Hole motion in the Ising antiferromagnet: An application of the recursion method

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We study hole motion in the Ising antiferromagnet using the recursion method. Using the retraceable path approximation we find the hole's Green's function as well as its wave function for arbitrary values of t/J_z. The effect of small transverse interaction also is taken into account. Our results provide some additional insight into the self-consistent Born approximation.

I. INTRODUCTION

The problem of the hole motion in a quantum antiferromagnet has become one of the central issues in developing the theory of high-T_c superconductivity. The problem is an old one,[1,2] but only recently has a good understanding of it been reached (for a review see, e.g., Ref. 3). The self-consistent Born approximation[4,5] turns out to be extremely successful in predicting the energy of the quasiparticle,[6,7] mainly due to the vanishing of the low-order vertex corrections.[6] Results obtained within this approximation agree well with exact diagonalization studies on small clusters.[8] The basic feature is that hole motion is strongly renormalized by the cloud of spin excitations (distortions) it causes and results in a narrow (of the order of the superexchange constant J) band with minima at the $(\pm\pi/2,\pm\pi/2)$ points on the boundary of the magnetic Brillouin zone. More recent work by one of us[9] has obtained the wave function of the hole within the same approximations and is also in agreement with exact diagonalization results.[10]

If the radius of the magnetic polaron is small enough, which happens at $J\sim t$, its wave function can be constructed variationally in direct space,[11,12] and the bandwidth becomes of order t^2/J.

It was found long ago that hole motion in an antiferromagnet with strong Ising anisotropy produces a string of the overturned spins, which confines the hole to the origin of its path,[2] making the hole's motion completely incoherent. The possibility of curing the spin background by the hole going one and a half times around an elementary loop on the lattice[13] does not change this picture significantly, as the effective mass of such coherent motion is extremely high. Surprisingly, the string picture seems to survive the limit of the isotropic Heisenberg model, where quantum fluctuations that can cure overturned spins are most effective. This shows up in the subleading peaks above the quasiparticle peak in the hole spectral function.[6,8]

The subject of the present work is to study the hole motion in an antiferromagnet with strong Ising anisotropy using the recursion method. The method, also known as Lanczos technique, was developed initially for the electronic structure calculations of disordered systems,[14,15] and later generalized for the finite-lattice calculations of the strongly correlated systems.[8,16] We show that the method is particularly well suited for the problem at hand and derive many of the known, as well as some new, results within its framework. To keep things as simple as possible, we treat the t-J model within the linear spin-wave approximation, which has no formal justification in the Ising limit. Nevertheless, we show that the physics of the problem remains essentially unchanged by this drastic approximation. Moreover, the spin-wave formalism permits us an important direct comparison with the results of the self-consistent Born approximation approach.[5]

The recursion method consists in the following.[14] Given the Hamiltonian and the initial vector $|1\rangle$, it generates a new basis of vectors according to the rule

$$|n+1\rangle=H|n\rangle-a_n|n\rangle-b_n^2|n-1\rangle, \tag{1}$$

with $|0\rangle=0$, and $b_1^2=\langle 1|1\rangle$. The coefficients in the recurrence are calculated from

$$a_n=\frac{\langle n|H|n\rangle}{\langle n|n\rangle},\quad b_n^2=\frac{\langle n|n\rangle}{\langle n-1|n-1\rangle}. \tag{2}$$

Remarkably, in the new basis the Hamiltonian has a tridiagonal form

$$H_{nn}=a_n,\quad H_{n-1,n}=H_{n,n-1}=b_n,$$
$$H_{nm}=0\quad \text{for all}\quad |n-m|>1, \tag{3}$$

and thus describes a fictitious semiline of ''atoms'' with local orbitals $|n\rangle$, diagonal energies a_n, and hopping elements b_n to the $n+1$ atom. Then the diagonal Green's function $G_{11}(\omega)=\langle 1|(\omega-H)^{-1}|1\rangle$ takes the form of a continued fraction

$$G_{11}(\omega)=\cfrac{b_1^2}{\omega-a_1-\cfrac{b_2^2}{\omega-a_2-\dots}}. \tag{4}$$

We derive analytical expressions for the hole's Green's function and wave function in the next section. The effect of a small transverse interaction $J_\perp$ is taken into account in Sec. III. We find that hole motion becomes coherent and describe it for different limits of t/J_z ratio. Comparison with previous works is done in the Conclusion.

