Fokker-Planck description of the transfer-matrix limiting distribution in the scattering approach to quantum transport

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The scattering approach to quantum transport through a disordered quasi-one-dimensional conductor in the insulating regime is discussed in terms of its transfer matrix $\boldsymbol{T}$. A model of N one-dimensional wires which are coupled by random hopping matrix elements is compared with the transfer matrix model of Mello and Tomsovic. We derive and discuss the complete Fokker-Planck equation which describes the evolution of the probability distribution of $\boldsymbol{TT}^\dagger$ with system length in the insulating regime. It is demonstrated that the eigenvalues of $\ln\boldsymbol{TT}^\dagger$ have a multivariate Gaussian limiting probability distribution. The parameters of the distribution are expressed in terms of averages over the stationary distribution of the eigenvectors of $\boldsymbol{TT}^\dagger$. We compare the general form of the limiting distribution with results of random matrix theory and the Dorokhov-Mello Pereyra-Kumar equation. [S0163-1829(96)04424-4]

I. INTRODUCTION

The statistical properties of phase coherent quantum transport in mesoscopic systems have received increasing attention during the past few years.[1] A variety of low temperature transport quantities of conductors which are coupled to ideal leads can be expressed in terms of their scattering properties.[2–4] Hence, their statistics may be studied in terms of the probability distribution of the scattering matrix. Several distinct approaches, including random matrix theory (RMT),[5–8] Fokker-Planck (FP) equations,[9–13] supersymmetry methods,[14–16] and diagrammatic techniques[17,18] have been employed. This led to considerable progress in the understanding of quasi-one-dimensional wires[19–23] whose width is of the order of the mean free path, which implies a structureless cross-section since no transverse diffusion takes place. The mean and the variance of the conductance of quasi-one-dimensional wires are now known for all length scales from the metallic to the localized regime.[24–26] The generalization of these results beyond the quasi-one-dimensional regime is of considerable interest and has been the subject of some recent work.[27–30] Having this goal in mind, we focus on wires in the localized regime which are still quasi-one-dimensional in the sense that they are much longer than wide but which are not structureless in the transverse direction. In this regime the FP description simplifies considerably and progress is possible.

This paper, which is the first of a series of two, deals mainly with the technical aspects of the problem and compares the general result which is obtained with previous results from random-matrix theory and the Dorokhov-Mello Pereyra-Kumar (DMPK) (Refs. 31 and 10) equation. It has some overlap with the pioneering work of Dorokhov[32] but goes beyond it by generalizing the derivation of the transfer matrix limiting distribution for the one-dimensional wire by Kree and Schmid[33] to the quasi-one-dimensional case. A preliminary account of the results that are presented here has been given in Ref. 34. In the second paper[35] we will investigate a model in which forward scattering is much stronger than backward scattering. We use the ratio of backward to forward scattering strength as a small expansion parameter and calculate the limiting distribution in the lowest two orders.

The transfer matrix transforms the amplitudes of the propagating wave modes (open channels) at the Fermi energy in the left lead into the amplitudes of the right lead. A convenient parametrization for conductors with time-reversal invariance and with no spin-orbit scattering is the polar decomposition[32,36]

$$\boldsymbol{T}=\begin{pmatrix}\boldsymbol{u} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{u}^*\end{pmatrix}\begin{pmatrix}\sqrt{\boldsymbol{1}+\boldsymbol{\lambda}} & \sqrt{\boldsymbol{\lambda}}\\ \sqrt{\boldsymbol{\lambda}} & \sqrt{\boldsymbol{1}+\boldsymbol{\lambda}}\end{pmatrix}\begin{pmatrix}\boldsymbol{v} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{v}^*\end{pmatrix}, \tag{1}$$

where $\boldsymbol{u}$, $\boldsymbol{v}$ are unitary $N\times N$ matrices and $\boldsymbol{\lambda}$ is diagonal with real and positive diagonal elements λ_i. The two-terminal conductance in units of e^2/h is $g=\Sigma_i\mathcal{T}_i$, where $\mathcal{T}_i=1/(1+\lambda_i)$ are the transmission eigenvalues of $\boldsymbol{tt}^\dagger$ and $\boldsymbol{t}=\boldsymbol{u}(\boldsymbol{1}/\sqrt{\boldsymbol{1}+\boldsymbol{\lambda}})\boldsymbol{v}$ is the transmission matrix.

The transfer matrix of two samples that are joined together is the product of the transfer matrices of the individual samples. Building up a long wire by combining short samples thus leads to a transfer matrix which is a product of a large number of random matrices. The eigenvalues of $\ln(\boldsymbol{TT}^\dagger)/2L$ come in pairs $[\alpha_m(L),-\alpha_m(L)]$ where $1+2\lambda_m\equiv\cosh(2\alpha_mL)\equiv\cosh\Gamma_m$ and L is the system length. The corresponding eigenvectors are $(\vec{u}_m^T,\vec{u}_m^{*T})^T/\sqrt{2}$ and $(\vec{u}_m^T,-\vec{u}_m^{*T})^T/\sqrt{2}$, where $\vec{u}_m$ is the mth column vector of $\boldsymbol{u}$.

From Oseledec's theorem[37] for random matrix products it is known that the $\alpha_m(L)$ are self-averaging and distinct if L goes to infinity. The limiting values $\alpha_m^\infty\equiv\lim_{L\to\infty}\alpha_m(L)$ are the Lyapunov exponents. They characterize the rate of exponential growth of the λ_m with system length. Furthermore there are central limit theorems[38,39] which show that $\boldsymbol{u}$ and $\boldsymbol{v}$ have stationary distributions and that the quantities $(\Gamma_m-2\alpha_m^\infty L)/\sqrt{L}$ have Gaussian limiting distributions if L goes to infinity. Oseledec's theorem implies that the Γ_m can be ordered as $1\ll\Gamma_1\ll\Gamma_2\cdots\ll\Gamma_N$ if $2\alpha_1^\infty L\gg1$. In this regime $g=\Sigma_m2/(2+\cosh\Gamma_m)\approx4\exp(-\Gamma_1)=4\exp(-2\alpha_1L)$ and the sample is strongly insulating. The decay length $\xi = 1/2\alpha_1^\infty$ of the typical conductance is usually identified with the localization length. Johnston and Kunz[40] applied the central limit theorems to the Anderson model. We derive the Gaussian limiting distribution within a FP approach, which will establish a link between the parameters of the limiting distribution and the stationary distribution of $\boldsymbol{u}$.

The determination of the Lyapunov exponents of random matrix products is a problem which arises often in the context of disordered systems. At present there is no powerful method to calculate them analytically. Only special cases like sparse random matrices have been solved.[41] The weak disorder expansions which have been developed cannot be successfully applied to quasi-one-dimensional conductors because of the problem of degenerate eigenvalues.[42–45] The full limiting distribution has been mainly studied in numerical simulations.[46,47] Apart from the numerical data, there are only two analytical approaches which make quantitative predictions, RMT (Refs. 5,6, and 48) and the DMPK equation.[22,23]

The RMT ensemble maximizes the information entropy of the transfer matrix probability distribution subject to the constraint of a given density $\langle \rho(\Gamma) \rangle \equiv \langle \Sigma_m \delta(\Gamma - \Gamma_m) \rangle$. As a consequence, $\boldsymbol{\Gamma}$ and the unitary matrices are stochastically independent. The unitary matrices are isotropically distributed and the probability distribution of $\boldsymbol{\Gamma}$ has the form

$$p(\boldsymbol{\Gamma}) = \mathcal{N}^{-1} \exp\{-\beta H(\boldsymbol{\Gamma})\}, \tag{2}$$

where

$$H(\boldsymbol{\Gamma}) = -\sum_{m<n} \ln|\cosh\Gamma_m - \cosh\Gamma_n| - \frac{1}{\beta}\sum_m \ln(\sinh\Gamma_m) + \sum_m V(\Gamma_m) \tag{3}$$

and $\mathcal{N}$ is a normalization factor. The parameter β is determined by the symmetry of the transfer matrix ensemble. The orthogonal $(\beta=1)$, unitary $(\beta=2)$, and symplectic $(\beta=4)$ ensembles correpond to conductors with time reversal symmetry, broken time reversal symmetry, and strong spin-orbit scattering, respectively. The potential $V(\lambda)$ has to be determined from $\langle \rho(\Gamma) \rangle$. RMT is known to be a good but not exact description of quasi-one-dimensional conductors without transverse structure.[7] For such conductors and large N, $\langle \rho(\Gamma) \rangle \approx Nl/2L$ if $0 \leqslant \Gamma < 2L/l$ and $\langle \rho(\Gamma) \rangle \approx 0$ if $\Gamma > 2L/l$, where l is the mean free path. The resulting potential is quadratic $V(\Gamma) \approx Nl\Gamma^2/4L$.[49] In the insulating regime where $1 \ll \Gamma_1 \ll \Gamma_2 \cdots \ll \Gamma_N$ the Hamiltonian (3) simplifies, since $\ln(\sinh\Gamma_m) \approx \Gamma_m$ and $\ln|\cosh\Gamma_m - \cosh\Gamma_n| \approx \Gamma_n$ if $m<n$. This leads to the Gaussian probability distribution

$$p(\boldsymbol{\Gamma}) = \prod_{m=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(\Gamma_m - \langle \Gamma_m \rangle)^2}{2\sigma^2} \right\}, \tag{4}$$

where $\langle \Gamma_m \rangle = (m-1+1/\beta)2L/(lN)$ and $\sigma^2 = 2L/(\beta l N)$. Note that any potential must have the form $V(\Gamma) \approx a\Gamma + b\Gamma^2$ if $L \gg \xi$ and $\Gamma \gg 1$ in order to be consistent with the Gaussian limiting distribution. This implies always equidistant mean values Γ_m and equal variances for the fluctuations around them. Numerical simulations of conductors with transverse structure show that this is in general not true.[50,46,47] Therefore RMT cannot describe such conductors.

The DMPK equation

$$\frac{\partial p(L;\boldsymbol{\Gamma})}{\partial L} = \frac{2}{l\gamma} \sum_{m=1}^{N} \frac{\partial}{\partial \Gamma_m} \left(\frac{\partial p}{\partial \Gamma_m} + \beta p \frac{\partial \Omega(\boldsymbol{\Gamma})}{\partial \Gamma_m} \right), \tag{5}$$

where

$\Omega(\boldsymbol{\Gamma}) = -\Sigma_{m<n} \ln|(\cosh\Gamma_m - \cosh\Gamma_n)/2| - 1/\beta \Sigma_m \ln|\sinh\Gamma_m|$

and $\gamma = \beta N + 2 - \beta$, constitutes an exact description of quasi-one-dimensional wires without transverse structure. Its solution

$$p(\boldsymbol{\Gamma}) \propto \prod_{m<n} |\cosh\Gamma_m - \cosh\Gamma_n|^{\beta/2} |\Gamma_m^2 - \Gamma_n^2| \times \prod_m [\exp(-\gamma\Gamma_m^2 l/8L)\Gamma_m(\sinh\Gamma_m)^{1/2}] \tag{6}$$

in the insulating regime[22,23] can be as well approximated by a Gaussian distribution of the form (4) if $1 \ll \Gamma_1 \ll \Gamma_2 \cdots \ll \Gamma_N$, where $\langle \Gamma_m \rangle = [1+\beta(m-1)]2L/[l(\beta N + 2 - \beta)]$ and $\sigma^2 = 4L/[l(\beta N + 2 - \beta)]$. Note that the mean values of RMT and of the DMPK equation coincide for large N whereas the variances differ by a factor of 2.

The content of the paper is organized as follows. In Sec. II a Hamiltonian model of N one-dimensional wires which are coupled by random hopping matrix elements is compared to the transfer matrix model of Mello and Tomsovic.[27,28] In Sec. III we derive the FP equation, which describes the evolution of the probability distribution of $\boldsymbol{\Gamma}$ and $\boldsymbol{u}$ with system length in the localized regime. In Sec. IV we generalize the derivation for the transfer matrix limiting distribution of a one-dimensional wire by Kree and Schmid[33] to the quasi-one-dimensional wire. A first application of this approach is presented in Sec. V, where we investigate the equivalent channel model (ECM) of Mello and Tomsovic. The joint probability distribution $p(L;\boldsymbol{\Gamma})$ of this model is known to be identical to the distribution of the DMPK equation for $\beta=1$. We recover the Gaussian distribution (4) and show that the stationary distribution of $\boldsymbol{u}$ is isotropic.

