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Magnetostrictive bending of a film-substrate system

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Relations are derived between the curvatures produced by magnetization of a very thin epitaxial film on a substrate of finite thickness and the magnetoelastic coefficients of the film material. Relations are found both for a free system and for one constrained to be flat in one direction. The derivation includes the important effects of the discontinuity in strain at the interface, which interacts with the changing strain in the film as the system bends. This interaction, which has been omitted in previous derivations, produces curvatures twice as large as previous calculations for given magnetoelastic coefficients. The derivation is based on a specific simple form for the magnetoelastic energy as a function of strains and magnetization direction and proceeds by minimizing the total energy with respect to the two curvatures. Quantitative application is made to a measured system and compared to the results of previous theories.

I. INTRODUCTION

An important technique for determining magnetostrictive strains when a ferromagnetic material is magnetized measures the bending of a substrate to which a film of the unmagnetized material has been bonded.[1,2] The film-substrate combination will be referred to as the system. The bending and the strains are produced when the bonded film is magnetized to saturation by a magnetic field. Two curvatures are produced with opposite signs, one along the direction of the magnetization and one perpendicular to it, as well as a volume strain. The theory of the bending relates the curvatures to dimensionless coefficients of the magnetoelastic energy, i.e., the energy of interaction of the strains and magnetization in the material. When the simplest suitable form for the magnetoelastic energy is assumed, which is linear in the strains, phenomenological coefficients λ_0 for the magnitude of the isotropic volume strain and λ_1 for the anisotropic strain are introduced. By evaluating the difference in bending for the magnetization along and perpendicular to the length of a film-substrate system, the volume magnetostriction cancels out, and the difference of the curvatures gives the value of λ_1 by itself.

Derivations by Klokholm[1] and by du Trémolet and Peuzin[3] have led to different forms of the relation between the difference of the curvatures and λ_1. The present paper gives still a third form of the relation, and includes an important effect which is omitted in Refs. 1 and 3. Namely the derivation takes account of the presence of a discontinuity of strain at the interface, which interacts with the change in magnitude of the strains in the film due to the bending. The effect of this interaction is to double the calculated curvatures for a given λ_1 compared to the previous theories. The total-energy minimization procedure applied in Ref. 3 is also used here. This procedure minimizes the total energy with respect to three parameters: two curvatures and the position of the unstrained layer in the bent substrate. In addition to finding the curvatures and deflections for a free system, these quantities are also found for a system which is constrained to remain flat in the width, which indicates the effects of clamping a cantilevered system.

Section II gives the magnetoelastic energy expression and the magnetostrictive strains produced by that energy in an isolated film. Section III formulates the total energy of the bent film-substrate system and derives the relations for curvatures and deflections by minimization. Section IV gives a quantitative application of the new relations to a measured magnetostrictive system and compares with results from the previous theories. Section V discusses the results and the approximations in the new formulation.

II. MAGNETOSTRICTION IN THE ISOLATED MAGNETIC FILM

The model of a magnetic material adopted here assumes a simple form for the magnetoelastic energy $E_{\rm mel}$ of the magnetic material, which has the correct symmetry and invariance properties for cubic or isotropic materials. The form has two terms—a totally symmetric isotropic term linear in the volume strain and an anisotropic term linear in the strain components.

$$E_{\rm mel}=-V[\lambda_0(c_{11}+2c_{12})(\varepsilon_1+\varepsilon_2+\varepsilon_3)+\lambda_1(c_{11}-c_{12})\times(\varepsilon_1-\varepsilon_2/2-\varepsilon_3/2)]. \tag{1}$$

The elastic constants in each term of $E_{\rm mel}$ make λ_0 and λ_1 dimensionless and lead to simple forms for the magnetostrictive strains, as will be shown. This choice of the anisotropic term is the same as in Ref. 3. The coefficient λ_1 is the λ of Ref. 1 and is $(2/3)b^{\gamma,2}/(c_{11}-c_{12})$, where $b^{\gamma,2}$ is the coefficient used in Ref. 3. The form (1) also agrees with the form given by Vonsovskii and others,[4] i.e., the term in a_1 is proportional to $\Sigma_{i=1}^{3}(\alpha_i^2-1/3)\varepsilon_i$, where the α_i are the direction cosines of the magnetization direction. Then (1) corresponds to taking the magnetization along x_1, i.e., $\alpha_1=1$, $\alpha_2=\alpha_3=0$.

