# **Disorder-induced decoupling of pancake vortices in a layered superconductor**

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We consider the effect of pancake vortices induced by a magnetic field on the Josephson coupling in a disordered, layered superconductor. Due to the random displacements of vortices, the magnetic field suppresses the Josephson coupling. We find the characteristic value of the ''decoupling'' field. The effective Josephson energy does not drop to zero above this field, but saturates at some small value and remains field independent in the regime of individual pinning. Crossover to the collective pinning regime at higher fields and temperatures leads to partial restoration of the Josephson energy. We studied also the influence of the pancake wandering on the field distribution inside the superconductor. We found that the feature which is most sensitive to the pancake misalignment is the large field tail in the distribution function. We show that the asymmetry of the field distribution induced by this tail disappears at fields substantially smaller than the decoupling field.

#### **I. INTRODUCTION**

A number of macroscopic properties of high-*T<sub>c</sub>* superconductors (HTSC) carry a signature of the layered structure of these materials. The properties of a mixed state are affected particularly strongly. In a layered material, vortex lines consist of separate ''pancakes,'' and interaction between the pancakes belonging to different layers is relatively weak. Random displacements of these pancakes may suppress the interlayer coherence. Thermal fluctuations of pancakes cause such a suppression at temperatures that may be well below the vortex lattice melting transition.<sup>1</sup> This suppression may be translated into an exponentially strong temperature dependence of the critical current in  $c$  direction.<sup>2</sup> At even lower temperatures, vortex pancakes in different layers are aligned if there is no disorder, and the interlayer coherence is barely affected.

Inhomogeneities of the material create a random potential that leads to pancake displacements even at  $T=0$ ; these disorder-induced displacements suppress the phase coherence between the layers. The analysis of the pinning-induced suppression of the interlayer Josephson current for the simplest model of strong identical pointlike pinning centers was considered by Daemen *et al.*<sup>2</sup> In this model the suppression is determined by a single parameter, the concentration of pinning sites *n*. The field dependence of the effective Josephson coupling energy was found to be  $E_J^{\text{eff}}(B)$  $=E_J \exp(-\pi B/2\Phi_0 n)$ . A similar model was used to describe influence of Abrikosov vortices on the critical current of a single Josephson junction. $3$ 

Several recent experiments suggest the importance of pinning for the interlayer coupling in layered HTSC. Measurements of the internal magnetic field distribution in  $Bi_2Sr_2CaCu_2O_x$  (BiSCCO) by means of  $\mu$ <sup>+</sup>SR method<sup>4</sup> demonstrated that at low temperatures alignment of pancake vortices decreases substantially at fields 500–700 Oe. In the absence of thermal fluctuations, it is natural to attribute this misalignment at low temperatures to wandering of the vortex lines induced by the pinning potential. Random displacements of pancake vortices can also explain the field induced suppression of the critical current  $J_{c,c}(B)$  in *c* direction. Cho *et al.*<sup>5</sup> found that at low temperatures and fields  $B \approx 1$  T, current  $J_{c,c}$  becomes much smaller than its zero-field value. However, further suppression of the critical current at higher fields is much slower than suggested by the exponential formula of Ref. 2. Moreover, an increase of temperature at high fields may lead to a partial restoration of  $J_{c,c}$ .<sup>5,6</sup> More recently the effective Josephson coupling has been probed by the interplane plasma resonance.<sup>7</sup> The frequency of the plasma resonance, which measures the effective Josephson coupling energy, $\delta$  has been measured in a wide range of fields and temperatures. At low temperatures the plasma frequency has been found to decrease with field and increase with temperature in agreement with behavior of the Josephson critical current.