There are four appendixes. The derivation of the FP equation in Sec. III is based on a simplified version of the general Langevin equations for $\boldsymbol{\Gamma}$ and $\boldsymbol{u}$, which are obtained in Appendix A. The coefficients of the FP operator are derived in Appendix B. In Appendix C we show that a measure for the unitary group which has been introduced in the text is the invariant measure. An alternative derivation of the FP equation is presented in Appendix D. The summation convention is used throughout the whole paper.

II. HAMILTONIAN VERSUS TRANSFER MATRIX MODELS

The FP approach to disordered conductors has been pioneered by Dorokhov.[32] He started from a microscopic model of disordered coupled chains which led to a quite complicated FP equation. Similar models were also studied by other techniques.[51–55] Recently Mello and Tomsovic proposed a class of models which was formulated on the level of the transfer matrix.[27,28] On the one hand, these models lead to simpler FP equations, but on the other hand, the underlying Hamiltonian is not known. In this section we propose a microscopic model, which is simpler than the one that has been used by Dorokhov, and compare it to the model class of Mello and Tomsovic.

Consider the scattering of electrons at a quasi-1D disordered conductor with a $(d-1)$-dimensional cross section which is connected to perfectly ordered leads. The conductor consists of N 1D-wires which are only coupled by random hopping matrix elements. It is described by the Hamiltonian

$$H_{nn'}=-\delta_{nn'}\frac{\hbar^2}{2m_0}\partial_x^2+V_{nn'}(x), \tag{7}$$

where $V_{nn'}(x)$ is real and symmetric in its indices and $n=1,\dots,N$. The potential $V_{nn'}(x)$ is zero in the leads and stochastic in the disordered system of length L. It describes on-wire disorder for $n=n'$ and random hopping between the wires for $n\neq n'$. The independent matrix elements of $\boldsymbol{V}(x)$ are chosen to be uncorrelated and Gaussian distributed with zero average

$$\langle V_{nn'}(x)\rangle=0,$$

$$\langle V_{nn'}(x)V_{mm'}(x')\rangle=U_{nn'}\delta(x-x')(\delta_{nm}\delta_{n'm'}+\delta_{nm'}\delta_{n'm}), \tag{8}$$

where $U_{nn'}=U_{n'n}$. The special case $U_{nn'}=U/N$ can be interpreted as a continuous one-dimensional N-orbital model,[56] which is connected to ideal leads with no exponential decaying modes.

The solution $\Psi(xn;E)$ of the scattering problem with the incoming waves

$$\Psi^{in}(xn;E)=\sqrt{\frac{m_0}{\hbar k}}(a_n^l\exp\{ikx\}+b_n^r\exp\{-ikx\}) \tag{9}$$

is an eigenfunction of the Schrödinger equation with energy $E=\hbar^2k^2/2m_0$. Its form in the left and the right lead, respectively, is

$$\Psi^{l/r}(xn;E)=\sqrt{\frac{m_0}{\hbar k}}(a_n^{l/r}\exp\{ikx\}+b_n^{l/r}\exp\{-ikx\}), \tag{10}$$

where the amplitudes $a_n^{l/r}$ and $b_n^{l/r}$ have been normalized in such a way that the probability current in the x-direction is $j_x=\Sigma_n|a_n|^2-|b_n|^2$. The $\boldsymbol{S}$-matrix transforms the amplitudes of the incident waves into the amplitudes of the scattered waves

$$\begin{pmatrix}\boldsymbol{b}^l\\ \boldsymbol{a}^r\end{pmatrix}=\boldsymbol{S}\begin{pmatrix}\boldsymbol{a}^l\\ \boldsymbol{b}^r\end{pmatrix},\quad \boldsymbol{S}=\begin{pmatrix}\boldsymbol{r} & \boldsymbol{t}'\\ \boldsymbol{t} & \boldsymbol{r}'\end{pmatrix}. \tag{11}$$

Current conservation and time reversal invariance imply that $\boldsymbol{S}$ is unitary and symmetric, respectively. The transfer matrix by contrast transforms the amplitudes in the left lead into the amplitudes in the right lead,

$$\begin{pmatrix}\boldsymbol{a}^r\\ \boldsymbol{b}^r\end{pmatrix}=\boldsymbol{T}\begin{pmatrix}\boldsymbol{a}^l\\ \boldsymbol{b}^l\end{pmatrix},\quad \boldsymbol{T}=\begin{pmatrix}\boldsymbol{t}-\boldsymbol{r}'\boldsymbol{t}'^{-1}\boldsymbol{r} & \boldsymbol{r}'\boldsymbol{t}'^{-1}\\ -\boldsymbol{t}'^{-1}\boldsymbol{r} & \boldsymbol{t}'^{-1}\end{pmatrix}. \tag{12}$$

Here, current conservation and time reversal invariance leads to the form

$$\boldsymbol{T}=\begin{pmatrix}\boldsymbol{\alpha} & \boldsymbol{\beta}\\ \boldsymbol{\beta}^* & \boldsymbol{\alpha}^*\end{pmatrix}, \tag{13}$$

where $\boldsymbol{\alpha}\boldsymbol{\alpha}^\dagger-\boldsymbol{\beta}\boldsymbol{\beta}^\dagger=\boldsymbol{1}$.[9,10] Apart from the polar decomposition (1) there is another useful parametrization of the transfer matrix which has been introduced by Mello and Tomsovic[28] and has the form

$$\boldsymbol{T}=\begin{pmatrix}\exp\boldsymbol{\vartheta} & \boldsymbol{0}\\ \boldsymbol{0} & \exp\boldsymbol{\vartheta}^*\end{pmatrix}\begin{pmatrix}\sqrt{\boldsymbol{1}+\boldsymbol{\eta}\boldsymbol{\eta}^*} & \boldsymbol{\eta}\\ \boldsymbol{\eta}^* & \sqrt{\boldsymbol{1}+\boldsymbol{\eta}^*\boldsymbol{\eta}}\end{pmatrix}, \tag{14}$$

where $\boldsymbol{\vartheta}$ and $\boldsymbol{\eta}$ are complex $N\times N$ matrices and $\boldsymbol{\vartheta}^\dagger=-\boldsymbol{\vartheta}$ and $\boldsymbol{\eta}^T=\boldsymbol{\eta}$. The wave amplitudes $\boldsymbol{a}_{l/r}$ and $\boldsymbol{b}_{l/r}$ fix the values of $\Psi(x,n;E)$ and $\partial_x\Psi(x,n;E)$ at the edges of the sample. This implies that the transfer matrix of two samples which are matched continuously together is

$$\boldsymbol{T}(L+L',0)=\boldsymbol{T}(L',L)\boldsymbol{T}(L,0). \tag{15}$$

Hence, the transfer matrix of a sample of length L can be obtained by dividing it into short segments of length δL and multiplying the transfer matrices of the segments. The evolution of the transfer matrix with the system length is a multiplicative stochastic process. It can be described by a Langevin equation since the model is continuous in the scattering direction. The Langevin equation has the form

$$\frac{d\boldsymbol{T}(x,0)}{dx}=\begin{pmatrix}\boldsymbol{\gamma}^{11}(x) & \boldsymbol{\gamma}^{12}(x)\\ \boldsymbol{\gamma}^{21}(x) & \boldsymbol{\gamma}^{22}(x)\end{pmatrix}\boldsymbol{T}(x,0) \tag{16}$$

with the noise $\gamma^{ij}(x)$. The symmetries

$$\boldsymbol{\gamma}^{22}=\boldsymbol{\gamma}^{11\,*},$$

$$\boldsymbol{\gamma}^{21}=\boldsymbol{\gamma}^{12\,*}, \tag{17}$$

and

$$\boldsymbol{\gamma}^{11\,\dagger}=-\boldsymbol{\gamma}^{11},$$

$$\boldsymbol{\gamma}^{12\,T}=\boldsymbol{\gamma}^{12}, \tag{18}$$

which will be derived below ensure time reversal invariance and flux conservation. Iterative integration of the Langevin equation yields

$$\boldsymbol{T}(x_0+\delta L,x_0)=\boldsymbol{1}+\begin{pmatrix}\boldsymbol{\varepsilon}^{11} & \boldsymbol{\varepsilon}^{12}\\ \boldsymbol{\varepsilon}^{21} & \boldsymbol{\varepsilon}^{22}\end{pmatrix}, \tag{19}$$

where

$$\boldsymbol{\varepsilon}^{ij}=\int_{x_0}^{x_0+\delta L}dx\,\boldsymbol{\gamma}^{ij}(x)+\int_{x_0}^{x_0+\delta L}dx\int_{x_0}^{x}dx'\,\boldsymbol{\gamma}^{jk}(x)\boldsymbol{\gamma}^{kj}(x')+\cdots. \tag{20}$$

For uncorrelated noise the first term of this expansion is of order $(\delta L)^{1/2}$ and the second term is of order δL. For the derivation of the symmetries (17) and (18), however, it is convenient to start with a finite correlation length of the noise. Taking $\delta x\leqslant\delta L$ and δL to be smaller than this correlation length one may expand $\boldsymbol{\gamma}^{ij}(x_0+\delta x)=\boldsymbol{\gamma}^{ij}(x_0)+\partial_x\boldsymbol{\gamma}^{ij}(x_0)\delta x+O((\delta x)^2)$, which leads to

$$\boldsymbol{\varepsilon}^{ij}=\boldsymbol{\gamma}^{ij}(x_0)\delta L+O((\delta L)^2). \tag{21}$$

Equation (13) enforces the symmetries $\boldsymbol{\gamma}^{22}=\boldsymbol{\gamma}^{11\,*}$ and $\boldsymbol{\gamma}^{21}=\boldsymbol{\gamma}^{12\,*}$. Comparing the expansion (21) with the parametrization (14) of the transfer matrix one finds

$$\boldsymbol{\vartheta}(x_0+\delta L,x_0)=\boldsymbol{\gamma}^{11}(x_0)\,\delta L+O(\delta L^2),$$

$$\boldsymbol{\eta}(x_0+\delta L,x_0)=\boldsymbol{\gamma}^{12}(x_0)\,\delta L+O(\delta L^2), \tag{22}$$

which implies $\boldsymbol{\gamma}^{11\,\dagger}=-\boldsymbol{\gamma}^{11}$ and $\boldsymbol{\gamma}^{12\,T}=\boldsymbol{\gamma}^{12}$. These symmetries remain valid in the limit of zero correlation length.