II. THE ISING LIMIT

Let us consider the motion of the hole coupled to the localized spins on the lattice.[1,2] In the linear spin-wave ap-

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proximation, the Hamiltonian of the t-J model reads[5–7]

$$H=zt\sum_{k,q}\phi(k,q)(c_k^+c_{k-q}a_q+c_{k-q}^+c_ka_q^+)$$

$$+J_z\sum_k\omega(k)a_k^+a_k,$$

$$\phi(k,q)=u_q\gamma_{k-q}+v_q\gamma_k,\quad \omega(k)=\sqrt{1-\alpha^2\gamma_q^2},$$

$$\gamma_q=\frac{1}{2}[\cos(q_x)+\cos(q_y)], \tag{5}$$

where we absorbed a factor zS in our definition of J (z is a number of nearest neighbors), u_q and v_q are the usual Bogolubov coefficients

$$u_q=\sqrt{\frac{1+\omega(q)}{2\omega(q)}},\quad v_q=-\mathrm{sgn}(\gamma_q)\sqrt{\frac{1-\omega(q)}{2\omega(q)}}, \tag{6}$$

and $\alpha\equiv J_\perp/J_z$. Each hopping of the hole produces emission or absorption of the spin excitations.

For the problem of single-hole motion in an antiferromagnet, it is natural to choose as a starting vector the state with one hole and no spin deviations, i.e., $|1\rangle=c_p^+|\mathrm{vac}\rangle$, where $|\mathrm{vac}\rangle$ denotes vacuum for both hole and magnon operators. Thus $|\mathrm{vac}\rangle$ is simply the Néel state. The next vector is then

$$|2\rangle=H|1\rangle=H_t|1\rangle=zt\sum_q\phi(p,q)c_{p-q}^+a_q^+|\mathrm{vac}\rangle. \tag{7}$$

Clearly, $\langle 1|2\rangle=0$, and $b_2^2=\langle 2|2\rangle/\langle 1|1\rangle=(zt)^2\Sigma_q\phi^2(p,q)$. Acting by H on $|2\rangle$, one finds

$$H_t|2\rangle=(zt)^2\sum_q\phi^2(p,q)c_p^+|\mathrm{vac}\rangle+(zt)^2$$

$$\times\sum_{q,q_1}\phi(p,q)\phi(p-q,q_1)c_{p-q-q_1}^+a_{q_1}^+a_q^+|\mathrm{vac}\rangle,$$

$$H_J|2\rangle=ztJ\sum_q\omega(q)\phi(p,q)c_{p-q}^+a_q^+|\mathrm{vac}\rangle, \tag{8}$$

$$a_2=J\frac{\Sigma_q\omega(q)\phi^2(p,q)}{\Sigma_q\phi^2(p,q)}.$$

Following the formula described in the Introduction, one obtains

$$|3\rangle=(zt)^2\sum_{q,q_1}\phi(p,q)\phi(p-q,q_1)c_{p-q-q_1}^+a_{q_1}^+a_q^+|\mathrm{vac}\rangle$$

$$+ztJ\sum_q\left(\omega(q)-\frac{\Sigma_q\omega(q)\phi^2(p,q)}{\Sigma_q\phi^2(p,q)}\right)$$

$$\times\phi(p,q)c_{p-q}^+a_q^+|\mathrm{vac}\rangle. \tag{9}$$

The procedure becomes untractable very quickly because of the branching: each action of H on a state with n magnons produces again a state with n magnons (H_J term) as well as states with $(n+1)$ and $(n-1)$ magnons (H_t term), whose coefficients have a complicated momentum dependence. The exception occurs when $\omega(q)$ is momentum independent, i.e., in the Ising ($\alpha=0$) case. In that case one has $a_2=J_z$ and