The addition of $E_{\rm mel}$ to the energy of an isolated film implies that stresses are produced when the film is magnetized, which strain the film. Although the forces are generated in the material of the film, they may be thought of as applied forces and used to find the corresponding strains from the elastic equations. Assuming that the elastic constants are not changed by the magnetization, the strains are the solutions of the elastic equations, which are, for a material with cubic symmetry,

$$c_{11}\varepsilon_1+c_{12}(\varepsilon_2+\varepsilon_3)=-\frac{\partial(E_{\rm mel}/V)}{\partial\varepsilon_1}=(c_{11}+2c_{12})\lambda_0+(c_{11}-c_{12})\lambda_1,$$

$$c_{11}\varepsilon_2+c_{12}(\varepsilon_3+\varepsilon_1)=-\frac{\partial(E_{\rm mel}/V)}{\partial\varepsilon_2}=(c_{11}+2c_{12})\lambda_0-(c_{11}-c_{12})\lambda_1/2,$$

$$c_{11}\varepsilon_3+c_{12}(\varepsilon_1+\varepsilon_2)=-\frac{\partial(E_{\rm mel}/V)}{\partial\varepsilon_3}=(c_{11}+2c_{12})\lambda_0-(c_{11}-c_{12})\lambda_1/2. \quad (2)$$

These equations are equivalent to minimizing the total energy of the film E_t with respect to the strain components, where

$$E_t=E_{\rm el}+E_{\rm mel}, \quad (3)$$

$$E_{\rm el}=V\left[\frac{c_{11}}{2}(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2)+c_{12}(\varepsilon_2\varepsilon_3+\varepsilon_3\varepsilon_1+\varepsilon_1\varepsilon_2)\right], \quad (4)$$

and $E_{\rm mel}$ is given by (1). Shear strains are omitted since they are not present in the symmetrical systems to be studied. By symmetry for a cubic material with the magnetization along the x_1 cubic axis

$$\varepsilon_2=\varepsilon_3. \quad (5)$$

Then the solutions of (2) are

$$\varepsilon_1=\lambda_0+\lambda_1,\qquad \varepsilon_2=\varepsilon_3=\lambda_0-\lambda_1/2. \quad (6)$$

III. THE MAGNETIC FILM-SUBSTRATE SYSTEM

The film is deposited and bonded to the substrate in an unmagnetized state. A magnetic field is applied along a cubic axis to magnetize the film to saturation. If the film were isolated, the magnetized equilibrium state of the film would differ from the unmagnetized state by the strains in (6). The substrate acts to restore the original dimensions of the film, but if the substrate can bend, the substrate will strain and the original film dimensions will not be completely restored. In epitaxial terms the magnetized film is mismatched to the substrate. Hence the equilibrium state of the film-substrate system can be found by elastic analysis of the strains produced by a mismatch between a film and substrate when the film is in pseudomorphic epitaxy on the substrate. The mismatch is defined as the equilibrium dimension of the substrate minus the equilibrium dimension of the magnetized film, which is divided by the dimension of the substrate. Hence the mismatch is the negative of the magnetostrictive strain,

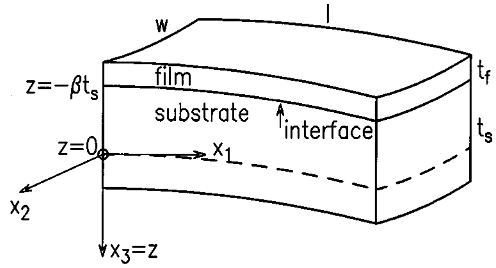

FIG. 1. Film-substrate system bent under magnetostrictive action of film magnetized along x_1 with $\lambda_1>0$. Cubic(001) surface with length l along cubic axis x_1, width w, film thickness t_f, substrate thickness t_s. The unstrained layer (dashed) is at a distance βt_s from the interface. The origin is at the unstrained layer; the $x_3=z$ axis points downward; the x_1 axis points to the right. The radius of curvature in the x_1-x_3 plane is $R_1>0$, and in the x_2-x_3 plane is $R_2<0$.

