Current fluctuations in mesoscopic systems with Andreev scattering

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Using a scattering theory approach we study the zero-frequency current fluctuations of the normal terminals of a phase-coherent mesoscopic structure with a superconducting region. We find that for devices where the potential of the superconducting region is externally fixed (Fig. 1), the expression for current fluctuations is a simple generalization of the corresponding expression obtained in Buttiker [Phys Rev. B **46**, 12 485 (1992)] for purely normal mesoscopic systems. In contrast to purely normal mesoscopic systems, we find that the current fluctuations between two different contacts can be positive in these devices. We apply this formula to derive a simple expression for the shot noise in a normal superconducting (NS) junction and study the noise to current ratio both as a function of the applied bias and a potential barrier at the NS interface. For devices with a floating superconductor (Fig. 2), a self-consistent calculation of the current fluctuations is necessary, and here we derive an approximate formula valid in the small bias limit. We show that two similar devices with identical average currents can exhibit very different fluctuations depending on whether the superconductor is held at a fixed potential or is left floating. [S0163-1829(96)00123-3]

I. INTRODUCTION

Recently, there have been many experimental[2–5] and theoretical[6–14] studies of mesoscopic devices with superconducting regions. These experiments typically measure the conductance as a function of the phase variation of the superconducting region. Some of the interesting effects observed are periodic oscillations of the conductance as a function of the phase difference between the superconducting regions in an Andreev interferometer,[3] and enhanced conductance oscillations in an Aharanov-Bohm ring with superconducting regions.[2] Some recent predictions in these structures include Anderson localization in a normal-superconducting-normal (NSN) junction due to a variation in the phase of the order parameter,[6] and the doubling of shot noise in a weakly transmitting NS junction.[12]

The scattering theory of transport (often referred to as the Landauer-Buttiker formalism) has been very successful in explaining normal mesoscopic phenomena.[15,16] It should be noted that, unlike the tunneling Hamiltonian formalism, the Landauer-Buttiker formalism does not assume weak coupling and can be applied even to ballistic conductors. It has recently been applied to mesoscopic superconductors by several authors.[6–8,12,13,17] Reference 1 shows that the scattering theory of transport can be used to calculate not only average current but also the current fluctuations. The purpose of this paper is to show with examples, how the results of Ref. 1 can be extended in a straightforward manner to apply to superconducting structures. Although Refs. 12 and 13 have applied the scattering theory of transport to calculate the noise in NS junctions, we are not aware of a general formulation of the type presented here. The expressions we derive can be used to calculate the current fluctuations in arbitrary multiterminal structures and configurations, as long as the superconducting regions are all maintained at the same electrochemical potential.

There are two distinct experimental configurations in these structures. In the first configuration there is a superconducting contact at an *externally fixed* potential[12,13] (Fig. 1).

In the second configuration, referred to as the *floating superconductor* case, the superconducting region is floating (Figs. 2).[3,6] This means that the chemical potential of the condensate (μ_S) floats to a value which is determined self-consistently by the condition that the sum of the steady-state currents flowing through the various contacts is zero.[9] Here the chemical potential of the superconductor can fluctuate with time about its steady-state value.

We first consider the configurations in Fig. 1, where there is a superconducting contact kept at an externally fixed potential. Using scattering theory formalism, we derive an expression for the current fluctuations. We find that the final expression for current fluctuations could have been obtained from the corresponding expression for current fluctuations in the purely normal case by (i) associating an additional index representing electron (e) and hole (h) channels with every contact (j) index, i.e., $j \rightarrow (je)$ and (jh); and (ii) correctly accounting for the sign of the electron and hole currents. To the best of our knowledge, the general expression [Eq. (38)] for current fluctuations presented here has not appeared in the literature before.

Next we consider the case of a floating superconductor (Fig. 2). Here the chemical potential of the condensate (μ_S) is determined self-consistently from the condition that the sum of all currents flowing in the contacts is zero, $\Sigma_i I_i = 0$.[9] As a result of this requirement, fluctuations in current cause the chemical potential of the condensate to fluctuate. This in turn affects the current fluctuations. We derive an expression for the current fluctuations in the floating superconductor case by properly accounting for the fluctuations in the chemical potential of the condensate. Our discussion in this case is valid only at small biases because it is based on the method of Langevin forces.

Finally, we consider two applications of the expressions for current fluctuations. In the first example, we derive an expression for shot noise in a NS junction valid at arbitrary applied voltages. Using this expression, we verify that a bal-

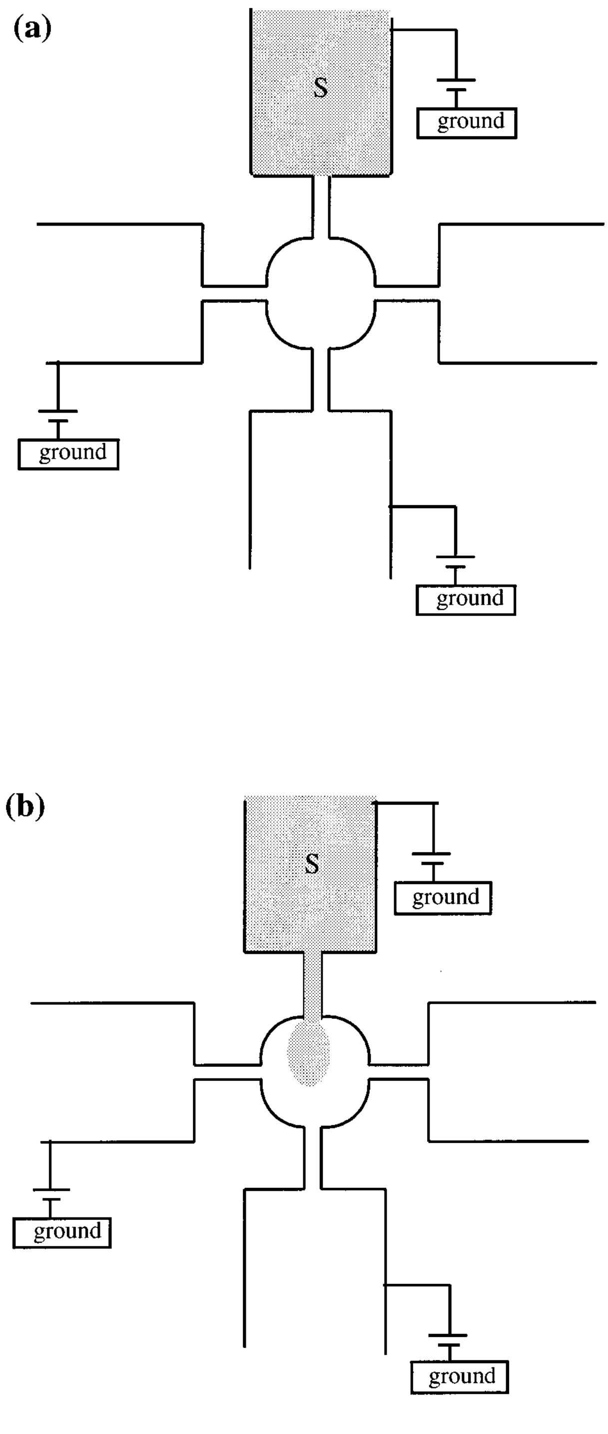

FIG. 1. A multiterminal mesoscopic device with one superconducting contact. Two of the normal contacts and the superconducting contact are at an externally fixed potential. The third normal contact is not kept at an externally fixed potential, i.e., it is floating. The exact position of the NS interface can be as in (a), where only the contact is superconducting, or (b), where the superconducting region extends continuously from the contact into the device.

listic NS junction has a nonzero shot noise at voltages larger than the superconducting gap. This result was predicted in Ref. 18 using a more complicated Keldysh Green's-function theory. Another related study predicts the doubling of noise to current ratio in the small voltage limit in weakly conducting NS junctions.[12] Using the expression for shot noise derived here, we extend this study to finite biases and arbitrary reflection coefficients due to a δ-function potential barrier at the NS interface. We find that a NS junction with a small normal reflection coefficient exhibits a peak in the noise/current ratio at a voltage larger than Δ [Fig. 3(a)]. For junctions with a large reflection coefficient, the noise/current ratio has a value four times the electronic charge at small applied voltages as predicted in Ref. 12, and the noise/current ratio decreases to the value of two times the electronic charge at voltages much larger than that of the superconducting gap [Fig. 3(b)]. This behavior is intuitively expected because at energies smaller than the superconducting gap an electron incident from the normal region is reflected as a hole at the NS interface, resulting in the flow a Cooper pair with charge $2e$ in the superconductor. At energies much larger than the superconducting gap, an electron incident from the normal region is transmitted as an electron-like quasiparticle in the superconductor. Reference 13 recently calculated the distribution function for the shot noise in a NS junction using the scattering theory approach. We do not address this issue in this paper.

The second example we consider is illustrated in Fig. 4. The purpose of this example is to illustrate the differences in the current fluctuations between the case of a superconductor kept at a fixed external potential and the case of a floating superconductor. The devices in Fig. 4 consist of a normal ballistic region connected to two normal contacts ($N1$ and $N2$), and has two superconducting boundaries maintained at phases ϕ_1 and ϕ_2. The device in Fig. 4(a) (device A) is connected to a single superconductor whose potential floats to a value which is determined by the currents flowing in the normal terminals. The device in Fig. 4(b) (device B) is similar to device A, except that the superconductor is maintained at an externally fixed potential. This potential is chosen to be equal to the potential the superconductor floats to in device A. Both the average current and current fluctuations are calculated at the normal terminals as a function of the phase difference $(\phi_1-\phi_2)$. We find that, while the average current is the same in the two devices, the current fluctuations are very different (Fig. 5).