$$|3\rangle=(zt)^2\sum_{q,q_1}\phi(p,q)\phi(p-q,q_1)c_{p-q-q_1}^+a_{q_1}^+a_q^+|\mathrm{vac}\rangle, \tag{10}$$

where now $\phi(p,q)=\gamma_{p-q}$, and $u_q=1$, $v_q=0$. The norm of this state is

$$\langle 3|3\rangle=(zt)^4\sum_{q,q_1}\sum_{l,l_1}\phi(p,q)\phi(p-q,q_1)\phi(p,l)$$

$$\times\phi(p-l,l_1)\langle\mathrm{vac}|a_la_{l_1}a_{q_1}^+a_q^+|\mathrm{vac}\rangle$$

$$=(zt)^4\sum_{q,q_1}\phi^2(p,q)\phi^2(p-q,q_1) \tag{11}$$

because of the property $\Sigma_{q_1}\gamma_{p-q_1}\gamma_{p-q-q_1}^2=0$. Thus, among two possible pairings of a operators in (11) only the "diagonal" one ($l_1=q_1, l=q$) contributes. Note also that $\Sigma_{q_1}\gamma_{p-q-q_1}^2=1/z$, and hence $b_3^2=zt^2$. Continuing one finds

$$a_3=J_z\frac{\Sigma_{q,q_1}[\omega(q)+\omega(q_1)]\phi^2(p,q)\phi^2(p-q,q_1)}{\Sigma_{q,q_1}\phi^2(p,q)\phi^2(p-q,q_1)}=2J_z,$$

$$|4\rangle=(zt)^3\sum_{q,q_1,q_2}\phi(p-q-q_1,q_2)\phi(p-q,q_1)$$

$$\times\phi(p,q)c_{p-q-q_1-q_2}^+a_{q_2}^+a_{q_1}^+a_q^+|\mathrm{vac}\rangle. \tag{12}$$

Now, however, in addition to the "diagonal" pairing ($l_i=q_i$) a new one ($l=q_2, l_1=q_1, l_2=q$) appears, for example,

$$\langle 4|4\rangle=(zt)^6\left\{\left(\frac{1}{z}\right)^3+\sum_{q,q_1,q_2}\gamma_{p-q}\gamma_{p-q_2}\gamma_{p-q_1-q_2}\gamma_{p-q-q_1}\gamma_{p-q-q_1-q_2}^2\right\}. \tag{13}$$

This pairing describes a hole jumping around an elementary loop (a plaquette on a square lattice) 1.5 times.[13] The triple integral in (13) is calculated to give $(1/z)^3\frac{1}{16}[2+\cos(p_x-p_y)][2+\cos(p_x+p_y)]$. It describes an effective hole hopping along the same sublattice. The hopping is maximal for $p=(0,0)$ and minimal for $p=(\pi,0)$ or $(0,\pi)$. Propagation via closed loops was studied in great detail in Ref. 17. For $J_z\gg t$ the effective hopping corresponds to the tunneling through a potential barrier (it scales as $\exp[-4.6(J_z/t)^{1/2}]$[17]) and leads to the band with minima at $p=(0,0)$. In the opposite limit ($t\gg J_z$) the hole can travel very far before noticing the confining potential because of the finite J_z, and the weight of the closed-loop paths among the long self-retraceable paths is very small. This point of view is supported by the results of Ref. 18, where it was found that the ground-state energy of the one-hole system scales as $J_z^{2/3}$ in a wide range of J_z/t ($1>J_z/t>5\times10^{-3}$), which is a characteristic feature of the retraceable-path approximation (see our discussion below).