$$m_1=-\varepsilon_1=-\lambda_0-\lambda_1,\qquad m_2=-\varepsilon_2=-\lambda_0+\lambda_1/2. \quad (7)$$

Positive λ_1 means that the film has positive strain along x_1 when magnetized and expands along x_1, hence the mismatch with the substrate is negative and equal to the negative of the strain produced by magnetization. The substrate then exerts compressive forces on the film and the film exerts tensile forces on the substrate, which bends down if the film is on top, as in Fig. 1.

The problem of finding the bending of an epitaxial film-substrate combination under isotropic strain was solved in 1925 by Timoshenko[5] in connection with the equivalent problem of the bending of a bimetallic strip, where the mismatch is created by temperature change and a difference in the coefficients of thermal expansion. The present problem is to generalize that solution to consider bending produced by anisotropic strain and must take account of two curvatures. The film thickness is assumed very small compared to the substrate thickness for simplicity and because it corresponds to the usual experimental situation.

Assume the film-substrate system is rectangular with length l along the cubic crystal axis x_1 as in Fig. 1, and width w along the orthogonal axis x_2; the film thickness is t_f, the substrate thickness is t_s. The x_3 or z axis points down in Fig. 1 and the origin is at the unstrained layer (dashed line), which is a distance βt_s from the interface. The radius of curvature in the x_1-x_3 plane is R_1, which is positive in Fig. 1, and in the x_2-x_3 plane is R_2, which is negative.

The strains produced in the substrate by bending are functions of z in the form[6]

$$\varepsilon_1^s(z)=-\frac{z}{R_1},\qquad \varepsilon_2^s(z)=-\frac{z}{R_2}. \quad (8)$$

These forms vanish on the unstrained layer and correspond to the change of arc length with z, divided by the arc length of a spherical layer at $z=0$ with radius R_1 or R_2, respectively, i.e., correspond to a strain. The strain component ε_3 in either film or substrate is obtained from the condition that there are no external applied forces on the magnetized film and substrate, hence

$$\sigma_3 = c_{11}\varepsilon_3 + c_{12}(\varepsilon_1+\varepsilon_2) = 0 \tag{9}$$

which gives

$$\varepsilon_3 = -\frac{c_{12}}{c_{11}}(\varepsilon_1+\varepsilon_2). \tag{10}$$

Putting (10) into (4) gives the elastic energy density as a function of just ε_1 and ε_2

$$E_{\mathrm{el}} = V\frac{(c_{11}-c_{12})}{2c_{11}}[(c_{11}+c_{12})(\varepsilon_1^2+\varepsilon_2^2)+2c_{12}\varepsilon_1\varepsilon_2]. \tag{11}$$

Unlike the case of the isolated film, ε_2 is not equal to ε_3 because forces are exerted on the film by the substrate in the x_2 direction.

Equation (11) will first be applied to the substrate, and (8) will give the z dependence of the elastic strain energy of each layer. The layer energies must be integrated over z from $-\beta t_s$ to $(1-\beta)t_s$ to give the elastic strain energy of the substrate E_{el}^s, namely

$$\begin{aligned}E_{\mathrm{el}}^s &= lw\frac{(c_{11}^s-c_{12}^s)}{2c_{11}^s}\left[(c_{11}^s+c_{12}^s)\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}\right)+\frac{2c_{12}^s}{R_1R_2}\right]\\&\quad\times\int_{-\beta t_s}^{(1-\beta)t_s} z^2dz\\&= V_s\frac{(c_{11}^s-c_{12}^s)}{2c_{11}^s}\left(\beta^2-\beta+\frac{1}{3}\right)\\&\quad\times[(c_{11}^s+c_{12}^s)(\alpha_1^2+\alpha_2^2)+2c_{12}^s\alpha_1\alpha_2],\end{aligned} \tag{12}$$

where α_1 and α_2 are dimensionless measures of curvature defined by

$$\alpha_i \equiv \frac{t_s}{R_i}, \quad i=1,2, \tag{13}$$

and $V_s = lwt_s$ is the volume of the substrate.

