Quantized levitation states of superconducting multiple-ring systems

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The quantized levitation, trapped, and suspension states of a magnetic microsphere held in equilibrium by two fixed superconducting (SC) microrings are calculated by minimizing the free energy of the system. Each state is a discrete function of two independent fluxoid quantum numbers of the rings. When the radii of the SC rings are of the same order as the Ginzburg-Landau coherence length $\xi(T)$, the system exhibits a small set of gravity and temperature-dependent levels. The levels of a weakly magnetized particle are sensitive functions of the gravitational field, indicating potential application as an accelerometer, and for trapping small magnetic particles in outer space or on Earth. The equilibrium states of a SC ring levitated by another SC ring are also calculated.

I. INTRODUCTION

Self-stabilizing magnetic levitation is an eclectic phenomenon that generally couples electromagnetics, superconductivity, and the gravitational field. The necessary ingredients for magnetic levitation are a magnetic-field source and a magnetic-field shaping, or trapping device.[1] The simplest system that satisfies these requirements is a magnetic dipole and a simply connected superconductor, a system analyzed many times in the literature, typically applying magnetic image methods.[1–5] For a temperature T_m below the superconducting (SC) critical temperature T_c, a unique equilibrium levitation height, independent of temperature and SC material properties, is obtained. The levitation force on macroscopic magnets of various shapes placed above a SC plane have been calculated using the London theory, which also gives a temperature-independent dipole height as long as it is much greater than the London penetration depth λ.[6] In the perfect Meissner limit $(\lambda \rightarrow 0)$ the force on the probe tip of a magnetic force microscope,[7] modeled by a linear superposition of magnetic dipoles, above a macroscopic SC ring has also been calculated.[8] The latter may be useful in interpreting the surface roughness effect on the probe.

Using high-temperature type-II oxide superconductors both repulsive levitation, based on partial flux exclusion and flux pinning, and attractive levitation (suspension) based on flux pinning, have been observed in macroscopic systems. A mesoscopic SC ring circuit is a simple, basic device that exhibits not only the necessary flux trapping, but also fluxoid quantization. In a recent study of a magnetic microsphere levitated, and suspended, by a superconducting microring, it was shown that the equilibrium value of the relative coordinate of the magnet is a member of a small set of quantized, weakly temperature-dependent levels.[9,10] This discrete nature of the levitation levels is a direct consequence of fluxoid quantization in a multiply connected superconductor.

Here we extend our analysis of quantized levitation in the mesoscopic regime to the study of the equilibrium levitation states generated by microscopic multiple-ring superconducting (SC) systems. In Sec. II we investigate the levitation, trapped, and suspension states of a magnetic microsphere, in a gravitational field, held in equilibrium by two superconducting microring circuits. In this system there is a strong nonlinear interaction between the gravitational and magnetic fields that can actually lift the magnet as gravity increases. The effect of weightless and small gravity environments, relevant to space applications, e.g., satellites, is also investigated. Charging the magnetic particle, and applying a uniform electric field provides a convenient mechanism for manipulating it among the quantized levitation levels. Since the effect of the electric field is identical to that of the gravitational field, variation of only one field is necessary to study the effect of either field. Using the approach developed in Refs. 3 and 4, the total free energy of the system is minimized, subject to the constraints imposed by single-valuedness of the complex superconducting order parameter in each ring, and mechanical equilibrium. In Sec. III we analyze the levitation of a SC ring by a fixed SC ring current. The resulting quantized states of both systems are investigated in detail and compared with the single ring results in Refs. 3 and 4. Section III is devoted to our conclusions, and a quantized, linear theory is developed in the Appendix.

II. A MAGNETIC MICROSPHERE TRAPPED BETWEEN TWO SC RINGS

Consider two superconducting microrings of radii a and b, and separation d, as shown in Fig. 1. (Throughout the article, parameters characterizing ring a are subscripted with a and those characterizing ring b are subscripted with b.) The cross sections are $s=\pi r^2$. It is assumed that the SC rings are mounted on nonmagnetic insulators which are fixed. The magnet trapped between the SC rings, at distance h from the lower ring b and $d-h$ from the upper ring a, is a uniformly magnetized sphere with saturation magnetization M_s, radius r_m, and density ρ. The magnetic moment of the magnet is $\mathbf{M}=M_0\hat{z}$, with $M_0=M_s 4\pi r_m^3/3$, and the weight $W=(4\pi r_m^3/3)\rho g$, where g is the gravitational acceleration constant, with nominal value $g_0=9.8\ \mathrm{m/s}^2$. The SC rings carry an induced current I, and have self-inductance L. The SC ring material is characterized by the temperature-dependent magnetic-field penetration depth $\lambda(t)$ and

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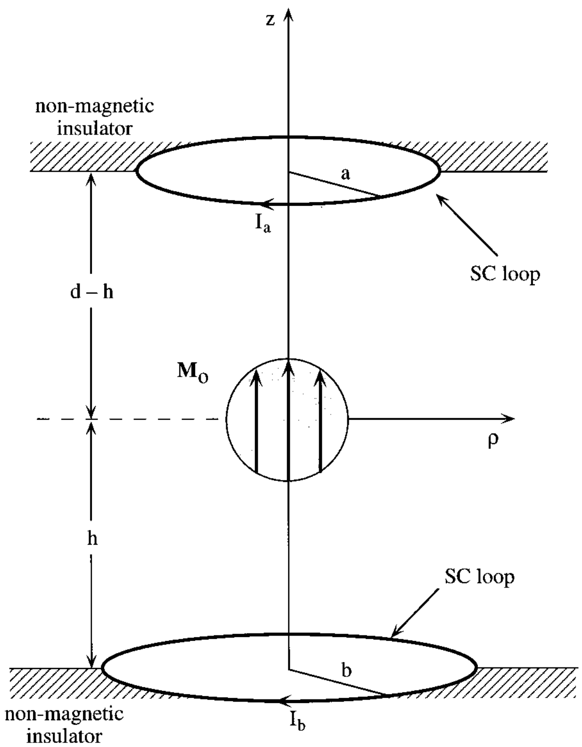

FIG. 1. The magnetic sphere of radius a is trapped between two superconducting rings of radii a and b and fixed separation d, carrying induced currents I_a and I_b. The levitation height h is measured from ring b.

Ginzburg-Landau (GL) coherence length $\xi(t)$. The variable $t = T/T_c$, with T the temperature, and T_c the critical temperature of the SC–normal phase transition in zero magnetic field.

