Crossover from three-dimensional to two-dimensional vortex fluctuations in the irreversibility line of the $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ ($0\leq x\leq 0.55$) system

C. C. Almasan* and M. B. Maple
Department of Physics and Institute for Pure and Applied Physical Sciences, University of California, San Diego, La Jolla, California 92093
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It is shown that the temperature dependence of the irreversibility line $B(T^*)$ of the $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ ($0\leq x\leq 0.55$) system, inferred from magnetoresistance measurements, can be described by a Lindemann-type model of a vortex-solid–vortex-fluid phase transition triggered by vortex fluctuations. In this model, the previously observed transition in $B(T^*)$ from a power-law temperature dependence near T_c to a more rapid dependence below $T^*/T_c\approx 0.6$ can be accounted for in terms of a crossover from three-dimensional (3D) to two-dimensional (2D) vortex fluctuations. For different x values, a lower limit for the anisotropy ratio γ and an upper limit for the crossover magnetic induction B_{cr} above which 2D vortex fluctuations dominate were determined.

The presence of a boundary $H(T^*)$ in the applied magnetic field H temperature T plane of the high transition temperature T_c oxide superconductors which delineates the superconducting region in which the critical current density $J_c(H,T)\neq 0$ and the magnetization $M(H,T)$ exhibits irreversible behavior was observed in polycrystalline $La_{2-x}Sr_xCuO_{4+\delta}$.[1] Since then, there has been an ongoing debate about whether the irreversibility line $H(T^*)$ represents a crossover from flux creep to flux flow[2–4] or is caused by glassy vortex kinetics[5–7] or vortex lattice melting.[8–11] An interesting development in the study of the irreversibility line is the observation in a large number of cuprate materials (both polycrystalline and single-crystal samples) of a transition from an approximate $(1-T^*/T_c)^{3/2}$ temperature dependence near T_c to a more rapid dependence at lower temperatures.[12–18] Recently, it was shown that the $H(T^*)$ curves of $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ ($0\leq x\leq 0.55$), $Sm_{1.85}Ce_{0.15}CuO_{4-y}$, $YBa_2Cu_3O_y$, $Bi_2Sr_2CaCu_2O_{8+\delta}$, $Bi_2Sr_2CuO_y$, and $Tl_2Ba_2CuO_6$ appear to obey a universal scaling relation characterized by an $m=3/2$ power law near T_c, with a change to a more rapid temperature dependence below $T^*/T_c\approx 0.6$.[19,20] Schilling *et al.*[21] observed that the magnetic induction $B(T^*)$, measured with $H\parallel c$ on single crystals of $Bi_2Sr_2CaCu_2O_8$, changes from a parabolic temperature dependence, $B=B_0(1-T^*/T_c)^2$, near T_c to an exponential dependence, $B\sim\exp(\text{const}/T^*)$, at larger B and $T^*/T_c\leq 0.6$. They argued that the two regimes reflect a crossover at an induction B_{cr} from essentially three-dimensional (3D) vortex fluctuations near T_c to quasi-two-dimensional (2D) vortex fluctuations at lower temperatures and higher fields.

In this paper, we apply the analysis proposed by Schilling *et al.* to the magnetoresistance data for the $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ ($0\leq x\leq 0.55$) system[19] and show that, even for this system which has a lower anisotropy ratio than $Bi_2Sr_2CaCu_2O_8$, $B(T^*)$ is consistent with a vortex-solid–vortex-fluid phase transition triggered by vortex fluctuations, and that the departure of $B(T^*)$ from the power-law temperature dependence for $T^*/T_c<0.6$ can be explained by a crossover from 3D to 2D vortex fluctuations. A Lindemann-type melting criterion yields quantitative expressions for $B(T^*)$ in the two regimes which reproduce the experimental data very well. We obtained reasonable values of the anisotropy ratio γ for different Pr concentrations by fitting the experimental data to a Lindemann-type 2D melting line; as expected, γ increases with increasing x.