Now consider the strains in the epitaxial magnetized film due to the mismatch (7) with the substrate. If the substrate remained flat, as is the case for substrates in the limit $t_s \to \infty$, all the strain would be in the film, which would have strains $\varepsilon_1^f = m_1$, $\varepsilon_2^f = m_2$. When the substrate bends, the change in strain in the substrate at the interface is given by (8) with $z=-\beta t_s$, i.e., $\beta\alpha_1$ along x_1 and $\beta\alpha_2$ along x_2. The same change in strain must occur in the film since the film is bonded to the substrate, hence in the bent system the film strains are

$$\begin{aligned}\varepsilon_1^f &= m_1+\beta\alpha_1,\\ \varepsilon_2^f &= m_2+\beta\alpha_2.\end{aligned} \tag{14}$$

Note that the strains ε_i^f in (14) are measured from the equilibrium state of the magnetized film, not the unmagnetized film. From (14) and (11) the film elastic energy can be written as

$$\begin{aligned}E_{\mathrm{el}}^f = V_f\frac{(c_{11}^f-c_{12}^f)}{2c_{11}^f}\{&(c_{11}^f+c_{12}^f)[(m_1+\beta\alpha_1)^2\\&+(m_2+\beta\alpha_2)^2]+2c_{12}^f(m_1+\beta\alpha_1)(m_2+\beta\alpha_2)\}.\end{aligned} \tag{15}$$

In (15) the film is assumed thin enough so that the strain can be considered homogeneous over the film volume.

The third component of the total energy of the system is the magnetoelastic energy E_{mel}^f in the film. Using the form (1), with ε_3 expressed as a function of ε_1 and ε_2 from (10) and ε_1^f, ε_2^f given by (14), E_{mel}^f becomes

$$\begin{aligned}E_{\mathrm{mel}}^f &= -V_f\frac{(c_{11}^f-c_{12}^f)}{c_{11}^f}X,\\ X &= (c_{11}^f+2c_{12}^f)(m_1+\beta\alpha_1+m_2+\beta\alpha_2)\lambda_0+[(2c_{11}^f+c_{12}^f)\\&\quad\times(m_1+\beta\alpha_1)-(c_{11}^f-c_{12}^f)(m_2+\beta\alpha_2)]\lambda_1/2.\end{aligned} \tag{16}$$

The total energy of the system E_t is given by the sum of (12), (15), and (16)

$$E_t = E_{\mathrm{el}}^f + E_{\mathrm{el}}^s + E_{\mathrm{mel}}^f. \tag{17}$$

The variables α_1, α_2, β must now be determined by minimizing E_t. From the conditions $\partial E_t/\partial\alpha_1 = \partial E_t/\partial\alpha_2 = 0$ come the equations for α_1 and α_2

$$\sum_{j=1}^{2} A_{ij}\alpha_j = B_i, \quad i=1,2, \tag{18}$$

where

$$\begin{aligned}A_{11} = A_{22} &= r\beta\frac{(c_{11}^f-c_{12}^f)}{c_{11}^f}(c_{11}^f+c_{12}^f)\\&\quad+\left(\beta-1+\frac{1}{3\beta}\right)\frac{(c_{11}^s-c_{12}^s)}{c_{11}^s}(c_{11}^s+c_{12}^s),\\ A_{12} = A_{21} &= r\beta\frac{(c_{11}^f-c_{12}^f)}{c_{11}^f}c_{12}^f\\&\quad+\left(\beta-1+\frac{1}{3\beta}\right)\frac{(c_{11}^s-c_{12}^s)}{c_{11}^s}c_{12}^s,\\ B_1 &= 2r\frac{(c_{11}^f-c_{12}^f)}{c_{11}^f}[(c_{11}^f+2c_{12}^f)\lambda_0+(2c_{11}^f+c_{12}^f)\lambda_1/2],\\ B_2 &= 2r\frac{(c_{11}^f-c_{12}^f)}{c_{11}^f}[(c_{11}^f+2c_{12}^f)\lambda_0-(c_{11}^f-c_{12}^f)\lambda_1/2].\end{aligned} \tag{19}$$

