Conductance through a quantum dot in an Aharonov-Bohm ring

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Using scattering theory and the multichannel Landauer-Büttiker formula, we calculate the conductance of an Aharonov-Bohm (AB) ring with a quantum dot embedded in one of its arms. The electron-electron interaction within the dot is treated in a self-consistent mean-field approximation. An analytical expression is derived for the AB oscillations of the entire device. This expression displays explicitly the dependence on temperature and on the voltage applied to the dot. It is shown that the amplitude of the AB oscillations with period h/e vanishes close to a conductance resonance of the quantum dot. This leads to a sudden phase change by π in these oscillations, in agreement with a recent experiment [Yacoby *et al.*, Phys. Rev. Lett. **74**, 4047 (1995)]. We also find that the total width and the partial width amplitudes of each conductance resonance are oscillatory functions of flux. This leads to oscillations in the excitation spectrum of the dot which may be observable in further experiments. [S0163-1829(96)00224-X]

I. INTRODUCTION

As a function of the electron density or the applied voltage, the conductance across a small and nearly isolated region of an electron gas (a quantum dot) displays sharp resonances. Within the Coulomb blockade model, this property has been explained as a manifestation of the Coulomb repulsion of electrons and the quantization of the electrical charge.[1] Indeed, the addition of a single electron to a dot increases the electrostatic energy by an amount which at low temperature and for a sufficiently small dot is large enough to suppress the conductance through the system. This Coulomb blockade can be compensated by changing the voltage applied to the dot, leading to the above-mentioned sharp resonances.

The Coulomb blockade mechanism has been investigated intensively in the past years.[1–4] The spacing of the conductance resonances was found to be essentially regulated by the Coulomb repulsion of electrons (a two-body interaction). However, other features such as the shape and the amplitude of the resonances could be understood in the framework of single-particle models. Within this framework, the strong fluctuations of the amplitudes of adjacent resonances have been explained in terms of the chaotic dynamics inside the dot. Such dynamics may be caused, for instance, by the irregular shape of the boundary.[5]

While all previous experiments measured the conductance of a quantum dot directly connected to leads, a recent experiment[6] introduced a novel feature: By embedding the quantum dot into one arm of an Aharanov-Bohm (AB) two-arm interferometer, and coupling the latter to two external leads (see Fig. 1 for a schematic representation), Yacoby *et al.* claimed to have measured both *amplitude* and *phase* of the transmission through the quantum dot.[6] Any shift in the transmission phase, they argued, would be reflected in a similar phase shift of the AB oscillations. Hence a standard measurement of the latter would reveal the behavior of the transmission phase through the dot. The paper of Yacoby *et al.* has initiated a number of theoretical[7–9] and experimental[10] studies.

In the experiment,[6] the conductance of the AB ring with the quantum dot (ABQD) was measured as a function of an electrostatic potential U_p applied to the dot. A series of conductance resonances was observed and for each of them the AB oscillations were investigated. From the oscillations with period h/e a phase was extracted which displayed the following features. (i) At each conductance resonance, the phase suddenly jumped by π on a scale much smaller than the scale extracted from the conductance line shape. (ii) All resonances investigated were in phase. Neither behavior had been expected from naive application of single-particle resonance theory.[10]

The fundamental reason for the sharp jump of the measured phase was quickly understood. It was pointed out[7–10] that any continuous shift of the AB pattern was forbidden by standard reciprocity symmetries known from the work of Büttiker.[11] These symmetries state that the linear conductance of any two-probe measurement is even as a function of the magnetic flux Φ. Hence the Fourier expansion of the conductance $G=\Sigma_{n=0}^{\infty}A_n\cos(2\pi n\Phi/\Phi_0)$, with $\Phi_0=h/e$, contains only terms even in flux, and any modification of the ring only affects the amplitudes A_n rather than shifting continuously the phases of the oscillatory terms. These latter phases stay constant unless the corresponding amplitude A_n happens to switch sign. Such a switch may be interpreted as a sudden phase change by π precisely as observed in the experiment. This reasoning, while basically explaining observation (i), also shows that the original idea of measuring the transmission phase through the dot by observing the shift of the AB pattern does not apply.

In the present paper we show that, nevertheless, important information about the transport through the quantum dot can be extracted from the AB interference experiment. This information, however, resides in the amplitudes A_n of the AB

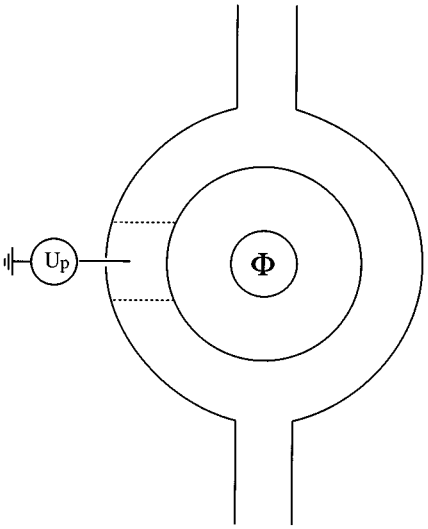

FIG. 1. Schematic representation of the system studied in this paper. A mesoscopic ring threaded by the magnetic flux Φ is connected to two external leads. A quantum dot controlled by the plunger voltage U_p is coupled to the ring via tunnel barriers.

harmonics rather than in the phases. Applying single-particle scattering theory for the ABQD device and using the standard Landauer-Büttiker formalism, we derive analytical expressions for these amplitudes as functions of temperature and of the plunger voltage on the dot. We allow for the existence of several channels both in the leads and in the arms of the AB ring. The electrons may be subject to weak disorder within the ring. The electron-electron interaction in the quantum dot is included in a self-consistent mean-field approximation. We find that for almost all voltages the AB oscillations of the current are dominated by the lowest harmonic with period Φ_0. The amplitude of this harmonic is shown to vanish generically close to a conductance resonance of the dot. This produces an apparent shift by π of the Φ_0-periodic AB oscillations. We predict that the amplitude changes sign over an energy interval of order k_BT. Moreover, our formulas for the scattering matrix of the system show that the widths of the conductance resonances as well as the occupation probabilities of states on the dot oscillate periodically as functions of the magnetic flux. This leads to oscillations of the energy of the excited levels of the dot. Our predictions can be tested by further experiments.