The present analysis was based on magnetoresistance $R(B,T)$ data taken on polycrystalline $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ ($0\leq x\leq 0.55$) samples, reported in Ref. 19. Due to the large anisotropy of the material, the onset of dissipation in $R(B,T)$ is determined by the grains which have their c axes parallel to the applied magnetic field H, while the grains with their ab planes parallel to H are still superconducting and, hence, do not contribute to $R(B,T)$. As described in Ref. 19, the curves in the B-T plane for several other systems inferred from the magnetoresistance and the real component of the ac magnetic susceptibility have the same temperature dependence (with slightly different magnetic field scales) as the true irreversibility line determined from measurements of the dc magnetization; it is in this sense that we denote $B(T^*)$ extracted from $R(B,T)$ data as the "irreversibility line." This approach is justifiable since the main focus in this work is on the *temperature dependence* of $B(T^*)$ and the uncertainty in the scale of H only introduces a small error in the value of γ, as discussed later. In particular, we were not able to extract the irreversibility line from dc magnetization measurements. For $x\geq 0.3$, the determination of $B(T^*)$ from magnetization measurements can only be made in low fields ($B\leq 0.8$ T) since the Pr ions carry localized magnetic moments and the resultant strong paramagnetism at higher fields dominates the diamagnetic superconducting response, making it difficult to extract the small diamagnetic signal from the large paramagnetic background.

Feigel'man, Geshkenbein, and Larkin[22] and Glazman and Koshelev[23] predicted the existence of a crossover from 3D to 2D vortex fluctuations at a magnetic field $B_{cr}\approx 4\phi_0/s^2\gamma^2$, where ϕ_0 is the magnetic-flux quantum, s is the distance between two superconducting layers, and $\gamma=\lambda_c/\lambda_{ab}$ is the anisotropy ratio, with λ_{ab} and λ_c the magnetic penetration depths. For $B\gg B_{cr}$ and moderate anisotropy ($\xi_{ab}\ll\gamma\ll\lambda_{ab}$),

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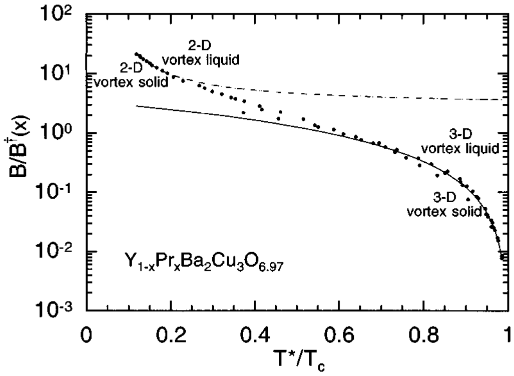

FIG. 1. $B/B^{\dagger}(x)$ vs T/T_c for $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$, on a logarithmic field scale. The solid line corresponds to a fit according to Eq. (4). The dashed line is a power-law fit to Eq. (5) with $m=1.45$.

2D vortex fluctuations are expected with a mean-squared thermal vortex fluctuation displacement $\langle u^2\rangle_{\rm th}$ in Josephson-coupled layers given by[23]

$$\langle u^2\rangle_{\rm th}=\frac{8\pi\lambda_{ab}^2 k_B T}{\phi_0 s B}\ln\left(\frac{Bs^2\gamma^2}{\phi_0}\right), \qquad (1)$$

where k_B is the Boltzmann constant, while, for $B \ll B_{\rm cr}$, 3D vortex fluctuations are expected with[8,9]

$$\langle u^2\rangle_{\rm th}=4\pi\gamma\lambda_{ab}^2 k_B T\left(\frac{4\pi}{B\phi_0^3}\right)^{1/2}. \qquad (2)$$

The phenomenological Lindemann melting criterion[24] predicts a melting transition when the displacement amplitude grows to a substantial fraction of the lattice constant a [for a vortex lattice $a\approx(\phi_0/B)^{1/2}$], i.e.,

$$\langle u^2\rangle = c_L^2 a^2 \approx c_L^2\phi_0/B, \qquad (3)$$

where c_L is the Lindemann number. This criterion is assumed to provide a quantitative description of the experimentally observed $B(T^*)$ boundary in the magnetic phase diagram which is considered as a vortex lattice melting line.[8,10,25]