In (18) and (19) the derivative equations have been divided by V_s and β; also m_1, m_2 have been replaced by their values in terms of λ_0 and λ_1 given in (7). The small parameter r is defined by

$$r \equiv \frac{t_f}{t_s} = \frac{V_f}{V_s}. \qquad (20)$$

The elastic constants in (19) can be expressed in terms of Young's modulus Y and the Poisson ratio ν by the relations

$$c_{11} = \frac{Y(1-\nu)}{(1+\nu)(1-2\nu)}, \quad c_{12} = \frac{Y\nu}{(1+\nu)(1-2\nu)}. \qquad (21)$$

Note that the factor 2 in B_1 and B_2 comes from the cross terms $\beta m \alpha$ in E^f_{el} in (15). These terms contribute a constant to the derivatives $\partial E_t/\partial \alpha_i$ and add to the constants in the B_i that come from E^f_{mel}. Without the misfit in E^f_{el}, these contributions would vanish, but the constants from E^f_{mel} would still be present.

The equation $\partial E_t/\partial \beta = 0$ is not needed because a general proof can be given that $\beta = 2/3$ for a thin epitaxial film exerting stresses on the substrate. The proof does not require knowledge of the stresses in the film or of the curvatures and uses the moment balance equation around the interface. The moments around the interface have no contribution from the stresses in the thin film because the moment arm is negligible. The stresses in the substrate are linear in z from (8) and the elastic equations. Thus using (8) for ε^s_1 and ε^s_2 and finding ε^s_3 from (10) gives

$$\sigma^s_1 = \frac{c^s_{11} - c^s_{12}}{c^s_{11}} [(c^s_{11} + c^s_{12})\varepsilon_1 + c^s_{12}\varepsilon_2] = Cz. \qquad (22)$$

The moment balance equation in the x_1 direction around the interface then gives a geometrical condition independent of the elastic constants

$$\int_{-\beta t_s}^{(1-\beta)t_s} Cz(z+\beta t_s)dz = Ct_s^3\left(\frac{1}{3} - \frac{\beta}{2}\right) = 0, \qquad (23)$$

hence β must be 2/3, and this value can be put directly into (19). Moments about the interface in the x_2 direction also give $\beta = 2/3$, so that β is isotropic when the film is very thin.

Dropping the small term with factor r in the A_{ij}, putting $\beta = 2/3$, introducing Y and ν from (21) and solving (18) for α_1, α_2 gives

$$\alpha_1 = \frac{12r}{(1-\nu_f)} \frac{Y_f}{Y_s} \left\{ (1-\nu_s)\lambda_0 + \frac{\lambda_1}{2} \frac{[(2-\nu_f) + \nu_s(1-2\nu_f)]}{(1+\nu_f)} \right\},$$
$$\alpha_2 = \frac{12r}{(1-\nu_f)} \frac{Y_f}{Y_s} \left\{ (1-\nu_s)\lambda_0 - \frac{\lambda_1}{2} \frac{[(1-2\nu_f) + \nu_s(2-\nu_f)]}{(1+\nu_f)} \right\}. \qquad (24)$$

Then (24) determines the difference $\alpha_1 - \alpha_2$, which will give the difference deflection, i.e., the deflection of the system for the magnetization along x_1 minus the deflection of the system for the magnetization along x_2,

$$\alpha_1 - \alpha_2 = 18r\lambda_1 \frac{Y_f/(1+\nu_f)}{Y_s/(1+\nu_s)}. \qquad (25)$$

The above solution for the curvatures is for a free system. But measurements are made on a system which is cantilevered by clamping one end, leaving the other end free. The clamping would constrain the specimen to remain flat across the width, at least near the clamped end. Some indication of the effect of clamping is obtained by constraining the system to be flat in the x_2 direction. Then α_1 is obtained by putting $\alpha_2 = 0$, i.e., keeping the specimen flat in the x_2 direction, into the equation for bending along x_1 with magnetization along x_1, which is (18) with $i = 1$, and solving for α_1. To obtain α_2 observe that the equation for bending along x_1 with magnetization along the perpendicular direction x_2 and the specimen kept flat along x_2 is the same as the equation for bending along x_2 with magnetization along x_1 and the specimen kept flat along x_1, i.e., take $i = 2$ in (18), put $\alpha_1 = 0$ and solve for α_2. The result for the difference $\alpha_1 - \alpha_2$ is

$$\alpha_1 - \alpha_2 = 18r\lambda_1 \frac{Y_f/(1+\nu_f)}{Y_s/[(1-\nu_s)(1+\nu_s)]}. \qquad (26)$$