The height h is determined self-consistently by minimizing the total free energy of the system consisting of the magnet and two SC rings, subject to fluxoid quantization and mechanical equilibrium constraints. Since the magnetic field at the SC rings is not a controllable external variable, the Helmholtz free energy F is the apppropriate functional to be minimized.[11] The difference between the SC and normal state free energies of the system is

$$\Delta F_{\rm sc} = \Delta F_{\rm sc} + \Delta F_m + Wh, \tag{2.1}$$

The term $\Delta F_{\rm sc}$ is the energy of the SC rings, and ΔF_m is the stored magnetic energy. The last term, Wh, is the gravitational potential energy of the magnet, relative to the plane of the lower SC ring b. The general form of the SC energy is

$$\Delta F_{\rm sc} = \frac{\Lambda(t)}{V} \int dv \left[-N + \frac{1}{2} N^2 + N Q^2 + (\xi \nabla \sqrt{N})^2 \right], \tag{2.2}$$

where $\Lambda(t) = \mu_0 V H_c^2(t)$, with V the volume the superconductor, and $H_c(t)$ is the thermodynamic critical magnetic field. The first two terms in the integrand are the normalized condensation energy of the superconductor, with $N = |\psi|^2$, where $\psi = \Psi/\Psi_{\rm bulk} = \sqrt{N}\, \exp(i\theta)$ is the normalized complex order parameter. The function $\mathbf{Q}$ is defined by $\mathbf{Q} = \xi[\nabla\theta + (2\pi/\phi_0)\mathbf{A}]$, with $\mathbf{A}$ the magnetic vector potential. The last two terms comprise the kinetic energy $K = \xi^2 |\mathbf{p}\psi|^2$, where $\mathbf{p} = -i\nabla + (2\pi/\phi_0)\mathbf{A}$. The thermodynamic critical magnetic field is given by

$$H_c(t) = \frac{\phi_0}{2\sqrt{2}\pi\mu_0 \lambda(t) \xi(t)} = H_c(0)(1 - t^2), \tag{2.3}$$

where $\phi_0 = h/(2|e|) = 2.07 \times 10^{-15}$ Weber is the fluxoid quantum.

The wire cross section of each ring is assumed small, so the transverse variations of N, $\mathbf{Q}$, and magnetic field $\mathbf{H}$ in the SC ring are small. In this limit, all integrals involving these quantities may be replaced by their mean values. Consistent with the thin wire approximation, and the cylindrical symmetry of the system, we also neglect the integral of $(\xi \nabla \sqrt{N})^2$, giving

$$\Delta F_{\rm sc} = \Lambda_a(t) \left(-N_a + \frac{1}{2} N_a^2 + N_a Q_a^2 \right) + \Lambda_b(t) \left(-N_b + \frac{1}{2} N_b^2 + N_b Q_b^2 \right), \tag{2.4}$$

For each ring of radius ρ and wire radius r, the constant $\Lambda(t) = \mu_0 V_\rho H_c^2(t)$, with $V_\rho = 2\pi^2 \rho r^2$ the volume of the SC ring. The flux coupling energy between the magnet of moment $\mathbf{M}$ and a SC ring of radius ρ is $-\mathbf{M} \cdot \mathbf{B} = I \phi_{m\rho}(z)$, where $\mathbf{B}$ is the flux density generated by the induced current I in the ring, at the magnet on the ring axis at distance z, measured from the ring, and $\phi_{m\rho}(z)$ is the "applied" magnetic flux enclosed by the SC ring due to $\mathbf{M}$. The total stored magnetic energy ΔF_m is

$$\Delta F_m = \frac{1}{2} L_a I_a^2 + \frac{1}{2} L_b I_b^2 - I_a I_b M_{ab}(d) + I_a \phi_{ma}(h - d) + I_b \phi_{mb}(h), \tag{2.5}$$

where M_{ab} is the mutual inductance between the rings. The currents are defined such that $I_a > 0$ and $I_b > 0$, as referenced in Fig. 1. In the trapped configuration shown, both rings repel the magnet.

Taking a contour integral of $\mathbf{Q}$ around a SC ring, requiring single valuedness of the complex order parameter, gives the flux quantization constraint

$$\oint \mathbf{dl} \cdot \mathbf{Q} = 2\pi\xi \left(\frac{\phi}{\phi_0} + n \right), \tag{2.6}$$

where the phase winding number n is an integer or zero, and ϕ is the total flux enclosed by the contour. Writing out the contributions to the total flux in each ring due to the magnet, the mutual inductance between the rings, and the self-inductance of each ring, the fluxoid quantization constraints, obtained from contour integrations of Q around each ring, are

$$Q_a=\frac{\xi_a}{a}\left\{\frac{1}{\phi_0}[\phi_{ma}(h-d)-I_bM_{ab}(d)-L_aI_a]+n_a\right\},$$
$$Q_b=\frac{\xi_b}{b}\left\{\frac{1}{\phi_0}[\phi_{mb}(h)-I_aM_{ab}(d)-L_bI_b]+n_b\right\}. \tag{2.7}$$

At this point the Q's are mean values in the rings. The flux linking a ring of radius ρ, due to the magnetic dipole, with magnetic moment $\mathbf{M}=M_0\hat{z}$, located on the ring axis at distance z from the ring is

$$\phi_{m\rho}(z)=\frac{\mu_0M_0}{2\rho}\left[1+\left(\frac{z}{\rho}\right)^2\right]^{-3/2}, \tag{2.8}$$

and the mutual inductance between the two coaxial rings separated by distance d is

$$M_{ab}=\mu_0\sqrt{ab}g_1[m_0(d)], \tag{2.9}$$

where the function g_1 is defined as

$$g_1=\frac{1}{\sqrt{m_0}}[(2-m_0)K(m_0)-2E(m_0)],$$
$$m_0=4\frac{a}{b}\left[\left(1+\frac{a}{b}\right)^2+\left(\frac{d}{b}\right)^2\right]^{-1},$$

with K and E complete elliptic integrals of the first and second kind, respectively. The self-inductance of a ring of radius ρ and wire radius $r\ll\rho$ is approximated by

$$L=\mu_0\rho\left[\ln\left(\frac{8\rho}{r}\right)-1.75\right].$$

Noting that the magnetic-flux density $\mathbf{B}$ on the axis of a current loop, of radius ρ, carrying current I, is proportional to the flux $\phi_{m\rho}(z)$, Eq. (2.8), the z component of the force on the dipole is

$$F_z=-\frac{\partial}{\partial z}(-\mathbf{M}\cdot\mathbf{B})=-I\frac{\partial\phi_{m\rho}(z)}{\partial z}. \tag{2.10}$$

Using this result, the equilibrium mechanical constraint on the magnet in Fig. 1 is

$$W=-I_a\phi'_{ma}(h-d)-I_b\phi'_{mb}(h). \tag{2.11}$$

The prime denotes differentiation with respect to z, and the argument of $\phi'_{m\rho}(z)$ is the value of z after differentiation.