The paper is organized as follows. In Sec. II the scattering problem for the ABQD device coupled to two leads is solved. The Hamiltonian of the system is introduced in Sec. II A. In Sec. II B the S matrix is derived under the (physically unrealistic) assumption that the interaction of electrons in the dot can be neglected. This shortcoming is removed in Sec. II C where we account for the electron-electron interaction in the mean-field approximation. The main result of Sec. II is the scattering matrix of the ABQD ring given in Eqs. (18) and (28). In Sec. III we use this result and the standard Landauer-Büttiker formalism to derive the conductance of the ABQD device. Our results are compared with the experimental observations (i) and (ii). The results are summarized in Sec. IV. Various details are given in four appendices. Fundamental symmetry properties of the scattering matrix are derived in Appendix A. Appendix B gives a relation between the total resonance width and the partial width amplitudes. A particular symmetry of the partial width amplitudes is derived in Appendix C. Finally, the flux dependence of the various terms contributing to the conductance is given in Appendix D.

II. THE SCATTERING PROBLEM

A. The model

We write the Hamiltonian H for the ABQD system as the sum of two terms,

$$H=H_0+H_T. \tag{1}$$

Here, H_0 describes the following four disconnected subsystems: The two leads (without coupling to the AB ring), the part of the AB ring not containing the dot, and the quantum dot. The ''tunneling Hamiltonian'' H_T (Ref. 12) describes the couplings between these four subsystems.

Explicitly, H_0 is given by

$$H_0=\sum_{a,r}\int dE\ Ec_{aE}^{r\dagger}c_{aE}^{r}+\sum_i \epsilon_i d_i^\dagger d_i+\sum_j E_j q_j^\dagger q_j+U. \tag{2}$$

Here, $r=1,2$ labels the two leads, a the channels in either lead (defined by the transverse modes below the Fermi surface), and i (j) the single-particle states in the AB ring (the quantum dot, respectively). We note that the spectrum of single-particle states in the leads is continuous while it is discrete in both the ring and the dot. The corresponding single-particle energies are denoted by E (the longitudinal energy in a channel), ϵ_i, and E_j, respectively, and the annihilation (creation) operators by c_{aE}^r, d_i, and q_j (by $c_{aE}^{r\dagger}$, $d_i^\dagger$, and $q_j^\dagger$, respectively). The quantity U is the electrostatic charging energy[1] of the dot. The form of U will be specified below. For brevity, we write H_0 also in the obvious form

$$H_0=H_{\rm lead}+H_{\rm ring}+H_{\rm dot}+U. \tag{3}$$

To mimic the effect of an external potential (the plunger voltage U_p) on the electrons on the dot, we use the approximation usually made in studies of the Coulomb blockade:[3] We assume that U_p affects only the single-particle energies E_j of the dot. More specifically, we assume a linear dependence of the form

$$E_j=E_j^0+\alpha U_p, \tag{4}$$

where α is some function of the capacitance matrix elements of the system. We also assume that the energies E_j are not degenerate. This is obviously justified for states of fixed spin. Moreover, the spin degeneracy is lifted by the magnetic field. We address this point again at the end of Sec. II C.

The tunneling Hamiltonian H_T has the form

$$H_T=\left(\sum_{a,i,r}\int dE\ W_{ai}^r(E)c_{aE}^{r\dagger}d_i+\text{H.c.}\right)+\left(\sum_{i,j,p}V_{ji}^p q_j^\dagger d_i+\text{H.c.}\right). \tag{5}$$

The matrix elements W describe the coupling between ring and leads, the matrix elements V the (much smaller) cou-

pling between ring and dot, and $p=L,R$ labels either side of the dot. In the experiment of Ref. 6, V can be controlled by two negatively charged tunable gates. In analogy to Eq. (3) and in keeping the order of terms in Eq. (5), H_T can be written as

$$H_T=(H_W+H_W^\dagger)+(H_V+H_V^\dagger). \tag{6}$$

As in previous work, we neglect the dependence of the coupling matrix elements W on the energy E. This is justified because we focus attention on an energy interval of the order of the Coulomb energy. Estimates show that the matrix elements of W vary significantly only over a much larger interval. Because of the high level density in the ring compared to that in the dot, the same statement holds *mutatis mutandis* for the dependence of the matrix elements of V on the index i.

It follows from time-reversal invariance that for vanishing magnetic flux through the ring, all matrix elements W, V can be taken real. To account for nonzero flux Φ (the total magnetic flux through the ABQD device), we attach to each matrix element V_{ij}^R a factor $\exp(i\phi)$, where $\phi\equiv 2\pi\Phi/\Phi_0$. Hence we write

$$V_{ij}^L=V_{ij}^{L*}=v_{ij}^L, \tag{7}$$

$$V_{ij}^R\exp(-i\phi)=V_{ij}^{R*}\exp(i\phi)=v_{ij}^R, \tag{8}$$

where v_{ij}^L and v_{ij}^R are real. This parametrization of the flux dependence in terms of a phase factor is adequate whenever the single-particle states both in the AB ring and in the dot do not change appreciably with flux, i.e., whenever the flux through each arm of the ring and through the dot is smaller than Φ_0.

We have chosen to put the flux dependence on the matrix elements V^R. We could equally have chosen the matrix elements V^L, or made any other choice which guarantees that upon moving an electron once around the AB ring, it picks up the correct phase factor. The resulting observables must not depend on this choice.