Based on these arguments we will omit all closed-loop contributions in the following. This corresponds to solving the problem on a Bethe lattice or in the limit of infinite dimensionality.[19] Diagrammatically this approximation consists in neglecting the vertex corrections.[6,7]

With this simplification in mind, the procedure can be iterated infinitely, and results in the following expression for the coefficients of the recursion

$$a_n=(n-1)J_z, \quad b_n^2=zt^2, \quad \text{for any } n\geqslant 2. \tag{14}$$

As should be clear by now, linear n dependence of a_n comes from the simple fact that $H_J|n\rangle=(n-1)J_z|n\rangle$. On the other hand, the n independence of b_n is caused by the particular form of the coupling γ_{p-q}. The strong momentum dependence of this coupling favors only one particular pairing out of $(n-1)!$ possible at the nth step of the recursion procedure. This particular pairing is nothing but the retraceable-path approximation of Brinkman and Rice:[1] first the last excited magnon is absorbed, then the one before last, etc. If the hole-boson coupling is momentum independent, as happens, for example, in the Holstein model, the outlined procedure leads to $b_n^2\sim n$,[20] and, as a result, to a different physics.

So far we have found that Hamiltonian H has a simple tridiagonal form in the space spanned by the vectors $|n\rangle$ [see Eq. (3)]. Then, for any vector $|\Psi\rangle=\Sigma_n\Psi_n|n\rangle$ of this space, the Schrodinger equation $H|\Psi\rangle=E|\Psi\rangle$ holds, or

$$(E-a_n)\Psi_n-b_{n-1}\Psi_{n-1}-b_{n+1}\Psi_{n+1}=0, \tag{15}$$

with boundary condition $\Psi_0=0$ (remember that $|0\rangle=0$ by construction). In the small-J (large-n) limit we may use a continuum approximation to find

$$\pm\sqrt{z}t\frac{\partial^2\Psi_n}{\partial n^2}\pm 2\sqrt{z}t\Psi_n+(n-1)J_z\Psi_n=E\Psi_n, \tag{16}$$

which is just the Schrödinger equation for a particle on a semiline in the linear potential. Interestingly, index n plays the role of coordinate along the path of the hole—this correspondence is exact because we neglected all closed loops. The $+$ $(-)$ sign in (16) corresponds to choosing energy E close to the upper (lower) edge of the band. At the bottom of the band one finds $J_z^{2/3}$ behavior:

$$E_n=-2\sqrt{z}t-J_z+\beta_n J_z^{2/3}(\sqrt{z}t)^{1/3}. \tag{17}$$

Here $-\beta_n$ are the zeros of the Airy function. It is known that the spin-wave approximation overestimates the bandwidth,[5] and more accurate treatment of the constraint leads to the replacement of $\sqrt{z}$ by the $\sqrt{z-1}$ in the formulas above. The result obtained is by no means new and goes back to the 1960s.[2]

We would like to demonstrate a different approach here. Consider the one-hole Green's function, which has a continued fraction form [see Eq. (4)].

$$G_{11}(\omega)=\cfrac{1}{\omega-\cfrac{zt^2}{\omega-J_z-\cfrac{zt^2}{\omega-2J_z-\cdots}}}. \tag{18}$$

A little thinking shows that it is equivalent to

$$G(\omega)[\omega-zt^2G(\omega-J_z)]=1, \tag{19}$$

which coincides with the self-consistent Born approximation,[5] in agreement with our expectations. Let us try the ansatz[21]

$$G(\omega)=-\frac{1}{\sqrt{z}t}\frac{\Upsilon(\omega)}{\Upsilon(\omega+J_z)}. \tag{20}$$

Then Eq. (19) takes the form of the difference equation

$$\Upsilon(\omega-J_z)+\Upsilon(\omega+J_z)=-\frac{\omega}{\sqrt{z}t}\Upsilon(\omega), \tag{21}$$

which is a well-known recursion relation for the Bessel functions,[22] and hence $\Upsilon(\omega)=AJ_{-\omega/J_z}(2\sqrt{z}t/J_z)$. This immediately gives, for the Green's function,

$$G_{11}(\omega)=-\frac{1}{\sqrt{z}t}\frac{J_{-\omega/J_z}(2\sqrt{z}t/J_z)}{J_{-(\omega+J_z)/J_z}(2\sqrt{z}t/J_z)}. \tag{22}$$

This form is a direct consequence of the linear dependence of a_n on n (see Ref. 23), which, in turn, is an intrinsic feature of the retraceable-path approximation. A similar expression was obtained within the retraceable-path approximation (without using spin-wave transformation) in Ref. 24. Notice that energy appears only in the indexes of the Bessel functions.