Approximations

The basic approximation we make is to neglect the current fluctuations in the pair potential $\Delta(\mathbf{r})$. In the Bogoliubov–de Gennes equations [Eq. (6)], the order parameter $\Delta(\mathbf{r})$ is calculated self-consistently. As a result there are fluctuations in $\Delta(\mathbf{r})$ due both to the stochastic nature of the occupancy factors for electrons and holes [the factors f_n in Eq. (8)], and to the stochastic nature of the transmission coefficients. These fluctuations in $\Delta(\mathbf{r})$ are neglected in this paper. It is not clear to us if these fluctuations can be included in the context of a scattering theory approach. Neglecting fluctuations in $\Delta(\mathbf{r})$ is similar to neglecting the effect of fluctuations in the effective potential seen by an electron due to all other electrons in purely normal mesoscopic systems. Most of the calculations of current fluctuations in purely normal mesoscopic systems in the literature are in this limit.

At low temperatures we expect the fluctuations in $\Delta(\mathbf{r})$ to be insignificant for the structures in Figs. 1(a) and 2(b). This is because the current density flowing in the contact is so small that it plays an insignificant role in determining the

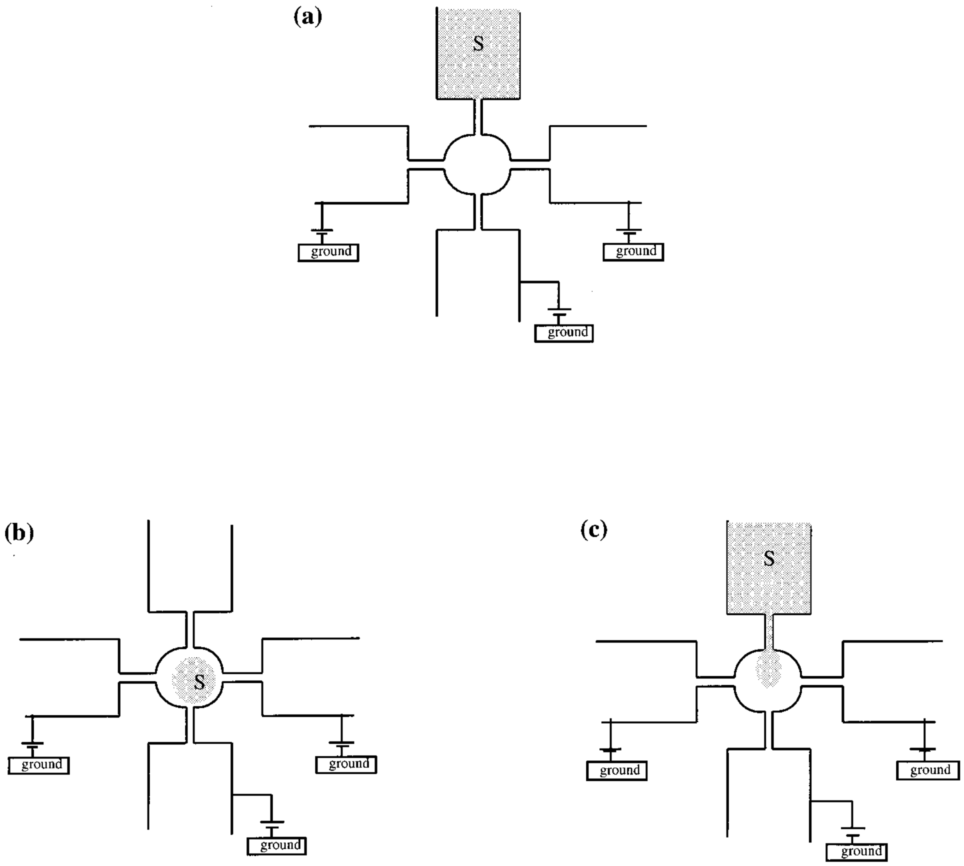

FIG. 2. A multiterminal mesoscopic device where the superconducting region is not externally kept at a constant potential; i.e., the superconductor is floating. All normal contacts are at an externally fixed potential. The structures can be as in (a), where the superconducting region is a contact, (b), where the superconducting region is part of the device, or (c), which is a combination of (a) and (b).

self-consistent value of $\Delta(\mathbf{r})$. Thus fluctuations in the current will not contribute significantly to fluctuations in $\Delta(\mathbf{r})$. On the other hand, the superconducting regions inside the devices in Figs. 1(b), 2(b), and 2(c) are smaller and can have an appreciable current density flowing in them. Thus self-consistency in $\Delta(\mathbf{r})$ is very important because fluctuations in the current could lead to significant fluctuations in $\Delta(\mathbf{r})$.

Outline

The remainder of the paper is arranged as follows. We begin Sec. II with a brief description of the Bogoliubov–de Gennes equations, and the picture adopted in this paper (Sec. II A). In Sec. II C, we derive an expression [Eq. (38)] for the current fluctuations in the case of a superconductor at an externally fixed potential. In Sec. II D, we discuss the sign of the current fluctuations. We discuss the floating superconductor case in Sec. III. In Sec. IV, we discuss two examples: (i) a NS junction and (ii) the device in Fig. 4. We present our conclusions in Sec. V.

II. CURRENT FLUCTUATIONS: SUPERCONDUCTOR AT A FIXED EXTERNAL POTENTIAL

A. Bogoliubov–de Gennes equations

Consider a mesoscopic device connected to one superconducting contact and an arbitrary number of normal contacts as shown in Fig. 1. Here an up-spin electron incident in contact j can either be transmitted as an up-spin electron or a down-spin hole to other contacts. The equation which describes the motion of quasiparticles under nonequilibrium conditions when all interactions involving spins are negligible is[19]

$$\begin{pmatrix} [H(x)+U(x)-\mu_S] & \Delta(x) \\ \Delta(x)^* & -[H(x)^*+U(x)-\mu_S] \end{pmatrix} \begin{pmatrix} u_n(x) \\ v_n(x) \end{pmatrix} = i h \frac{\partial}{\partial t} \begin{pmatrix} u_n(x) \\ v_n(x) \end{pmatrix}, \tag{1}$$

where

$$H(x)=\frac{1}{2m}\left(-i\hbar\boldsymbol{\nabla}-e\,\frac{\mathbf{A}(x)}{c}\right)^2+V_s(x), \tag{2}$$

$$\Delta(x)=+V(x)\sum_n v_n^*(x)u_n(x)(1-2f_n), \tag{3}$$

$$U(x)=-V(x)\sum_n |u_n(x)|^2 f_n+|v_n(x)|^2(1-f_n). \tag{4}$$

Here $x\equiv(\mathbf{r},t)$, $V_s(x)$ is the scalar potential (the potential at equilibrium plus the potential resulting from the applied bias), $V(x)$ is the local attractive electron-electron interaction, $\mathbf{A}(x)$ is the vector potential, and μ_S is the chemical potential of the superconducting region. f_n is the occupation factor for state n. We consider only devices where all superconducting regions have the same chemical potential μ_S. The self-consistent potentials $\Delta(x)$ and $U(x)$ are then time independent. We also assume that $V_s(x)$ and $\mathbf{A}(x)$ are time independent. Equation (1) can then be written as

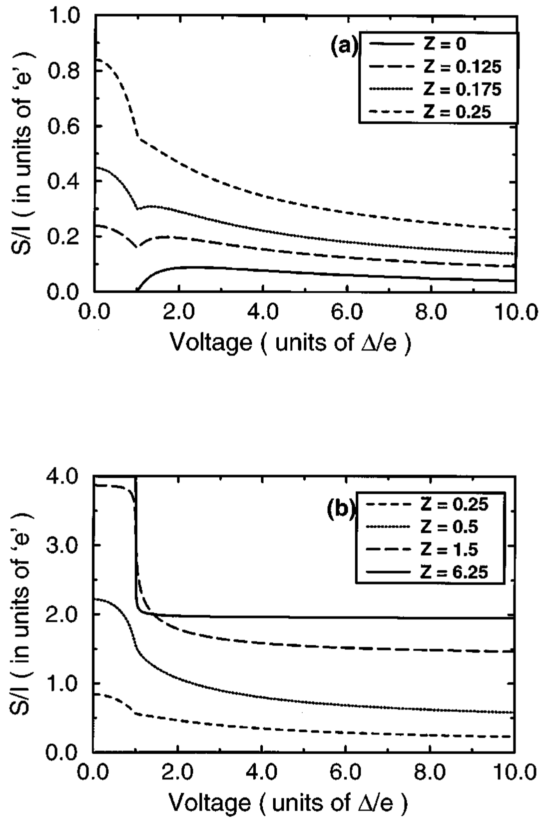

FIG. 3. A plot of the noise to current ratio (S/I) of a NS junction as a function of the applied voltage. This ratio has a peak at a voltage larger than Δ/e. Note that a ballistic NS junction has a nonzero noise for $eV>\Delta$. (b) Same as (a) but for larger barrier strengths. For large barriers, the S/I ratio approaches a value of $4e$ at small applied biases, as predicted in Ref. 12. The S/I ratio, however, decreases to a value $2e$ at voltages larger than Δ/e. $Z=k_F U/2\epsilon_F$ is a dimensionless barrier strength (Ref. 26), and $U\delta(x)$ represents the potential of the barrier placed at the NS interface. Plots are for zero temperature.