By assuming that $\langle u^2\rangle_{\rm th}$ dominates possible quantum fluctuations, i.e., $\langle u^2\rangle\approx\langle u^2\rangle_{\rm th}$, we determined the melting line via the Lindemann criterion and compared this result with our $B(T^*)$ data. For the 2D vortex fluctuation region

$$\frac{B(T^*)}{B^{\dagger}(x)}=\frac{\phi_0}{s^2\gamma^2 B^{\dagger}(x)}\exp\left(\frac{\phi_0^2 c_L^2 s}{8\pi\lambda_{ab}^2(0)k_B T^*}\right), \qquad (4)$$

where $\lambda_{ab}(T)\approx\lambda_{ab}(0)$ at low temperatures, while for the 3D vortex fluctuation region

$$\frac{B(T^*)}{B^{\dagger}(x)}=\frac{\phi_0^5 c_L^4}{16\pi^3\lambda_{ab}^4(0)\gamma^2 B^{\dagger}(x)(k_B T_c)^2}\left(1-\frac{T^*}{T_c}\right)^m \qquad (5)$$

with $m=2$.

Plotted in Fig. 1 on a logarithmic field scale is $B/B^{\dagger}(x)$ vs T^*/T_c for the $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ system, where $B^{\dagger}(x)$ is a scaling induction defined to be the value of $B(T^*)$ at $T^*/T_c=0.6$, and $T^*(B)$ was taken to be the temperature at which $R(B,T)$ drops to 90% of its extrapolated normal-state value. We obtained similar results by extracting the irreversibility line from the magnetoresistance data corresponding to different voltage criteria. This indicates that the crossover in the irreversibility line is not due to changes in dynamics. Also, the magnetoresistance measurements were performed at very low dissipation levels ($I\approx 2$ μA) where the system is presumably in equilibrium and, hence, any changes in the I-V curves would be due the a phase transition and not due to dynamical effects.

In Fig. 1, the dashed line represents a fit of Eq. (4) to the data at low temperatures and high fields, while the solid line is a power-law fit of the data to Eq. (5) with $m=1.45$ in the vicinity of T_c. Recently, Blatter and Ivlev[25] have shown that by including the quantum fluctuations along with thermal fluctuations in $\langle u^2\rangle$ [Eq. (3)], the shape of the melting line can be approximated by a power law with an exponent $m=1.45$ instead of $m=2$. Hence, the fact that the best fit of the data to Eq. (5) is obtained for $m=1.45$, rather than $m=2$ for thermal fluctuations alone, could indicate that the quantum fluctuations are important in the $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ system. On the other hand, in the Bi- and Tl-based compounds, the thermal fluctuations are enhanced due to the weak interlayer coupling, while the quantum fluctuations remain essentially unchanged;[25] hence, the quantum correction is less important and the data for these materials are better described by a power law with exponent $m=2$.

The scaling behavior of $B(T^*)$ in the $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ system for various values of x allowed us to determine the anisotropy ratio γ even for those x values for which the low-T ($T^*/T_c\leqslant 0.2$) and high-B regime is not experimentally accessible; we assumed that, if high enough fields would be experimentally attainable, all of the data (even the data at low-x values) would follow the same behavior in the low-T and high-B regime. Hence, we determined γ from the prefactor to the exponential function [Eq. (4)] by fitting it with all of the experimental data available in this region, for different x values. Assuming that the two adjacent CuO_2 planes are strongly coupled so they can be treated as a single superconducting layer,[26] and that the occurrence of induced superconductivity on the chains results in an enhanced coupling between the CuO_2 planes, the distance $s=4.5$ Å between the weakly coupled CuO_2 and chain layers was chosen to be the relevant distance.[27] Even though the fitting of the data to Eq. (4) was done over a rather small range of fields and temperatures, we obtained reasonable values for γ which, as expected, increase with increasing Pr concentration. The values of γ for different Pr concentrations are given in Table I. The very good agreement between the irreversibility line and the melting line, obtained by applying a Lindemann criterion, as well as the reasonable values of the anisotropy for different Pr concentrations obtained by applying this analysis indicate that a Lindemann criterion is applicable in this case (even though it does not take the dynamic nature of the measurements into account) and that the presence of the melting transition reveals its signature in dynamics.