In the sequel we derive $\boldsymbol{\gamma}^{11}(x_0)$ and $\boldsymbol{\gamma}^{12}(x_0)$ for the Hamiltonian model (7). The stationary solution of the scattering problem obeys the Lippmann-Schwinger equation

$$\Psi(xn;E)=\Psi^{in}(xn;E)+\int dx_1\sum_{n_1,n_1'} G_0^+(xn,x_1n_1;E)$$
$$\times V_{n_1n_1'}(x)\Psi(x_1,n_1';E), \tag{23}$$

where

$$G_0^+(xn,x'n';E)=\frac{-im_0\delta_{nn'}}{\hbar^2k(E)}\exp\{ik(E)|x-x'|\} \tag{24}$$

is the free retarded Green's function. Iteration of the Lippmann-Schwinger yields the Born series

$$\Psi(xn;E)=\Psi^{in}(xn;E)+\int_{x_0}^{x_0+\delta L}dx_1\sum_{n_1,n_1'} G_0^+(xn,x_1n_1;E)$$
$$\times V_{n_1n_1'}(x_1)\Psi^{in}(x_1,n_1';E)+\cdots \tag{25}$$

which can be translated into series for the transmission and reflection matrices by Eq. (9) and Eq. (11). The nth orders of these series are at least of the order $(\delta L)^n$ since they contain n integrations from x_0 to $x_0+\delta L$. Thus, only the first orders can contribute to the terms of order δL of the expansions $\boldsymbol{t}=\boldsymbol{1}+\boldsymbol{t}^1\delta L+\cdots$, $\boldsymbol{r}=\boldsymbol{r}^1\delta L+\cdots$, $\boldsymbol{t}'=\boldsymbol{1}+\boldsymbol{t}'^{\,1}\delta L+\cdots$, $\boldsymbol{r}'=\boldsymbol{r}'^{\,1}\delta L+\cdots$. Inserting these contributions into the relations $\boldsymbol{\gamma}^{11}=\boldsymbol{t}^1$ and $\boldsymbol{\gamma}^{12}=\boldsymbol{r}'^{\,1}$ which follow from Eq. (12) and Eq. (21) yields

$$\gamma^{11}_{nn'}(x_0)=\frac{-im_0}{\hbar^2k}V_{nn'}(x_0)$$

$$\gamma^{12}_{nn'}(x_0)=\frac{-im_0\exp(-i2kx_0)}{\hbar^2k}V_{nn'}(x_0). \tag{26}$$

The phase $\exp(-i2kx_0)$, which appears in $\boldsymbol{\gamma}^{12}(x_0)$, is a consequence of the transformation rule

$$\boldsymbol{T}(L+x_0,x_0)=\begin{pmatrix} e^{-ikx_0}\boldsymbol{1} & \boldsymbol{0}\\ \boldsymbol{0} & e^{ikx_0}\boldsymbol{1}\end{pmatrix}\boldsymbol{T}(L,0)\begin{pmatrix} e^{ikx_0}\boldsymbol{1} & \boldsymbol{0}\\ \boldsymbol{0} & e^{-ikx_0}\boldsymbol{1}\end{pmatrix}, \tag{27}$$

which accounts for a shift of the disordered region by x_0.

Now we are in the position to compare the Hamiltonian model (8) with the transfer matrix model of Mello and Tomsovic.[27,28] They divided a sample of length L into $n=L/\delta L$ uncorrelated scattering units with identical statistical properties. Specifying the first two moments of ϑ_{mn} and η_{mn} for one scatterer

$$\langle\vartheta_{mn}\rangle_{\delta L}=\langle\eta_{mn}\rangle_{\delta L}=0,$$
$$\langle\vartheta_{mn}\vartheta_{m'n'}\rangle_{\delta L}=\kappa^{11,11}_{mn,m'n'},$$
$$\langle\vartheta_{mn}\vartheta^*_{m'n'}\rangle_{\delta L}=\kappa^{11,22}_{mn,m'n'},$$
$$\langle\eta_{mn}\eta_{m'n'}\rangle_{\delta L}=\kappa^{12,12}_{mn,m'n'},$$
$$\langle\eta_{mn}\eta^*_{m'n'}\rangle_{\delta L}=\kappa^{12,21}_{mn,m'n'},$$
$$\langle\vartheta_{mn}\eta_{m'n'}\rangle_{\delta L}=\kappa^{11,12}_{mn,m'n'},$$
$$\langle\vartheta_{mn}\eta^*_{m'n'}\rangle_{\delta L}=\kappa^{11,21}_{mn,m'n'}, \tag{28}$$

and taking the continuum limit of a high number of weak scattering units such that

$$\lim_{\delta L\to 0}\frac{1}{\delta L}\kappa^{ij,i'j'}_{mn,m'n'}=\sigma^{ij,i'j'}_{mn,m'n'} \tag{29}$$

and that $1/\delta L$ times higher moments gives zero in the same limit determined completely the stochastic evolution of the transfer matrix. As a consequence one finds

$$[\varepsilon^{11}_{mn}]=(\sigma^{11,11}_{mk,kn}+\sigma^{12,21}_{mk,kn})/2,$$
$$[\varepsilon^{12}_{mn}]=\sigma^{11,12}_{mk,kn},$$
$$[\varepsilon^{ij}_{mn}\varepsilon^{i'j'}_{m'n'}]=\sigma^{ij,i'j'}_{mn,m'n'}, \tag{30}$$

where $[\cdots]\equiv\lim_{\delta L\to 0}\langle\cdots\rangle_{\delta L}/\delta L$. The same limit for higher moments of ε^{ij}_{mn} is zero. Mello and Tomsovic have chosen the following simple model for one scattering unit. Assume that the independent matrix elements of $\boldsymbol{\vartheta}$ and $\boldsymbol{\eta}$ are uncorrelated and that their phases are randomly distributed. Averaging over the arbitrary distribution of their modulus then leads to

$$[\varepsilon^{ij}_{mn}]=\delta_{ij}\delta_{mn}\left(\frac{1}{l^b}-\frac{1}{l^f}\right),$$

$$[\varepsilon^{11}_{mn}\varepsilon^{11}_{m'n'}]=-\delta_{mn'}\delta_{nm'}\frac{1}{l^f_{mn}},$$

$$[\varepsilon^{11}_{mn}\varepsilon^{22}_{m'n'}]=\delta_{mm'}\delta_{nn'}\frac{1}{l^f_{mn}},$$

$$[\varepsilon^{12}_{mn}\varepsilon^{21}_{m'n'}]=\frac{\delta_{mm'}\delta_{nn'}+\delta_{mn'}\delta_{nm'}}{1+\delta_{mn}}\frac{1}{l^b_{mn}},$$

$$[\varepsilon^{11}_{mn}\varepsilon^{12}_{m'n'}]=0,$$
$$[\varepsilon^{11}_{mn}\varepsilon^{21}_{m'n'}]=0,$$
$$[\varepsilon^{12}_{mn}\varepsilon^{12}_{m'n'}]=0, \tag{31}$$

where l^f_{mn} and l^b_{mn} are the mean free paths for forward and backward scattering from channel m into channel n and $1/l^{f/b}=\Sigma_n 1/l^{f/b}_{mn}$ are the total inverse mean free paths. The inverse mean free paths $1/l^f_{mn}$ and $1/l^b_{mn}$ are defined by the probabilities per length $[|t_{mn}-\delta_{mn}|^2]$ and $[|r_{mn}|^2]$ for a forward and a backward scattering process, respectively. By Eq. (12) and Eq. (19) $[|t_{mn}-\delta_{mn}|^2]=[|\varepsilon^{11}_{mn}|^2]$ and $[|r_{mn}|^2]=[|\varepsilon^{12}_{mn}|^2]$, which leads to the above identification of the model parameters with the mean free paths.

Now we calculate $[\varepsilon^{ij}_{mn}]$ and $[\varepsilon^{ij}_{mn}\varepsilon^{i'j'}_{m'n'}]$ for the Hamiltonian model (8). Inserting $\boldsymbol{\gamma}^{11}(x_0)$ and $\boldsymbol{\gamma}^{12}(x_0)$ from Eq. (26) into Eq. (20) and averaging over the Gaussian white noise yields

$$[\varepsilon^{ij}_{mn}]=0,$$

$$[\varepsilon^{ij}_{mn}\varepsilon^{i'j'}_{m'n'}]=\frac{c(ij,i'j')}{l_{mn}}\,\frac{\delta_{mm'}\delta_{nn'}+\delta_{mn'}\delta_{nm'}}{1+\delta_{mn}}, \qquad (32)$$

where $1/l_{mn}\equiv 1/l^f_{mn}=1/l^b_{mn}=(m_0/(\hbar^2k))^2U_{mn}(1+\delta_{mn})$ and $1/l\equiv\Sigma_m 1/l_{mn}$. Note that l^f_{mn} and l^b_{mn} are not independent as in Eq. (31). The coefficients $c(ij,i'j')$, which are not related through the symmetries of $\boldsymbol{\gamma}^{ij}(x_0)$, are $c(11,11)=-c(11,22)=-c(12,21)=-1$, $c(11,21)=-c(11,12)^*=-\exp(i2kx_0)$, $c(12,12)=-\exp(-i4kx_0)$. Hence, the moments $[\varepsilon^{ij}_{mn}\varepsilon^{i'j'}_{m'n'}]$ which vanish in model (31) oscillate with x_0 in Eq. (32). This will cause the coefficients of the FP equation (39) for the probability distribution of $\boldsymbol{TT}^\dagger$ to oscillate with the system length. In the limit of weak disorder $(kl\gg 1)$ these oscillations are very fast on the scale of the mean free path l, which is the characteristic length over which the probability distribution changes. Then, it is justified to average over the oscillations, which amounts to replace the oscillating moments in Eq. (32) by zero. The resulting model is very similar to the model (31) if $l^f_{mn}=l^b_{mn}$ but not equivalent. It would be equivalent if the phases of η_{mn} were not random but had been chosen to take the values $\exp(i\pi/2)$ and $\exp(-i\pi/2)$ with equal probability. For stronger disorder it is no longer justified to average over the oscillations.

As an alternative to the continuum limit of Mello and Tomsovic one may specify directly the statistics of $\gamma^{ij}_{mn}(x)$. Choosing Gaussian white noise such that

$$\langle\gamma^{ij}_{mn}(x)\rangle=0,$$

$$\langle\gamma^{ij}_{mn}(x)\gamma^{i'j'}_{m'n'}(x')\rangle=\sigma^{ij,i'j'}_{mn,m'n'}(x)\,\delta(x-x'), \qquad (33)$$

leads to

$$[\varepsilon^{ij}_{mn}(x_0)]=\sigma^{ik,kj}_{ml,ln}(x_0)/2,$$

$$[\varepsilon^{ij}_{mn}(x_0)\varepsilon^{i'j'}_{m'n'}(x_0)]=\sigma^{ij,i'j'}_{mn,m'n'}(x_0). \qquad (34)$$

The Hamiltonian model (8) and the model (31) are special cases of this class of models. Note, however, that $[\varepsilon^{ij}_{mn}(x_0)]$ differs in general from the result (30) of the continuum limit.

III. LANGEVIN AND FOKKER-PLANCK EQUATIONS

The evolution of the transfer matrix with the system length is a stochastic process which can be described by Langevin and FP equations. Dorokhov[32] recognized that the stochastic equations for the matrix

$$\boldsymbol{M}=\boldsymbol{TT}^\dagger=\begin{pmatrix}\boldsymbol{u} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{u}^*\end{pmatrix}\begin{pmatrix}\cosh\boldsymbol{\Gamma} & \sinh\boldsymbol{\Gamma}\\ \sinh\boldsymbol{\Gamma} & \cosh\boldsymbol{\Gamma}\end{pmatrix}\begin{pmatrix}\boldsymbol{u}^\dagger & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{u}^T\end{pmatrix} \qquad (35)$$

are closed, which allows us to eliminate the degrees of freedom of $\boldsymbol{v}$. We follow Dorokhov and start from his Langevin equations (A10) for $\boldsymbol{\Gamma}$ and $\boldsymbol{u}$, which are derived in Appendix A for the sake of completeness. Due to the self-averaging of the α_m one expects that in the insulating regime where $L\gg\xi=1/(2\alpha_1^\infty)$ the Γ_m can be ordered: $1\ll\Gamma_1\ll\cdots\ll\Gamma_N$. This ordering justifies the neglect of exponentially small contributions to the terms

$$\coth\Gamma_j=1+O(\exp(-2\Gamma_j)),$$

$$\frac{\sinh\Gamma_n}{\cosh\Gamma_n-\cosh\Gamma_j}=\begin{cases}0+O(\exp(\Gamma_n-\Gamma_j)) & n<j\\ 1+O(\exp(\Gamma_j-\Gamma_n)) & n>j,\end{cases} \qquad (36)$$

of the general Langevin equations (A10), which leads to the considerable simplification

$$\frac{d\Gamma_m}{dL}=E_{mm}+E^*_{mm},$$

$$\frac{d\boldsymbol{u}}{dL}=\boldsymbol{\gamma}^{11}\boldsymbol{u}+\boldsymbol{uP}. \qquad (37)$$

The matrix elements of $\boldsymbol{P}$ are $P_{mn}=\theta(n-m)E_{mn}-\theta(m-n)E^*_{m-n}$, where

$$\theta(n)=\begin{cases}1, & n>0\\ \frac{1}{2}, & n=0\\ 0, & n<0\end{cases} \qquad (38)$$

and $\boldsymbol{E}=\boldsymbol{u}^\dagger\boldsymbol{\gamma}^{12}\boldsymbol{u}^*$. The symmetries $\boldsymbol{\gamma}^\dagger=-\boldsymbol{\gamma}$ and $\boldsymbol{P}^\dagger=-\boldsymbol{P}$ imply that $d(\boldsymbol{uu}^\dagger)/dL|_{\boldsymbol{uu}^\dagger=\boldsymbol{1}}=0$. Thus, the simplified Langevin equations (37) still conserve the unitarity of $\boldsymbol{u}$. The stochastic process described by them leads to a limiting distribution for $(\Gamma_m-2\alpha^\infty_m L)/\sqrt{L}$ which is independent of the initial conditions. Hence, this limiting distribution must be identical to the one which is produced by the original Langevin equations (87) as long as $1\ll\Gamma_1\ll\cdots\ll\Gamma_N$. Therefore, it is possible to use the simplified Langevin equations together with convenient initial conditions to determine the form of the limiting distribution in this parameter range.