IV. APPLICATION OF THE BENDING EQUATIONS

The solutions of (18) for $\alpha_1 - \alpha_2$ will now be applied to a measured case of magnetostrictive bending. First note that the deflection of one end with respect to the tangent at the other end of a specimen of length l along x_1 and curvature $\alpha_1 = t_s/R_1$ is

$$\Delta_{l1} = \frac{l^2}{2t_s} \alpha_1, \qquad (27)$$

which gives the deflection when the magnetization is along x_1. The curvature in the x_2 direction when the magnetization is along x_1 is $\alpha_2 = t_s/R_2$, which is independent of the dimensions of the specimen, hence is the same as the curvature along x_1 when the magnetization is along x_2, and is given by

$$\Delta_{l2} = \frac{l^2}{2t_s} \alpha_2. \qquad (28)$$

The difference deflection is then, using (25),

$$\Delta_l = \Delta_{l1} - \Delta_{l2} = \frac{l^2}{2t_s}(\alpha_1 - \alpha_2) = 9\frac{l^2}{t_s} r\lambda_1 \frac{Y_f/(1+\nu_f)}{Y_s/(1+\nu_s)}, \qquad (29)$$

which is independent of λ_0 and measures λ_1 directly.

Equations (24), (25), and (26) will now be applied to a set of values of the system parameters for a permalloy (Ni/Fe) film on a Corning 7059 glass substrate for which Δ_l has been measured.[7] Namely,

$$t_f = 720 \text{ Å} = 0.72 \times 10^{-5} \text{ cm},$$
$$t_s = 0.04 \text{ cm},$$
$$l = 4 \text{ cm},$$
$$Y_f = 2.18 \text{ Mbar}, \quad \nu_f = 0.28, \tag{30}$$
$$Y_s = 0.68 \text{ Mbar}, \quad \nu_s = 0.22,$$
$$\lambda_1 = 20.4 \times 10^{-6}.$$

The value of λ_1 has been fixed to give the observed deflection difference Δ_l of 1300 Å using the formula of Klokholm given below. From (24) and (25) the following numerical values are obtained;

$$\alpha_1 = 1.39 \times 10^{-7}, \quad \alpha_2 = -0.63 \times 10^{-7},$$
$$\Delta_l = 4040 \text{ Å} = 4.040 \times 10^{-5} \text{ cm},$$
$$\frac{E_{\text{el}}^f}{V_s} = 8.517 \times 10^{-14}, \quad \frac{E_{\text{el}}^s}{V_s} = 0.077 \times 10^{-14}, \tag{31}$$
$$\frac{E_{\text{mel}}^f}{V_s} = 17.111 \times 10^{-14}, \quad \frac{E_t}{V_s} = 25.704 \times 10^{-14} \text{ Mbar}.$$

Direct calculation has verified that E_t is a minimum at the above values of α_1, α_2, and β=2/3.

From (26) is obtained the deflection difference for a system held flat in the width

$$\Delta_l^{\text{flat}} = 9 \frac{l^2}{t_s} r \lambda_1 \frac{Y_f/(1+\nu_f)}{Y_s/[(1-\nu_s)(1+\nu_s)]} = 3151 \text{ Å}. \tag{32}$$

The results in (31) show that the elastic energy in the substrate is only 0.9% of the elastic energy in the film, which itself is 50% of the magnetoelastic energy in the film. If the substrate is very thick, so that it does not bend, $E_{\text{el}}^s = 0$, E_{el}^f is 0.9% larger than in the bent system, and E_{mel}^f is 0.45% larger. The bending produces a radius of curvature along the direction of magnetization of 2.88×10^5 cm and of -6.39×10^5 cm in the perpendicular direction.