It is interesting to evaluate the forces on the rings. To do this, assume that the rings have weight W_a and W_b and are tied together by stiff rods, and that the "birdcage" system is suspended by a string. Let $\mathbf{T_0}=T_0\hat{z}$ be the tension in the string, and $\mathbf{T}=\pm T\hat{z}$ be the tension in the rods necessary to hold the rings in place. The total force on each ring is

$$\mathbf{F}_a=\hat{z}[T_0-T-W_a+I_aI_bM'_{ab}(d)+I_a\phi'_{ma}(h-d)],$$
$$\mathbf{F}_b=\hat{z}[T-W_b+I_aI_bM'_{ab}(-d)+I_b\phi'_{mb}(h)]. \tag{2.12}$$

In equilibrium $\mathbf{F}_a=\mathbf{F}_b=0$. Noting that $M'_{ab}(d)=-M'_{ab}(-d)<0$, Eqs. (2.12) and (2.11) yield

$$T_0=W_a+W_b+W.$$

Thus, the rope holds up the total weight of the system, including the floating magnet. No further information is obtained from Eq. (2.12).

The free energy ΔF in Eq. (2.1), is a function of five unknown variables, N_a,N_b,I_a,I_b,h. Minimizing E with respect to variation in N_a and N_b gives

$$N_a=1-Q_a^2,\quad N_b=1-Q_b^2, \tag{2.13}$$

which, in view of the flux constraint Eq. (2.7) leaves us with three unknowns, I_a, I_b, and h. The only other contraint that reduces the number of unknowns is Eq. (2.11), which we will use to eliminate I_b. The remaining problem is an unconstrained minimization of the two parameter function $\Delta F(I_a,h)$. For numerical purposes, we intoduce normalized energy E and a normalized current density J defined by

$$E=\frac{\Delta F}{\sqrt{\Lambda_a(0)\Lambda_b(0)}}, \tag{2.14}$$

and

$$J=\frac{\lambda(t)}{\sqrt{2}sH_c(t)J_c}I=\beta I, \tag{2.15}$$

where $J_c=2/\sqrt{27}$ is the critical current density of a SC ring in the presence of a magnetic reservoir. Using Eqs. (2.1) and (2.11) leads to the normalized energy function $E(J_a,h)$ given by

$$E(J_a,h)=E_{sc}(t)+C[E_2(h)J_a^2+E_1(h)J_a+E_0(h)], \tag{2.16}$$

where the SC contribution is

$$E_{\rm sc}(t)=-\frac{1}{2}\left[\eta(1-Q_a^2)^2+\frac{1}{\eta}(1-Q_b^2)^2\right](1-t^2)^2, \tag{2.17}$$

and

$$E_2(h)=\frac{L_a}{2\beta_a^2}\left[1+\frac{\phi'_{ma}}{\phi'_{mb}}\left(\frac{L_b}{L_a}\frac{\phi'_{ma}}{\phi'_{mb}}+2\frac{M_{ab}}{L_a}\right)\right],$$

$$E_1(h)=\frac{\phi_{ma}}{\beta_a}\left[\frac{WL_b}{\phi_{ma}\phi'_{mb}}\left(\frac{\phi'_{ma}}{\phi'_{mb}}+\frac{M_{ab}}{L_b}\right)+1-\frac{\phi_{mb}}{\phi_{ma}}\frac{\phi'_{ma}}{\phi'_{mb}}\right],$$

$$E_0(h)=W\frac{\phi_{mb}}{\phi'_{mb}}\left(\frac{WL_b}{2\phi_{mb}\phi'_{mb}}-1\right)+Wh,$$

$$\eta=\sqrt{\frac{\Lambda_a(0)}{\Lambda_b(0)}}=\sqrt{\frac{b}{a}\left[\frac{\lambda_b(0)}{r_b}\frac{\xi_b(0)}{b}\right]\left[\frac{\lambda_a(0)}{r_a}\frac{\xi_a(0)}{a}\right]^{-1}},$$

$$C=\frac{1}{\sqrt{\Lambda_a(0)\Lambda_b(0)}}=\frac{4\mu_0\sqrt{ab}}{\phi_0^2}\frac{\lambda_b(0)}{r_b}\frac{\lambda_a(0)}{r_a}\frac{\xi_b(0)}{b}\frac{\xi_a(0)}{a}.$$

From Eqs. (2.7) and (2.11)

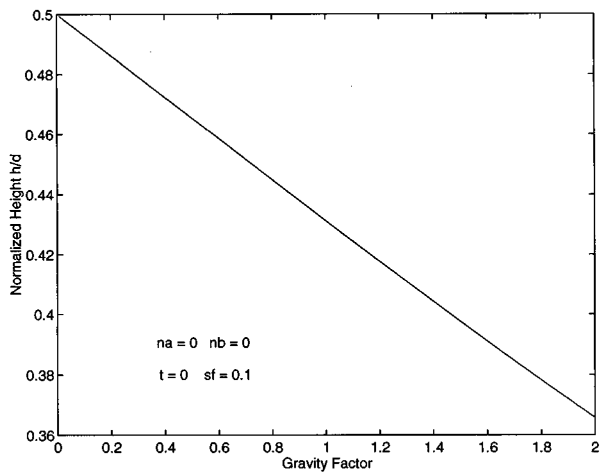

FIG. 2. Normalized trapped height h/d as a function of the gravity factor $\mathrm{gf}=g/g_0$ for aluminum SC rings of radius $a=b=2$ μm, separation $d=4$ μm, at zero temperature, and fluxoid quantum numbers $n_a=n_b=0$. The magnet is YIG, with 10% saturation (sf = 0.1).