Due to the neglect of exponential small terms in (36), Γ_m can become negative and it is natural to extend the range of Γ_m to $-\infty$, which is justified because the probability to find a negative value of Γ_m will turn out to be exponentially small. For similar reasons we relax the strict ordering of the Γ_m. A parametrization of $\boldsymbol{u}$ by a set of N^2 independent parameters seems to be rather complicated. Instead, we extend the range of the matrix elements $u_{mn}=x_{mn}+iy_{mn}$ to arbitrary complex numbers thereby obtaining a stochastic process on a higher dimensional Cartesian space. The standard derivation technique[57] for the FP equation of such a process yields

$$\frac{\partial p(L;\boldsymbol{\Gamma},\boldsymbol{u},\boldsymbol{u}^*)}{\partial L}=(\partial_{\Gamma_m}\partial_{\Gamma_n}\hat{A}_{mn}+\partial_{\Gamma_m}\hat{B}_m+\hat{C})p(L;\boldsymbol{\Gamma},\boldsymbol{u},\boldsymbol{u}^*), \tag{39}$$

where $\Pi_m d\Gamma_m \Pi_{m'n'} dx_{m'n'} dy_{m'n'}$ is the measure of the Cartesian space. The operators $\hat{A}_{mn}$, $\hat{B}_m$, and $\hat{C}$ are

$$\hat{A}_{mn}=\frac{1}{2}[\Delta\Gamma_m\Delta\Gamma_n],$$

$$\hat{B}_m=-[\Delta\Gamma_m]+\partial_{u_{m'n'}}[\Delta\Gamma_m\Delta u_{m'n'}]+\partial_{u^*_{m'n'}}[\Delta\Gamma_m\Delta u^*_{m'n'}],$$

$$\hat{C}=-\partial_{u_{mn}}[\Delta u_{mn}]-\partial_{u^*_{mn}}[\Delta u^*_{mn}]+\partial_{u_{mn}}\partial_{u^*_{m'n'}}[\Delta u_{mn}\Delta u^*_{m'n'}]$$
$$+\frac{1}{2}\partial_{u_{mn}}\partial_{u_{m'n'}}[\Delta u_{mn}\Delta u_{m'n'}]$$
$$+\frac{1}{2}\partial_{u^*_{mn}}\partial_{u^*_{m'n'}}[\Delta u^*_{mn}\Delta u^*_{m'n'}], \tag{40}$$

where $\Delta\Gamma_m=\Gamma_m(L+\delta L)-\Gamma_m(L)$ and $\Delta u_{mn}=u_{mn}(L+\delta L)-u_{mn}(L)$. The brackets $[\cdots]$ define the coefficients of the FP operator and stand for $\lim_{\delta L\to 0}\langle\cdots\rangle_{\delta L}/\delta L$, where $\langle\cdots\rangle_{\delta L}$ is the average over the disorder in the region between L and $L+\delta L$. The explicit form of the coefficients is derived in Appendix B.

The multiplication of a probability distribution on the Cartesian space by the δ function

$$\delta(\mathbf{1},\boldsymbol{u}\boldsymbol{u}^\dagger)=\prod_{m=1}^{N}\delta\left(1-\sum_n u_{mn}u^*_{mn}\right)$$
$$\times\prod_{m'<n'}\delta\left(\sum_n \mathrm{Re}(u_{m'n}u^*_{n'n})\right)$$
$$\times\delta\left(\sum_n \mathrm{Im}(u_{m'n}u^*_{n'n})\right) \tag{41}$$

restricts it to the unitary group. Since the Langevin equations (37) conserve unitarity, one expects that the FP operator commutes with the δ-function. In fact, the operators $\hat{A}_{mn}$, $\hat{B}_m$, and $\hat{C}$ commute with every function of the type $f(\boldsymbol{u}\boldsymbol{u}^\dagger)$ for *arbitrary* complex matrices $\boldsymbol{u}$. A lengthy but straightforward calculation with the coefficients (B2) which exploits the symmetries (18) proves that this is true. Thus, the restriction to the unitary group may be incorporated into the new measure $\Pi_m d\Gamma_m d\mu(\boldsymbol{u})$, where $d\mu(\boldsymbol{u})=\mathcal{V}^{-1}(N)\delta(\mathbf{1},\boldsymbol{u}\boldsymbol{u}^\dagger)\Pi_{m'n'}dx_{m'n'}dy_{m'n'}$ and $\mathcal{V}(N)$ is the volume of the unitary group. It is shown in Appendix C that $d\mu(\boldsymbol{u})$ is the invariant measure of $U(N)$. We note that an integral of the type $\int d\mu(\boldsymbol{u})\hat{X}g(\boldsymbol{u},\boldsymbol{u}^*)$ ($\hat{X}=\hat{A}_{mn}$, $\hat{B}_m$, or $\hat{C}$) with respect to the invariant measure is evaluated in two steps. First, the δ-function is commuted with $\hat{X}$. Second, the integrations are carried out yielding only contributions for terms which do not have derivatives with respect to u_{mn} or u^*_{mn} in front of them.

It is worth emphasizing that the operators $\hat{A}_{mn}$, $\hat{B}_m$, and $\hat{C}$ do not depend on $\boldsymbol{\Gamma}$. This is the great simplification which has been achieved by the neglect of the exponential small terms in (36). However, they can still depend on the system length L as is the case for the Hamiltonian model (8). There, the factor $\exp(-ikx_0)$ of $\boldsymbol{\gamma}^{12}(x_0)$ in Eq. (26) leads to terms which oscillate with the system length. For weak disorder ($kl\gg 1$) the oscillations are fast on the scale of the mean free path l which justifies replacing the oscillating terms by their averages. For stronger disorder the oscillations can be absorbed into the new variable $\tilde{u}_{mn}=u_{mn}\exp(ikL)$. The transformation of the FP equation to this variable leads to the additional term $-ik(\tilde{u}\partial\tilde{u}_{mn}-\tilde{u}^*\partial\tilde{\mu}^*_{mn})$ in the FP operator. The resulting FP equation is very similar to the FP equation of Kree and Schmid.[33] It is the generalization from $N=1$ to arbitrary channel numbers.

IV. THE LIMITING DISTRIBUTION OF THE TRANSFER MATRIX

Kree and Schmid discussed thoroughly the asymptotic probability distribution for the Landauer conductance $g=|t|^2$ of a long one-dimensional wire.[33] The transmission and reflection amplitude for incident waves from the right are $t=u(2/(1+\cosh\Gamma))^{1/2}v$ and $r=u^2((\cosh\Gamma-1)/(\cosh\Gamma+1))^{1/2}$, where u and v are simply phases. They showed that $(\Gamma-2\alpha^\infty L)/\sqrt{L}$ has a Gaussian limiting distribution if $L\to\infty$, which implies a log-normal distribution for the conductance. The parameters of the Gaussian distribution were expressed in terms of averages over the stationary distribution of u. Similarly we expect that the corresponding quantity $(\boldsymbol{\Gamma}-2\boldsymbol{\alpha}^\infty L)/\sqrt{L}$ for the quasi-one-dimensional wire has a multivariate Gaussian limiting distribution whose parameters can be expressed in terms of averages over the stationary distribution of $\boldsymbol{u}$.

It is useful to look at the first moments $\langle\Gamma_m\rangle_L$ in some detail before deriving the general form of the limiting distribution. Integrating Eq. (39) with respect to $\boldsymbol{\Gamma}$ leads to the closed FP equation

$$\frac{\partial q(L;\boldsymbol{u},\boldsymbol{u}^*)}{\partial L}=\hat{C}q(L;\boldsymbol{u},\boldsymbol{u}^*) \tag{42}$$

for $\boldsymbol{u}$. We expect that the stationary distribution of $\boldsymbol{u}$ is the unique stationary solution $q_{\mathrm{stat}}(\boldsymbol{u},\boldsymbol{u}^*)$ of this equation. Hence, the spectrum of $\hat{C}$ should consist of one eigenvalue ν_0 which is zero and others ν_i with negative real parts. The smallest absolute value of these real parts is henceforth called ν. In the sequel it will be assumed that the eigenfunctions $q_i(\boldsymbol{u},\boldsymbol{u}^*)$ of $\hat{C}$ form a complete system where $q_0(\boldsymbol{u},\boldsymbol{u}^*)=q_{\mathrm{stat}}(\boldsymbol{u},\boldsymbol{u}^*)$. Then, any probability distribution may be expanded into $\Sigma_i c_i q_i(\boldsymbol{u},\boldsymbol{u}^*)$, where conservation of probability implies $\int d\mu(\boldsymbol{u})q_i(\boldsymbol{u},\boldsymbol{u}^*)=0$ for $i\neq 0$ and $c_0=1$. Multiplying Eq. (39) with Γ_m and integration by parts yields

$$\frac{\partial\langle\Gamma_m\rangle_L}{\partial L}=-\int d\mu(\boldsymbol{u})\hat{B}_m q(L;\boldsymbol{u},\boldsymbol{u}^*)$$
$$=\int d\mu(\boldsymbol{u})[\Delta\Gamma_m]q(L;\boldsymbol{u},\boldsymbol{u}^*). \tag{43}$$

Expanding the initial distribution into eigenfunctions $q(0;\boldsymbol{u},\boldsymbol{u}^*)=q_{\mathrm{stat}}(\boldsymbol{u},\boldsymbol{u}^*)+\Sigma_{i\neq 0}c_i q_i(\boldsymbol{u},\boldsymbol{u}^*)$ gives $q(L;\boldsymbol{u},\boldsymbol{u}^*)$

$=q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*)+\Sigma_{i\neq 0}c_i\exp(\nu_i L)q_i(\boldsymbol{u},\boldsymbol{u}^*)$. This leads to the large length asymptotic behavior

$$\langle\Gamma_m\rangle_L-\langle\Gamma_m\rangle_0\approx L\int d\mu(\boldsymbol{u})[\Delta\Gamma_m]q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*)+[\text{const}+O(\exp\{-\nu L\})], \tag{44}$$

where the terms in brackets result from the crossover of the initial into the stationary distribution of $\boldsymbol{u}$. The self-averaging $\alpha_m^\infty\equiv\lim_{L\to\infty}\alpha_m(L)=\lim_{L\to\infty}\langle\alpha_m(L)\rangle_L$ of the Lyapunov exponents implies $\alpha_m^\infty=\lim_{L\to\infty}\langle\Gamma_m\rangle_L/2L$, which leads to

$$\alpha_m^\infty=\frac{1}{2}\int d\mu(\boldsymbol{u})[\Delta\Gamma_m]q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*). \tag{45}$$

This relation has been first derived by Dorokhov.[32] Using Eq. (B2) to calculate $[\Delta\Gamma_m]$ for the model (31) yields

$$\alpha_m^\infty=\frac{\theta(m-k_3)}{2l^b_{k_1k_2}(1+\delta_{k_1k_2})}\int d\mu(\boldsymbol{u})(u_{k_1m}u_{k_2k_3}u^*_{k_1k_3}u^*_{k_2m}+u^*_{k_1m}u^*_{k_2k_3}u_{k_1k_3}u_{k_2m}+2u_{k_1m}u_{k_2k_3}u^*_{k_1m}u^*_{k_2k_3})q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*). \tag{46}$$

The same formula has been obtained by Chalker and Bernhardt[29] for the special case that there is only back-scattering into the same channel. They discussed also the consequences of this relation in the context of the Anderson transition.