In this paper we develop a model that leads to results consistent with the experimental findings. We consider the influence of a relatively weak pinning on the Josephson coupling between layers and on the random wandering of vortex lines. Unlike the case of strong pinning centers which unconditionally "trap" the pancakes, $\frac{2}{3}$  here behavior of the vortex system is more complicated. At a small field, when vortex lines can be treated separately ("individual" pinning<sup>9</sup>), adjustment of pancakes to the pinning potential in the layers

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FIG. 1. Pinning-induced wandering of a vortex line in a layered superconductor.

leads to a random wandering of the lines. This wandering can be characterized quantitively by the elemental wandering distance,  $r_w$ , which can be defined as the average in-plane distance between two same-line pancakes belonging to the adjacent layers. It can be determined from the balance condition between the interlayer coupling and the strength of disorder. The estimate of the length  $r_w$  allows us to define the characteristic "decoupling" field  $B_w$  at which the vortex lines are destroyed by pinning, and the Josephson coupling between layers is suppressed significantly. Above  $B_w$ , the residual Josephson coupling energy does not drop to zero, but saturates at some value, which is much smaller than  $E_I$ until the pinning remains individual. This result is in agreement with the experiment of Cho *et al.*<sup>5</sup> Crossover to the collective pinning regime at higher fields leads to the partial restoration of Josephson coupling. In this regime Josephson coupling energy increases with the field; also, anomalous temperature dependence of  $J_{c,c}$  is possible here.

In addition to the suppression of the interplane critical current, we consider also the influence of random vortex line wandering on the field distribution in superconductors. We find that the asymmetry of the field distribution, which is characteristic for the vortex lattice, vanishes at fields substantially smaller than  $B_w$ .

## **II. SUPPRESSION OF THE INTERLAYER COUPLING BY THE DISORDER-INDUCED PANCAKE DISPLACEMENTS**

In an ideal layered superconductor at low temperatures, pancakes in different layers are aligned. Point defect disorder produces a random potential acting on each pancake. This leads to a distortion of vortex lines. The natural measure of the pinning effect is the typical in-plane shift  $r_w$  of a vortex line between the adjacent layers (see Fig. 1). To estimate  $r_w$ , we consider first a pair of pancakes separated by a distance *r*. The corresponding energy can be estimated as  $\epsilon(r) = \epsilon_{\text{ran}}(r) + \epsilon_{\text{coup}}(r)$ . The loss of coupling energy between the layers due to the pancake displacements  $\epsilon_{\text{coup}}(r)$ consists of the Josephson and magnetic contributions,

$$
\epsilon_{\text{coup}}(r) = \frac{\pi}{2} E_J r^2 \ln \frac{r_J}{r} + E_M r^2 \ln \frac{a}{r}.
$$
 (1)

Here  $E_J = \Phi_0^2 / \pi (4 \pi \lambda_c)^2 s$  is the Josephson energy per unit area,  $r_j = \gamma s$  is the Josephson length, *s* is the interlayer spacing, and  $\gamma = \lambda_c / \lambda_{ab}$  is the penetration depths anisotropy ratio. The magnetic coupling contribution in  $(1)$  is proportional to the energy  $E_M = s\bar{\Phi}_0^2/32\pi^2\lambda_{ab}^4$ , where  $\lambda_{ab}$  is the in-plane London penetration depth; *a* is the typical spacing between vortices. The energy  $\epsilon_{\text{ran}}(r)$  is the gain in the pinning energy corresponding to the adjustment of the pancake position to a minimum of the random potential within the area  $\pi r^2$ .<sup>10</sup> The latter function is determined by the distribution of the pinning energies  $P(\epsilon)$ . If the pinning is produced by a large number of point defects, this distribution is Gaussian,

$$
P(\epsilon) = \frac{1}{\sqrt{\pi}U_p} \exp\left(-\frac{\epsilon^2}{U_p^2}\right),\tag{2}
$$

where  $U_p$  is the pinning energy. The function  $\epsilon_{\text{ran}}(r)$  is determined by the condition that the probability to find a location with pinning energy less than  $\epsilon_{\text{ran}}$  in an area  $\sim r^2$  is of the order of unity, i.e.,

$$
\frac{r^2}{r_p^2} \int_{-\infty}^{\epsilon_{\text{ran}}} d\epsilon P(\epsilon) \approx 1.
$$
 (3)

Here  $r_p$  is the range of the pinning potential; for the shortrange potential that we consider further,  $r_p \approx \xi$ . For large displacements,  $r \gg r_p$ , the condition (3) yields

$$
\epsilon_{\rm ran} = -U_p \ln^{1/2} \left( \frac{r^2}{2\sqrt{\pi}r_p^2} \right). \tag{4}
$$