$$\begin{pmatrix} [H(\mathbf{r})+U(\mathbf{r})-\mu_S] & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -[H(\mathbf{r})^*+U(\mathbf{r})-\mu_S] \end{pmatrix} \begin{pmatrix} u_n(\mathbf{x}) \\ v_n(\mathbf{x}) \end{pmatrix} = ih \frac{\partial}{\partial t} \begin{pmatrix} u_n(\mathbf{x}) \\ v_n(\mathbf{x}) \end{pmatrix} \tag{5}$$

in the superconducting regions. In the normal regions, the same equation holds with the self-consistent potentials $U(\mathbf{r})$ and $\Delta(\mathbf{r})$ set equal to zero [since $V(x)=0$]. Equation (5) can now be written in a time-independent form by assuming a solution of the form $u_n(x)\rightarrow e^{-iEt}u_n(\mathbf{r})$ and $v_n(x)\rightarrow e^{-iEt}v_n(\mathbf{r})$:

$$\begin{pmatrix} [H(\mathbf{r})+U(\mathbf{r})-\mu_S] & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -[H(\mathbf{r})^*+U(\mathbf{r})-\mu_S] \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix} = E \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}, \tag{6}$$

where

$$H(\mathbf{r}) = \frac{1}{2m}\left(-i\hbar\boldsymbol{\nabla} - e\,\frac{\mathbf{A}(\mathbf{r})}{c}\right)^2 + V_s(\mathbf{r}), \tag{7}$$

$$\Delta(\mathbf{r}) = +V(\mathbf{r})\sum_n v_n^*(\mathbf{r})u_n(\mathbf{r})(1-2f_n), \tag{8}$$

$$U(\mathbf{r}) = -V(\mathbf{r})\sum_n |u_n(\mathbf{r})|^2 f_n + |v_n(\mathbf{r})|^2(1-f_n). \tag{9}$$

Here $V_s(\mathbf{r})$ is the scalar potential (the potential at equilibrium plus the potential resulting from the applied bias), and $\mathbf{A}(\mathbf{r})$ is the vector potential. Note that while solving Eq. (6), μ_S is a position-independent constant throughout the device. Equation (6) explicitly involve diagonal sub-Hamiltonians for an up-spin electron band and a down-spin hole band. The off-diagonal term $\Delta(\mathbf{r})$ known as the order parameter, represents a coupling between the up-spin electron band and the down-spin hole band: $\Delta(\mathbf{r})=\mathbf{0}$ in the normal regions.

B. Scattering states and occupation factors in the contacts

In the following discussion of scattering states, we assume the Hamiltonian in the contacts to be separable in the x and (y,z) directions (the Hamiltonian will not be separable inside the device, which may have an arbitrary shape). We further assume for simplification that the vector potential $A(\mathbf{r})=\mathbf{0}$ and that the single-particle potential $V_s(\mathbf{r})$ has the following form in the contacts:

$$V_s(\mathbf{r}) = V_i(y,z) \quad \text{in contact } i. \tag{10}$$

Then the Hamiltonian H appearing in Eq. (6) is separable into x and (y,z) components in contact i as follows:

$$H(\mathbf{r}) = H(x) + H(y,z), \tag{11}$$

where

$$H(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \tag{12}$$

and

$$H(y,z) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) + V_i(y,z). \tag{13}$$

Applying a bias in contact i would change the value of V_i by an amount equal to the applied bias. The assumption made regarding the form of $V_s(\mathbf{r})$ is made only to simplify the calculation. As in the purely normal case, the final answer for the average current and current fluctuations does not depend on the detailed shape of the contacts.[1]

Normal contacts

We will now discuss Eq. (6) in a normal contact, discussing both the scattering states and the occupation factors for electrons and holes in the contact. The off-diagonal potential $\Delta(\mathbf{r})$ is zero, and so Eq. (6) simplifies to

$$+(H-\mu_S)u_n = Eu_n, \tag{14}$$

$$-(H-\mu_S)v_n = Ev_n, \tag{15}$$

where H is given by Eq. (11). Equation (14) represents an electron band which is shifted by a constant energy $-\mu_S$. Similarly Eq. (15) represents a hole band which is shifted by a constant energy $+\mu_S$.

Inside normal contact i, the solutions to Eqs. (14) and (15) at energy $E=E_x+E_{xy}$ can be written as a product of a

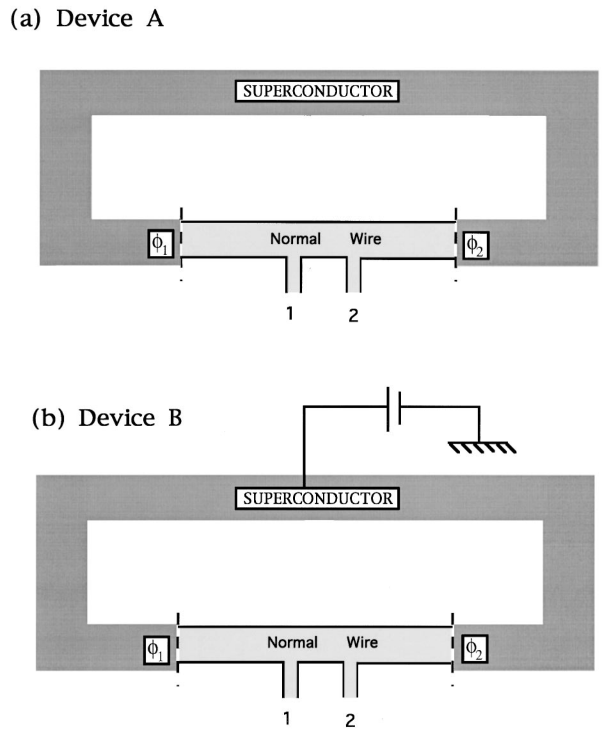

FIG. 4. Andreev interferometer: (a) The superconducting region floats to a value (μ_S) determined by current conservation for this two-terminal device. (b) The superconducting region is held externally at a chemical potential μ_S identical to that in (a).

state with energy E_x along the x direction and a state with energy E_{yz} along the (y,z) direction. Incident electrons and holes from normal contact i have the following form:

$$e^{\pm ik_{ie}(E_x)x}\begin{pmatrix} u_n(i,y,z) \\ 0 \end{pmatrix} \tag{16}$$

and

$$e^{\pm ik_{ih}(E_x)x}\begin{pmatrix} 0 \\ v_n(i,y,z) \end{pmatrix}. \tag{17}$$

$k_{ie}(E_x)$ and $k_{ih}(E_x)$ are the x components of the wave vectors in contact i corresponding to electrons and holes with an x component of energy equal to E_x. These wave vectors are equal to

$$k_{ie}(E_x)=\left(\frac{2m(\mu_s+E_x-V_i)}{\hbar^2}\right)^{1/2}$$

and

$$k_{ih}(E_x)=\left(\frac{2m(\mu_s-E_x-V_i)}{\hbar^2}\right)^{1/2}. \tag{18}$$

Note that energies E_x, $E_{y,z}$, and E are all measured with respect to μ_S, which is equal to the chemical potential of the superconducting region. The plus and minus signs in Eqs. (16) and (17) correspond to incoming and outgoing waves. $u_n(i,y,z)$ and $v_n(i,y,z)$ are the solutions to the (y,z) components of Eqs. (14) and (15).

The occupancy factors for the electron and hole states incident from contact j are[9,17]

$$f_{ie}(E)=\left[1+\exp\left(\frac{E-(\mu_i-\mu_S)}{kT}\right)\right]^{-1}$$

and

$$f_{ih}(E)=\left[1+\exp\left(\frac{E+(\mu_i-\mu_S)}{kT}\right)\right]^{-1}. \tag{19}$$

An important feature of these occupation factors is that the chemical potentials for the electron and hole bands are different.

Superconducting contacts

In the superconducting contacts, we only discuss the case where the single-particle potential $V_s(\mathbf{r})$ is a constant in the bulk of the contact, and abruptly goes to infinity at the extremities of the contacts the (y,z) direction. Then $\Delta(\mathbf{r})$ can be approximated by a constant Δ_j in the bulk of the super-

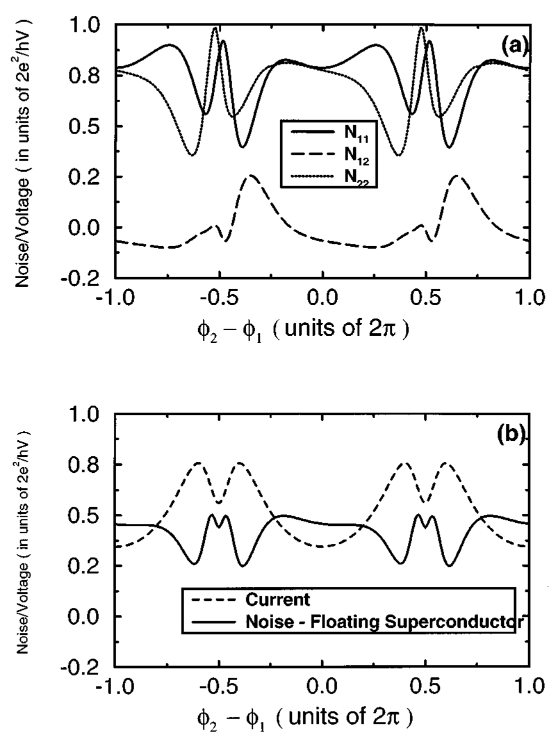

FIG. 5. (a) A plot of the shot noise per unit applied voltage (N_{11}, N_{12}, N_{22}) for the device in Fig. 4(b) as a function of the phase difference between the superconducting boundaries. Note that $N_{11}\neq N_{22}\neq N_{12}$, and that N_{12} can be either positive or negative unlike normal mesoscopic systems. (b) When the superconducting region is floating, $N_{11}=N_{22}=-N_{12}$ (solid line), as expected for a two-terminal device. The dashed line is the average conductance as a function of the phase difference between the superconducting boundaries for the devices in both Figs. 4(a) and 4(b). Both plots are for zero temperature.

conducting contact labeled j. An incident particle at energy E from the superconducting contact j has the following form:

$$\begin{pmatrix} e^{\pm ik_{jx}x}u_n(j,y,z) \\ e^{\pm ik_{jx}x}v_n(j,y,z) \end{pmatrix}, \tag{20}$$

where k_{jx} is the x component of the wave vector in contact j, $k_j=\sqrt{[2m(\mu_s-V_j-E\pm\sqrt{|E^2-\Delta_j^2|})]/\hbar^2}$. V_j is the constant single-particle potential $V_s(\mathbf{r})$ which represents the bottom of the band in superconducting contact j. The plus and minus signs in Eq. (20) correspond to incoming and outgoing waves.