The model defined by Eqs. (1)–(8) has the following physical content. For $U=0$ and for vanishing coupling matrix elements W and V, the eigenstates of H are bound single-particle states (localized in either the dot or the AB ring) and single-particle scattering states (the channel states in the leads). Because of the relative smallness of the quantum dot, the mean spacing of the single-particle levels in the AB ring is much smaller than that in the dot. For $W=0$ and nonzero V, the single-particle states in dot and ring are mixed but remain bound states. For $V=0$ and nonzero W, the bound single-particle states in the AB ring turn into scattering (and conductance) resonances with a finite lifetime (due to the coupling W to the leads) while the single-particle states in the quantum dot remain bound. Passage from one lead to the other is now possible through that arm of the AB ring which does not contain the quantum dot. We will assume throughout the rest of the paper that the resonances generated for $V=0$ and nonzero W are so narrowly spaced, and have such large widths, that the resulting scattering matrix is in fact a smooth function of energy. (Actually, we employ the construction using bound single-particle states in the AB ring only in order to obtain an explicit analytical expression for the scattering matrix.) When both sets of matrix elements W and V do not vanish, transmission through both arms of the AB ring is possible. The single-particle states in the quantum dot also turn into resonances. These resonances appear in the scattering matrix of the system as distinct structures superposed on a smooth background, the latter being due to the states in the ring. It remains to display these features by explicit construction of the scattering matrix.

B. *S* matrix for noninteracting electrons

In the present section we construct the S matrix and use the result to display some of the physical features of our model. We neglect the interaction U (its effect is investigated in Sec. II C).

To derive the scattering matrix $S_{ab}=\delta_{ab}-2\pi i T_{ab}$ at energy E and (dimensionless) flux ϕ, we solve the Lippmann-Schwinger equation

$$T=H_T+H_T(E-H_0+i\eta)^{-1}T \tag{9}$$

for the transition operator T. Here, η is positive infinitesimal. This equation can be solved algebraically (see Ref. 13). Alternatively, the solution can be obtained by summing up the Born expansion

$$T=H_T+H_T\frac{1}{E-H_0+i\eta}H_T + H_T\frac{1}{E-H_0+i\eta}H_T\frac{1}{E-H_0+i\eta}H_T+\cdots. \tag{10}$$

To this end, we introduce the Green functions of the leads, the ring, and the dot, respectively,

$$G_{\rm lead}=\frac{1}{E-H_{\rm lead}+i\eta}, \tag{11}$$

$$G_{\rm ring}=\frac{1}{E-H_{\rm ring}+i\eta}, \tag{12}$$

$$G_{\rm dot}=\frac{1}{E-H_{\rm dot}+i\eta}. \tag{13}$$

We are interested in transitions between states in either lead and consider the projection $T_{\rm lead}$ of T onto the lead states. Using Eq. (6), we obtain for the Born series for $T_{\rm lead}$ the expression

$$T_{\rm lead}=H_WG_{\rm ring}H_W^\dagger+H_WG_{\rm ring}H_W^\dagger G_{\rm lead}H_WG_{\rm ring}H_W^\dagger + H_WG_{\rm ring}H_V^\dagger G_{\rm dot}H_VG_{\rm ring}H_W^\dagger+\cdots. \tag{14}$$

We collect first all terms not containing $G_{\rm dot}$ and then all remaining terms. The result has the form

$$T_{\rm lead}=H_WD_{\rm ring}^{-1}H_W^\dagger+H_WD_{\rm ring}^{-1}H_V^\dagger D_{\rm dot}^{-1}H_VD_{\rm ring}^{-1}H_W^\dagger, \tag{15}$$

where

$$D_{{\rm ring},ik}=(E-\epsilon_i)\delta_{ik}+i\pi[W^\dagger W]_{ik}, \tag{16}$$

$$D_{{\rm dot},jl}=(E-E_j)\delta_{jl}-[VD_{\rm ring}^{-1}V^\dagger]_{jl} \tag{17}$$

are the propagators for the ring and the dot, respectively. In Eq. (17) we used the notation $V \equiv V^L + V^R$. Moreover, in Eq. (16) we dropped a principal-value integral which is known not to be important if the energy dependence of the matrix elements W is smooth. This is the assumption made in Sec. II A.

The resulting form for $T_{\rm lead}$ allows for a straightforward physical interpretation, in keeping with the remarks at the end of Sec. II A. We note first [see Eqs. (16) and (17)] that due to the coupling to the leads, both the states in the ring and those in the dot have acquired a finite width. Moreover, Eq. (15) shows that the passage of electrons through the ABQD device may take place in either one of the following two ways. (i) The electrons may not enter the quantum dot at all [first term on the right-hand side of Eq. (15)]. This contribution should and does not depend on the properties of the dot. In particular, it is independent both of the plunger voltage U_p and of the magnetic flux Φ. (ii) The electrons enter the dot at least once [second term on the right-hand side of Eq. (15)]. This term also accounts for multiple scattering of electrons around the ring. [To see this, expand the inverse propagator $D_{\rm dot}^{-1}$ in powers of the tunneling amplitudes V and obtain a power series containing only even powers of V. Each term proportional to V^{2n} corresponds to an $(n+1)$-fold passage of the electron through the dot and contributes up to the $(n+1)$st harmonic of the AB oscillations.]