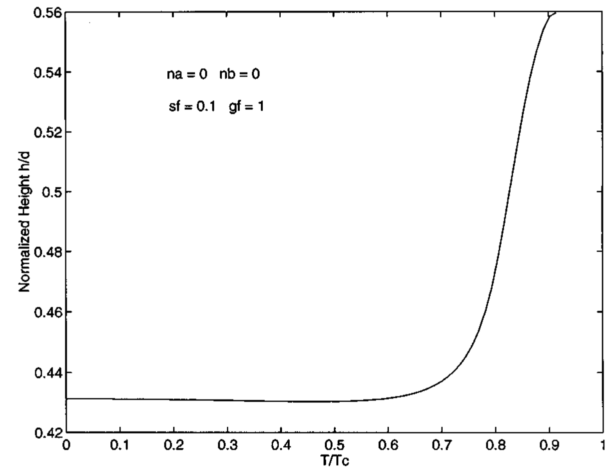

FIG. 3. Normalized height h/d as a function of $t=T/T_c$, with gf = 1, for the data of Fig. 2.

$$Q_a=\frac{\xi_a}{a}\left[\frac{1}{\phi_0}\left(\phi_{ma}+\frac{WM_{ab}}{\phi'_{mb}}\right)+\frac{L_a}{\phi_0\beta_a}\left(-1+\frac{M_{ab}}{L_a}\frac{\phi'_{ma}}{\phi'_{mb}}\right)J_a+n_a\right], \quad (2.18)$$

$$Q_b=\frac{\xi_b}{a}\left[\frac{1}{\phi_0}\left(\phi_{mb}+\frac{WL_b}{\phi'_{mb}}\right)+\frac{L_b}{\phi_0\beta_a}\left(\frac{\phi'_{ma}}{\phi'_{mb}}-\frac{M_{ab}}{L_b}\right)J_a+n_b\right].$$

All flux and flux derivatives, $\phi_{m\rho}$ and $\phi'_{m\rho}$, are evaluated at $h-d$ for $\rho=a$ and h for $\rho=b$. Since the discovery of high-T_c superconductors there is considerable interest in refrigerationless superconducting electronics in space applications, such as satellites. The trapped states of a magnetic particle in a weightless environment are obtained by setting $W=0$, in Eqs. (2.16) – (2.18).

All calculations and figures for the double ring-magnet system are based on the following data: The magnetic particle is an yttrium iron garnet (YIG) sphere of radius $r_m=0.4$ μm, saturation magnetization $M_s=2\times10^5$ A/m and density 5.2×10^3 kg/m^3, with resultant magnetic moment $M_0=5.362\times10^{-14}$ A m^2, and weight $W=1.366\times10^{-14}$ N. Both SC rings have wire cross section $s=1.0\times10^{-15}$ m^2, and radius $a=b=2$ μm, and the ring separation is $d=4$ μm. The SC rings are Al with experimental values of $H_c(0)=0.79\times10^4$ A/m, and GL parameter $\kappa=\lambda(0)/\xi(0)=0.015$. Using Eq. (2.3) the zero-temperature values of the penetration depth and coherence length are calculated to be $\lambda(0)=0.0188$ μm, and $\xi(0)=1.25$ μm. The temperature dependence used is $\lambda(t)=\lambda(0)/\sqrt{1-t^2}$ and $\xi(t)=\xi(0)/\sqrt{1-t^2}$; thus $\kappa=\lambda/\xi$ is temperature independent. Furthermore, we assume that the magnetic sphere is only 10% saturated, reducing the magnetic forces relative to the gravitational. In all the figures below we keep the magnetic moment of the sphere constant and vary the gravitational factor gf $\equiv g/g_0$, where $g_0=9.8$ m/s, at constant temperature, or vary the temperature at constant gravitational factor.

Our object is to find the minimum energy of Eq. (2.16) by varying the current I_a in the upper ring (Fig. 1) and the height h of the magnet above $(h>0)$ or below $(h<0)$ the lower ring. When the magnet finds a minimum in the free energy with $h<0$, we call this solution a suspension (S) state. In that case, there is a net attraction between the ring currents I_a and I_b and the magnet, balanced by the weight of the magnet. When $h>d$, the separation between the rings (Fig. 1), there is a net repulsion between the ring currents and magnet, which is balanced by the gravitational force on the magnet. This is a levitation (L) state. When the particle finds a minimum in the free energy with h in between the two rings $(0<h<d)$ the solution corresponds to a trapped (T) state. In this case the net magnetic force on the magnet is upward directed, and balanced by the weight. The net magnetic force can be composed of repulsive forces from each ring current, attractive forces from each ring current, or a repulsive force due to ring current I_b and an attractive force due to ring current I_a. As the gravitational force on the magnet decreases, the magnet is pushed toward the center between the rings. Depending, however, on the fluxoid quantum numbers n_a and n_b (initial magnetic flux state of the rings), there can be more than one trapped level not necessarily near $0.5d$. We find with the above data that, with one exception discussed shortly, the overall lowest-energy states are described by fluxoid quantum numbers $n_a=n_b=0$. In this case, there exist three equilibrium position levels: one S, one T, and one L level. The L level behaves similar to the results reported in Refs. 9 and 10.

Figure 2 shows the trapped height level as a function of the gravity factor, gf, at $T=0$ K for $n_a=n_b=0$ with a magnetic saturation factor sf = 0.1 as referred to YIG. A very light particle tends to be trapped in the center between the magnets, while a heavy particle of the same magnetic strength finds a minimum energy height pushed toward the lower ring. In this particular case, the magnet is being pushed away from both rings, with the current in the lower ring being larger than that in the upper ring.

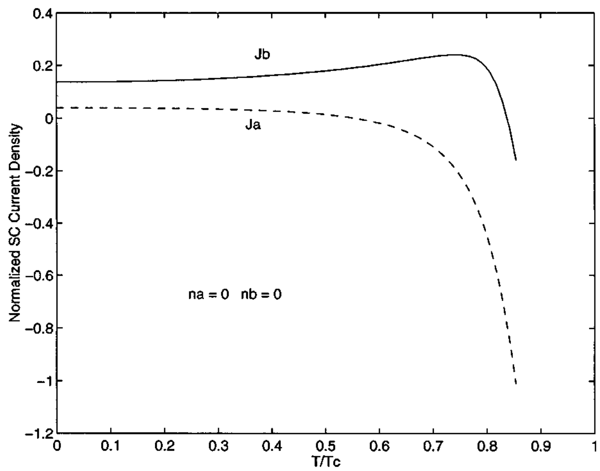

FIG. 4. Normalized current densities, J_a in the upper and J_b in the lower ring, corresponding to the data of Fig. 3.