Let us look at the $t/J_z\gg 1$ limit first. Here we need the double asymptotic expansion of the Bessel function at a large value of the argument and index[22]

$$J_{\mu\cos\theta}(\mu)\sim\sqrt{\frac{2}{\pi\mu\sin\theta}}\cos\left\{\mu(\sin\theta-\theta\cos\theta)-\frac{\pi}{4}\right\},$$

$$\mu>0, \quad 0<\theta<\pi/2. \tag{23}$$

Somewhat lengthy calculations give an expected answer

$$G_{11}(\omega)=\frac{J_z}{\sqrt{z}t}\sum_{\nu=0}^{\infty}\frac{1}{\omega-\omega_\nu},$$

$$\omega_\nu=-2\sqrt{z}t-J_z+\beta_\nu J_z^{2/3}(\sqrt{z}t)^{1/3}, \tag{24}$$

in complete agreement with the results of the Schrödinger equation (17). Notice that residue of the poles is ν independent and is given by $J_z/\sqrt{z}t$, supporting the "dominant-pole" approximation of Ref. 5. We note in passing that the quasiclassical expression for the zeros of the Airy function $\beta_\nu=[3\pi/2(\nu+\frac{3}{4})]^{2/3}$ works extremely well even for $\nu=0$: it gives ~ 2.32, compared to the exact value $\beta_0=2.34$.

In the opposite $t/J_z \ll 1$ limit one can use the standard small argument expansion

$$J_\eta(x)=\left(\frac{x}{2}\right)^\eta \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(\eta+k+1)}\left(\frac{x}{2}\right)^{2k}, \tag{25}$$

to find the perturbation theory result $G_{11}(\omega)=(\omega+zt^2/J_z)^{-1}$.

Recently, one of us[9] derived a wave function that corresponds exactly to the self-consistent Born approximation (SCBA),

$$|\Psi_k^{\rm SCBA}\rangle=z_k\Big(c_k^+ + N^{-1/2}\sum_q zt\phi(k,q)G_{k-q}(\epsilon_k-\omega_q)c_{k-q}^+ a_q^+ + \cdots + N^{-n/2} \times \sum_{q,q_1,\cdots,q_{n-1}} (zt)^n \phi(k,q)G_{k-q}(\epsilon_k-\omega_q)\cdots\phi(k-q-q_1-\cdots-q_{n-2},q_{n-1}) \times G_{k-q-q_1-\cdots-q_{n-1}}(\epsilon_k-\omega_q-\cdots-\omega_{q_{n-1}})c^+_{k-q-q_1-\cdots-q_{n-1}}a_q^+\cdots a_{q_{n-1}}^+\Big)|{\rm vac}\rangle. \tag{26}$$

Here $\epsilon_k=\Sigma_k(\epsilon_k)$ is the quasiparticle energy and Σ_k is the self-energy in the SCBA. The quasiparticle spectral weight is given by $(z_k)^2=(1-\partial\Sigma_k(\omega)/\partial\omega|_{\omega=\epsilon_k})^{-1}$. Clearly, in our case the role of the momentum k is played by the index ν. The specific form of the Green's function (22) immediately gives us that the chain product of the Green's function in the last equation has a simple form

$$G(\omega-J_z)G(\omega-2J_z)\cdots G(\omega-nJ_z)=\left(\frac{-1}{\sqrt{z}t}\right)^n J_{-(\omega-nJ_z)/J_z}\left(\frac{2\sqrt{z}t}{J_z}\right) \Big/ J_{-\omega/J_z}\left(\frac{2\sqrt{z}t}{J_z}\right).$$