Using the relation $S_{ab} = \delta_{ab} - 2\pi i T_{ab}$, we find for the S-matrix element S_{ab}^{rs} (connecting channel b in lead s with channel a in lead r) the final expression

$$S_{ab}^{rs}(E,\phi) = \delta^{rs}\delta_{ab} - 2\pi i [W D_{\rm ring}^{-1} W^\dagger]_{ab}^{rs} - i[\gamma D_{\rm dot}^{-1} \overline{\gamma^\dagger}]_{ab}^{rs}, \tag{18}$$

where

$$\gamma_{aj}^r = \sqrt{2\pi}[W (D_{\rm ring})^{-1} V^\dagger]_{aj}^r, \tag{19}$$

$$\overline{\gamma_{bl}^s} = \sqrt{2\pi}[W (D_{\rm ring}^\dagger)^{-1} V^\dagger]_{bl}^s, \tag{20}$$

and $r,s = 1,2$. The term in Eq. (18) involving the Kronecker deltas accounts for reflection from channel b back into the same channel (and not for transmission of electrons from one lead into the other). In the light of the remarks on the T matrix made above, the physical interpretation of the remaining two terms on the right-hand side of Eq. (18) is obvious. [For the sake of completeness, we mention that in Eq. (18), we have suppressed an overall phase factor. This factor accounts for elastic scattering in the channels in the absence of the coupling matrix elements V and W and is totally immaterial for all that follows.] In scattering theory,[14] the quantities γ_{aj}^s and $\overline{\gamma_{aj}^s}$ are known as partial width amplitudes. We note that in our case, the entire dependence on the flux ϕ of the S matrix resides in these partial width amplitudes, and in the inverse propagator $D_{\rm dot}^{-1}$. Both quantities contain factors V^R and are therefore oscillatory functions of flux.

Equation (18) demonstrates the nontrivial quantum-coherence properties of the ABQD system. The total transmission amplitude through the ABQD system is the sum of two terms. Naively, one might have expected that these two terms correspond, respectively, to transmission through either arm of the AB ring, and to differ in phase by a factor $\exp(i\phi)$. Equation (18) shows that this naive expectation is wrong: the dependence on ϕ is more complex.

We show in Appendix A that the S matrix (18) has the well-known[15] symmetry properties

$$S(\phi) S^\dagger(\phi) = S^\dagger(\phi) S(\phi) = 1, \tag{21}$$

$$S(\phi) S^*(-\phi) = S^*(\phi) S(-\phi) = 1, \tag{22}$$

required by the fundamental symmetries of the Hamiltonian. The unitarity relation (21) for S is a consequence of current conservation. Equation (22) is required by full time-reversal invariance (including a reversal of the magnetic field). As shown by Büttiker,[11] both symmetries together imply that the linear-response form of the conductance of the noninteracting electron system is an even function of magnetic flux.

Both nontrivial contributions to S have the form of a sum over resonances. As explained earlier, the resonances due to the single-particle states in the ring are narrowly spaced and overlap strongly. We accordingly consider the second term on the right-hand side of Eq. (18) and the partial width amplitudes as smooth functions of energy. In contradistinction, the resonances in the dot are expected to be isolated (their spacing is much larger than their total widths) because the dot is both smaller and better isolated from the outside than the ring. In such a case, a single-level approximation for the third term on the right-hand side of Eq. (18) is justified. We accordingly assume that the matrix $D_{\rm dot}$ is diagonal, with diagonal elements given by

$$(D_{\rm dot})_{jl} \equiv (E - E_j - \Delta E_j + i\Gamma_j/2)\,\delta_{jl}. \tag{23}$$

In scattering theory,[14] ΔE_j is known as the energy shift (of the resonance position with respect to the bound state energy of the isolated system), and Γ_j as the total width of the resonance with index j. Combining Eqs. (17) and (23), we find that ΔE_j and Γ_j are given by

$$\Delta E_j = {\rm Re}[V D_{\rm ring}^{-1} V^\dagger]_{jj}, \tag{24}$$

$$\Gamma_j = -2\ {\rm Im}[V D_{\rm ring}^{-1} V^\dagger]_{jj}. \tag{25}$$

Using unitarity, we can show (see Appendix B) that for isolated resonances

$$\Gamma_j = \sum_{c,t} |\gamma_{cj}^t|^2 = \sum_{c,t} |\overline{\gamma_{cj}^t}|^2. \tag{26}$$

We emphasize that because of the oscillatory behavior of the partial width amplitudes, both ΔE_j and Γ_j are also oscillating with flux.

According to Eqs. (24) and (25) both the energy shift and the typical resonance width have the same order of magnitude. When the single-level approximation applies, the energy shift therefore is much smaller than the resonance spacing and the resonance position $E_j + \Delta E_j$ is found to be only weakly flux dependent.

C. Self-consistent approximation to the Coulomb interaction between electrons on the quantum dot

The model used in Sec. II B is unrealistic because it neglects the electrostatic charging energy, i.e., the Coulomb interaction between electrons on the quantum dot, the fundamental cause for the Coulomb blockade. In the present section we remedy this shortcoming. We do so in the framework of the mean-field approximation. At the end of the section we discuss possible modifications of our results due to genuine many-body effects beyond the mean-field approximation (quantum fluctuations).

As far as we know, the Coulomb blockade problem has been addressed in the literature in two ways. (i) The interacting electrons on the quantum dot are brought into contact with a heat bath. A recent example is provided by the work of Kamenev and Gefen.[16] The solution obtained by these authors applies to the entire regime between weak and strong coupling. It does not, however, address explicitly the coupling of the system to external leads. (ii) The coupling of the quantum dot to the leads is explicitly taken into account; at zero temperature, the electrons in the leads fill all states up to the Fermi energy. This is the approach taken by Beenakker[3] and by Ng and Lee.[17] Here, we follow this second approach because it permits us to calculate the conductance directly via the Landauer formula. For simplicity, it is assumed throughout this section that both temperature and the widths of the resonances are smaller than the single-particle mean resonance spacing.

We use a formulation first introduced by Anderson[18] in the context of localized magnetic states in metals, and also used by Ng and Lee[17] for quantum dots coupled to two leads. We go beyond these works in two respects. As explained above, we (i) couple the quantum dot to an AB ring threaded by a magnetic flux (this gives rise to the oscillatory behavior of the partial and total resonance widths with flux displayed above), and we (ii) allow for an arbitrary number of open channels in either lead coupled to the ring.