The optimization of  $\epsilon(r)$  with respect to *r* gives the following equation:

$$
r_w^2 = U_p \left( \pi E_J \ln \frac{r_J}{r_w} + 2E_M \ln \frac{a}{r_w} \right)^{-1} \ln^{-1/2} \left( \frac{r_w^2}{2\sqrt{\pi}r_p^2} \right), \quad (5)
$$

which is valid if  $r_w \ge \xi$ . The weak dependence of  $r_w$  on the magnetic field comes through the intervortex distance  $a \approx (\Phi_0 / B)^{1/2}$ . Equation (5) can be solved by iterations; a rough estimate given by the zeroth-order iteration is  $r_w^2 \approx U_p / (\pi E_J + 2E_M).$ 

The vortex displacements produce the phase difference between the layers,

$$
\delta\phi(\mathbf{r}) = \sum_{i} \mathbf{r}_{wi} \nabla_{\mathbf{r}} \phi_{v}(\mathbf{r} - \mathbf{R}_{i}),
$$
 (6)

and therefore cause suppression of the Josephson coupling [here  $\phi_n(\mathbf{r})$  is the phase distribution around the vortex core, and  $\mathbf{r}_{wi}$  is the relative displacement of neighbor pancakes in the *i*th vortex line]. Averaging with respect to  $\mathbf{r}_{wi}$  gives

$$
\langle (\delta \phi)^2 \rangle = r_w^2 \sum_i |\nabla_{\mathbf{r}} \phi_v(\mathbf{r} - \mathbf{R}_i)|^2 \approx 2 \pi n_v r_w^2 \ln \frac{r_j}{a}, \quad (7)
$$

where  $n_v = B/\Phi_0$  is the vortex density. The resulting effective Josephson energy is  $E_J^{\text{eff}} = E_J[1 - \langle (\delta \phi)^2 \rangle/2]$ . The last equation together with  $(5)$  and  $(7)$  gives

$$
E_J^{\text{eff}} - E_J = -E_J \frac{B}{B_w},\tag{8}
$$

where the characteristic field  $B_w$  is introduced. This field can be found as the solution of the following equation:

$$
B_w = \frac{\Phi_0}{\pi r_w^2 \ln(r_J/a_w)},
$$
\n(9)

with  $a_w = \sqrt{\Phi_0 / B_w}$ . The right-hand side of (9) only weakly depends on  $B_w$  through the field dependence of *a* and  $r_w$ , and this equation can be solved easily by iterations. Up to logarithmic factors the field  $B_w$  can be estimated as  $B_w \approx (E_J + E_M)\Phi_0 / U_p$ . Using the relation  $U_p \approx \Phi_0 s j_c \xi / c$ between the pinning potential and the critical current  $j_c$  and taking the parameters typical for BiSCCO compound,  $j_c = 10^6$  A/cm<sup>2</sup>,  $\lambda_{ab} = 1800$ Å,  $\gamma = 200$ ,  $\xi = 15$  Å, we obtain  $r_w \approx 50$  Å,  $B_w \approx 3$  T solving (5) and (9) numerically.

The estimate (8) is valid at weak fields,  $B \ll B_w$ . The behavior at higher fields depends upon the relation between  $B_w$  and the typical field, at which the crossover to the collective pinning regime takes place,  $B_{cp}$ . We assume that  $B_w \leq B_{cp}$ , which means that at  $B \approx B_w$  the interaction between vortices still only weakly affects their adjustment to the pinning potential. This assumption is justified for strong enough pinning potential,  $U_p > \Phi_0[(E_J + E_M)\xi^3]^{1/2}/(4\pi\lambda)$ . At fields  $B \ge B_w$  the coupling between layers can be treated as a perturbation. If we neglect the coupling completely, the vortices adjust to the pinning potential, and in this state the average Josephson energy

$$
\mathcal{E}_J = -E_J \sum_n \int d^2 \mathbf{r} \cos[\phi_{n+1}(\mathbf{r}) - \phi_n(\mathbf{r})]
$$
 (10)