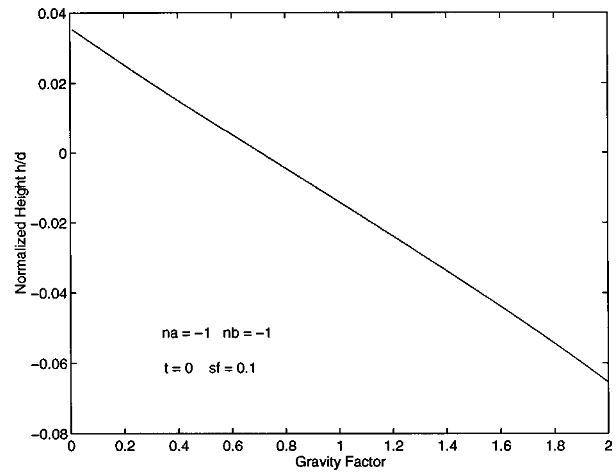

FIG. 6. Normalized height h/d changing from a trapped to a suspension state as a function of the gravity factor gf $= g/g_0$ for SC rings of radius $a = b = 2\,\mu$m, separation $d = 4\,\mu$m, at zero temperature, and fluxoid quantum numbers $n_a = n_b = -1$. The magnet is YIG, with 10% saturation (sf = 0.1).

Figure 3 shows the magnet height at normal weight, gf = 1, as a function of temperature with the above assumed temperature dependences for $\xi(t)$ and $\lambda(t)$. Figure 4 shows the corresponding currents. At low temperatures both ring currents push the magnet away, while for $t > 0.84$ both currents attract the magnet, the upper stronger than the lower. Between $0.55 < t < 0.84$ the lower current pushes the magnet away, while the upper attracts it. As a consequence the magnet exhibits a rapid transition from below the midpoint position at low temperatures to above the midpoint as the temperature increases above $t = 0.55$.

Figure 5 shows the suspension height as a function of weight. Lighter magnets of the same magnetic strength find a minimum energy position further away from the lower ring than heavier magnets, which require a larger magnetic force to keep them suspended. To keep a heavier magnet suspended requires a larger current. In this particular instance almost all of the attraction is due to the current in the lower ring, while the current in the upper ring is practically zero.

A moderate increase in the quantum numbers n_a and n_b increases the number of levels in general. However, for larger values of the quantum numbers no minima in the free energy are found, and all quantized levels disappear. For all quantum numbers the bulk normalized superelectron densities $N = |\psi|^2$ remain close to unity, except near the critical temperature T_c, where N of the "dominant" ring falls to zero.

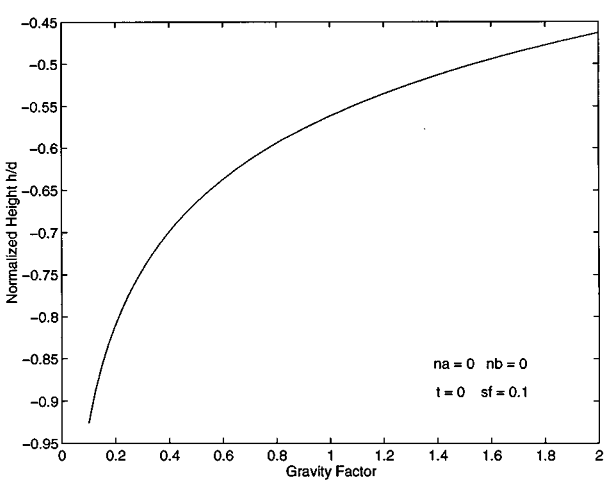

FIG. 5. Normalized suspension height as a function of the gravity factor gf for the data of Fig. 2.

As an interesting locked-in flux state, we choose $n_a = n_b = -1$ with a sf = 0.1. For this set of quantum numbers some T states have energy minima lower than those for $n_a = n_b = 0$, in the same height range. We find one L level, four T levels and two S levels, one of which passes through the lower ring and becomes a T level as the gravitational force is decreased. The L levels behave similarly to those shown in Refs. 9 and 10. There is a S level at $h/d \approx -0.9$ for gf = 1, which rises as temperature and gravity factors (as in Fig. 5) are increased. The height of the other S level is shown in Fig. 6 as a function of the gravity factor. At normal weight the magnet is suspended below the lower ring. As the gravity factor is decreased the magnet finds an equilibrium position, corresponding to a minimum free energy, as the magnet rises through the lower ring and becomes a T level for gf < 0.73. For this level the functional variation of h versus gf is opposite to that found for the lower S level, discussed above. The explanation for this behavior can be found in the slope of the force on the dipole, Eq. (2.10) which is of opposite sign near and far away from the current carrying ring circuits. The magnet position shown in Fig. 6 is near the center of the lower ring where the lift forces due to the current in the lower ring are small. We find that over the gravity range shown in Fig. 6 the magnetic sphere is mainly held in position by the upper ring current. It should be mentioned that when the magnet finds itself in the T level both upper and lower currents pull on it. Over the gf range shown, the current varies only slightly, and for gf = 1 the height is decreasing with increasing temperature, but only very slightly on the scale shown in Fig. 6.

Besides the S level which becomes a T level, mentioned above, there are three additional T levels possible over vari-

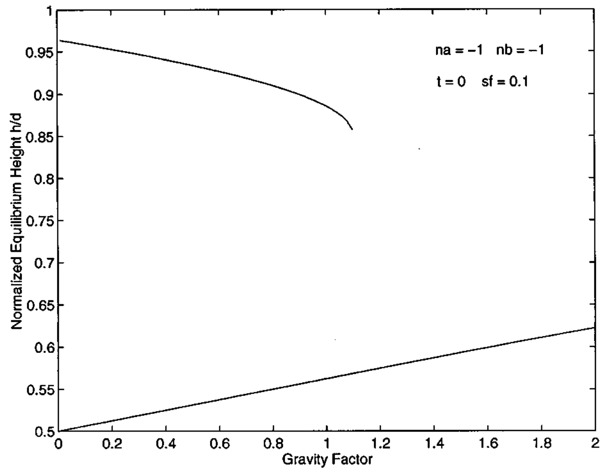

FIG. 7. Two trapped levels, one near the center between the rings, the other near the upper ring, as a function of the gravity factor gf. The upper level exists only for gf <1.1. Data is the same as in Fig. 6.