We complete the definition of the Hamiltonian given in Sec. II A by defining the electrostatic charging energy U as

$$U = U_0 \sum_{j<j'} n_j n_{j'}, \tag{27}$$

where $n_j = q_j^\dagger q_j$. The energy needed to add an electron to the quantum dot occupied by K electrons is obviously given by $U_0 K$. We neglect the Coulomb interaction between electrons in the AB ring, and in the leads. This is legitimate because ring and leads are very much larger than the dot, and the ring is strongly coupled to the leads. As a consequence, the electronic states on ring and leads are more spread out, and the electrons are better screened, than those on the dot.

We calculate the scattering matrix $S_{ab}(E,\phi)$ at the Fermi energy E_F in the Hartree-Fock approximation. A similar calculation for the one-channel case is described in Ref. 18; for the many-channel case, the methods described in Ref. 13 can be used. Therefore we do not give any details and focus attention on the results which are anyway very similar in form to the ones obtained in Sec. II B. The Hartree-Fock approximation essentially amounts to solving the scattering problem for an effective Hamiltonian with the correlation term $U_0 \Sigma_{j<j'} \langle n_j \rangle n_{j'}$. The angle brackets indicate an expectation value which is taken with respect to the Hartree-Fock wave function. This correlation term leads to a shift of the single-particle energies $E_j \to E_j + U_0 \Sigma_{j' \neq j} \langle n_{j'} \rangle$. A corresponding shift is found for the positions of the resonances of the S matrix, which otherwise has the same form as in Eq. (18).

To discuss the physical content of the resulting expression, we again use the approximation that the resonances on the quantum dot are isolated (diagonal approximation for $D_{\rm dot}$). We also assume that initially there are no electrons on the quantum dot. Moreover, it is assumed that the plunger voltage U_p is chosen in such a way that the first empty single-particle level labeled 1 on the quantum dot is located right above the Fermi energy E_F. Increasing $-U_p$, we follow this and the next higher single-particle level labeled 2, neglecting all levels with $j>2$. We expect that as the first level moves into the Fermi sea, it turns into a resonance of the scattering matrix and therefore of the conductance through the dot. At the same time, this level should become filled with an electron (more precisely, the occupation probability of the resonant state should increase gradually from zero to one). As this happens, the second level should get shifted upwards by the Coulomb repulsion.

In the limit of nonoverlapping resonances (the resonance spacing is large compared to the resonance width), the scattering matrix has the form

$$S_{ab}^{rs}(E_F,\phi) = \delta^{rs}\delta_{ab} - 2\pi i [W D_{\rm ring}^{-1} W^\dagger]_{ab}^{rs} - i \sum_j \gamma_{aj}^r (E_F - \mathbf{E}_j + i\Gamma_j/2)^{-1} \overline{\gamma_{jb}^s}, \tag{28}$$

where γ_{aj}, $\overline{\gamma_{bj}}$, and Γ_j are defined in Eqs. (19), (20), and (25), respectively. We have replaced the energy E by the Fermi energy E_F as required by the Kubo formula for zero temperature. The energies $\mathbf{E}_j$ depend on the plunger voltage U_p. Our result (28) closely resembles Eq. (18) which was obtained in the absence of interactions. The only effect of the interaction is to shift the resonance positions: the resonance energies $\mathbf{E}_j$ are given by

$$\mathbf{E}_1 = E_1 + \Delta E_1 + U_0 \langle n_2 \rangle, \tag{29}$$

$$\mathbf{E}_2 = E_2 + \Delta E_2 + U_0 \langle n_1 \rangle, \tag{30}$$

where, at zero temperature, the average occupation probabilities $\langle n_j \rangle$ are defined self-consistently by the conditions

$$\langle n_j \rangle = \frac{1}{\pi} \int_{-\infty}^{E_F} dE \, \frac{\Gamma_j}{|E - \mathbf{E}_j + i\Gamma_j/2|^2} \tag{31}$$

$$= \frac{1}{\pi} \tan^{-1}[2(E_F - \mathbf{E}_j)/\Gamma_j], \quad j = 1,2. \tag{32}$$

The solutions of these equations are easily found and have the following qualitative feature. (For simplicity, we denote the sums $E_j + \Delta E_j$ by E_j'.) For $E_F - E_j'$ negative and large compared to Γ_j, we have $\langle n_j \rangle \approx 0$. The occupation probability $\langle n_j \rangle$ grows monotonically with increasing $E_F - E_j'$ and reaches asymptotically the value unity. The increase essen-

tially starts at $E_F - E_j' \approx -\Gamma_j$ and essentially occurs within an energy interval of width U_0. At finite temperature T, the integrand in Eq. (31) has to be multiplied by the Fermi function, and the domain of integration extends over the entire real axis. Then, the increase of $\langle n_j \rangle$ essentially starts at $E_F - E_j' \approx -k_B T$ and is otherwise similar to the case $T=0$.

The resulting influence of the Coulomb energy on the scattering matrix depends crucially on whether $k_B T$ is small compared to or of the same size as the level separation $|E_1 - E_2|$. In the first case, we choose U_p initially in such a way that both E_1' and E_2' lie above E_F. Then Eqs. (29), (30), and (32) show that both $\langle n_j \rangle \approx 0$, so that $\mathbf{E}_2 - \mathbf{E}_1 \approx E_2' - E_1'$. As we increase $-U_p$, the energy E_1' dives into the Fermi sea, and $\langle n_1 \rangle$ starts to increase according to Eq. (32), while $\langle n_2 \rangle$ remains essentially zero. As a consequence, the spacing $\mathbf{E}_2 - \mathbf{E}_1 \approx E_2' - E_1' + U_0 \langle n_1 \rangle$ grows, too, until it reaches the asymptotic value $E_2' - E_1' + U_0$. As a result, the higher level is pushed up by the Coulomb energy as the lower one dives into the Fermi sea. In the second case (where $k_B T$ is at least comparable to the spacing $|E_1 - E_2|$), a rather more complicated pattern may result. The first case is the one applicable to the experiment where a series of resonances roughly separated by the Coulomb energy is observed. In this case, the Coulomb interaction leads only to a shift of the levels, and for any one resonance observed experimentally the considerations of subsections II A and II B apply without modification.