equals zero. Finite coupling induces small Josephson ( $f_{ni}^J$ ) and magnetic  $(f_{ni}^M)$  forces acting on vortices. To estimate  $\mathbf{f}_{ni}^{J}$  we calculate the variation of the Josephson energy  $\delta \mathcal{E}_{J}$ caused by small vortex displacements **u***ni* ,

$$
\delta \mathcal{E}_j = E_j \sum_n \int d^2 \mathbf{r} [\phi_{n+1}^{(1)}(\mathbf{r}) - \phi_n^{(1)}(\mathbf{r})]
$$
  
 
$$
\times \sin[\phi_{n+1}^{(0)}(\mathbf{r}) - \phi_n^{(0)}(\mathbf{r})], \qquad (11)
$$

where  $\phi_n^{(1)} = \sum_i \mathbf{u}_{ni} \nabla_{\mathbf{r}} \phi_v(\mathbf{r} - \mathbf{R}_{ni})$  are the phase changes produced by vortex displacements. Varying Eq.  $(11)$  with regard to  $\mathbf{u}_{ni}$ , we find the Josephson forces acting on vortices:

$$
\mathbf{f}_{ni}^{J} = -E_{J} \int d^{2} \mathbf{r} \nabla_{\mathbf{r}} \phi_{v}(\mathbf{r} - \mathbf{R}_{ni}) \{ \sin[\phi_{n}^{(0)}(\mathbf{r}) - \phi_{n+1}^{(0)}(\mathbf{r})] + \sin[\phi_{n}^{(0)}(\mathbf{r}) - \phi_{n-1}^{(0)}(\mathbf{r})] \}.
$$
 (12)

In the regime of individual pinning, the increase of the pinning energy due to vortex displacements  $\mathbf{u}_{ni}$  can be estimated as

$$
\mathcal{E}_{\text{pin}} = \frac{1}{2} \sum_{i,n} K_{ni} \mathbf{u}_{ni}^2, \qquad (13)
$$

where the Labush constants  $K_{ni} \approx U_p / \xi^2$  do not depend on the interaction between vortices. Small displacements of the vortices under the Josephson and magnetic forces

$$
\mathbf{u}_{ni} = (\mathbf{f}_{ni}^J + \mathbf{f}_{ni}^M)/K_{ni} \tag{14}
$$

produce a finite Josephson energy

$$
\delta \mathcal{E}_J = -\sum_{i,n} (\mathbf{f}_{ni}^J)^2 / K_{ni} \,. \tag{15}
$$

Functions  $(\mathbf{f}_{ni}^J)^2$  and  $1/K_{ni}$  can be averaged over the realizations of the random potential separately, because the forces  $\mathbf{f}_{ni}^{J}$  are determined by interaction with a large number of pancakes. We also neglect correlations between the Josephson and magnetic forces. Assuming randomly placed pancakes we obtain the averaged square of the Josephson force  $\langle (\mathbf{f}_{ni}^J)^2 \rangle = 2 \pi E_J^2 / n_v$ . Substituting this into Eq. (15), we obtain the effective Josephson coupling energy

$$
E_J^{\text{eff}} = 2 \pi \frac{E_J^2}{K_0} \ll E_J,
$$

where  $1/K_0 = \langle 1/K_{ni} \rangle$ . It is important to note that in the individual pinning regime  $E_J^{\text{eff}}$  is field independent,

$$
E_J^{\text{eff}} \sim 2\,\pi E_J \frac{E_J \xi^2}{U_p}.\tag{16}
$$

The above consideration is valid if the condition  $E_J \ll U_p / \xi^2$  is satisfied, which means that  $E_J^{\text{eff}} \ll E_J$  at fields  $B \ge B_w$ .