The occupancy factor for an incident state from superconducting contact j is[9,17]

$$f_j(E)=\left[1+\exp\left(\frac{E}{kT}\right)\right]^{-1}. \tag{21}$$

We stress again that all energies E are measured with respect to μ_S, which is the chemical potential of the superconducting regions.

We use a picture consisting of both positive and negative energy states.[20,17] Reference 17 contains a detailed description of this picture. All results in this paper can, however, be obtained from the conventional picture which considers only the positive energy states.

Given the states in the various contacts, the next question is what is the scattering state in normal contact i as a result of a particle incident in any of the contacts. A particle incident from contact j can either be transmitted as an electron or a hole to contact i. The resultant scattering state in contact i is of the form

$$\begin{pmatrix} e^{ik_{ie}x}\delta_{ij}+\dfrac{\sqrt{v_{ie}}}{\sqrt{v_{je}}}\,s_{ij}^{\mathrm{ee}}(E)e^{-ik_{ie}x} \\ \dfrac{\sqrt{v_{ih}}}{\sqrt{v_{jh}}}\,s_{ij}^{\mathrm{he}}(E)e^{ik_{ih}x} \end{pmatrix}$$

and

$$\begin{pmatrix} \dfrac{\sqrt{v_{ie}}}{\sqrt{v_{jh}}}\,s_{ij}^{\mathrm{eh}}(E)e^{-ik_{ie}x} \\ e^{ik_{ih}x}\delta_{ij}+\dfrac{\sqrt{v_{ih}}}{\sqrt{v_{jh}}}\,s_{ij}^{\mathrm{hh}}(E)e^{ik_{ih}x} \end{pmatrix}. \tag{22}$$

Here $k_{j\beta}$ and $v_{j\beta}=\hbar k_{j\beta}/m$ are the wave vector and velocity, respectively, of $\beta\in e,h$ at energy E in contact j. s is the scattering matrix of the device including the superconducting region. $s_{ij}^{\alpha\beta}$ represents the scattering coefficient for a particle of type β incident from contact j which is transmitted to contact i as a particle of type α (α, $\beta\in e,h$). Note that Eq. (22) has been written down only for a single mode; the generalization to many modes is straightforward.

The scattering matrix can be obtained by solving Eq. (6) in the various regions of the device and then matching the wave functions u_n and v_n at suitable spatial locations (like interfaces between two different regions), in a manner similar to that in normal mesoscopic systems. An iterative procedure would be required for a self-consistent solution of Eqs. (6).

When a bias is applied to a contact, it changes both the chemical potential and the energy of the band bottom V_i. If a bias voltage V_a is applied to contact i, the changes in μ_i and V_i are

$$\mu_i\to\mu_i+eV_a \quad \text{and} \quad V_i\to V_i+V_a\,.$$

A change in V_i would cause a change in the exact form of the scattering states throughout the device, and would also cause a change in the occupancy of the scattering states at a given energy in contact i.

The field operator $\hat{\Psi}(i,x)$ in contact i is a linear combination of the states in Eq. (22),

$$\hat{\Psi}(i,x)=\begin{pmatrix} \hat{\Psi}_e(i,x) \\ \hat{\Psi}_h(i,x) \end{pmatrix}=\sum_{j\in N,S;\beta\in e,h}\frac{1}{\sqrt{2\pi}}\int\frac{dE}{\sqrt{\hbar v_{j\beta}}}\times\begin{pmatrix} e^{ik_{ie}x}\delta_{ij}\delta_{e\beta}+\dfrac{\sqrt{v_{ie}}}{\sqrt{v_{j\beta}}}\,s_{ij}^{e\beta}(E)e^{-ik_{ie}x} \\ e^{-ik_{ih}x}\delta_{ij}\delta_{h\beta}+\dfrac{\sqrt{v_{ih}}}{\sqrt{v_{j\beta}}}\,s_{ij}^{h\beta}(E)e^{ik_{ih}x} \end{pmatrix}\times e^{-iEt}\hat{a}_{j\beta}(E). \tag{23}$$

Throughout the manuscript latin alphabets correspond to the terminals and greek alphabets correspond to the electron (e) and hole (h) channels. N and S refer to the set of all normal and superconducting contacts respectively. $a_{j\beta}(E)$ is the annihilation operator in contact j for a particle of type $\beta\in e,h$ at energy E. $k'_{j\beta}$ and $\theta'_{j\beta}$ are the wave vector and velocity, respectively, of particle of type $\beta\in e,h$ at energy E' in contact j.

The field operators $\hat{\Psi}(i,x)$ and $\hat{a}_{j\beta}(E)$ obey the commutator rules for fermions,

$$[\hat{\Psi}(i,x),\hat{\Psi}(j,x')]_+=0, \quad [\hat{a}_{i\alpha}(E),\hat{a}_{j\beta}(E')]_+=0, \tag{24}$$

$$[\hat{\Psi}^\dagger(i,x),\hat{\Psi}(j,x')]_+=\delta_{ij}\delta(x-x')$$

and

$$[\hat{a}_{i\alpha}^\dagger(E),\hat{a}_{j\beta}(E')]_+=\delta_{ij}\delta_{\alpha\beta}\delta(E-E'). \tag{25}$$

The expectation value of $\hat{a}_{i\alpha}^\dagger(E)\hat{a}_{j\beta}(E')$ is nonzero only when $i=j$ and it is the distribution function in contact i,

$$\langle\hat{a}_{i\alpha}^\dagger(E)\hat{a}_{j\beta}(E')\rangle=\delta_{ij}\delta_{\alpha\beta}\delta(E-E')f_{i\alpha}(E). \tag{26}$$

C. Evaluation of the current fluctuations

In the evaluation of the current fluctuations presented here, we consider only a single-moded device. The generalization to many modes is straightforward and we only present the final result in Appendix A.

The current operator for electrons and holes in lead i are

$$\hat{I}_{i\alpha}(x,t)=\mathrm{sgn}(\alpha)\,\frac{e\hbar}{2mi}\,\mathrm{Tr}\left(\hat{\Psi}^{\dagger}_{\alpha}(i,x)\,\frac{d\hat{\Psi}_{\alpha}(i,x)}{dx}-\frac{d\hat{\Psi}^{\dagger}_{\alpha}(i,x)}{dx}\,\hat{\Psi}_{\alpha}(i,x)\right)$$

where $\alpha\in e,h$, and $\mathrm{sgn}(\alpha)=+1$ for $\alpha=e$ and -1 for $\alpha=h$. Then the total-current operator in lead i which is the sum of the electron, and the hole components can be compactly written as

$$\hat{I}_{i}(x,t)=\frac{e\hbar}{2mi}\,\mathrm{Tr}\left(\hat{\Psi}^{\dagger}(i,x)\sigma_z\,\frac{d\hat{\Psi}(i,x)}{dx}-\frac{d\hat{\Psi}^{\dagger}(i,x)}{dx}\,\sigma_z\hat{\Psi}(i,x)\right), \tag{27}$$

where Tr denotes trace and the Pauli spin matrix σ_z correctly accounts for the sign of the electron and hole components. Substituting Eq. (23) into Eq. (27), we find

$$\hat{I}_i(x,t)=\frac{e\hbar}{2m}\sum_{j,k\in N,S;\alpha\beta,\gamma\in e,h}\mathrm{sgn}(\alpha)\,\frac{1}{2\pi}\int\frac{dE}{\sqrt{\hbar v_{j\beta}}}\int\frac{dE'}{\sqrt{\hbar v'_{j\beta}}}\,e^{-i(E-E')t}\hat{a}^{\dagger}_{j\beta}(E)\hat{a}_{k\gamma}(E')$$
$$\times\Bigg\{(k'_{i\alpha}+k_{i\alpha})\Bigg[\delta_{ij}\delta_{\alpha\beta}\delta_{ik}\delta_{\alpha\gamma}e^{-i\,\mathrm{sgn}(\alpha)(k_{i\alpha}-k'_{i\alpha})x}-\frac{\sqrt{\hbar v_{j\beta}}}{\sqrt{\hbar v_{i\alpha}}}\frac{\sqrt{\hbar v'_{k\gamma}}}{\sqrt{\hbar v'_{i\alpha}}}\,s^{\alpha\beta\dagger}_{ij}(E)s^{\alpha\gamma}_{ik}(E')e^{i\,\mathrm{sgn}(\alpha)(k_{i\alpha}-k'_{i\alpha})x}\Bigg]$$
$$+(k'_{i\alpha}-k_{i\alpha})\Bigg[\delta_{ij}\delta_{\alpha\beta}e^{-i\,\mathrm{sgn}(\alpha)(k'_{i\alpha}+k_{i\alpha})x}\,\frac{\sqrt{\hbar v'_{k\gamma}}}{\sqrt{\hbar v'_{i\alpha}}}\,s^{\alpha\gamma}_{ik}(E')-\delta_{ik}\delta_{\alpha\gamma}e^{i\,\mathrm{sgn}(\alpha)(k'_{i\alpha}+k_{i\alpha})x}\,\frac{\sqrt{\hbar v_{j\beta}}}{\sqrt{\hbar v_{i\alpha}}}\,s^{\alpha\beta\dagger}_{ij}(E)\Bigg]\Bigg\}, \tag{28}$$

where $\mathrm{sgn}(\alpha)=+1$ for $\alpha=e$ and $\mathrm{sgn}(\alpha)=-1$ for $\alpha=h$. Also, $k_{i\alpha}$ ($v_{i\alpha}$) and $k'_{i\alpha}$ ($v'_{i\alpha}$) correspond to the wave vector (velocity) of a particle of type α in contact i at energies E and E'. Equation (28) does not account for displacement currents due to charging, and is hence valid only for the low-frequency components of the current, for which the the capacitive component can be neglected.