This result can easily be extended to a situation where there are K electrons on the dot, and where more single-particle levels are taken into account. We conclude that the charging energy, at least on the level of the mean-field approximation, merely leads to a renormalization of the resonance positions. This allows us to simplify notation and to denote the resonance positions again by E_j' rather than by $\mathbf{E}_j$, keeping in mind that the former are derived from a set of self-consistent equations taking account of the charging energy.

It may be argued that since we consider a case where the Coulomb energy has the only effect of increasing the spacing of the resonances, the present subsection is altogether redundant, because for a single resonance, the situation is exactly the same as without any Coulomb interaction. However, our result has a peculiar feature which is specific for the AB geometry and hence absent in previous calculations for dots directly coupled to leads: the occupation probabilities as well as the resonance energies are oscillatory functions of flux, and the oscillatory behavior of the resonance spacings is amplified by the Coulomb interaction. These statements follow from the oscillatory flux dependence of the resonance widths Γ_j. To explore the consequence of this feature, we denote the minimum (maximum) values of the Γ_j by $\Gamma_{\min,j}$ and $\Gamma_{\max,j}$, respectively. Let us imagine that E_1' is slightly above E_F with $\Gamma_{\min,1} < E_1' - E_F < \Gamma_{\max,1}$. Equations (29), (30), and (32) show that the resonance spacing $\mathbf{E}_2 - \mathbf{E}_1$ oscillates with flux between a value close to $E_2' - E_1' + U_0/2$ (for $\Gamma_1 \approx \Gamma_{\max,1}$) and $E_2' - E_1' + U_0$ (for $\Gamma_1 \approx \Gamma_{\min,1}$). A similar reasoning shows that $\mathbf{E}_2 - \mathbf{E}_1$ oscillates between $E_2' - E_1'$ and $E_2' - E_1' + U_0/2$ when E_1' is slightly below E_F. Such oscillations of the excited levels of the dot should be observable in experiment. This is true even taking into account that our treatment is unrealistic in the sense that it neglects all inelastic scattering. In the experiment the scattering through the quantum dot is predominantly inelastic. Nonetheless, even a small coherent component will lead to strong oscillations of the excited levels since the charging energy is typically much larger than the level spacing.

The self-consistent picture used above is known to break down for pairs of degenerate levels at very low temperature. In this case, quantum fluctuations dominate the problem, which acquires much similarity with the Kondo problem. Arguments put forward in Ref. 17 strongly suggest that for the situation considered in the present paper (nondegenerate levels at finite temperature), such quantum fluctuations play a negligible role, and that the treatment given in the present section adequately describes the physical situation. As a result, we essentially recover the picture of isolated resonances developed in Sec. II B, modified by the ''stretching'' of the single-particle spectrum due to the Coulomb interaction. This justifies our use of the results of Sec. II B in analyzing the experiment of Yacoby *et al.*[6] in the next section.

III. CONDUCTANCE OF THE ABQD DEVICE

The dimensionless conductance $g = (h/e^2)G$ of the ABQD device is obtained from the multichannel Landauer formula[19]

$$g = 2 \int dE \left(-\frac{\partial f}{\partial E} \right) \sum_{a,b=1}^{N} |t_{ab}(E)|^2 . \tag{33}$$

Here $t_{ab}(E) = S_{ab}^{12}(E)$ is the transmission amplitude through the ring for an electron entering the ring via channel b in lead 2, and leaving it via channel a in lead 1. The derivative of the Fermi function f is given by $-(\partial f/\partial E) = (4k_B T)^{-1} \cosh^{-2}[(E - E_F)/2k_B T]$, and E_F is the Fermi energy in the leads. A factor 2 accounts for the spin degeneracy of the electron.

In the following it is assumed that both the charging energy U_0 and $k_B T$ are much larger than the resonance widths Γ_j. Both these restrictions together imply that the conductance through the quantum dot is much smaller than e^2/h.[3] This is a necessary condition for the Coulomb blockade regime. We note that both conditions are met in the experiment of Yacoby *et al.* where $U_0 \approx 500$ μeV, $k_B T \approx 9$ μeV, and $\Gamma_j \approx 0.2$ μeV.[6]

The assumption $U_0 \gg \Gamma_j$ implies that we work in the regime of isolated resonances. Hence an appreciable current can pass the dot only if a resonance in the dot is close to the Fermi energy, $E_j \approx E_F$. The contribution of other resonances $j' \neq j$ to the transmission amplitude is smaller by a factor of order $\Gamma_j / U_0 \ll 1$ and is therefore negligible. (We indicate qualitatively below how other resonances contribute to the current.) The index j is suppressed in the following to simplify notation. According to Eq. (18), the transmission amplitude has the form

$$t_{ab} = t_{\mathrm{ring},ab} - i \frac{\gamma_a^1 \overline{\gamma_b^2}^*}{E - E' + i\Gamma/2}, \tag{34}$$

where $t_{\mathrm{ring},ab} = -2i\pi \Sigma_{ik} W_{ai}^1 (D^0)_{ik}^{-1} W_{bk}^{2*}$ is the transmission through the part of the ring not containing the dot. This con-