In the sequel we will go beyond the first moment $\langle\Gamma_m\rangle_L$ and derive the general form of the limiting distribution of $(\boldsymbol{\Gamma}-2\boldsymbol{\alpha}^\infty L)/\sqrt{L}$. For the sake of simplicity we choose the initial distribution

$$p(0;\boldsymbol{\Gamma},\boldsymbol{u},\boldsymbol{u}^*)=\prod_{m=1}^{N}\delta(\Gamma_m)q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*), \tag{47}$$

which implies that $\langle\Gamma_m\rangle_L=2\alpha_m^\infty L$ [see Eq. (44)]. The formalism that will be developed below could be used to show that a different initial condition would not change the form of the limiting distribution but only the way it is approached. It is convenient to introduce Dirac notation

$$(0|\hat{C}=0,\quad \hat{C}|0)=0,$$
$$\hat{\mathcal{P}}=|0)(0|,\quad \hat{\mathcal{Q}}=1-\hat{\mathcal{P}}_1, \tag{48}$$

where $q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*)=(\boldsymbol{u},\boldsymbol{u}^*|0)$ and $(0|\boldsymbol{u},\boldsymbol{u}^*)=(0|0)=1$ so that the average $\int d\mu(\boldsymbol{u})\ \hat{X}q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*)$ may be simply expressed as $(0|\hat{X}|0)$.

The central quantity which will be used below to derive the limiting distribution is the generating function

$$P(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*)=\int\prod_{m'}d\Gamma_{m'}\exp\left\{i\sum_m\tau_m(\Gamma_m-\langle\Gamma_m\rangle_L)\right\}\times p(L;\boldsymbol{\Gamma},\boldsymbol{u},\boldsymbol{u}^*) \tag{49}$$

for the central moments

$$\begin{aligned}\mathcal{C}_L(m_1r_1,\ldots,m_kr_k)&=\langle(\Gamma_{m_1}-\langle\Gamma_{m_1}\rangle_L)^{r_1}\cdots(\Gamma_{m_k}-\langle\Gamma_{m_k}\rangle_L)^{r_k}\rangle_L\\&=\int d\mu(\boldsymbol{u})(-i\partial_{\tau_{m_1}})^{r_1}\cdots(i\partial_{\tau_{m_k}})^{r_k}P(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*)|_{\boldsymbol{\tau}=0}.\end{aligned} \tag{50}$$

The Fourier transform of the generating function gives back the probability distribution

$$p(L;\boldsymbol{\Gamma},\boldsymbol{u},\boldsymbol{u}^*)=\frac{1}{(2\pi)^N}\int\prod_{m=1}^{N}d\tau_mP(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*)\times\exp\left\{-i\sum_{n=1}^{N}(\Gamma_n-\langle\Gamma_n\rangle_L)\tau_n\right\}. \tag{51}$$

This implies the evolution equation

$$\frac{\partial P(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*)}{\partial L}=(-\tau_m\tau_n\hat{A}_{mn}-i\tau_m\hat{B}^0_m+\hat{C})P(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*), \tag{52}$$

where

$$\hat{B}^0_m=\hat{B}_m+\frac{d\langle\Gamma_m\rangle_L}{dL}=\hat{B}_m-(0|\hat{B}_m|0) \tag{53}$$

and $P(0;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*)=q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*)$. The formal solution of Eq. (52) is

$$P(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u})=\exp\{(-\tau_m\tau_n\hat{A}_{mn}-i\tau_m\hat{B}^0_m+\hat{C})L\}q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*). \tag{54}$$

We follow Kree and Schmid and reexpress it in terms of the operator generalization of the Cauchy formula $\exp(bL)=1/(2\pi i)\oint d\zeta\exp(i\zeta L)/(\zeta+ib)$,

$$P(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*)=\frac{1}{2\pi i}\oint d\zeta\exp\{i\zeta L\}\hat{R}(\boldsymbol{\tau},\zeta)q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*), \tag{55}$$

where

$$\hat{R}(\boldsymbol{\tau},\zeta)=[\zeta+i(-\tau_m\tau_n\hat{A}_{mn}-i\tau_m\hat{B}^0_m+\hat{C})]^{-1} \tag{56}$$

is the resolvent operator and the integration contour encircles all the eigenvalues of $-i(-\tau_m\tau_n\hat{A}_{mn}-i\tau_m\hat{B}^0_m+\hat{C})$ counter-clockwise. Equation (50) for the moments of $(\Gamma_m-\langle\Gamma_m\rangle_L)$ only requires the resolvent operator for infinitesimal values of τ. The spectrum of $-i(-\tau_m\tau_n\hat{A}_{mn}-i\tau_m\hat{B}^0_m+\hat{C})$ then lies in the neighborhood of the spectrum of $-i\hat{C}$, which consists of one zero eigenvalue and other eigenvalues in the upper half plane. Hence one can choose the integration to run just below the real line from minus to plus infinity and close the contour in the upper half plane.

We do not reconstruct the limiting distribution from the moments $\mathcal{C}_L(m_1r_1,\ldots,m_kr_k)$ but proceed indirectly. First consider the linear combination $\Omega\equiv c_m\Omega_m\equiv c_m(\Gamma_m-\langle\Gamma_m\rangle_L)$ with arbitrary coefficients c_m and study its moments which are given by

$$\langle\Omega^r\rangle=\int d\mu(\boldsymbol{u})(-i\partial_\tau)^r P(L;\tau\boldsymbol{c},\boldsymbol{u},\boldsymbol{u}^*)|_{\tau=0}$$
$$=\frac{1}{2\pi i}\oint d\zeta\exp\{i\zeta L\}(-i\partial_\tau)^r(0|\hat{R}(\tau\boldsymbol{c},\zeta)|0)|_{\tau=0}. \quad (57)$$

Since

$$\hat{R}(\tau\boldsymbol{c},\zeta)=[\zeta+i(-\tau^2c_mc_n\hat{A}_{mn}-i\tau c_m\hat{B}_m^0+\hat{C}]^{-1} \quad (58)$$

is similar to the resolvent operator of a one-dimensional wire one can employ the technique of Kree and Schmid to calculate the moments. Expanding $\hat{R}(\tau\boldsymbol{c},\zeta)$ into powers of τ yields

$$\hat{R}(\tau\boldsymbol{c},\zeta)=\hat{R}_0(\zeta)\sum_{k=0}^{\infty}[(i\tau^2c_mc_n\hat{A}_{mn}-\tau c_m\hat{B}_m^0)\hat{R}_0(\zeta)]^k, \quad (59)$$

where $\hat{R}_0(\zeta)=(\zeta+i\hat{C})^{-1}$ may be decomposed into a part which is singular at $\zeta=0$ and a nonsingular part

$$\hat{R}_0=\hat{R}_{0s}+\hat{R}_{0n}=\frac{\hat{\mathcal{P}}}{\zeta}+\frac{\hat{\mathcal{Q}}}{\zeta+i\hat{C}}. \quad (60)$$

Only the terms of order τ^r of the expansion (59) contribute to the rth moment of Ω. The first moment

$$\langle\Omega\rangle_L=\frac{1}{2\pi i}\oint d\zeta\exp\{i\zeta L\}(0|\hat{R}_0(\zeta)ic_m\hat{B}_m^0\hat{R}_0(\zeta)|0)$$
$$=\frac{c_m}{2\pi}(0|\hat{B}_m^0|0)\oint d\zeta\frac{\exp\{i\zeta L\}}{\zeta^2} \quad (61)$$

is zero because of Eq. (53) as it should be. Collecting the terms which contribute to the second moment yields

$$\langle\Omega^2\rangle=\frac{1}{2\pi i}\oint d\zeta\exp\{i\zeta L\}(-2c_mc_n)(0|i\hat{R}_0\hat{A}_{mn}\hat{R}_0+\hat{R}_0\hat{B}_m^0\hat{R}_0\hat{B}_n^0\hat{R}_0|0)$$
$$=\frac{1}{2\pi i}\oint d\zeta\exp\{i\zeta L\}\times\frac{-2c_mc_n}{\zeta^2}\left(0\left|i\hat{A}_{mn}+\hat{B}_m^0\frac{\hat{\mathcal{Q}}}{\zeta+i\hat{C}}\hat{B}_n^0\right|0\right). \quad (62)$$

The residues of the pole of second order at $\zeta=0$ are $2c_mc_n(0|\hat{A}_{mn}|0)L$ and $-2c_mc_n$ $(0|\hat{B}_m^0\hat{C}^{-1}\hat{B}_n^0L+\hat{B}_m^0\hat{C}^{-2}\hat{B}_n^0|0)$ for the first and the second term, respectively. The poles in the upper half plane of the second term only give rise to exponential decaying contributions. Hence

$$\langle\Omega^2\rangle=2\omega L+\text{const}+O(\exp\{-\nu L\}), \quad (63)$$

where

$$\omega=c_mc_n(0|\hat{A}_{mn}-\hat{B}_m^0\hat{C}^{-1}\hat{B}_n^0|0)=c_mc_n\mathcal{A}_{mn}. \quad (64)$$

It can be shown along the same lines as in work of Kree and Schmid that the higher moments have the form

$$\langle\Omega^{2n}\rangle=\frac{(2n)!}{n!}(\omega L)^n+O(L^{n-1}),$$
$$\langle\Omega^{2n+1}\rangle=O(L^n). \quad (65)$$

For details of the proof we refer to Ref. 33. The form of the moments implies that $\overline{\Omega}\equiv\Omega/\sqrt{L}$ has the Gaussian limiting distribution

$$s_\infty(\overline{\Omega})=\frac{1}{\sqrt{4\pi\omega}}\exp\{-\overline{\Omega}^2/(4\omega)\}. \quad (66)$$

Now we discard the finite length corrections to the limiting distribution and keep only the universal parts $\langle\Omega^{2n}\rangle_L^u=(\omega L)^n(2n)!/n!$ and $\langle\Omega^{2n+1}\rangle_L^u=0$ of the moments (65). The corresponding universal part of the generating function $S(L;\boldsymbol{\tau})=\int d\mu(\boldsymbol{u})P(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*)$ is denoted by $S^u(L;\boldsymbol{\tau})$. Since $\langle\Omega^r\rangle_L^u=(-i\partial_\tau)^rS^u(L;\tau\boldsymbol{c})|_{\tau=0}$ we find $S^u(L;\tau\boldsymbol{c})=\exp\{-\tau^2c_mc_n\mathcal{A}_{mn}L\}$ which implies

$$S^u(L;\boldsymbol{\tau})=\exp\{-\tau_m\tau_n\mathcal{A}_{mn}L\}. \quad (67)$$

The Fourier transform (51) of $S^u(L;\boldsymbol{\tau})$ yields the universal part of the probability distribution

$$s^u(L;\boldsymbol{\Gamma})=\frac{1}{(4\pi L)^{N/2}\sqrt{\det(\{\mathcal{A}_{mn}\})}}\times\exp\{(-\Gamma_m-\langle\Gamma_m\rangle_L)\mathcal{A}_{mn}^{-1}(\Gamma_n-\langle\Gamma_n\rangle_L)/4L\}, \quad (68)$$

where $\langle\Gamma_m\rangle_L=2\alpha^\infty L$. Hence, the limiting distribution of $\overline{\Omega}\equiv(\boldsymbol{\Gamma}-2\boldsymbol{\alpha}^\infty L)/\sqrt{L}$ is

$$s_\infty(\overline{\Omega})=\frac{1}{(4\pi)^{N/2}\sqrt{\det(\{\mathcal{A}_{mn}\})}}\exp\{-\overline{\Omega}_m\mathcal{A}_{mn}^{-1}\overline{\Omega}_n/4\}. \quad (69)$$

Note that the form of the correlator

$$\langle(\alpha_m-\alpha_m^\infty)(\alpha_n-\alpha_n^\infty)\rangle_L^u=\frac{1}{4L^2}(-i\partial_{\tau_m})(-i\partial_{\tau_n})S^u(L;\boldsymbol{\tau})|_{\tau=0}$$
$$=\frac{1}{4L}(\mathcal{A}_{mn}+\mathcal{A}_{nm}) \quad (70)$$

implies that the $\alpha_m\equiv\Gamma_m/2L$ are self-averaging and that the fluctuations around their limiting values are in general correlated. Such correlations are not predicted by RMT or the DMPK equation but are consistent with numerical simulations.[46,47]

The variance of the Gaussian distribution of $\ln g\approx-\Gamma_1+$const in the insulating regime which follows from $\text{var}(\ln g)\approx\text{var}(\Gamma_1)$ and Eq. (63) is

$$\text{var}(\ln g)\approx2\mathcal{A}_{11}L+O(1). \quad (71)$$

V. THE EQUIVALENT CHANNEL MODEL AS A SPECIAL CASE

Mello and Tomosovic have shown[27,28] that the joint probability distribution $s(L;\boldsymbol{\Gamma})$ of the ECM, which is the model

(31) with backscattering mean free paths of the form

$$\frac{1}{l^b_{mn}}=\frac{1+\delta_{mn}}{l^b(N+1)}, \tag{72}$$

obeys the DMPK equation for $\beta=1$. The form (4) of the solution for the DMPK equation in the insulating regime is a special case of the multivariate Gaussian distribution (68). Hence, we expect to recover this solution from our approach if we apply it to the ECM.