The average current is obtained by taking the expectation value of Eq. (28). Noting that $\langle\hat{a}^{\dagger}_{j\beta}(E)\hat{a}_{k\gamma}(E')\rangle=f_{j\beta}(E)\delta_{jk}\delta_{\beta\gamma}\delta(E-E')$, it is straightforward to verify that the average current in lead i is

$$I_i=\frac{e}{h}\sum_{\alpha,j\in NS,\beta}\mathrm{sgn}(\alpha)[\delta_{ij}\delta_{\alpha\beta}-T^{\alpha\beta}_{ij}(E)]f_{j\beta}(E), \tag{29}$$

where the transmission coefficients are related to the scattering matrix by $T^{\alpha\beta}_{ij}(E)=|s^{\alpha\beta}_{ij}(E)|^2$. Linearizing Eq. (29) and using Eq. (A2) from the Appendix, we obtain

$$I_i=\sum_{j\in N}g_{ij}(\mu_j-\mu_S), \tag{30}$$

where

$$g_{ij}=\frac{2e^2}{h}\int dE[\delta_{ij}-T^{\mathrm{ee}}_{ij}(E)+T^{\mathrm{he}}_{ij}(E)]\left(-\frac{\partial f_j(E)}{\partial E}\right)_{\mathrm{eq}}. \tag{31}$$

Equation (30) was originally derived in Ref. 9. $T^{\alpha\beta}_{ij}$ is the transmission coefficient of a particle of type β incident in contact j to be transmitted to contact i as a particle of type α. As before $\alpha,\beta\in e,h$.

The general expression for current fluctuation between contacts i and j is

$$S_{ij}(\tau)=\langle\Delta\hat{I}_i(t)\Delta\hat{I}_j(t+\tau)+\Delta\hat{I}_j(t+\tau)\Delta\hat{I}_i(t)\rangle, \tag{32}$$

where

$$\Delta\hat{I}_i(t)=\hat{I}_i(t)-\langle\hat{I}_i(t)\rangle. \tag{33}$$

The spectral function of the current fluctuations which is the Fourier transform of Eq. (32) is

$$S_{ij}(\omega)\delta(\omega+\omega_1)=\frac{1}{2\pi}\langle\Delta\hat{I}_i(\omega_1)\Delta\hat{I}_j(\omega)+\Delta\hat{I}_j(\omega)\Delta\hat{I}_i(\omega_1)\rangle, \tag{34}$$

where

$$\Delta\hat{I}_i(\omega)=\hat{I}_i(\omega)-\langle\hat{I}_i(\omega)\rangle. \tag{35}$$

In this paper we will calculate the fluctuations only in the zero-frequency limit because of the limitation of Eq. (28) mentioned above. Now, using the relationship $\int dt e^{i(E-E')t/\hbar}=2\pi\hbar\,\delta(E-E')$, in the zero-frequency limit Eq. (28) simplifies to

$$\hat{I}_i(\omega=0)=e\sum_{k,l\in N,S,\alpha,\gamma,\delta\in e,h}\mathrm{sgn}(\alpha)\int dE\; A_{k\gamma;l\delta}(i\alpha,E)\times\hat{a}^{\dagger}_{k\gamma}(E)\hat{a}_{l\delta}(E), \tag{36}$$

where

$$A_{k\gamma;l\delta}(i\alpha,E)=\delta_{ik}\delta_{il}\delta_{\alpha\gamma}\delta_{\alpha\delta}-s^{\alpha\gamma\dagger}_{ik}(E)s^{\alpha\delta}_{il}(E). \tag{37}$$

The difference between Eq. (36) and the corresponding expression for the current operator in normal mesoscopic systems[1] is that now every contact index k is generalized to $k\gamma$. γ is an index representing the electron and hole channels. The signum function accounts for the sign of the electron and hole currents.

Given the above information, the algebra involved in the evaluation of the spectral function $S_{ij}(\omega)$ is along the lines of Ref. 1. So we relegate it to Appendix B. The final result for the current fluctuation spectral function $S_{ij}(\omega)$ is

$$S_{ij}=\frac{2e^2}{h}\sum_{k,l\in N,S,\alpha,\beta,\gamma,\delta\in e,h}\mathrm{sgn}(\alpha)\mathrm{sgn}(\beta)\times\int dE\; A_{k\gamma;l\delta}(i\alpha,E)A_{l\delta;k\gamma}(j\beta,E)f_{k\gamma}(E)[1-f_{l\delta}(E)]. \tag{38}$$

Equation (38) is a multiterminal formula for current fluctuations between normal contacts i and j, and is valid in the presence of an applied bias. It is valid when the superconducting region is a contact kept at an externally fixed chemical potential. To the best of our knowledge, Eq. (38) has not appeared in the literature before. Note that Appendix A contains the expression for S_{ij} in the multi moded case.

We comment that Eq. (38) can be viewed as a simple generalization of the corresponding expression for current fluctuations in normal mesoscopic systems. The expression for current fluctuations in normal mesoscopic systems is[1]

$$S_{ij}(\omega=0)=\frac{e^2}{h}\sum_{k,l}\int dE\;\mathrm{Tr}[A_{kl}(i,E)A_{lk}(j,E)]\times f_k(E)[1-f_l(E)], \tag{39}$$

where

$$A_{lk}(j,E)=\delta_{jl}\delta_{kl}-s^{\dagger}_{jl}s_{jk}.$$

Now, (i) by associating an additional index representing electron and hole channels with every contact index [i.e., $j\to(je)$ and (jh)] in Eq. (39), and (ii) correctly accounting for the sign of the electron and hole currents in Eq. (39), it is easy to see that we obtain Eq. (38).

D. Sign of current noise in the presence of transport

In a normal mesoscopic device, an electron incident in contact j is always transmitted as an electron to any other contact i. As a result of this, the zero-frequency current fluctuations between two different contacts in a purely normal device is always negative, as proven in Ref. 1. However, in the presence of a superconducting region, an electron incident in contact i can result in either an electron or a hole leaving contact j. So we ask whether the current fluctuations between two different contacts be positive as a result of the Andreev processes. To check the sign of the current fluctuations, we can write the total current fluctuations between terminals i and j as

$$S_{ij}=S^{AA}_{ij}+S^{AB}_{ij}, \tag{40}$$

where

$$S^{AA}_{ij}=\langle\Delta\hat{I}_{ie}\Delta\hat{I}_{je}\rangle+\langle\Delta\hat{I}_{ih}\Delta\hat{I}_{jh}\rangle$$

and

$$S^{\mathrm{AB}}_{ij}=\langle\Delta\hat{I}_{ie}\Delta\hat{I}_{jh}\rangle+\langle\Delta\hat{I}_{ih}\Delta\hat{I}_{je}\rangle.$$

Using Eq. (38) and the orthogonality relations in Appendix A, we can show that

$$S^{AA}_{ii}=(+)\,\frac{2e^2}{h}\sum_{\alpha}\left\{[1-T^{\alpha\alpha}_{ii}(E)]^2f_{i\alpha}(E)[1-f_{i\alpha}(E)]+\sum_{(k\gamma l\delta)\neq(i\alpha i\alpha)}T^{\alpha\gamma}_{ik}(E)T^{\alpha\delta}_{il}(E)f_{k\gamma}[1-f_{l\delta}(E)]\right\}, \tag{41}$$

$$S^{AB}_{ii}=(+)\,\frac{2e^2}{h}\sum_{\alpha}\left\{2T^{\alpha\bar{\alpha}}_{ii}(E)f_{i\bar{\alpha}}(E)[1-f_{i\bar{\alpha}}(E)]+\sum_{k\gamma}s^{\bar{\alpha}\gamma}_{ik}(E)s^{\alpha\gamma\dagger}_{ik}(E)f_{k\gamma}(E)\sum_{l\delta}s^{\alpha\delta}_{il}(E)s^{\bar{\alpha}\delta\dagger}_{il}(E)\times f_{l\delta}(E)\right\}. \tag{42}$$

Here, if $\alpha=e$, then $\alpha=h$, and if $\alpha=h$, then $\bar{\alpha}=e$. $T^{\alpha\beta}_{ik}$ is the transmission coefficient of a particle of type β from contact j to contact i as a particle of type α. The last term of Eq. (42) is positive because it is of the form AA. The other terms of both S^{AA}_{ii} and S^{AB}_{ii} are clearly positive. Thus both at equilibrium and away from equilibrium, the current fluctuations in a single contact is always positive just as in a purely normal device.[20]

Using Eq. (38) and the orthogonality relations in Appendix A, we can also show that the current fluctuations between two different contacts i and j is

$$S^{AA}_{ij}|_{i\neq j}=(-)\,\frac{2e^2}{h}\sum_{\alpha}\left\{T^{\alpha\alpha}_{ij}(E)f_{j\alpha}(E)[1-f_{j\alpha}(E)]+T^{\alpha\alpha}_{ji}(E)f_{i\alpha}(E)[1-f_{i\alpha}(E)]+\sum_{k\gamma}s^{\alpha\gamma}_{jk}(E)s^{\alpha\gamma\dagger}_{ik}(E)f_{k\gamma}(E)\sum_{l\delta}s^{\alpha\delta}_{il}(E)s^{\alpha\delta\dagger}_{jl}(E)\times f_{l\delta}(E)\right\}, \tag{43}$$

$$S^{AB}_{ij}|_{i\neq j}=(+)\,\frac{2e^2}{h}\sum_{\alpha}\left\{T^{\alpha\bar{\alpha}}_{ij}(E)f_{j\bar{\alpha}}(E)[1-f_{j\bar{\alpha}}(E)]+T^{\bar{\alpha}\alpha}_{ji}(E)f_{i\alpha}(E)[1-f_{i\alpha}(E)]+\sum_{k\gamma}s^{\bar{\alpha}\gamma}_{jk}(E)s^{\alpha\gamma\dagger}_{ik}(E)f_{k\gamma}(E)\sum_{l\delta}s^{\alpha\delta}_{il}(E)s^{\bar{\alpha}\delta\dagger}_{jl}(E)\times f_{l\delta}(E)\right\}. \tag{44}$$

The last terms of both Eqs. (43) and (44) are of the form AA. Then clearly S_{ij}^{AA} is a negative definite quantity and S_{ij}^{AB} is a positive definite quantity. The total current fluctuations S_{ij} can either be a positive or a negative quantity depending on the relative strengths of the above two terms. Only the terms in S_{ij}^{AA} are present in the expression for current fluctuations of a purely normal device and, as a result, S_{ij} is always negative in a purely normal device.[1] We discuss an example in Sec. IV, where S_{ij} can be either positive or negative depending on the phase difference between the superconducting boundaries.