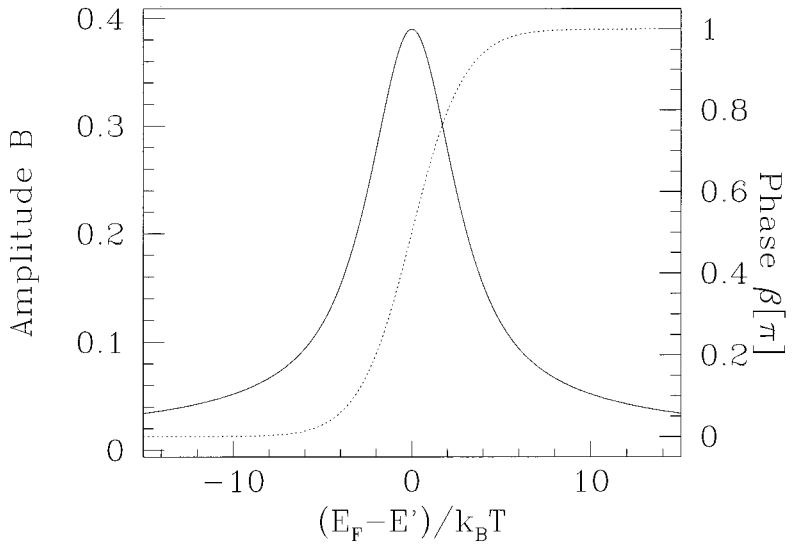

FIG. 2. The amplitude B (solid line) and the phase β (dashed line), both as a function of $(E_F-E')/k_BT$.

tribution is independent of ϕ and U_p and only weakly dependent on energy (this energy dependence will be neglected).

Inserting Eq. (34) into the Landauer formula (33) and summing over the channels yields several terms whose flux dependence is given in Appendix D. As a result, one finds

$$g=\int dE\left(-\frac{\partial f}{\partial E}\right)\left\{g_{\rm ring}+\frac{4y_0}{1+x_0\cos\phi}\sin\delta\sin(\xi_0-\delta)+\frac{4y_1}{1+x_0\cos\phi}\cos\phi\sin\delta\sin(\xi_1-\delta)+\frac{z_0+z_1\cos\phi+z_2\cos^2\phi}{(E-E')^2+\Gamma^2/4}\right\}. \quad (35)$$

We have used the following parametrizations:

$$\Gamma=(1+x_0\cos\phi)\overline{\Gamma}, \quad (36)$$

$$2\sum_{a,b} t_{{\rm ring},ab}\gamma_a^{1*}\overline{\gamma_b^2}=[y_0\exp(i\xi_0)+y_1\exp(i\xi_1)\cos\phi]\overline{\Gamma}, \quad (37)$$

$$2\sum_{a,b}|\gamma_a^1|^2|\overline{\gamma_b^2}|^2=[z_0+z_1\cos\phi+z_2\cos^2\phi]\overline{\Gamma^2}. \quad (38)$$

We have introduced positive amplitudes $\overline{\Gamma},y_0,y_1,z_0$, the real amplitudes x_0,z_1,z_2, and two real phase shifts ξ_x, ξ_y all of which do not depend on ϕ. The resonance denominator has been parametrized in terms of the angle δ defined as

$$\exp(i\delta)\equiv-\frac{E-E'-i\Gamma/2}{|E-E'-i\Gamma/2|}. \quad (39)$$

This angle takes the value $\pi/2$ at $E=E'$ and approaches 0 for $E\to-\infty$ and π for $E\to\infty$.

To average g over disorder, the phase shifts ξ_0 and ξ_1 are averaged over some suitable interval. This procedure only rescales the amplitudes y_0, y_1 and the phase shifts ξ_0, ξ_1, but leaves the *form* of Eq. (35) unchanged.

We integrate over energy and find for the terms linear in the resonance denominator the result

$$\int dE\left(-\frac{\partial f}{\partial E}\right)\sin\delta\sin(\xi-\delta)\equiv\frac{\Gamma}{k_BT}B\sin(\xi-\beta), \quad (40)$$

where the amplitude B and the phase β are defined by

$$B=[A_1^2+A_2^2]^{1/2}, \quad (41)$$

$$\tan\beta=A_2/A_1, \quad (42)$$

with

$$A_1=\int dE\left(-\frac{\partial f}{\partial E}\right)\frac{k_BT(E'-E)/2}{(E-E')^2+\Gamma^2/4}, \quad (43)$$

$$A_2=\int dE\left(-\frac{\partial f}{\partial E}\right)\frac{k_BT(\Gamma/4)}{(E-E')^2+\Gamma^2/4}. \quad (44)$$

In the regime $k_BT\gg\overline{\Gamma}$, one finds that both A_1 and A_2 become independent of Γ,

$$A_1\to-\frac{1}{8}{\rm P}\int_{-\infty}^{\infty}dE\left(\frac{1}{E'-E}\right)\cosh^{-2}\left(\frac{E_F-E}{k_BT}\right), \quad (45)$$

$$A_2\to\frac{\pi}{8}\cosh^{-2}\left(\frac{E_F-E'}{k_BT}\right), \quad (46)$$

where (45) is a principal-value integral. As a result, both B and β shown in Fig. 2 are independent of Γ and are functions only of the ratio $(E_F-E')/k_BT$. We note that B decreases algebraically with increasing $|E_F-E'|$. The integral over the fourth term in Eq. (35) which is quadratic in the resonance denominator can easily be done using Eq. (46). Collecting results, we finally obtain

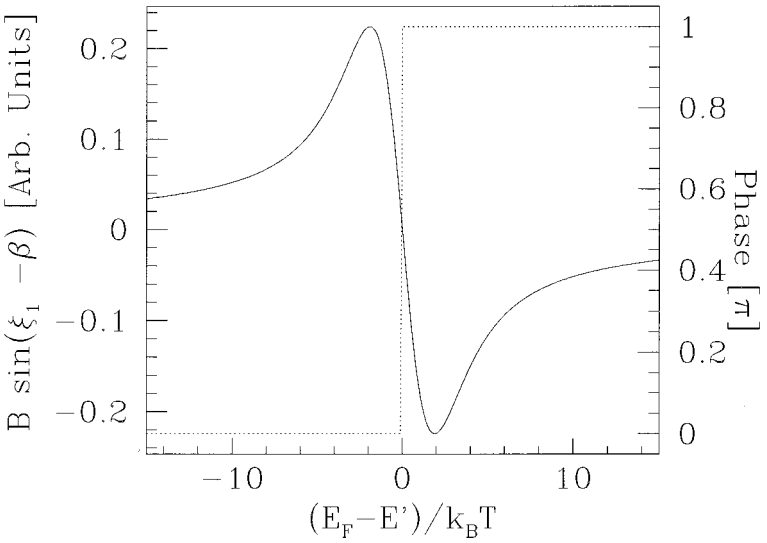

FIG. 3. Amplitude of the third term of Eq. (47) as a function of $(E_F-E')/k_BT$ for $\xi_1=\pi/2$. The dashed line shows the phase extracted from the sign of $B\sin(\xi_1-\beta)$.