Evaluating Eq. (B2) for the coefficients $[\Delta\Gamma_m\Delta\Gamma_n]$ and $[\Delta\Gamma_m]$ with the backscattering mean free paths (72) yields

$$[\Delta\Gamma_m\Delta\Gamma_n]=\frac{2}{l^b(N+1)}(u^*_{k_1m}u^*_{k_2m}u_{k_1n}u_{k_2n}+u_{k_1m}u_{k_2m}u^*_{k_1n}u^*_{k_2n}),$$

$$[\Delta\Gamma_m]=\frac{\theta(m-k_3)}{l^b(N+1)}(u^*_{k_1m}u^*_{k_2k_3}u_{k_1k_3}u_{k_2m}+u_{k_1m}u_{k_2k_3}u^*_{k_1k_3}u^*_{k_2m}+u^*_{k_1m}u^*_{k_2k_3}u_{k_2k_3}u_{k_1m}+u_{k_1m}u_{k_2k_3}u^*_{k_2k_3}u^*_{k_1m}). \tag{73}$$

Since the coefficients have the constant values $[\Delta\Gamma_m\Delta\Gamma_n]=4\delta_{mn}/[l^b(N+1)]$ and $[\Delta\Gamma_m]=2m/[l^b(N+1)]$ if $\boldsymbol{u}$ is unitary, one can integrate the evolution equation (52) for the generating function $P(L;\boldsymbol{\tau},\boldsymbol{u},\boldsymbol{u}^*)$ with respect to $\boldsymbol{u}$. The solution of the resulting equation

$$\frac{\partial S(L;\boldsymbol{\tau})}{\partial L}=-\frac{2\delta_{mn}}{l^b(N+1)}\tau_m\tau_n S(L;\boldsymbol{\tau}) \tag{74}$$

is $S(L;\boldsymbol{\tau})=\exp\{-2\Sigma_m\tau_m^2L/[l^b(N+1)]\}$, which leads to the Gaussian distribution

$$s(L;\boldsymbol{\Gamma})=\prod_{m=1}^{N}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(\Gamma_m-\langle\Gamma_m\rangle_L)^2}{2\sigma^2}\right\} \tag{75}$$

with $\langle\Gamma_m\rangle_L=2mL/[l^b(N+1)]$ and $\sigma^2=4L/[l^b(N+1)]$ as expected.

It is a specific property of ECM's that the evolution of the joint distribution $s(L;\boldsymbol{\Gamma})$ decouples from $\boldsymbol{u}$. Therefore, the limiting distribution does not depend on the stationary distribution of $\boldsymbol{u}$. Still, it is of interest to know the stationary distribution. Solving the equation $\hat{C}q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*)=0$ which determines it is in general a difficult problem. We demonstrate below that due to the simple form of the backscattering mean free paths the stationary probability measure $q_{\rm stat}(\boldsymbol{u},\boldsymbol{u}^*)\Pi_{m,n}d\mathrm{Re}(u_{mn})d\mathrm{Im}(u_{mn})$ can be found to be $\mathcal{V}^{-1}\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger)\Pi_{m,n}d\mathrm{Re}(u_{mn})d\,\mathrm{Im}(u_{mn})$ which is the invariant measure of the unitary group. Using the form (D10) of $\hat{C}$ in which the derivatives act directly on the distribution and the property that $\hat{C}$ commutes with the δ-function yields

$$\hat{C}\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger)=\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger)\left(\frac{1}{2}mn[\Delta\Gamma_m\Delta\Gamma_n]-m[\Delta\Gamma_m]\right)=\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger)g(\boldsymbol{u}\boldsymbol{u}^\dagger), \tag{76}$$

where $g(\boldsymbol{u}\boldsymbol{u}^\dagger)$ is zero if $\boldsymbol{u}$ is unitary. Applying higher powers of $\hat{C}$ to the δ-function gives $\hat{C}^k\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger)=\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger)g(\boldsymbol{u}\boldsymbol{u}^\dagger)^k$. Hence, the initial distribution $q(0;\boldsymbol{u},\boldsymbol{u}^*)=\mathcal{V}^{-1}\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger)$ evolves into

$$q(L;\boldsymbol{u},\boldsymbol{u}^*)=\mathcal{V}^{-1}\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger)\exp\{g(\boldsymbol{u}\boldsymbol{u}^\dagger)L\}=\mathcal{V}^{-1}\delta(\boldsymbol{1},\boldsymbol{u}\boldsymbol{u}^\dagger), \tag{77}$$

which shows that it is the stationary distribution.

VI. CONCLUSION

The general form (68) of the limiting distribution and the link between its parameters and the stationary distribution of $\boldsymbol{u}$ are the main results of this paper. This form implies that the RMT probability distribution (2) is not sufficient to describe quasi-one-dimensional conductors with transverse structure. The generalization of RMT to such conductors remains a challenging problem.[58] Beenakker[22] has shown that a correct description of quasi-one-dimensional wires without transverse structure requires a modification of the interaction in the Hamiltonian (3). It is not clear whether a modification of the interaction is sufficient to describe conductors with transverse structure or if three and more eigenvalue interactions are needed. Since any generalization must be consistent with the form (68) it is of considerable interest to have explicit results for $\langle\Gamma_m\rangle$ and $\mathcal{A}_{mn}$ which go beyond RMT and the DMPK equation. Numerical simulations[46,47] showed that the correlations between a pair of α_m are rather weak. So it might be that the correlations vanish in the thermodynamic limit leading to a diagonal form of $\mathcal{A}_{mn}$. A perturbative calculation of the limiting distribution for strong forward scattering will be published in a subsequent paper.[35] It will shed some light on these questions.

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APPENDIX A: DERIVATION OF THE LANGEVIN EQUATIONS

The Langevin equations for $\boldsymbol{\Gamma}$ and $\boldsymbol{u}$ which describe the stochastic evolution of the matrix

$$\boldsymbol{T}\boldsymbol{T}^\dagger\equiv\boldsymbol{M}=\begin{pmatrix}\boldsymbol{M}^{11} & \boldsymbol{M}^{12}\\ \boldsymbol{M}^{21} & \boldsymbol{M}^{22}\end{pmatrix}=\begin{pmatrix}\boldsymbol{u}(\cosh\boldsymbol{\Gamma})\boldsymbol{u}^\dagger & \boldsymbol{u}(\sinh\boldsymbol{\Gamma})\boldsymbol{u}^T\\ \boldsymbol{u}^*(\sinh\boldsymbol{\Gamma})\boldsymbol{u}^\dagger & \boldsymbol{u}^*(\cosh\boldsymbol{\Gamma})\boldsymbol{u}^T\end{pmatrix} \tag{A1}$$

were already given by Dorokhov in his pioneering work.[32] Since he did not derive them explicitly we derive them in this appendix. The multiplicative nature (15) of the transfer matrix implies that

$$\boldsymbol{M}(L+\delta L)=\boldsymbol{T}(L+\delta L,L)\boldsymbol{M}(L)\boldsymbol{T}^\dagger(L+\delta L,L) \tag{A2}$$

if a short segment of length δL is added to a sample of length L. The change $\Delta\boldsymbol{M}=\boldsymbol{M}(L+\delta L)-\boldsymbol{M}(L)$ of $\boldsymbol{M}$ induces the changes $\Delta\boldsymbol{\Gamma}$ and $\Delta\boldsymbol{u}$ which can be calculated by perturbation theory. The eigenvalues of the Hermitean matrix $\boldsymbol{M}^{11}=\boldsymbol{u}(\cosh\boldsymbol{\Gamma})\boldsymbol{u}^{\dagger}$ are $\cosh\Gamma_m$. The corresponding eigenvector to $\cosh\Gamma_m$ is the mth column vector $\vec{u}_m$of $\boldsymbol{u}$. The change $\Delta\boldsymbol{u}$ may be expanded into these eigenvectors

$$\Delta\vec{u}_m=\sum_{k=1}^{N} c_{mk}\vec{u}_k. \tag{A3}$$

Nondegenerate first-order perturbation theory then yields

$$\Delta\Gamma_m=\frac{\{\boldsymbol{u}^{\dagger}\Delta\boldsymbol{M}^{11}\boldsymbol{u}\}_{mm}}{\sinh\Gamma_m}+O((\Delta\boldsymbol{M}^{11})^2) \tag{A4}$$

and

$$c_{mn}=\frac{\{\boldsymbol{u}^{\dagger}\Delta\boldsymbol{M}^{11}\boldsymbol{u}\}_{nm}}{\cosh\Gamma_m-\cosh\Gamma_n}+O((\Delta\boldsymbol{M}^{11})^2) \tag{A5}$$

for $n\neq m$. The expansion coefficient c_{mm} can be calculated from

$$\Delta\boldsymbol{M}^{12}=\Delta\boldsymbol{u}(\boldsymbol{F}+\Delta\boldsymbol{F})\boldsymbol{u}^T+\boldsymbol{u}(\boldsymbol{F}+\Delta\boldsymbol{F})\Delta\boldsymbol{u}^T+\boldsymbol{u}\Delta\boldsymbol{F}\boldsymbol{u}^T+\Delta\boldsymbol{u}(\boldsymbol{F}+\Delta\boldsymbol{F})\Delta\boldsymbol{u}^T, \tag{A6}$$

where $\boldsymbol{F}=\sinh\boldsymbol{\Gamma}$. Equation (A3) implies that $\boldsymbol{u}^{\dagger}\Delta\boldsymbol{u}=\boldsymbol{c}^T$. Multiplying Eq. (A6) with $\boldsymbol{u}^{\dagger}$ from the left and with $\boldsymbol{u}^*$ from the right thus yields

$$\boldsymbol{u}^{\dagger}\Delta\boldsymbol{M}^{12}\boldsymbol{u}^*=\boldsymbol{c}^T(\boldsymbol{F}+\Delta\boldsymbol{F})+(\boldsymbol{F}+\Delta\boldsymbol{F})\boldsymbol{c}+\Delta\boldsymbol{F}+\boldsymbol{c}^T(\boldsymbol{F}+\Delta\boldsymbol{F})\boldsymbol{c}. \tag{A7}$$