It appears from Eq.  $(16)$  that the effective Josephson energy is zero in the limit of infinitely strong pinning  $(U_p \rightarrow \infty)$ . In fact, a small but finite Josephson coupling exists at  $B > B_w$  even in this limit,<sup>11</sup> because the intralayer phase stiffness energy

$$
\mathcal{E}_0 = \sum_n \int d^2 \mathbf{r} \frac{J}{2} (\nabla \phi_n^{(1)})^2 \tag{17}
$$

limits the phase perturbations  $\phi_n^{(1)}(\mathbf{r})$ . Here the stiffness constant  $J = s \Phi_0^2 / [\pi (4 \pi \lambda)^2]$ . Optimizing the total energy  $\delta \mathcal{E}_J + \mathcal{E}_0$  with respect to  $\phi_n^{(1)}$  we obtain for restored Josephson energy

$$
E_J^{\text{eff}} = \frac{E_J^2 \Phi_0}{2\pi J B}.
$$
 (18)

Comparing the latter expression with Eq. (16) at  $B = B_w$ , we conclude that the in-plane phase adjustments can be neglected if pinning is weak enough

$$
U_p < 2\pi \xi \sqrt{J(E_J + E_M)}.
$$
\n(19)

The latter relation is equivalent to the condition  $B_w < \sqrt{B_c}B_{c2}$ , which is compatible with the requirement  $B_w \ge B_{cr}$  that allowed us to use the model of displaced pancakes.

The above estimates are valid for a not too large field when interaction between vortices can be neglected. At some typical field  $B_{cp}$ , a crossover to the collective pinning regime takes place. In the case of weak interlayer coupling, condition  $B_{cp} \ge B_w$  is satisfied. Discussing the behavior of the Josephson coupling energy in this regime, we consider the simplest case of orientationally ordered lattices with the same orientation direction in all the layers. However otherwise pancake vortices in different layers are pinned independently  $[two-dimensional (2D)$  collective pinning. The collective pinning state is characterized by the field of displacements  $\mathbf{u}(\mathbf{r})$  with respect to the positions of the ideal lattice sites. The displacements are produced by the random potential, and are slowly varying in space. The field  $\mathbf{u}(\mathbf{r})$  can be characterized by two typical lengths, namely, by collective pinning radius  $R_c$  (the range where displacement varies by the length  $r_p$ , which we still assume to be of the order of  $\xi$ ) and the range of lattice order  $R_a$  (the range where displacement changes by the intervortex spacing  $a$ <sup>9</sup>. We will follow the same scheme as for the individual pinning, i.e., we neglect first the interaction between the layers, calculate the Josephson forces acting on the lattice, and estimate the restored Josephson energy caused by additional displacements  $u<sub>I</sub>$  under the action of small Josephson forces. To estimate these forces acting on the vortex lattice within an area of a size less than  $R_a$ , we can assume that just a homogeneous shift **u** exists between the vortex lattices in the adjacent layers. The sine of the phase difference, which determines the spatial distribution of the Josephson forces [see Eq.  $(12)$ ] oscillates with the wave vector proportional to the shift,  $\sin(\phi_n^{(0)} - \phi_{n+1}^{(0)}) = \sin[2\pi n_v(u_x y - u_y x) + \varphi]$ . Here  $\varphi$  is the random phase determined by the lattice deformations at distances larger than  $R_a$ . This estimate is valid when the typical period of oscillations  $1/(n<sub>v</sub>u)$  is smaller than  $R<sub>a</sub>$ . Direct calculation using Eq. (12) yields for the area density  $F_J = f_J n_v$ of the Josephson force

$$
F_J = \frac{E_J}{u} \sin[2\pi n_v (u_x y - u_y x) + \varphi];
$$
 (20)

the displacement  $u$  should satisfy the conditions  $1/n_p R_a \le u \le (\pi n_p)^{-1/2}$ . The displacement of the lattice under an arbitrary periodic force,  $u(\mathbf{k}) = G(\mathbf{k})F(\mathbf{k})$ , is determined by the response function  $G(\mathbf{k})$  which can be approximated by the form:

$$
G(\mathbf{k}) \approx \frac{1}{C_{66}k^2 + n_v K}.
$$
 (21)