We now discuss the current fluctuations at equilibrium. At equilibrium, it is easy to verify that the expression for S_{ji} reduces to

$$(S_{ij})_{\rm eq}=\langle \Delta I_i \Delta I_j\rangle_{\rm eq}=2kT[g_{ij}+g_{ji}], \tag{45}$$

where g_{ij} are the conductance matrix elements appearing in Eq. (30). Equation (45) is simply a verification of the generalized fluctuation-dissipation theorem.[21] At equilibrium, the current fluctuations between contacts i and j are related only to the conductance matrix elements between these two contacts.

III. CURRENT NOISE: FLOATING SUPERCONDUCTOR

In Sec. II, we assumed that the superconductor is kept at an externally fixed chemical potential. In this section, we address current fluctuations in devices where the superconductor is a floating region in the device (Fig. 2). In the floating superconductor case, the expression for current in a normal contact is still given by Eqs. (29) and (30).[9] However, the expression for current fluctuations is very different from Eq. (38). To illustrate that the floating superconductor case is different, consider a *two-terminal* NSN device where the two normal regions widen into contacts and the superconducting region is floating. The two normal contacts are maintained at the same external potential. We consider the low-temperature limit where direct transmission of quasiparticles between the two normal regions is negligible. Then, at equilibrium, a straightforward application of Eq. (38) gives the following expressions for current fluctuations:

$$\langle \Delta I_1 \Delta I_1\rangle_{\rm eq}=4kTg_{11}, \quad \langle \Delta I_2 \Delta I_2\rangle_{\rm eq}=4kTg_{22}, \quad \langle \Delta I_1 \Delta I_2\rangle_{\rm eq}=0 \tag{46}$$

Equation (46) is clearly wrong because at equilibrium any two-terminal device should obey the Johnson-Nyquist relationship

$$\langle \Delta I_1 \Delta I_1\rangle_{\rm eq}=\langle \Delta I_2 \Delta I_2\rangle_{\rm eq}=-\langle \Delta I_1 \Delta I_2\rangle_{\rm eq}=4kTG, \tag{47}$$

where G is the linear-response conductance of the device. For a NSN device, the linear response conductance can be calculated using Eq. (30), and is given by

$$G=\frac{g_{11}g_{22}}{g_{11}+g_{22}} \tag{48}$$

The reason for this apparent violation of the Johnson-Nyquist relationship in Eq. (46) is now described. In the floating superconductor case, the chemical potential of the condensate (μ_S) should be determined self-consistently from the condition[9] that the sum of the currents flowing in the various contacts is zero, $\Sigma_i I_i=0$. As a result of this requirement, fluctuations in the current causes the chemical potential of the condensate (μ_S) to fluctuate, in turn affecting the current flowing in the contacts. These processes were not accounted for in the derivation of Eq. (38) because we assumed the superconductor to be held at a fixed potential in Sec II. In the above example of the NSN device, the average value of the chemical potential of the superconducting region (μ_S) is the same as that of the two normal contacts. μ_S can, however, fluctuate with time, and this fluctuation was not taken into account in Sec. II.

We will now derive an expression for the current fluctuations in the floating superconductor case by accounting for fluctuations in the chemical potential of the condensate. Fluctuations in the chemical potential of the condensate can be included using the method of Langevin forces.[1,22] This method is valid only at small biases, and consists of writing the current operator in contact i as the sum of the expression obtained for average current and a generalized Langevin force which causes the fluctuations in the chemical potential μ_S

$$I_i=\sum_j g_{ij}(\mu_j-\mu_S)+\delta I_i \tag{49}$$

The average value of δI_i is zero, and fluctuations in δI_i are given by Eq. (38) with μ_S set equal to the steady-state value $\overline{\mu_S}$. Now, making use of the point that at zero frequency,

$$\sum_i I_i=0, \tag{50}$$

the chemical potential of the superconductor can be written as the sum of its steady-state value and a fluctuation term

$$\mu_S=\overline{\mu_S}+\Delta\mu_S=\frac{\sum_{i,j} g_{i,j}\mu_j}{\sum_k x_k}+\frac{\sum_i \delta I_i}{\sum_k x_k}, \tag{51}$$

where $x_i=\Sigma_j g_{ij}$. Noting that the average value of the fluctuation in the chemical potential of the superconductor ($\overline{\Delta\mu_S}$) is equal to 0, the fluctuation in the total current ΔI_i is

$$S_{ij}=\langle \Delta I_i \Delta I_j\rangle$$
$$=\langle \delta I_i \delta I_j\rangle+x_i x_j\sum_{k,l}\langle \Delta\mu_S \Delta\mu_S\rangle-x_j\langle \delta I_i \Delta\mu_S\rangle-x_i\langle \Delta\mu_S \delta I_j\rangle$$

Substituting the value of $\Delta\mu_S$ from Eq. (51) into the previous equation,

$$S_{ij}=\frac{1}{[\sum_k x_k]^2}\sum_{m,n}\{x_m x_n\langle\delta I_i\delta I_j\rangle - x_j x_m\langle\delta I_i\delta I_n\rangle - x_m x_i\langle\delta I_n\delta I_j\rangle + x_i x_j\langle\delta I_m\delta I_n\rangle\}, \quad (52)$$

where $\langle\delta I_i\delta I_j\rangle$ are the current fluctuations given by Eq. (38) with the chemical potential of the superconductor set equal to its steady-state value $(\overline{\Delta\mu_S})$. Equation (52) is the expression for current fluctuations in the floating superconductor case. It expresses the total current fluctuation in the floating superconductor case in terms of the current fluctuations of the same system, with μ_S held at its steady-state value. From the above discussion we see that there are similarities between the floating superconductor case and a purely normal device with a floating voltage probe, which has been discussed in Ref. 23. In fact, the expression for the linear-response current [Eq. (30) of this paper] derived in Ref. 9 is similar to the expression for the linear-response current in a normal device with a floating voltage probe. The floating superconductor, however, may only be a part of the device and not a contact as in the case of a floating voltage probe.

We now verify that if Eq. (52) is used to calculate the current fluctuations, the Johnson-Nyquist relationship is indeed valid for the two-terminal NSN device discussed at the beginning of this section. For a two-terminal device, Eq. (52) has the form

$$S_{11}=\frac{1}{(x_1+x_2)^2}[x_2^2\langle\delta I_1\delta I_1\rangle + x_1^2\langle\delta I_2\delta I_2\rangle - 2x_1x_2\langle\delta I_1\delta I_2\rangle]. \quad (53)$$

Also, $S_{22}=-S_{12}=-S_{21}=S_{11}$. Now, substituting the equilibrium values of $\langle\delta I_i\delta I_j\rangle$ from Eq. (45) in Eq. (53), we verify the Johnson-Nyquist relationship

$$\langle\Delta I_1\Delta I_1\rangle_{\rm eq}=\langle\Delta I_2\Delta I_2\rangle_{\rm eq}=-\langle\Delta I_1\Delta I_2\rangle_{\rm eq}=4kTG,$$

where G is the two-terminal conductance given by Eq. (48).

IV. EXAMPLES

A. NS junction with an applied bias

Using Eq. (38), we find the shot noise of a NS junction at zero temperature to be

$$\langle\delta I_1\delta I_1\rangle=\frac{4e^2}{h}\int_{\mu_S}^{\mu_N}dE\{T_{11}^{\rm ee}(E)[1-T_{11}^{\rm ee}(E)] + T_{11}^{\rm he}(E)[1-T_{11}^{\rm he}(E)]+2T_{11}^{\rm ee}(E)T_{11}^{\rm he}(E)\}, \quad (54)$$

where 1 refers to the normal terminal. This expression is valid in the presence of a bias larger than the superconducting gap also. In the small bias limit, where $T_{11}^{\rm ee}+T_{11}^{\rm he}=1$, Eq. (54) agrees with the result in Ref. 12.