We overestimated the value of $T^*(B)$ by defining it as the temperature corresponding to a 90% drop in $R(B,T)$ instead of defining it as the irreversibility temperature which lies at lower T, in the flux-creep regime. As a result, this analysis gives an upper limit for $B^{\dagger}(x)$ and $B_{\rm cr}$ (given in Table I) and

TABLE I. Superconducting transition temperature T_c, magnetic penetration depth λ (from Ref. 28), scaling induction $B^{\dagger}(x)$, anisotropy ratio γ, crossover induction B_{cr}, and Lindemann numbers c_L^{2D} and c_L^{3D} determined from the Eqs. (4) and (5), respectively, for $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ with various values of x between 0 and 0.53. The uncertainty in $B^{\dagger}(x)$ is the standard deviation of the data from fits to $B(T^*)=B_0(1-T^*/T_c)^{1.45}$.

x	T_c (K)	λ (nm)	$B^{\dagger}(x)$ (T)	γ	B_{cr} (T)	c_L^{2D}	c_L^{3D}
0	91.9	143	66.28±15	7.4	745	0.091	0.18
0.1	88.5	172	51.78±13	8.4	579	0.11	0.22
0.2	73.3	193	17.68±7.0	14.3	200	0.11	0.22
0.3	62.5	219	13.48±4.0	16.4	152	0.11	0.23
0.4	45.3	293	8.4±0.5	20.8	94.4	0.13	0.26
0.5	31.1		2.7	36.6	30.5		
0.55	22.1		0.1	190.3	1.13		
0.53	12.7		0.5	85.1	5.64		

a lower limit for γ. However, as reported earlier,[19] the temperature dependence of $B(T^*)$ does not depend on choice of T^*.

Also given in Table I are the values of the Lindemann numbers c_L^{2D} and c_L^{3D} determined by fitting the data to Eqs. (4) and (5), respectively. For low x, the values for c_L^{2D} are somewhat smaller than the expected lower limit, $0.1\leqslant c_L \leqslant 0.3$.[29] However, Brandt has offered several reasons for why the Lindemann number might be lower than $c_L=0.1$.[9] Also the values of c_L^{2D} determined by fitting the data in the high-field regime, are smaller than the values of c_L^{3D}, extracted from the data in the low-field regime; the same tendency was obtained by Schilling *et al.* for $Bi_2Sr_2CaCu_2O_8$.[19] Both results disagree with the prediction by Ryu *et al.*[29] who, by applying Monte Carlo calculations, obtained a Lindemann number which decreases with decreasing field.

In summary, we have shown that the temperature dependence of the irreversibility line $B(T^*)$ of the $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ $(0\leqslant x \leqslant 0.55)$ system, inferred from magnetoresistance measurements, can be described by a Lindemann-type model of a vortex-solid–vortex-fluid phase transition triggered by vortex fluctuations. In this model, the previously observed transition in $B(T^*)$ from a power-law temperature dependence near T_c to a more rapid dependence below $T^*/T_c \approx 0.6$ can be accounted for in terms of a crossover from 3D to 2D vortex fluctuations. For different Pr concentrations, we determined a lower limit for the values of the anisotropy ratio γ and an upper limit for the crossover induction B_{cr}, above which 2D vortex fluctuations are expected to occur, by fitting the experimental data to Eq. (4). The field dependence of the Lindemann numbers c_L^{2D} and c_L^{3D} determined by fitting the data to Eqs. (4) and (5), respectively, disagree with the prediction by Ryu *et al.*[29] that c_L decreases with decreasing field.

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[*]Present address: Department of Physics, Kent State University, Kent, OH 44242.

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