Here  $n_v K \approx C_{66} \pi^2 / R_c^2$  is the Labush constant. Using Eqs.  $(20)$ ,  $(21)$  we can estimate the Josephson energy restored by the force  $F_J$  as

$$
E_J^{\text{eff}} = \langle F_J u_J \rangle = \pi n_v E_J^2 \int_{1/n_v R_a}^{1/\sqrt{\pi n_v}} \frac{u \ du}{u^2} \frac{1}{C_{66} (2 \pi n_v u)^2 + n_v K}
$$
  
=  $\frac{\pi E_J^2}{2K} \ln \left( 1 + \frac{R_a^2}{R_c^2} \right).$  (22)

The Labush constant may be related to the field-dependent critical current,  $K \approx \Phi_0 j_c(B)/c \xi$ . In the 2D collective pinning regime *K* decreases with field as  $K \approx K_0 B_{cp}/B$ , <sup>12</sup> which means that the effective Josephson energy in this regime increases proportional to the field. Schematic field dependence of the effective Josephson energy at low temperatures is shown in Fig. 2.

Equation  $(22)$  also gives a natural explanation for partial restoration of the Josephson coupling with temperature observed experimentally. $6,5$  Above the depinning temperature  $T_{dp}$  thermal fluctuations start to suppress pinning which



FIG. 2. Schematic field dependence of the effective Josephson coupling energy. The characteristic field value  $B_w$  is given by Eq.  $(9)$ ; crossover to the regime of collective pinning occurs at  $B \approx B_{cn}$ .

leads to decrease of the Labush parameter *K* and an increase of  $E_J^{\text{eff}}$ . For the well developed 2D collective pinning regime above  $T_{dp}$  the Labush parameter  $K(T)$  is expected to decrease with *T* as  $K(T,B) = K(0,B)(T_{dp}/T)^{3}$  with  $K(0,B)$  $\propto$  1/*B* and  $T_{dp} \propto B$  (see Ref. 12). Therefore  $E_J^{\text{eff}}$  in this regime is expected to change with field and temperature as  $E_J^{\text{eff}}$  $\propto T^3 B^{-2}$ . At higher temperatures thermal suppression of the Josephson coupling becomes important, and  $E_J^{\text{eff}}$  should decrease again.

### **III. THE EFFECT OF PANCAKE MISALIGNMENT ON THE FIELD DISTRIBUTION INSIDE A SUPERCONDUCTOR**

Measurement of the field distribution inside a superconductor provides an independent probe for the pancake alignment. The influence of different kinds of lattice deformations on the field distribution was studied by Brandt.<sup>13</sup> In this paper we are especially interested in the influence of the pancake misalignments on the field distribution. The signature of perfectly aligned vortices is a stretched tail on the strongfield side arising from the singularities in  $B(r)$  at the vortex cores. The misalignment of pancakes within a vortex line affects mostly the tail of the field distribution. An indication of the suppression of the tail was found in the recent experiments<sup>4,14</sup> using  $\mu$ <sup>+</sup> SR technique. Moreover, asymmetric field distribution was observed at weak magnetic fields, but the asymmetry abruptly decreases with the increase of the field.<sup>4</sup> We relate this phenomenon to the wandering of vortex lines in the regime of individual pinning.

For a perfectly aligned vortex lattice the field at distance *r* from a vortex core,  $\xi \le r \le a$ , is given by

$$
B(r) = \bar{B} + 2B_{\lambda} \ln \frac{a}{r}.
$$
 (23)

Here  $\bar{B}$  is the average field,  $B_{\lambda} = \Phi_0/4\pi\lambda^2$ . Equation (23) defines the field distribution

$$
p(B) = \frac{1}{B_{\lambda}} \exp\left(-\frac{B-\bar{B}}{B_{\lambda}}\right)
$$
 (24)

for  $B - \overline{B} > B_{\lambda}$ . This exponential tail ends abruptly at the maximum field at the vortex core,

$$
B_{\text{core}} \approx \bar{B} + 2B_{\lambda} \ln(a/\xi). \tag{25}
$$

We estimate now the suppression of the tail  $(24)$  in the field distribution by the random wandering of vortex lines. We assume that the density of pancakes in each layer is almost homogeneous. In this case the excess magnetic field  $\delta B = B - B$  in the vicinity of a wandering vortex line is determined by pancakes belonging to this line at distances smaller than *a* from the given point. Using the known result<sup>15,16</sup> for the z component of the field of a single pancake,  $B_{pz} = B_{\lambda} s/r$ , we obtain for the field in the core of a pancake:

$$
\delta B \approx B_{\lambda} s \sum_{r_n < a} \frac{1}{r_n}, \quad r_n = \sqrt{z_n^2 + [u(z_n)]^2}.\tag{26}
$$