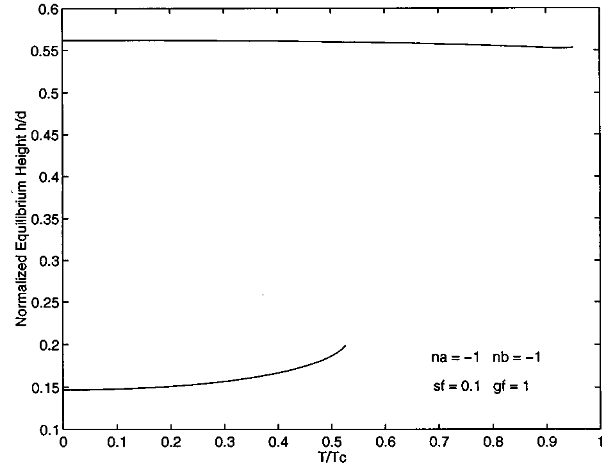

FIG. 9. Normalized height h/d as a function of $t=T/T_c$, for gf $=1$ and data of Fig. 6, for two trapped levels, one near the center between the rings, the other near the lower ring. The lower level exists only for $t<0.52$.

ous gravity and temperature ranges shown in Figs. 7–10. The level located just above the midpoint between the two rings exists for all temperatures and gravity factors shown. Its energy minimum is lower than those of the other two levels for equal (T, gf) conditions. The currents for the center level (curves near -0.4) and the level near the upper ring are shown in Fig. 8. Both ring currents attract the magnet with the upper ring generating the larger force. The upper level disappears when the gravity factor exceeds 1.1. Similarly, Figs. 9 and 10 show there exists a T level near the lower ring for low temperature, which disappears at $T/T_c \approx 0.52$. Magnetic particles trapped in the higher energy states near the upper or lower ring will most likely make a transition to the lower energy center level by tunneling out of the energy well before the corresponding energy minimum disappears. The currents for both levels, shown in Fig. 9, attract the magnetic particle, in contrast to the case $n_a = n_b = 0$ (Fig. 4) where the force is repulsive over all gravity factors and temperatures below $0.55T_c$.

For other combinations of n_a and n_b we find similar quantized levels. Since all the equations used are strongly nonlinear it is not possible to reduce our result to a common variable, like M_0/W or similar. However, for a given set of experimental parameters the levitation, trapped, and suspension levels can be uniquely calculated from the above equations for various fluxoid quantum numbers n_a and n_b.

III. LEVITATION OF ONE SUPERCONDUCTING MICRO-RING BY ANOTHER

Here we analyze the two SC ring system depicted in Fig. 11. Assume that SC ring b is fixed, and carries a persistent current I_b, i.e., ring b is an energy source with $n_b \neq 0$. If a second SC ring, of weight W_a, is placed on the axis of ring b, a current I_a will be induced as shown in Fig. 11, and it will levitate at equilibrium height h. The free energy of this system, and the flux quantization constraints are obtained from those given in Sec. II with all magnet terms $\phi_{m\rho}$ and $\phi'_{m\rho}$ set

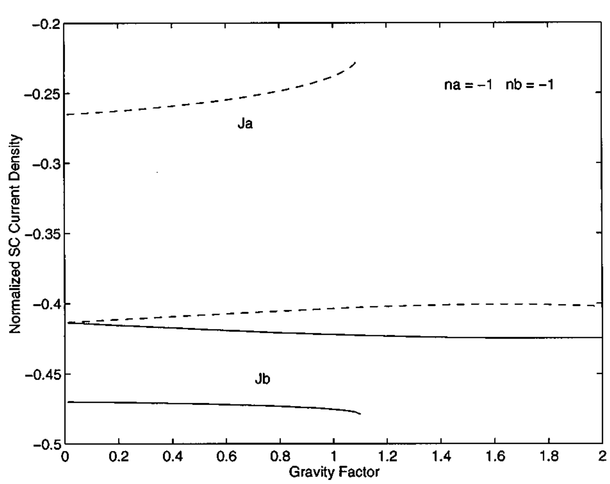

FIG. 8. Normalized current densities, J_a in the upper and J_b in the lower ring corresponding to the data of Fig. 7.

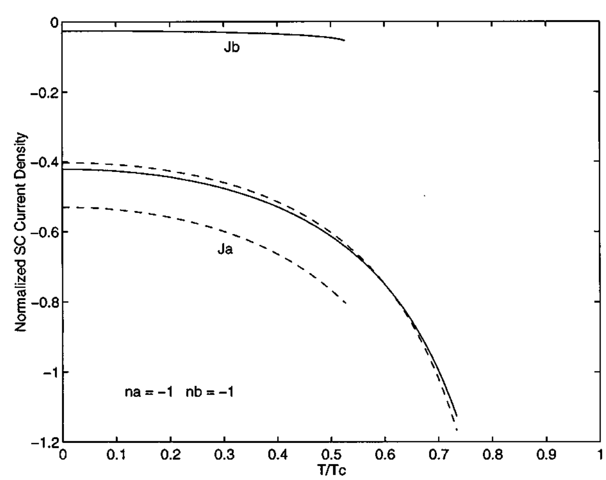

FIG. 10. Normalized current densities, J_a in the upper and J_b in the lower ring corresponding to the data of Fig. 9.

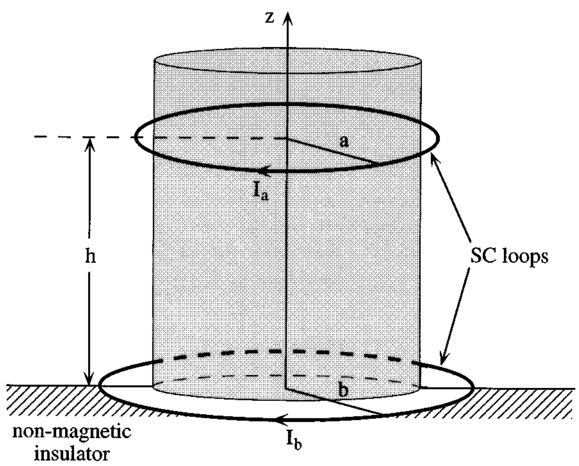

FIG. 11. Superconducting ring current I_a levitated by a fixed SC ring with current I_b.