Hence

$$c_{mm}=\frac{\{\boldsymbol{u}^{\dagger}\Delta\boldsymbol{M}^{12}\boldsymbol{u}^*\}_{mm}-\coth\Gamma_m\{\boldsymbol{u}^{\dagger}\Delta\boldsymbol{M}^{11}\boldsymbol{u}\}_{mm}}{\sinh\Gamma_m}+O((\Delta\boldsymbol{M}^{12})^2). \tag{A8}$$

Inserting the expansion (21) of $\boldsymbol{T}(L+\delta L,L)$ into powers of δL into Eq. (A2) gives

$$\Delta\boldsymbol{M}^{ij}=(\boldsymbol{\gamma}^{ik}\boldsymbol{M}^{kj}+\boldsymbol{M}^{ik}\boldsymbol{\gamma}^{kj\dagger})\,\delta L+O(\delta L^2). \tag{A9}$$

Collecting results and taking the limit $\delta L\to 0$ finally leads to the Langevin equations

$$\frac{d\Gamma_m}{dL}=E_{mm}+E^*_{mm},$$

$$\frac{du_{mn}}{dL}=\sum_k \gamma^{11}_{mk}u_{kn}+\sum_{k\neq n}\left(\frac{E_{kn}\sinh\Gamma_n+E^*_{kn}\sinh\Gamma_k}{\cosh\Gamma_n-\cosh\Gamma_k}\right)u_{mk}+\left(\frac{1}{2}\coth\Gamma_n(E_{nn}-E^*_{nn})\right)u_{mn}, \tag{A10}$$

where $\boldsymbol{E}=\boldsymbol{u}^{+}\boldsymbol{\gamma}^{12}\boldsymbol{u}^*$. Note that the symmetries $\boldsymbol{\gamma}^{11\,\dagger}=-\boldsymbol{\gamma}^{11}$ and $\boldsymbol{\gamma}^{12\,T}=\boldsymbol{\gamma}^{12}$ imply that $d(\boldsymbol{u}\boldsymbol{u}^{\dagger})/dL|_{\boldsymbol{u}\boldsymbol{u}^{\dagger}=\boldsymbol{1}}=0$, which ensures that unitarity is conserved.

APPENDIX B: THE COEFFICIENTS OF THE FP OPERATOR

The coefficients of the FP operator can be calculated by iterative integration of the Langevin equations (37) and subsequent averaging over the disorder.[57] Integration of the Langevin equation yields

$$\Gamma_m(x)=\Gamma_m(L)+\int_L^x dx'\{E(x')+E^*(x')\}_{mm},$$

$$u_{mn}(x)=u_{mn}(L)+\int_L^x dx'\{\boldsymbol{\gamma}^{11}(x')\boldsymbol{u}(x')+\boldsymbol{u}(x')\boldsymbol{P}(x')\}_{mn}. \tag{B1}$$

The matrix elements of $\boldsymbol{u}(x')$ and $\boldsymbol{u}^*(x')$ which appear in the integrands can again be expressed by the second of Eqs. (B1). Iterating this procedure leads to an increasing number of terms with polynomials of $\boldsymbol{\gamma}^{ij}$ of increasing degree r. For the Gaussian white noise model (33) only polynomials of even degree have nonzero disorder averages. The integration over the δ-functions of the disorder averaged polynomials then leads to terms of order $\delta L^{r/2}$. Hence, only polynomials of degree 2 contribute to the limit $[\cdots]\equiv\lim_{\delta L\to 0}\langle\cdots\rangle_{\delta L}/\delta L$. Collecting these polynomials leads to the following result for the coefficients of the FP operator:

$$[\Delta\Gamma_m]=\left[\int_L^{L+\delta L}dx\int_L^{L+\delta L}dx'\{\boldsymbol{u}^{\dagger}\boldsymbol{\gamma}^{12}(x)\boldsymbol{\gamma}^{11\,*}(x')\boldsymbol{u}^*+\mathscr{E}(x)\mathscr{P}^*(x')+\text{c.c.}\}_{mm}\right], \tag{B2}$$

$$[\Delta\Gamma_m\Delta\Gamma_n]=\left[\int_L^{L+\delta L}dx\int_L^{L+\delta L}dx'\{\mathscr{E}(x)+\mathscr{E}^*(x)\}_{mm}\{\mathscr{E}(x')+\mathscr{E}^*(x')\}_{nn}\right],$$

$$[\Delta\Gamma_m\Delta u_{m'n'}]=\left[\int_L^{L+\delta L}dx\int_L^{L+\delta L}dx'\{\mathscr{E}(x)+\mathscr{E}^*(x)\}_{mm}\{\boldsymbol{\gamma}^{11}(x')\boldsymbol{u}+\boldsymbol{u}\mathscr{P}(x')\}_{m'n'}\right],$$

$$\begin{aligned}[\Delta u_{mn}]=\frac{1}{2}\Bigg[&\int_L^{L+\delta L}dx\int_L^{L+\delta L}dx'\{\boldsymbol{\gamma}^{11}(x)\boldsymbol{\gamma}^{11}(x')\boldsymbol{u}\\&+2\boldsymbol{\gamma}^{11}(x)\boldsymbol{u}\mathscr{P}(x')+\boldsymbol{u}\mathscr{P}(x')\mathscr{P}(x)\}_{mn}\\&+\sum_j u_{mj}\theta(n-j)\{\boldsymbol{u}^{\dagger}\boldsymbol{\gamma}^{11\,\dagger}(x')\boldsymbol{\gamma}^{12}(x)\boldsymbol{u}^*\\&+\boldsymbol{u}^{\dagger}\boldsymbol{\gamma}^{12}(x)\boldsymbol{\gamma}^{11\,*}(x')\boldsymbol{u}^*+\mathscr{P}^{\dagger}(x')\mathscr{E}(x)\\&+\mathscr{E}(x)\mathscr{P}^*(x')\}_{jn}-\sum_j u_{mj}\theta(j-n)\\&\times\{\boldsymbol{u}^{\dagger}\boldsymbol{\gamma}^{11\,\dagger}(x')\boldsymbol{\gamma}^{12}(x)\boldsymbol{u}^*+\boldsymbol{u}^{\dagger}\boldsymbol{\gamma}^{12}(x)\boldsymbol{\gamma}^{11\,*}(x')\boldsymbol{u}^*\\&+\mathscr{P}^{\dagger}(x')\mathscr{E}(x)+\mathscr{E}(x)\mathscr{P}^*(x')\}^*_{jn}\Bigg],\end{aligned}$$

$$[\Delta u_{mn}\Delta u_{m'n'}]=\left[\int_L^{L+\delta L}dx\int_L^{L+\delta L}dx'\{\boldsymbol{\gamma}^{11}(x)\boldsymbol{u}+\boldsymbol{u}\mathscr{P}(x)\}_{mn}\{\boldsymbol{\gamma}^{11}(x')\boldsymbol{u}+\boldsymbol{u}\mathscr{P}(x')\}_{m'n'}\right],$$

$$[\Delta u_{mn}\Delta u^*_{m'n'}]=\left[\int_L^{L+\delta L}dx\int_L^{L+\delta L}dx'\{\boldsymbol{\gamma}^{11}(x)\boldsymbol{u}+\boldsymbol{u}\mathscr{P}(x)\}_{mn}\{\boldsymbol{\gamma}^{11\,*}(x')\boldsymbol{u}^*+\boldsymbol{u}^*\mathscr{P}^*(x')\}_{m'n'}\right],$$

where

$$\mathscr{E}(x)=\boldsymbol{u}^\dagger(L)\boldsymbol{\gamma}^{12}(x)\boldsymbol{u}^*(L),$$

$$\mathcal{P}_{mn}(x)=\theta(n-m)\mathcal{E}_{mn}(x)-\theta(m-n)\mathcal{E}^*_{mn}(x) \qquad \text{(B3)}$$

and integrals of the type $\int_L^{L+\delta L}dx\int_L^x dx'\,\delta(x-x')f(x')$ have been replaced by $\int_L^{L+\delta L}dx\,\int_L^{L+\delta L}dx'\,\delta(x-x')f(x')/2$.

APPENDIX C: THE INVARIANT MEASURE OF THE UNITARY GROUP

The invariant measure $d\mu(\boldsymbol{u})$ of the unitary group is invariant under multiplication with an arbitrary element $\boldsymbol{u}_0$ of the group from the left and the right

$$d\mu(\boldsymbol{u})=d\mu(\boldsymbol{u}_0\boldsymbol{u})=d\mu(\boldsymbol{u}\boldsymbol{u}_0). \qquad \text{(C1)}$$

As claimed in Sec. III we show in this appendix that the invariant measure has the form

$$d\mu(\boldsymbol{u})=\mathcal{V}^{-1}(N)\,\delta(\mathbf{1},\boldsymbol{u}\boldsymbol{u}^\dagger)\prod_{m,n}dx_{mn}dy_{mn}, \qquad \text{(C2)}$$

where $u_{mn}=x_{mn}+iy_{mn}$, $\mathcal{V}(N)$ is the volume of the unitary group, and the δ-function $\delta(\mathbf{1},\boldsymbol{u}\boldsymbol{u}^\dagger)$ has been defined in Eq. (41). Since the unitary group is compact, invariance under multiplication from the right implies invariance under multiplication from the left (cf. p. 316 in Ref. 59). Therefore it is sufficient to show the right invariance. Let us write $\boldsymbol{u}'=\boldsymbol{u}\boldsymbol{u}_0$. Then

$$d\mu(\boldsymbol{u}')=\mathcal{V}^{-1}(N)\,\delta(\mathbf{1},\boldsymbol{u}'\boldsymbol{u}'^\dagger)\prod_{m,n}dx'_{mn}dy'_{mn}$$
$$=\mathcal{V}^{-1}(N)\,\delta(\mathbf{1},\boldsymbol{u}\boldsymbol{u}^\dagger)\left|\det\frac{\partial(x_{m'n'},y_{m'n'})}{\partial(x_{mn},y_{mn})}\right|\times\prod_{m,n}dx_{mn}dy_{mn}. \qquad \text{(C3)}$$

Thus $d\mu(\boldsymbol{u})$ is right invariant, if the absolute value of the Jacobi determinant of the linear transformation $\boldsymbol{u}'=\boldsymbol{u}\boldsymbol{u}_0$ equals one. This transformation is equivalent to

$$\begin{pmatrix}\vec{\boldsymbol{x}}'\\ \vec{\boldsymbol{y}}'\end{pmatrix}=\begin{pmatrix}\mathbf{1}_N\otimes\boldsymbol{x}_0^T & -\mathbf{1}_N\otimes\boldsymbol{y}_0^T\\ \mathbf{1}_N\otimes\boldsymbol{y}_0^T & \mathbf{1}_N\otimes\boldsymbol{x}_0^T\end{pmatrix}\begin{pmatrix}\vec{\boldsymbol{x}}\\ \vec{\boldsymbol{y}}\end{pmatrix}=\frac{\partial(\vec{\boldsymbol{x}}',\vec{\boldsymbol{y}}')}{\partial(\vec{\boldsymbol{x}},\vec{\boldsymbol{y}})}\begin{pmatrix}\vec{\boldsymbol{x}}\\ \vec{\boldsymbol{y}}\end{pmatrix}, \qquad \text{(C4)}$$

where $\boldsymbol{u}=\boldsymbol{x}+i\boldsymbol{y}$ has been written in the vector form

$$(\vec{\boldsymbol{x}}^T\quad\vec{\boldsymbol{y}}^T)=(x_{11}\quad x_{12}\quad\cdots\quad x_{1N}\quad x_{21}\quad\cdots\quad x_{NN}\quad y_{11}\quad\cdots\quad y_{NN}). \qquad \text{(C5)}$$

The multiplication rule $(\boldsymbol{A}\otimes\boldsymbol{B})(\boldsymbol{C}\otimes\boldsymbol{D})=(\boldsymbol{AC})\otimes(\boldsymbol{BD})$ for cross products of matrices and the unitarity of $\boldsymbol{u}$ imply that $\partial(\vec{\boldsymbol{x}}',\vec{\boldsymbol{y}}')/\partial(\vec{\boldsymbol{x}},\vec{\boldsymbol{y}})$ is an orthogonal matrix. Therefore the absolute value of the Jacobi determinant is 1 and the measure is right invariant.

