may serve to justify our analysis below by two average relaxation rates τ^{-1} and τ_s^{-1} representing some average, respectively, of $[\tau(s)]^{-1}$ and $\beta(s,s')$, i.e., the normal and skew scattering functions, that occur in Eqs. (3.7)–(3.9). In any case, it would be pointless at the present stage of the microscopic theory to take into account the detailed momentum dependence of the scattering rates, nor do we believe will it change the qualitative nature of the calculated anomalies. Thus we will work only with the average rates, $\tau^{-1}(T)$, $\tau_s^{-1}(T)$ which capture the overall temperature dependence of $\tau(s)$ and $\beta(s,s')$:

$$\tau\equiv\int ds\ l(s)\cos^2\phi(s)\Big/\int ds\ v(s)\cos^2\phi(s), \tag{4.1}$$

$$1/E_F\tau_s$$
$$\equiv1/\tau\sigma_{xx}\int ds\ ds'\beta(s,s')l(s)\cos\phi(s)l(s')\sin\phi(s'). \tag{4.2}$$

The complicated angular integrals in (3.3)–(3.5) give rise to numerical coefficients a,b, etc., which depend on the shape of the Fermi surface, deviation from particle-hole symmetry, etc., and which have a very weak temperature dependence, if any.

With these simplifications, the essential physical points of Eqs. (3.3)–(3.5) and (3.8)–(3.10) can be summarized as

$$\sigma_{xx}\simeq v_F^2\tau, \tag{4.3}$$

$$\sigma_{xy}\simeq(v_F^2\tau)(\tau\omega_c)\left[a+\frac{b}{E_F\tau_s}\right]. \tag{4.4}$$

The term proportional to a represents the conventional behavior; $a=0$ at particle-hole symmetry. Thus the Hall angle is

$$\Theta_H\approx(\tau\omega_c)\left[a+\frac{b}{E_F\tau_s}\right], \tag{4.5}$$

and the magnetoresistance

$$\frac{\Delta\rho}{\rho}\approx(\omega_c\tau)^2\left[c+\frac{d_1}{E_F\tau_s}+\left(\frac{d_2}{E_F\tau_s}\right)^2\right], \tag{4.6}$$

where c and d_1 and d_2 are again coefficients depending on the Fermi surface and $c=0$ for an isotropic Fermi surface.

In comparing with experimental results, we first note that the term proportional to a in (4.5) is the usual contribution to the Hall angle which is absent when the Fermi surface has particle-hole symmetry. Hall-effect calculations based on band structure[11] of 123 at the ideal composition yields a very Θ_H but do yield a sizable Θ_H for 214 at near the ideal composition $\approx$15% Sr. Also magnetoresistance measurements are available only for 123. So we compare (4.5) and (4.6) only with results in (123) where $a\approx 0$.

Next, we note that if the skew scattering rate τ_s^{-1} diverges as T^{-1}, Θ_H from (4.5) is $\sim T^{-2}$ and the last term in (4.6), which is the dominant term at low temperatures is $\sim T^{-4}$ as observed.[3,6] We can also make a quantitative check. For $a\approx 0$, and if the terms proportional to c and d_1 are unimportant in the temperature range of measurements,

$$\theta_H^2/(\Delta\rho/\rho)\approx b^2/d^2. \tag{4.7}$$

Harris and co-workers[3,6] find $\Theta_H^2\approx 1630T^{-4}$ and $\Delta\rho/\rho\approx 4140T^{-4}$ at fields of 1 T where T is in degrees. We would then deduce $b/d\approx 0.6$, a sensible sort of result for different Fermi-surface anisotropies occurring in the expression for Θ_H and $\Delta\rho/\rho$. If we use the deduction from the resistivity that $\tau^{-1}\approx k_BT$, we can further deduce that $(E_F\tau_s)^{-1}\approx 40/T$ where T is in degrees.

With these numbers, we find that if $c=d_1=d_2^2$, the last term in (4.6) dominates only below 100 K. Experimentally,[6] a unique power law appears not to fit the data taken between 100 and 400 K; the uncertainties in the measurement do not exclude a smooth crossover to lower exponents than 4 at higher temperatures. We can understand the results if c/d_1 and d_1/d_2^2 are less than about 1/4. As mentioned, they depend on details of the Fermi surface and are very hard to estimate. We would predict in any case a crossover to a lower temperature dependence if the data could be extended to higher temperatures.

Spielman *et al.*[7] have measured the frequency dependence of the Hall effect. Although most of the results presented are for the superconducting state, they have clearly deduced that in the normal state the experimental results are consistent with

$$\Theta_H/(\omega,T)\approx\frac{bv_F^2\omega_c}{E_F\tau_s}[\mathrm{Re}(i\omega+\tau^{-1})]^{-1}. \tag{4.8}$$

This equation can be deduced from Eqs. (2.2) and (2.5) as the straightforward modification of the results derived to finite frequencies. The important point is the replacement $\tau^{-1}\rightarrow\tau^{-1}+i\omega$ with τ_s^{-1} appearing in the same form as at $\omega=0$. It should be noted that (4.6) as well as all other results in this paper are based on the low-field expansion (2.13). Since C_s of (2.13) is effectively proportional to (ω_c/T) this procedure breaks down at low enough temperature or large enough fields. We have checked that the expansion is quite valid in the experiments quoted, primarily because the normal phase is unstable to superconductivity at fairly high temperatures. It would be interesting to carry out Hall-effect and magnetoresistance experiments in the anomalous copper oxide metals with low T_c's.