Using Eq. (54), we now discuss the prediction in Ref. 18 that a ballistic NS junction ($T_{11}^{\rm ee}=T_{11}^{\rm hh}=0$) has a nonzero shot noise when $V>\Delta$. In a ballistic NS junction, for $E<\Delta$, every incident electron in the normal region results in the reflection of a hole in the normal region, and hence the flow of a Cooper pair in the superconductor with unity probability ($T_{11}^{\rm he}=1$ and $T_{11}^{\rm ee}=0$). Then, at zero temperature, it follows trivially from Eq. (54) that the shot noise is *zero* for $V<\Delta$. At energies larger than Δ, the physics is very different in the two limits $\Delta<E<$fewΔ and $E>\Delta$. For energies $\Delta<E<$fewΔ, when an electron is incident from the normal region to the superconductor, there are two competing processes which contribute to current transport; (i) the electron can be reflected as a hole in the normal region, resulting in the flow of a Cooper pair with charge $2e$ at the Fermi energy in the superconductor; and (ii) the electron is transmitted to the superconducting region as an electronlike quasiparticle. For energies $\Delta<E<$few Δ, $T_{11}^{\rm he}\neq\{0,1\}$ and $T_{11}^{\rm ee}=0$. It then follows from Eq. (54) that the competition between the two different transmission processes causes a quick increase in the shot noise at voltages larger than Δ. For energies, $E\gg\Delta$, every incident electron from the normal region is transmitted as an electronlike quasiparticle to the superconducting region (i.e., $T_{11}^{\rm he}\sim 0$, $T_{11}^{\rm ee}\sim 0$, and $T_{11}^{\rm ee}\sim 1$). Then it follows from Eq. (54) that transport of electrons at these energies does not contribute to shot noise. As a result of this, the shot noise tends to saturate at large applied voltages.

The noise-to-current ratio (S/I) is a quantity of interest in noise studies.[12,24,25] Using Eq. (54), we discuss the S/I ratio in a NS junction with a δ-function potential barrier at the NS interface, both as a function of the applied bias and the barrier strength. The potential barrier is a δ function with strength $U\delta(x)$.[26] In the previous paragraph we saw that the current fluctuation saturates with the applied voltage in the case of a ballistic NS junction. The average current, however, continues to increase as the applied voltage is increased. As a result of this the S/I ratio is peaked at a voltage large than Δ/e. Now as the barrier strength is increased we find that for small values of the barrier strength, the peak in the S/I ratio survives. This peak is, however, washed out for large barrier strengths. For large values of the normal reflection coefficient, the S/I ratio approaches the value $4e$ in the small bias limit [Fig. 3(a)] because transport is only due to reflection of holes in the normal region (and hence a flow of a Cooper pair with charge $2e$ in the superconductor). This was predicted in Ref. 12. We find that as the voltage is increased, to values larger than Δ/e, the S/I ratio approaches the value $2e$ [Fig. 3(b)], as now most of the transport is due to quasiparticles with the charge of a single electron.

B. Current fluctuations in a floating superconductor

The devices considered in this example are shown in Fig. 4. The purpose of this example is to illustrate the difference in shot noise between the case of a superconductor kept at a fixed external potential and the case of a floating superconductor. The devices in Fig. 4 consist of a normal region connected to two normal terminals $N1$ and $N2$. Further they have two superconducting boundaries whose phases ϕ_1 and ϕ_2 can be changed. Experimental and theoretical studies involving the conductance of devices where the phase difference between two superconducting boundaries can be changed in a controlled fashion is being actively pursued now.[2–5,14,17,27]

The device in Fig. 4(a) (device A) is connected to a single superconductor whose potential floats to a value which is determined by the current flowing in the normal terminals. The device in Fig. 4(b) (device B) is similar to device A, except that the superconductor is maintained at an externally fixed potential. The potential of the superconductor in device B is chosen to be equal to the potential of the superconductor in device A.

The average currents in devices A device B are identical. This is because the average current depends only on the steady-state value of the potential of the superconductor, and is not sensitive to whether the superconductor is floating or not. The current fluctuation is, however, sensitive to this detail. We illustrate this point by computing the current fluctuations in the normal terminals of devices A and B as a function of the phase difference $(\phi_1-\phi_2)$ between the two superconducting boundaries.

We model the devices by a single moded ballistic channel, and assume that the two NS junctions are perfectly ballistic. We further assume zero temperature and the small bias limit. The scattering matrix of the two couplers (the couplers are the two T-shaped regions connecting leads 1 and 2 to the normal wire) are taken to be

$$s_i=\begin{pmatrix} \sqrt{1-2\epsilon_i} & \sqrt{\epsilon_i} & \sqrt{\epsilon_i} \\ \sqrt{\epsilon_i} & \frac{1}{2}(1-\sqrt{1-2\epsilon_i}) & \frac{1}{2}(1+\sqrt{1-2\epsilon_i}) \\ \sqrt{\epsilon_i} & \frac{1}{2}(1+\sqrt{1-2\epsilon_i}) & \frac{1}{2}(1-\sqrt{1-2\epsilon_i}) \end{pmatrix}, \tag{55}$$

where $i\in 1,2$ are the two couplers, and s_i is the scattering matrix of coupler i. The scattering matrix element $s_i(2,1)=\sqrt{\epsilon_i}$ represents the strength of the scattering amplitude of an electron incident in lead 1 to scatter to the left side of coupler i. $s_i(3,1)$ represents a similar amplitude to scatter to the right of coupler i. The matrix elements $s_i(1,1)$, $s_i(2,2)$, and $s_i(3,3)$ are the amplitudes for reflection of a wave incident in contact 1, incident from the left of coupler i and incident from the right of coupler i, respectively. The other matrix elements are defined similarly. As the NS junctions are assumed to be perfectly ballistic, an electron (hole) incident from the normal region is always reflected as a hole (electron). The reflection coefficients are given by $r_{\mathrm{eh}}=-ie^{i\phi}$ and $r_{\mathrm{he}}=-ie^{-i\phi}$, where ϕ is the phase of the superconducting region. We obtain the scattering matrix of the device numerically by cascading the scattering matrix of the individual elements. The values for the various parameters used in the calculation presented are $\epsilon_1=0.40$, $\epsilon_2=0.30$, $L_1=1.6$ μm, $L_{12}=1.8$ μm, and $L_2=1.6$ μm.

Using Eq. (30) we find that, for device A,

$$\mu_1-\mu_S=+\frac{(g_{12}+g_{22})}{g_{11}+g_{12}+g_{21}+g_{22}}(\mu_1-\mu_2) \tag{56}$$

and

$$\mu_2-\mu_S=-\frac{(g_{11}+g_{21})}{g_{11}+g_{12}+g_{21}+g_{22}}(\mu_1-\mu_2). \tag{57}$$

The Fermi functions for electrons and holes in the normal contacts are

$$f_{1e}=\Theta(\mu_1-\mu_S),\quad f_{1h}=\Theta(\mu_S-\mu_1),$$

$$f_{2e}=\Theta(\mu_2-\mu_S),\quad f_{2h}=\Theta(\mu_S-\mu_2), \tag{58}$$

where μ_1 and μ_2 are given by Eqs. (56) and (57). For device B, the potential of the superconducting region is externally chosen to have the same Fermi functions as those given above.

We first consider device B, where the superconductor is fixed at an external potential. The current fluctuations are computed by substituting the Fermi functions and the scattering matrix for the device in Eq. (38). The current fluctuation is plotted as a function of the phase difference $(\phi_1-\phi_2)$ in Fig. 5(a). Note that while current fluctuations at a single terminal are always positive, the current fluctuations between two different terminals can either be positive or negative, as discussed in Sec. II D.

For device A, the superconductor is floating. Here we use Eq. (53) to calculate the current fluctuations. As devices A and B have the same steady-state value for the chemical potential of the superconductor, the $\langle \delta I_i \delta I_j\rangle$ appearing in Eq. (53) are just those obtained for device B. The current fluctuation in this case is plotted in Fig. 5(b). Note that, as device A is a two-terminal device, the various current fluctuations obey $\langle \Delta I_1 \Delta I_1\rangle=\langle \Delta I_2 \Delta I_2\rangle=-\langle \Delta I_1 \Delta I_2\rangle=-\langle \Delta I_2 \Delta I_1\rangle$, with the current fluctuations in a single terminal always being positive and the current fluctuations between the two different terminals always being negative. As discussed above, the average conductance of both devices is the same, and this is plotted in Fig. 5(b).

V. CONCLUSIONS

In conclusion, we have presented a general expression [Eq. (38)] for current fluctuations in the normal terminals of a phase-coherent mesoscopic device with a superconducting region at an externally fixed potential (Fig. 1). Equation (38) can be viewed as a simple generalization of the corresponding expression derived by Buttiker[1] for a purely normal mesoscopic device (i) where every contact k is generalized to $k\gamma$, where γ represents the electron and hole channels, and (ii) correctly accounting for the sign of the electron and hole currents. We find that the current correlation between two different contacts of a device can be *either* positive or negative as a result of Andreev scattering. In contrast, in a purely normal mesoscopic device the current correlation between two different contacts is always negative.[1] Using Eq. (38), we derive an expression for the shot noise in a NS junction valid at voltages larger than Δ/e, where Δ is the superconducting gap energy. Using the Keldysh Green's-function theory, Ref. 18 predicted that a ballistic NS junction should exhibit a nonzero shot noise at applied voltages larger than Δ/e. This result is simple to understand from the scattering theory approach presented in this paper, and is discussed in Sec. IV. We have also studied the noise-to-current ratio as a function of both the bias and the strength of a δ-function barrier at the NS interface. We find that for junctions with a small reflection coefficient, the noise-to-current ratio is peaked at voltages larger than Δ/e [Fig. 3(a)]. As the strength of the barrier is increased, this peak disappears. Further, for the strong barrier limit, the noise to current ratio approaches the value of $4e$ in the small bias limit as predicted in Ref. 12. We find that as the voltage is increased to values larger than Δ/e, this ratio approaches the value of $2e$ [Fig. 3(b)] in the strong barrier limit. For devices with a floating superconducting region (Fig. 2), we derive an expression for the current fluctuations in the small bias limit [Eq. (52)]. That the floating superconductor case is distinctly different is illustrated using a simple example (Fig. 4). While the average current is the same for the two devices in Fig. 4, the current fluctuations are very different (Fig. 5). A floating superconductor acts in much the same way as a floating voltage probe[1] in normal mesoscopic devices even though the superconductor may only be a part of the device. We would like to comment that throughout this paper, we have assumed the order parameter of the superconductor to be fixed. However, the order parameter fluctuates, and this can be seen from the self-consistency requirement in Eq. (8). We leave this for future work.