APPENDIX D: ALTERNATIVE DERIVATION OF THE FP EQUATION

The FP equation (39) can be derived in an alternative way yielding immediately the form in which the derivatives act directly on the probability distribution and not on the coefficients of the FP operator. This implies useful relations between these two forms which are difficult to obtain by direct differentiation of the coefficients if the result is not known in advance. Therefore we describe the alternative derivation in this appendix. It is a generalization of the technique which was employed by Mello and co-workers to derive the FP equation of their isotropic model.[60,10]

Assume that the probability distribution of $\boldsymbol{M}$ for a system of length L is $\tilde{p}(L;\boldsymbol{M})$. Then add a statistically independent segment of length δL to the system. The distribution of the transfer matrix $\boldsymbol{T}_{\delta L}$ of the segment is denoted by $w(L,\delta L;\boldsymbol{T}_{\delta L})$. Averaging a function $f(\boldsymbol{M})$ over the disorder of the whole system yields

$$\langle f(\boldsymbol{M})\rangle_{L+\delta L}\equiv\int d\rho(\boldsymbol{M})\tilde{p}(L+\delta L;\boldsymbol{M})f(\boldsymbol{M})$$
$$=\int\int d\rho(\boldsymbol{M}')d\omega(\boldsymbol{T}_{\delta L})\tilde{p}(L;\boldsymbol{M}')\times w(L,\delta L;\boldsymbol{T}_{\delta L})f(\boldsymbol{T}_{\delta L}\boldsymbol{M}'\boldsymbol{T}^\dagger_{\delta L}), \qquad \text{(D1)}$$

where $\boldsymbol{M}=\boldsymbol{T}_{\delta L}\boldsymbol{M}'\boldsymbol{T}^\dagger_{\delta L}$ and $d\omega(\boldsymbol{T})$, $d\rho(\boldsymbol{M})$ are measures on the matrix spaces of $\boldsymbol{T}$ and $\boldsymbol{M}$. If the measure $d\rho(\boldsymbol{M})$ is chosen to be invariant under the transformation $\boldsymbol{T}_0\boldsymbol{M}\boldsymbol{T}_0^\dagger$ for any transfer matrix $\boldsymbol{T}_0$ one finds

$$\langle f(\boldsymbol{M})\rangle_{L+\delta L}=\int\int d\rho(\boldsymbol{M})d\omega(\boldsymbol{T}_{\delta L})\times\tilde{p}(L;\boldsymbol{T}_{\delta L}^{-1}\boldsymbol{M}(\boldsymbol{T}_{\delta L}^\dagger)^{-1})w(L,\delta L;\boldsymbol{T}_{\delta L})f(\boldsymbol{M}). \tag{D2}$$

A similar line of reasoning as in Ref. 10 yields that the invariant measure has the form

$$d\rho(\boldsymbol{M})=J(\boldsymbol{\Gamma})\prod_{m=1}^{N}d\Gamma_m d\mu(\boldsymbol{u}), \tag{D3}$$

where $J(\boldsymbol{\Gamma})=\Pi_{m<n}|\cosh\Gamma_m-\cosh\Gamma_n|\Pi_m\sinh\Gamma_m$ and $d\mu(\boldsymbol{u})$ is the invariant measure on the unitary group. Comparing Eq. (D1) with Eq. (D2) immediately shows that

$$\tilde{p}(L+\delta L;\boldsymbol{M})=\int d\omega(\boldsymbol{T}_{\delta L})\tilde{p}(L;\boldsymbol{T}_{\delta L}^{-1}\boldsymbol{M}(\boldsymbol{T}_{\delta L}^\dagger)^{-1})\times w(L,\delta L;\boldsymbol{T}_{\delta L}) \tag{D4}$$

or equivalently

$$\tilde{p}(L+\delta L;\boldsymbol{M})=\langle\tilde{p}(L;\boldsymbol{T}_{\delta L}^{-1}\boldsymbol{M}(\boldsymbol{T}_{\delta L}^\dagger)^{-1})\rangle_{\delta L} \tag{D5}$$

since the average of $\tilde{p}(L;\boldsymbol{T}_{\delta L}^{-1}\boldsymbol{M}(\boldsymbol{T}_{\delta L}^\dagger)^{-1})$ with the probability $w(L,\delta L;\boldsymbol{T}_{\delta L})d\omega(\boldsymbol{T}_{\delta L})$ is just the average $\langle\cdots\rangle_{\delta L}$ over the disorder of the segment. For a fixed value of $\boldsymbol{\Gamma}$ the distribution $\tilde{p}(L+\delta L;\boldsymbol{M})$ may be expanded into the irreducible representations of the unitary group. Since the irreducible representations are polynomials in u_{mn} and u_{mn}^*,[61,62] $\tilde{p}(L+\delta L;\boldsymbol{M})$ can be analytically continued to arbitrary complex matrix elements u_{mn}. This justifies writing Eq. (D5) in the form

$$\tilde{p}(L+\delta L;\boldsymbol{\Gamma},\boldsymbol{u},\boldsymbol{u}^*)=\langle\tilde{p}(L;\boldsymbol{\Gamma}+\Delta\boldsymbol{\Gamma},\boldsymbol{u}+\Delta\boldsymbol{u},\boldsymbol{u}^*+\Delta\boldsymbol{u}^*)\rangle_{\delta L}, \tag{D6}$$

where $\Delta\boldsymbol{\Gamma}$, $\Delta\boldsymbol{u}$, and $\Delta\boldsymbol{u}^*$ are the changes of the parameters of $\boldsymbol{M}$ which are induced by the transformation $\boldsymbol{T}_{\delta L}^{-1}\boldsymbol{M}(\boldsymbol{T}_{\delta L}^\dagger)^{-1}$. A Taylor expansion of the left side in powers of δL and of the right side in powers of $\Delta\Gamma_m$, Δu_{mn}, and Δu_{mn}^* yields

$$\begin{aligned}\frac{\partial\tilde{p}}{\partial L}=\Big\{&\frac{1}{2}[\Delta\Gamma_m\Delta\Gamma_n]\partial_{\Gamma_m}\partial_{\Gamma_n}+[\Delta\Gamma_m]\partial_{\Gamma_m}\\&+[\Delta\Gamma_m\Delta u_{m'n'}]\partial_{\Gamma_m}\partial_{u_{m'n'}}+[\Delta\Gamma_m\Delta u_{m'n'}^*]\partial_{\Gamma_m}\partial_{u_{m'n'}^*}\\&+[\Delta u_{mn}]\partial_{u_{mn}}+[\Delta u_{mn}^*]\partial_{u_{mn}^*}\\&+[\Delta u_{mn}\Delta u_{m'n'}^*]\partial_{u_{mn}}\partial_{u_{m'n'}^*}\\&+\frac{1}{2}[\Delta u_{mn}\Delta u_{m'n'}]\partial_{u_{mn}}\partial_{u_{m'n'}}\\&+\frac{1}{2}[\Delta u_{mn}^*\Delta u_{m'n'}^*]\partial_{u_{mn}^*}\partial_{u_{m'n'}^*}\Big\}\tilde{p}\end{aligned} \tag{D7}$$

in the limit $\delta L\to 0$. The coefficients $[\cdots]\equiv\lim_{\delta L\to 0}\langle\cdots\rangle_{\delta L}/\delta L$ could be determined by the calculation of $\Delta\Gamma_m$, Δu_{mn}, and Δu_{mn}^* with perturbation theory and subsequent averaging over the disorder. This would lead to expressions which involve $[\overline{\varepsilon}_{mn}^{ij}]$ and $[\overline{\varepsilon}_{mn}^{ij}\overline{\varepsilon}_{m'n'}^{i'j'}]$ where

$$\boldsymbol{T}(L+\delta L,L)^{-1}=\boldsymbol{1}+\begin{pmatrix}\overline{\boldsymbol{\varepsilon}}^{11} & \overline{\boldsymbol{\varepsilon}}^{12}\\ \overline{\boldsymbol{\varepsilon}}^{21} & \overline{\boldsymbol{\varepsilon}}^{22}\end{pmatrix}, \tag{D8}$$

$\overline{\boldsymbol{\varepsilon}}^{11}=\boldsymbol{\varepsilon}^{11\,\dagger}$, and $\overline{\boldsymbol{\varepsilon}}^{12}=-\boldsymbol{\varepsilon}^{12\,T}$. In order to include all the terms which contribute to the coefficients one has to go to the second order of the perturbation theory, which is quite involved. We proceed in a different way instead. For the Gaussian white noise model (33) one finds

$$[\overline{\varepsilon}_{mn}^{ij}]=[\varepsilon_{mn}^{ij}],$$

$$[\overline{\varepsilon}_{mn}^{ij}\overline{\varepsilon}_{m'n'}^{i'j'}]=[\varepsilon_{mn}^{ij}\varepsilon_{m'n'}^{i'j'}]. \tag{D9}$$

Hence, the coefficients may be as well evaluated with the changes $\Delta\Gamma_m$, Δu_{mn}, and Δu_{mn}^*, which are induced by the transformation $\boldsymbol{T}_{\delta L}\boldsymbol{M}\boldsymbol{T}_{\delta L}^\dagger$. These changes can be obtained by iterative integration of the Langevin equations (87) similar to that described in Appendix B for the simpler case of large system lengths.

Since $d\rho(\boldsymbol{M})/J(\boldsymbol{\Gamma})=\Pi_m d\Gamma_m d\mu(\boldsymbol{u})$ is the same measure that was used for the probability distribution of the FP equation (39), one expects that the distribution $J(\boldsymbol{\Gamma})\tilde{p}(L;\boldsymbol{\Gamma},\boldsymbol{u},\boldsymbol{u}^*)$ obeys this FP equation for large system lengths. In fact, the transformation of Eq. (D7) to this distribution and the neglect of the exponential small contributions to terms of the form (36) leads to Eq. (39), where the operators $\hat{A}_{mn}$, $\hat{B}_m$, and $\hat{C}$ appear in the form

$$\hat{A}_{mn}=\frac{1}{2}[\Delta\Gamma_m\Delta\Gamma_n],$$

$$\hat{B}_m=[\Delta\Gamma_m]-n[\Delta\Gamma_n\Delta\Gamma_m]+[\Delta\Gamma_m\Delta u_{m'n'}]\partial_{u_{m'n'}}+[\Delta\Gamma_m\Delta u_{m'n'}^*]\partial_{u_{m'n'}^*},$$

$$\begin{aligned}\hat{C}=&\frac{1}{2}mn[\Delta\Gamma_m\Delta\Gamma_n]-m[\Delta\Gamma_m]-m[\Delta\Gamma_m\Delta u_{m'n'}]\\&\times\partial_{u_{m'n'}}-m[\Delta\Gamma_m\Delta u_{m'n'}^*]\partial_{u_{m'n'}^*}+[\Delta u_{mn}]\partial_{u_{mn}}\\&+[\Delta u_{mn}^*]\partial_{u_{mn}^*}+[\Delta u_{mn}\Delta u_{m'n'}^*]\partial_{u_{mn}}\partial_{u_{m'n'}^*}\\&+\frac{1}{2}[\Delta u_{mn}\Delta u_{m'n'}]\partial_{u_{mn}}\partial_{u_{m'n'}}\\&+\frac{1}{2}[\Delta u_{mn}^*\Delta u_{m'n'}^*]\partial_{u_{mn}^*}\partial_{u_{m'n'}^*}.\end{aligned} \tag{D10}$$

The equivalence with the form of $\hat{A}_{mn}$, $\hat{B}_m$, and $\hat{C}$ in Eq. (40) can be shown by a lengthy but straightforward calculation which exploits only the symmetries $\boldsymbol{\gamma}^{11\,\dagger}=-\boldsymbol{\gamma}^{11}$ and $\boldsymbol{\gamma}^{12\,T}=\boldsymbol{\gamma}^{12}$.

[1] For reviews, see *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991); and *Quantum Coherence in Mesoscopic Systems*, Vol. 254 of *NATO Advanced Study Institute Series B: Physics*, edited by B. Kramer (Plenum, New York, 1991).
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