Here  $r_n$  is the distance of the pancake in the *n*th layer from the chosen pancake in zeroth layer,  $z_n = sn$ . The random relative displacement  $u(z_n)$  grows with  $z_n$  $u(z) = r_w(z/s)^{2}$ . For an elastic string traveling in 3D space the wandering exponent  $\zeta$ =3/5 (see, e.g., Ref. 9) is slightly larger than the diffusion exponent 1/2. The lower cutoff,  $n_{\text{cut}} \equiv z_{\text{cut}} / s$ , of the logarithmically divergent sum over *n* in Eq. (26) is determined by the condition  $u(z_{\text{cut}})=z_{\text{cut}}$  which gives

$$
z_{\text{cut}} = s \left(\frac{r_w}{s}\right)^{\beta}, \quad \text{with} \quad \beta = \frac{1}{1 - \zeta} \tag{27}
$$

(the latter formula is valid if  $z_{\text{cut}} \geq s$ ). For estimates we assume that the vortex lines wander independently, and  $\zeta$ =3/5,  $\beta$ =5/2 (the diffusion model  $\zeta$ =1/2 would give  $\beta$ =2). The large distance cutoff in summation over *n* is determined by the minimum of the lattice constant *a* and the entanglement length  $L_{en}$ . The latter is determined by the condition  $u(L_{en}) \approx a$ , or

$$
L_{\text{en}} = s \left( \frac{a}{r_w} \right)^{1/\zeta}.
$$
 (28)

For the parameters we consider  $L_{en} > a$ , therefore the average field in the core  $B_{\text{core}}$  that cuts the tail  $(24)$  can be estimated by replacing  $\xi$  in (25) with  $z_{\text{cut}}$ :

$$
B_{\text{core}} \approx \bar{B} + 2B_{\lambda} \ln \frac{a}{z_{\text{cut}}}.
$$
 (29)

The tail disappears if  $z_{\text{cut}} \approx a$ , and correspondingly  $B_{\text{core}} \approx \overline{B} + B_{\lambda}$ . Thus the distribution  $p(B)$  becomes symmetric if the external field exceeds the characteristic value  $B_{sym}$ ,

$$
B_{\text{sym}} = \frac{\Phi_0}{s^2} \left(\frac{s}{r_w}\right)^{2\beta} \approx B_w 2 \pi \left(\frac{s}{r_w}\right)^{2\beta - 2} \tag{30}
$$

with  $2\beta - 2 = 3$  (or  $2\beta - 2 = 2$  in the diffusion model). From Eq.  $(30)$  we can see that the field  $B_{sym}$  is substantially smaller than  $B_w$ , i.e., asymmetry in the field distribution should disappear when the pancake alignment is not destroyed yet by disorder. We ascribe  $B_{sym}$  to the field  $B \approx 700$  Oe that corresponds to the drastic changes in distribution  $p(B)$  observed in the experiment.<sup>4</sup> At  $\overline{B} \simeq B_{sym}$  the asymmetry of the distribution vanishes, but its width  $\sigma = (\bar{B}^2 - \bar{B}^2)^{1/2}$  remains of the order of the width of  $p(B)$ for an ideal lattice,  $\sigma \approx B_{\lambda}$ . For the field increasing in the interval  $B_{\text{sym}} \le B \le B_w$  the distribution continues to be symmetric, but its width shrinks and becomes  $\sigma \approx B_\lambda \sqrt{s/a}$  in the field  $B \approx B_w$ . The latter estimate of  $\sigma$  obtained in the model of independent ideal 2D vortex lattices, $17$  and it is in agreement with the experimental values reported in Ref. 17 for fields  $B \ge 1$  T.

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