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APPENDIX A

Some useful relations used in this paper are the following.

$$\sum_{l\in N,S;\delta\in e,h} s_{il}^{\alpha\delta\dagger} s_{jl}^{\beta\delta} = \sum_{l\delta} s_{li}^{\delta\alpha\dagger} s_{lj}^{\delta\beta} = \delta_{ij}\delta_{\alpha\beta} \quad \text{orthogonality,} \tag{A1}$$

$$\sum_{l\in N,S;\delta\in e,h} T_{il}^{\alpha\delta}(E) = 1 \quad \text{sum rule,} \tag{A2}$$

$$s_{ij}^{\alpha\beta}(E,B,\Delta) = s_{ji}^{\beta\alpha}(E,-B,\Delta^*) \quad \text{where} \quad \alpha,\beta\in e,h, \tag{A3}$$

$$g_{ij} = \frac{2e^2}{h}\int dE[\delta_{ij} - T_{ij}^{\text{ee}}(E) + T_{ij}^{\text{he}}(E)]\left(-\frac{\partial f_j(E)}{\partial E}\right)_{\text{eq}}, \tag{A4}$$

$$= \frac{2e^2}{h}\int dE[\delta_{ij} - T_{ij}^{\text{hh}}(E) + T_{ij}^{\text{eh}}(E)]\left(-\frac{\partial f_j(E)}{\partial E}\right)_{\text{eq}} \tag{A5}$$

$$= g_{ji}. \tag{A6}$$

Proof of

$$\sum_{k,\gamma,l,\delta,\alpha,\beta} \text{sgn}(\alpha)\text{sgn}(\beta) A_{k\gamma;l\delta}(i\alpha,E) A_{l\delta;k\gamma}(j\beta,E) f_{k\gamma}(E)$$

$$= \text{sgn}(\alpha)\text{sgn}(\beta) \sum_{k,\gamma,l,\delta,\alpha,\beta} A_{k\gamma;l\delta}(i\alpha,E) A_{l\delta,k\gamma}(j\beta,E) f_{l\delta}(E) \tag{A7}$$

is as follows:

$$\text{LHS} = \sum_{k,\gamma,l,\delta,\alpha,\beta} \text{sgn}(\alpha)\text{sgn}(\beta)[\delta_{ik}\delta_{il}\delta_{\alpha\gamma}\delta_{\alpha\delta} - s_{ik}^{\alpha\gamma\dagger} s_{il}^{\alpha\delta}] \times [\delta_{jk}\delta_{jl}\delta_{\beta\gamma}\delta_{\beta\delta} - s_{jl}^{\beta\delta\dagger} s_{jk}^{\beta\gamma}] f_{k,\gamma}(E),$$

$$\text{LHS} = \sum_{\alpha,\beta}\{\delta_{ij}\delta_{\alpha\beta} f_{i\alpha}(E) - \text{sgn}(\alpha)\text{sgn}(\beta)[T_{ji}^{\beta\alpha} f_{i\alpha} + T_{ij}^{\alpha\beta} f_{j\beta}(E)] + \text{sgn}(\alpha)\text{sgn}(\beta) s_{ik}^{\alpha\gamma\dagger} s_{il}^{\alpha\delta} s_{jl}^{\beta\delta\dagger} s_{jk}^{\beta\gamma} f_{k,\gamma}(E)\}. \tag{A8}$$

Using the orthogonality of the scattering matrix $\Sigma_{l\delta} s_{il}^{\alpha\delta} s_{jl}^{\beta\delta\dagger} = I_l$ in the third term of Eq. (A8), we obtain

$$\text{LHS} = \sum_{\alpha,\beta}\left\{\delta_{ij}\delta_{\alpha\beta} f_{i\alpha}(E) - \text{sgn}(\alpha)\text{sgn}(\beta)[T_{ji}^{\beta\alpha} f_{i\alpha} + T_{ij}^{\alpha\beta} f_{j\beta}(E)] + \sum_{k\gamma} \delta_{ij}\delta_{\alpha\beta} T_{ik}^{\alpha\gamma}(E) f_{k,\gamma}(E)\right\} = \text{RHS}. \tag{A9}$$

Equation (38) becomes the following equation when the various modes are included;

$$S_{ij} = \frac{e^2}{h} \sum_{\alpha,\beta,\gamma,\delta\in e,h \text{ and } k,l\in N,\ S \text{ contacts}} \text{sgn}(\alpha)\text{sgn}(\beta) \times \int dE\ \text{Tr}[A_{k\gamma;l\delta}(i\alpha,E) A_{l\delta;k\gamma}(j\beta,E)] \times f_{k\gamma}(E)[1 - f_{l\delta}(E)], \tag{A10}$$

where

$$\text{Tr}[A_{k\gamma;l\delta}(i\alpha,E) A_{l\delta,k\gamma}(j\beta,E)] = \sum_{mn} A_{km\gamma;ln\delta}(i\alpha,E) A_{ln\delta;km\gamma}(j\beta,E) \tag{A11}$$

and

$$A_{km\gamma;ln\delta}(i\alpha,E) = \sum_p [\delta_{ik}\delta_{il}\delta_{pm}\delta_{pn}\delta_{\alpha\gamma}\delta_{\alpha\delta} - s_{ip;km}^{\alpha\gamma\dagger} s_{ip;ln}^{\alpha\delta}]. \tag{A12}$$

Here p, m, and n correspond to the modes in contacts i, k, and l, respectively.

APPENDIX B

Derivation of Eq. (38): Using Eq. (36), it is straightforward to verify that, $\langle \hat{I}(\omega k=0) I(\omega'=0)\rangle$

$$\langle \hat{I}_i(\omega=0)\hat{I}_j(\omega'=0)\rangle = \frac{e}{\hbar^2}\sum_{\alpha,\beta,k\gamma,\lambda,\delta} \mathrm{sgn}(\alpha)\mathrm{sgn}(\beta)\int dE\int dE'\int dE'' \times\int dE'''\,\delta(E-E')\,\delta(E''-E''')A_{k\gamma;l\delta}(i\alpha,E) \times A_{l\delta;k\gamma}(j\beta,E'')\langle \hat{a}^\dagger_{k\gamma}(E)\hat{a}_{l\delta}(E)\hat{a}^\dagger_{m\zeta}(E'')\hat{a}_{n\eta}(E'')\rangle. \tag{B1}$$

Using Wick's theorem, the expectation value of the four operators in Eq. (B1) is

$$\langle \hat{a}^\dagger_{k\gamma}(E)\hat{a}_{l\delta}(E)\hat{a}^\dagger_{m\zeta}(E'')\hat{a}_{n\eta}(E'')\rangle = \delta_{kl}\delta_{\gamma\delta}\delta_{mn}\delta_{\zeta\eta}\delta(E-E')\,\delta(E''-E''')f_{k\gamma}(E)f_{m\zeta}(E'') + \delta_{kn}\delta_{\gamma\eta}\delta_{ml}\delta_{\zeta\delta}\delta(E-E''')\,\delta(E'-E'') \times f_{k\gamma}(E)[1-f_{m\zeta}(E'')], \tag{B2}$$

Using Eq. (B2) and the identity $\delta(\hbar\omega)=(1/\hbar)\delta(\omega)$, it is straightforward to verify that

$$\langle \Delta\hat{I}_i\Delta\hat{I}_j\rangle = \frac{e^2}{\hbar}\,\delta(0)\sum_{\alpha,\beta,k\gamma,l,\delta m\zeta,n,\eta}\mathrm{sgn}(\alpha)\mathrm{sgn}(\beta) \times\int dE\; A_{k\gamma;l\delta}(i,E)A_{l\delta;k\gamma}(j,E) \times f_{k\gamma}(E)[1-f_{l\delta}(E)]. \tag{B3}$$

Similarly, it can be verified that

$$\langle \Delta\hat{I}_j\Delta\hat{I}_i\rangle = \frac{e^2}{\hbar}\,\delta(0)\sum_{\alpha,\beta,k\gamma,l,\delta m\zeta,n,\eta}\mathrm{sgn}(\alpha)\mathrm{sgn}(\beta) \times\int dE\; A_{k\gamma,l\delta}(i,E)A_{l\delta;k\gamma}(j,E) \times f_{l\delta}(E)[1-f_{k\gamma}(E)]. \tag{B4}$$

Equations (B3) and (B4) give

$$S_{ij} = \frac{e^2}{2h}\sum_{\alpha,\beta,k\gamma,l,\delta m\zeta,n,\eta}\mathrm{sgn}(\alpha)\mathrm{sgn}(\beta) \times\int dE\; A_{k\gamma;l\delta}(i,E)A_{l\delta;k\gamma}(j,E)\{f_{k\gamma}(E)[1-f_{l\delta}(E)] + f_{l\delta}(E)[1-f_{k\gamma}(E)]\}. \tag{B5}$$

The two terms in Eq. (B6) are identical to each other. While for $i=j$, it is straightforward to see this, it is not so obvious for $i\neq j$. When $i\neq j$, it is straightforward to see that the contribution from the terms bilinear in the Fermi factors are identical to each other. It is shown in Appendix A that terms linear in the Fermi factors are also identical to each other. The zero-frequency current fluctuations is then

$$S_{ij} = \frac{e^2}{h}\sum_{\alpha,\beta,k\gamma,l,\delta m\zeta,n,\eta}\mathrm{sgn}(\alpha)\mathrm{sgn}(\beta) \times\int dE\; A_{k\gamma;l\delta}(i,E)A_{l\delta;k\gamma}(j,E)f_{k\gamma}(E)[1-f_{l\delta}(E)]. \tag{B6}$$

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