to zero, W replaced by W_a, and d replaced by the variable h. The equilibrium force constraint is a distinct function of the currents, given by

$$W_a = I_a I_b M'_{ab}(h), \tag{3.1}$$

where $M'_{ab}(h)$ is the derivative with respect to z of the mutual inductance between the rings, evaluated at h. It is

$$M'_{ab}(h) = -\frac{\mu_0 h}{4\sqrt{ab}} g_2[m_0(h)],$$

$$g_2(m_0) = \sqrt{m_0}\left[\frac{2-m_0}{1-m_0}E(m_0) - 2K(m_0)\right],$$

with the parameter m_0 is defined after Eq. (2.9). Eliminating I_b from the free energy, the normalized function to be minimized is

$$E(J_a,h) = E_{\rm sc}(t) + CW_a b\left[\alpha J_a^2 + \frac{1}{\alpha}\left(\frac{\sqrt{L_a L_b}}{2bM'_{ab}}\right)^2 \frac{1}{J_a^2} + \frac{h}{b} - \frac{M_{ab}}{M'_{ab}}\right], \tag{3.2}$$

where $\alpha = L_a/(2bW_a\beta_a^2)$, with β_a defined in Eq.(2.15). The SC contribution $E_{\rm sc}$ is given by Eq. (2.17) and the superfluid velocities are

$$Q_a = \frac{\xi_a}{a}\left[-\frac{W_a M_{ab}\beta_a}{\phi_0 M'_{ab}}\frac{1}{J_a} - \frac{L_a}{\phi_0}J_a + n_a\right],$$
$$Q_b = \frac{\xi_b}{b}\left[-\frac{W_a L_b \beta_a}{\phi_0 M'_{ab}}\frac{1}{J_a} - \frac{M_{ab}}{\phi_0}J_a + n_b\right]. \tag{3.3}$$

Minimization of Eq. (3.2) for various ring radii and fluxoid quantum numbers leads to the following general observations. A very restricted number of levitation and suspension states exist for a fixed source ring of radius b greater than the radius a of the levitated ring, and no states exist when $b \leqslant a$. As an example, aluminum rings with $b = 2a = 2$ μm (other material parameters are given in Sec. II) have only one distinct levitation (suspension) state with $n_a = 0$, $n_b = \pm 1$. Energy minima appear for other quantum numbers, but the value of the pair density N is negative. Figure 12 shows the levitation level as a function of temperature, and Fig. 13 shows the corresponding SC pair densities. The source ring b rapidly loses its energy as T increases, with $N_b = 0$ at $T = 0.73T_c$. As T increases, the equilibrium levitation height increases because the source ring, with reduced energy, can exert a larger force further from the plane of the ring with smaller current for the height range of this state.

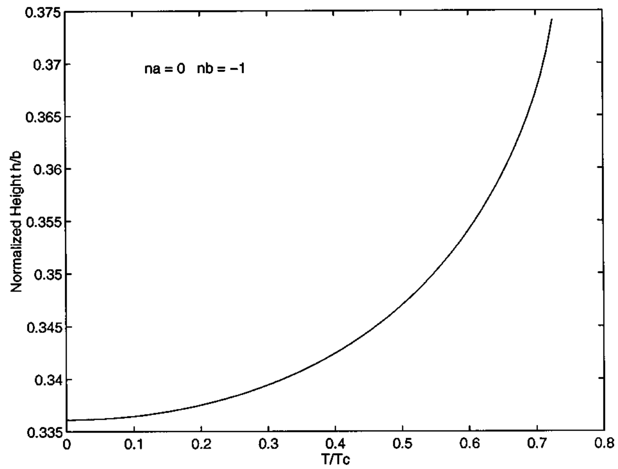

FIG. 12. Normalized height h/d as a function of $t = T/T_c$ for the situation shown in Fig. 11, with aluminum rings of radius $a = 1$ μm, $b = 2$ μm, and fluxoid quantum numbers $n_a = 0, n_b = -1$.

IV. CONCLUSIONS

We have analyzed a fixed two ring current system with a floating magnetic sphere, and a system consisting of a fixed ring and a levitated ring current. In the latter case at least one

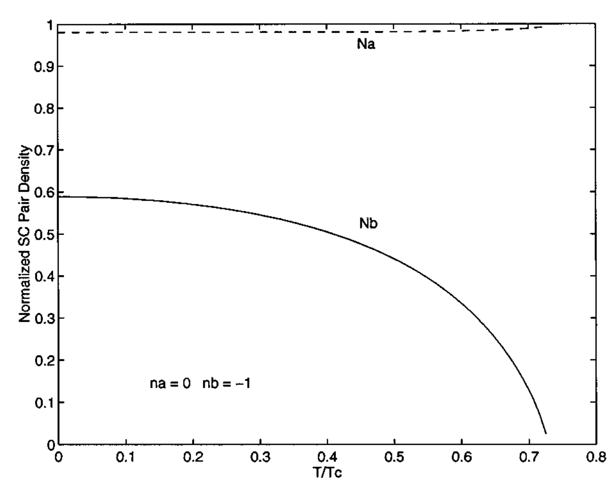

FIG. 13. Bulk normalized superelectron densities $N = |\psi|^2$ in the upper (N_a) and lower (N_b) rings corresponding to the data of Fig. 12.

of the quantum numbers n_a or n_b must be nonzero for a solution to exist. In general we find that levitation, trapped and suspension heights occur in discrete levels. These levels depend strongly on the fluxoid quantum numbers that lock-in flux in the system, apart from the geometric and material parameters of the system. For small, nonzero quantum numbers, e.g., $n_a = n_b = -1$, multiple trapped states may occur simultaneously over a range of temperatures and gravity factors. As the temperature T, or gravity factor gf is changed, some levels disappear at a critical value of T, or gf. A magnetic particle trapped in such a level makes an abrupt transition to another level. The transition probably occurs by tunneling out of the energy well before the level completely disappears.

Charging the particle and placing it in a uniform electric field provides a mechanism for manipulating the particle, which has exactly the same effect as changing the gravitational constant. It would be desirable to have experimental verification of our quantized levitation findings. The first system could be a very useful device for trapping very small magnetic particles on Earth or in outer space. In another related experiment, discussed in the Appendix, moving the probe of a magnetic force microscope along the ring axis would measure the discrete change in the force on the probe tip when the quantum number of a ring changes.