Using again asymptotic expansion of the Bessel functions, we find for the hole in lowest state $\nu=0$ $(t/J_z \gg 1)$

$$|\Psi_k^{\rm SCBA}\rangle \sim \sqrt{\frac{J_z}{\sqrt{z}t}}\Big(c_k^+ - N^{-1/2}\sum_q \sqrt{z}\gamma_{k-q}c_{k-q}^+a_q^+ + \cdots + N^{-n/2} \times \sum_{q,q_1,\cdots,q_{n-1}} (-1)^n z^{n/2} n\gamma_{k-q}\cdots\gamma_{k-q-\cdots-q_{n-1}}c^+_{k-q-q_1-\cdots-q_{n-1}}a_q^+\cdots a_{q_{n-1}}^+\Big)|{\rm vac}\rangle, \tag{27}$$

for $n \ll (\sqrt{z}t/J_z)^{1/3}$.

III. SMALL $J_\perp$ LIMIT

A virtue of the present approach is that we can include the effect of the small transverse interaction $J_\perp$, $J_\perp=\alpha J_z \ll J_z$. As was already discussed in Ref. 5, analytical consideration is possible if one restricts oneself to linear in α accuracy. Then $\omega(q)=1+O(\alpha^2)$, $u_q=1+O(\alpha^2)$, but $v_q=-\frac{1}{2}\alpha\gamma_q$. Hence,

$$\phi(p,q)=\gamma_{p-q}-\frac{1}{2}\alpha\gamma_q\gamma_p+O(\alpha^2). \tag{28}$$

One finds for first a and b coefficients

$$a_1=0, \quad a_2=J_z, \quad b_2^2=(zt)^2\sum_q (\gamma_{p-q}^2-\alpha\gamma_p\gamma_q\gamma_{p-q})=zt^2(1-\alpha\gamma_p^2). \tag{29}$$

The key property $\Sigma_q\gamma_{p-q}\gamma_{k-q}^2=0$ ensures that the retraceable-path approximation is exact at low order, i.e., that

$$\sum_{q,q_1}\phi(p,q_1)\phi(p-q_1,q)\phi(p-q,q_1)\phi(p,q)=0+O(\alpha^2). \tag{30}$$

Moreover, one can find

$$I_n \equiv \sum_{q,q_1,\cdots,q_{n-1}} \phi^2(p,q)\phi^2(p-q,q_1)\cdots\phi^2(p-q-q_1-\cdots-q_{n-2},q_{n-1})$$
$$= \sum_{q,q_1,\cdots,q_{n-2}} \phi^2(p,q)\phi^2(p-q,q_1)\cdots\phi^2(p-q-q_1-\cdots-q_{n-3},q_{n-2})\sum_{q_{n-1}} \phi^2(p-q-q_1-\cdots-q_{n-2},q_{n-1})$$
$$= \frac{1}{z}I_{n-1} - \frac{\alpha}{z}\left(\frac{1}{z}\right)^{n-2}\sum_q \gamma_q^4. \tag{31}$$

As a result

$$b_n^2 = zt^2(1-\alpha zB) + O(\alpha^2), \quad n \geqslant 3,$$

$$B = \sum_q \gamma_q^4 = \frac{9}{64}. \tag{32}$$

At the same time, there are no changes in a_n, $a_n = (n-1)J_z$. Because of the difference between b_2^2 and $b_{n\neq 2}^2$, the continued fraction expression for the Green's function is now equivalent to the following system of two equations:

$$G_{11}(\omega) = \frac{1}{\omega - zt^2(1-\alpha\gamma_p^2)G_{22}(\omega)}, \tag{33}$$

$$G_{22}(\omega) = \frac{1}{\omega - J_z - z\tilde{t}^2 G_{22}(\omega - J_z)}.$$

We introduced $\tilde{t}^2 = t^2(1-\alpha zB)$ here. Comparing the last equation of (33) with Eqs. (19) and (22) of the preceding section one finds the answer

$$G_{22}(\omega) = -\frac{1}{\sqrt{z}\tilde{t}}\frac{J_{-(\omega-J_z)/J_z}(2\sqrt{z}\tilde{t}/J_z)}{J_{-\omega/J_z}(2\sqrt{z}\tilde{t}/J_z)}. \tag{34}$$