In this paper we have considered only samples near the composition of the highest T_c and ignored the effect of impurities. Impurities lead to at least two kinds of changes in the theory: addition of a temperature-independent scattering rate in τ^{-1} and possible cutoff of the singularities leading to the linear T dependence in τ^{-1} and T^{-1} dependence in τ_s^{-1}. These lead to qualitatively different results. On the experimental side, the change in the Hall angle with impurity concentration is qualitatively different for addition of impurities[3] in $YBa_2Cu_3O_7$ and for changing Sr concentration[4] in $La_{2-x}Sr_xCuO_4$. Even in $YBa_2Cu_3O_7$, impurities in the plane and between planes seem to have different effects.[12] We hope to address these issues in the future.

V. IMPLICATIONS FOR A MICROSCOPIC THEORY

The normal state of copper oxide metals is not a Fermi liquid. Many of the transport anomalies could be understood by the phenomenological ansatz, Eq. (1.1) leading to a zero quasiparticle renormalization amplitude at the chemical potential and a scattering rate proportional to T. Such a "soft" breakdown of Landau theory implies that the scattering amplitude of fermions at the Fermi energy is singular or that there is a resonance at the chemical potential. As observed earlier, such a soft breakdown cannot explain the magnetotransport behavior. The principal result of this paper is that the skew scattering rate in the presence of a magnetic field needs to be singular $\sim T^{-1}$ to explain magnetotransport consistently. This is much more singular than the singularity in Eq. (1.1).

It is quite obvious that the skew scattering implied here is due to some intrinsic fluctuations and are not due to, for instance, spin-orbit scattering at magnetic impurities. These intrinsic fluctuations must be proportional to B and therefore are chiral in nature. They must be highly singular at the chemical potential to produce the deduced skew scattering rate. This implies in turn that the chiral response of the Cu-O metals at the ideal composition is singular at $(\omega,T)\rightarrow 0$. A quantum critical point has already been suggested[13] as necessary to produce Eq. (1.1). The present results suggest that fluctuations at $B=0$ acquire a more singular form at finite B. These fluctuations could be centered around $q\approx 0$ or finite q.[14] We suggest that such singular fluctuations may be observable in light scattering or neutron scattering in a magnetic field.

The singular chiral fluctuations suggested here should be compared to the suggestion of uniform chiral flux phases.[15] Such phases are predicted to have a spontaneous[16] (i.e., in zero B) Hall current. By contrast copper oxide materials appear to exhibit a Hall current proportional to B which diverges as T^{-1} suggestive of fluctuations to a chiral phase at $T=0$. This divergence is cut off by the intervention of superconductivity.

ACKNOWLEDGMENTS

We wish to thank B. Batlogg and N. P. Ong for a discussion of the experimental results. G.K. was supported by NSF Grant No. DMR 95-29138.

[1] For review of experimental properties, see for example, *Physical Properties of High Temperature Superconductors II*, edited by D. M. Ginzburg (World Scientific, Singapore, 1990).
[2] See for example, C. M. Varma, in *Los Alamos Symposium on Strong Correlations*, edited by K. Bedell *et al.* (Addison-Wesley, New York, 1994).
[3] T. R. Chien, Z. Z. Wang, and N. P. Ong, Phys. Rev. Lett. **67**, 2088 (1991); J. M. Harris, Y. F. Yan, and N. P. Ong, Phys. Rev. B **46**, 14 293 (1992).
[4] H. Y. Hwang *et al.*, Phys. Rev. Lett. **72**, 2636 (1994).
[5] C. M. Varma *et al.*, Phys. Rev. Lett. **63**, 1996 (1989).
[6] J. M. Harris *et al.*, Phys. Rev. Lett. **75**, 1387 (1995).
[7] S. Spielman *et al.*, Phys. Rev. Lett. **73**, 1537 (1994).
[8] P. W. Anderson, Phys. Rev. Lett. **67**, 2992 (1991).
[9] A. Carrington *et al.*, Phys. Rev. Lett. **69**, 2855 (1992); B. Stojkovic and D. Pines (unpublished).
[10] N. P. Ong, Phys. Rev. B **43**, 193 (1991).
[11] W. E. Pickett, Rev. Mod. Phys. **61**, 433 (1989).
[12] P. Xiong *et al.*, Phys. Rev. B **47**, 5516 (1993).
[13] C. M. Varma, Phys. Rev. Lett. **75**, 898 (1995).
[14] Z. Wang, G. Kotliar, and X. Wang, Phys. Rev. B **42**, 8670 (1990).
[15] R. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988).
[16] Y. Chen *et al.*, Int. J. Mod. Phys. B **3**, 1001 (1989).