APPENDIX A QUANTIZED LINEAR THEORY

The analysis in Secs. II and III requires the minimization of the multiparameter energy function. To find the equilibrium states of a magnet supported by p superconducting rings, application of the flux and force constraints leaves one with a p-parameter minimization problem. This minimization can be avoided by linearizing the current-flux equations. To this end, we neglect the difference between the quantum current density $\mathbf{j}_\psi = -(\sqrt{2}H_c/\lambda)N\mathbf{Q}$ and the actual current density $\mathbf{j} = \nabla \times \mathbf{H}$ (See the discussion in Haley and Fink[10]), and assume that $N \approx 1$, i.e., $Q \ll 1$. In this limit one obtains

$$\mathbf{I} = s\mathbf{j} = -\frac{\phi_0}{2\pi\rho(\lambda/r)^2(\xi/\rho)}\mathbf{Q}. \tag{A1}$$

Equation (A1) is equivalent to starting with a "quantized London equation," with the current given in terms of a gauge transformed vector potential in the form

$$\mathbf{I} = -\frac{s}{\mu_0 \lambda^2(t)}\left[\frac{\phi_0}{2\pi}\nabla\theta + \mathbf{A}\right],$$

with the contour integral of $\nabla\theta$ defined to be $2\pi n$.

Using Eq. (A1), and noting the sign convention in Fig. 1, the superfuid velocity equations (2.7) give the coupled set of linear equations

$$\begin{aligned}
&[\zeta_a(t) + L_a]I_a + M_{ab}(d)I_b = \phi_{ma}(h-d) + n_a\phi_0, \\
&M_{ab}(d)I_a + [\zeta_b(t) + L_b]I_b = \phi_{mb}(h) + n_b\phi_0
\end{aligned} \tag{A2}$$

where

$$\zeta_a(t) = 2\mu_0 a\left(\frac{\lambda_a}{r_a}\right)^2, \quad \zeta_b(t) = 2\mu_0 b\left(\frac{\lambda_b}{r_b}\right)^2.$$

Solving (A2) yields

$$\begin{aligned}
f_a(h) = \frac{\mu_0\sqrt{ab}}{\phi_0}I_a = \Bigg[&\sqrt{\frac{b}{a}}c_b\left(\frac{\phi_{ma}}{\phi_0} + n_a\right) \\
&- g_1[m_0(d)]\left(\frac{\phi_{mb}}{\phi_0} + n_b\right)\Bigg]D^{-1},
\end{aligned}$$

$$\begin{aligned}
f_b(h) = \frac{\mu_0\sqrt{ab}}{\phi_0}I_b = \Bigg[&\sqrt{\frac{a}{b}}c_a\left(\frac{\phi_{mb}}{\phi_0} + n_b\right) \\
&- g_1[m_0(d)]\left(\frac{\phi_{ma}}{\phi_0} + n_a\right)\Bigg]D^{-1},
\end{aligned} \tag{A3}$$

where $D = c_a c_b - g_1^2(m_0)$, with g_1 defined in Eq. 2.9, and

$$c_a = \frac{\zeta_a + L_a}{\mu_0 a} = 2\left(\frac{\lambda_a}{r_a}\right)^2 + \ln\frac{8a}{r_a} - 1.75,$$

$$c_b = \frac{\zeta_b + L_b}{\mu_0 b} = 2\left(\frac{\lambda_b}{r_b}\right)^2 + \ln\frac{8b}{r_b} - 1.75.$$

Substituting (A2) in the equilibrium force Eq. (2.11) gives the levitation height equation for the magnet. It is

$$\frac{\mu_0\sqrt{ab}W}{\phi_0^2} + f_a(h)\frac{\phi'_{ma}(h-d)}{\phi_0} + f_b(h)\frac{\phi'_{mb}(h)}{\phi_0} = 0. \tag{A4}$$

Equation (A4) can be useful in obtaining approximate initial estimates for J_a and h for the minimization of the energy $E(J_a, h)$. For some quantized levels the linear approximation is quite accurate. However, unfortunately, the linearized equations do not always yield all solutions possible for a given set of quantum numbers. It is evident that the linearized analysis can readily be extended to systems with more than two rings. The number of coupled equations equals the number of rings. As noted in Ref. 10, if a ring of radius ρ is thick, the levitation states are obtained from the linearized equations by neglecting $\zeta_\rho(t)$ for all thick rings. Using linearized equations to determine the levitation height of the SC ring in Fig. 11 is not reasonable since N is not close to unity.

In contrast to the levitation application, the linearized theory should be quite satisfactory for determining the force on the probe tip of a magnetic force microscope.[7] Since the probe is a controllable magnetic source, the current density j is identical to the quantum current density j_ψ. As the probe is moved along the ring axis, a discrete change in the force on the tip should be detectable as the quantum number in a SC ring changes. Assuming a cylindrical model for the probe, the flux in a ring of radius ρ is $\phi_p = 2\pi\rho A_p$, where A_p is the magnetic vector potential obtained by integrating the point dipole contribution[10] over the volume of the probe. Equation (A4) is directly applicable by replacing ϕ_m for a dipole with ϕ_p due to the probe, and replacing W with F_z, the magnetic force on the probe.

[1] F. C. Moon, *Superconducting Levitation: Application to Bearings and Magnetic Transportation* (Wiley, New York, 1994).
[2] L. C. Davis, E. M. Logothetis, and R. E. Soltis, J. Appl. Phys. **64**, 4212 (1988).
[3] F. Hellman, E. M. Gyorgy, D. W. Johnson, H. M. O'Bryan, and R. C. Sherwood, J. Appl. Phys. **63**,447 (1988).
[4] Z. J. Yang, T. H. Johansen, H. Bratsberg, G. Helgesen, and A. T. Skjeltorp, Physica C **165**, 397 (1990).
[5] T. P. Orlando and D. A. Delin, *Foundations of Applied Superconductivity* (Addison-Wesley, Reading, MA, 1991), p.199.
[6] Z. J. Yang, J. Supercond. **5**, 259 (1992).
[7] Y. Martin and H. K. Wickramasinghe, Appl. Phys. Lett. **50**, 1455 (1987); J. J. Saenz *et al.*, J. Appl. Phys. **62**, 4293 (1987).
[8] Z. J. Yang, J. Appl. Supercond. **2**, 559 (1994).
[9] S. B. Haley, Phys. Rev. Lett. **74**, 3261 (1995).
[10] S. B. Haley and H. J. Fink, following paper, Phys. Rev. B **53**, 3506 (1996).
[11] T. Van Duzer and C. W. Turner, *Principles of Superconductive Devices and Circuits* (Elsevier, Amsterdam, 1981), p. 266.