The above values of the deflection Δ_l may be compared with the values calculated by the formula proposed by Klokholm,[1] which is, in the present notation,

$$\Delta_l = \frac{9}{2} \frac{l^2}{t_s} r \lambda_1 \frac{Y_f/(1+\nu_f)}{Y_s/(1-\nu_s)} = 1291 \text{ Å}, \tag{33}$$

and with the formula proposed by du Trémolet and Peuzin,[3] namely

$$\Delta_l = \frac{9}{2} \frac{l^2}{t_s} r \lambda_1 \frac{Y_f/(1+\nu_f)}{Y_s/(1+\nu_s)} = 2020 \text{ Å}. \tag{34}$$

In the formula (34) Δ_l is half the value given by (29) and is exactly the result of assuming that the bending moments due to the magnetized film remain constant at the values for a flat substrate, while the system bends. The formula (33) gives a smaller value because it attempts to take into account the effect of clamping of a cantilevered specimen by assuming the specimen remains flat across the width; (33) like (34) is missing the factor 2 due to the mismatch. There are some flaws in the derivation of (33), which are corrected in (32). The result is to introduce a factor $(1+\nu_s)$ into (33) as well as the factor 2 due to the mismatch.

V. DISCUSSION

The principal contribution of this paper on magnetostrictive bending by magnetization of an epitaxial film is a formulation that includes the discontinuity in strain at the interface. That discontinuity is a consequence of the misfit between the dimensions of the magnetized and unmagnetized films. The substrate is in equilibrium with the unmagnetized film to which it is bonded, hence there is a misfit between the substrate and the magnetized film. The idea that due to misfit a discontinuity in parallel strain exists at the interface between an epitaxial film and a substrate is well known in epitaxy theory. There the discontinuity is obvious because, in the usual case, the thick substrate has negligible strain, and the film has the full misfit strain.

The idea introduced here is that the magnetized film is in a new equilibrium state, effectively a new crystalline phase with new dimensions, which must be strained toward the dimensions of the old equilibrium state of the unmagnetized film by the action of the substrate. The calculation of the strain is then a problem of plate-bending elasticity theory, but with the complication that anisotropic strain must be considered, resulting in anisotropic curvature. To solve this problem, which does not involve any concentrated applied forces, the simple powerful method of minimization of the total energy introduced by du Trémolet and Peuzin[3] is very suitable. The derivation here parallels their analysis, but adds the misfit effects and extends the discussion to the case of a specimen constrained to be flat in one direction.

The essential reason for the importance of the misfit in determining the curvatures is the interaction of the misfits with the curvature variables α_i, as noted explicitly after (21). A substantial increase in curvatures for given λ_1 is shown in the Δ_l values in (31) and (32) compared to the values in (33) and (34). The increase is somewhat less for (32), which applies to a specimen kept flat in one direction. However the real problem of a cantilevered specimen has clamped boundary conditions at just one end and free boundary conditions at the other end and is a difficult inhomogeneous elasticity problem. In the absence of a solution with these mixed boundary conditions, a reasonable estimate would be the average of Δ_l in (31) and (32). This average still increases the deflections for a given λ_1 and would require a reduction of λ_1 for Fe/Ni from 20.4×10^{-6} to 7.3×10^{-6} to fit the observed deflection. There would be less uncertainty in the value of λ_1 if the curvature of a free specimen could be measured and (29) applied to fix λ_1.

The analysis here assumes linear elastic relations, which is a good assumption for the small deflections and small magnetostrictive strains that occur. The film is assumed very thin compared to the substrate, which is a necessary condition for β to equal 2/3. This assumption could be removed by including the inhomogeneous strain in the film produced by bending, although β would then be anisotropic. The epitaxy is assumed to occur on a cubic(001) or isotropic surface, but this assumption could also be removed by using the elastic constants appropriate for the crystal structure and surface involved.

Note added in proof. Strictly, relation (9) applies only to the substrate, whereas the film has from (1) a stress $\sigma_3 = -\partial(E_{\mathrm{mel}}/V)/\partial\varepsilon_3$ linear in λ_0 and λ_1. However, this magnetostrictive stress merely adds small constant terms quadratic in λ_0 and λ_1 to E_{el} and E_{mel}, but does not affect the curvature relations.

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