In the $J_z \ll \tilde{t}$ limit, expression (34) takes the familiar form

$$G_{22}(\omega) = \frac{J_z}{\sqrt{z}\tilde{t}}\sum_{\nu=0}^{\infty}\frac{1}{\omega - \bar{\omega}_\nu}, \tag{35}$$

$$\bar{\omega}_\nu = -2\sqrt{z}t + \beta_\nu J_z^{2/3}(\sqrt{z}\tilde{t})^{1/3}.$$

We want to consider the lowest hole state and thus may approximate

$$\sum_{\nu=0}^{\infty}\frac{1}{\omega-\bar{\omega}_\nu} \simeq \frac{1}{\omega-\bar{\omega}_0} + \sum_{\nu\neq 0}\frac{1}{\bar{\omega}_0 - \bar{\omega}_\nu}$$
$$= \frac{1}{\omega-\bar{\omega}_0} + \left(\frac{3\pi J_z}{2}\right)^{-2/3}(\sqrt{z}\tilde{t})^{-1/3}$$
$$\times \sum_{\nu\neq 0}\frac{1}{\nu^{1/3}(\nu+3/2)^{1/3}}. \tag{36}$$

The last sum can be approximated by the integral from 0 to some ν_m, which, in turn, is determined by the condition $\bar{\omega}_{\nu_m} \simeq 2\sqrt{z}\tilde{t}$ (upper edge of the incoherent band at $J_z = 0$). In this way we find that the last sum is given by $-4/\pi J_z$. We substitute the result just found for $G_{22}(\omega)$ into the first of Eqs. (33) and find, after some algebra,

$$G_{11}(\omega) = \frac{\pi^2}{4(\pi-2)}\frac{J_z}{\sqrt{z}\tilde{t}}\frac{1}{\omega - \tilde{\omega}}, \tag{37}$$

$$\tilde{\omega} = -2\sqrt{z}\tilde{t} + 2.34J_z^{2/3}(\sqrt{z}\tilde{t})^{1/3} - \frac{\pi J_z}{2(\pi-2)}(1-\alpha\gamma_p^2).$$

The pole at $\tilde{\omega}$ describes a narrow, coherent band of bandwidth $\sim 1.38J_\perp$ with minima at $\gamma_p = 0$, i.e., along the boundary of the magnetic Brillouin zone.[5] Note that residue of the pole remains unaffected. It is a matter of short calculation to find out that in the opposite limit ($J_z \gg \tilde{t}$) the answer is

$$G_{11}(\omega) = \frac{1}{\omega + (z\tilde{t}^2/J_z)(1-\alpha\gamma_p^2)}. \tag{38}$$

The bandwidth is much smaller, $z\tilde{t}^2 J_\perp / J_z^2$, and goes to the correct perturbative answer $\sim zt^2/J$ at the isotropic point $J_\perp = J_z$ ($\alpha = 1$).

IV. CONCLUSIONS

A simple recursion method was used to calculate a single-particle Green's function for the hole moving in the antiferromagnet with strong Ising anisotropy. By neglecting closed-loop contributions we reduced the problem to an exactly solvable one. In the Ising limit we find, in agreement with known results, that the hole is confined to the origin of its path by the effective linear potential due to the overturned spins. We then take into account the small transverse interaction between spins and find that the hole motion becomes coherent. We calculate the hole's spectrum and Green's function to the first order in $J_\perp/J_z$. This calculation explains the ''dominant-pole'' approximation of Ref. 5. We argue that the spin-wave approximation, employed in this paper, does not affect the essence of the problem, as can be seen, for example, from the comparison of our expression (22) for the hole's Green's function with formula (4) of Ref. 24. Overall, our approach can be considered as another way to arrive at the self-consistent Born approximation[5] at least in the case of strong Ising anisotropy. Our results become exact in the limit of infinite dimension.

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