$$g=g_{\rm ring}+4y_0\frac{\overline{\Gamma}}{k_BT}B\sin(\xi_0-\beta)+4y_1\frac{\overline{\Gamma}}{k_BT}B\sin(\xi_1-\beta)\cos\phi+\frac{\pi\overline{\Gamma}}{2k_BT}\frac{z_0+z_1\cos\phi+z_2\cos^2\phi}{1+x_0\cos\phi}\cosh^{-2}\left(\frac{E_F-E'}{k_BT}\right). \tag{47}$$

Equation (47) allows us to discuss in detail the dependence of the current through the ABQD device on energy and magnetic flux. To fix notation, we refer to the first term on the right-hand side of Eq. (47) as background term, to the second and third terms as interference terms, and to the fourth term as resonance term. The background term $g_{\rm ring}$ comprises the bulk part of the current. This term is independent of both energy and flux. It is due to transmission of electrons through that arm of the AB ring which does not contain the quantum dot. In order to focus on the resonance peaks, this term is subtracted in the experimental analysis from the total current through the device. All remaining terms are smaller than the background term by a factor $\overline{\Gamma}/k_BT\ll 1$. The interference terms are due to the interference of the background amplitude with the transmission amplitude involving passage through the quantum dot. These terms display resonant behavior (through the amplitude B) and involve the phase shifts ξ_0 and ξ_1. The latter quantities depend on the geometry of the ring and the distribution of disorder. One of the interference terms is flux independent, while the other displays Φ_0-periodic AB oscillations. This particular dependence on flux is the result of the temperature average. At zero temperature, both interference terms give rise to higher AB harmonics; see the integrand of Eq. (35). The amplitude of the resonance term has the usual temperature dependence known for thermally broadened resonances[3] of quantum dots directly coupled to leads. However, in the AB geometry this term is flux modulated and contributes Φ_0-periodic and $\Phi_0/2$-periodic AB oscillations as well as all higher harmonics.

We now analyze the AB oscillation described by Eq. (47). In a multichannel ring the dominant oscillatory contribution is due to the Φ_0-periodic interference term. This term is both larger than the resonance term by a factor $\sqrt{g_{\rm ring}}$ and falls off more gently than the latter with increasing $|E_F-E'|$. But at each resonance, β changes by π with increasing E_F-E', with ξ_1 essentially staying constant. Therefore the oscillating interference term always changes sign in the interval $-\infty<E_F-E'<\infty$ (compare Fig. 3). As a result, the amplitude of the Φ_0-periodic oscillations generically vanishes for any value of the phase shift ξ_1 somewhere in the vicinity of each resonance. The precise location of the zero of the amplitude depends on ξ_1 as well as on the relative magnitude of the third and fourth terms on the right-hand side of Eq. (47). The change of sign of the amplitude of the Φ_0-periodic oscillations occurs over an interval of order k_BT. Indeed, this interval is determined by the behavior of β, and Fig. 2 shows that β increases from zero to π over an interval of order k_BT. At the zero of the Φ_0-periodic oscillations one finds AB oscillations of shorter period in the flux caused by the higher harmonics contained in the resonance term. Their amplitudes reach maximum value at $E_F=E'$ (independent of ξ_1). In conclusion, Eq. (47) predicts a zero of the lowest harmonic close to (but not necessarily at) the maximum of the higher harmonics. This is in agreement with the results of a numerical study of the ABQD device.[10] Experimentally, the higher harmonics have not yet been investigated.

In deriving Eq. (47) we have neglected far away resonances of the dot. Their contribution to the current can easily be found by including in Eq. (34) the sum over resonances $j'\neq j$. Performing the same manipulations as before one finds that the far away resonances lead to AB oscillations with a *plunger voltage independent* amplitude of order $\sqrt{g_{\rm ring}}\Gamma/U_0\ll 1$, where Γ is a typical total resonance width of the dot.

Inelastic scattering in the quantum dot destroys the unitarity of the S matrix. Phenomenologically, this increases the widths of the transmission resonances Γ by the addition of an inelastic width[20,11] and thereby reduces the amplitudes of both the interference terms and the resonance term. The fun-

damental property of the current to be an even function of the applied flux remains unchanged (at least within the present phenomenological models for inelastic scattering[11,7]). We conclude that with properly rescaled amplitudes, Eq. (47) accounts for the current through the ABQD device even in the presence of inelastic scattering. The basic conclusions concerning the temperature and flux dependence of the various terms remain valid.

As the last point, we investigate the relative phase of the Φ_0-periodic AB oscillations at different conductance resonances. The amplitude multiplying $\cos\phi$ in the interference term in Eq. (47) is clearly real. Therefore two different resonances are either in phase, or completely out of phase (phase difference 0 or π). Which of these two values is realized depends on the signs of the matrix elements (''tunneling amplitudes'') V^L_{kj} and V^R_{kj} connecting states in the dot and in the AB ring. These amplitudes are given[12] by the overlap integrals

$$V^s_{kj}=\left(\frac{\hbar^2}{2m}\right)\int_{A(s)}(\psi_k\nabla\phi_j^*-\phi_j^*\nabla\psi_k)d\mathbf{S}. \tag{48}$$

The integral extends over a surface in the barrier region separating the dot from the ring (the index $s=L,R$ selects one of the two barriers). The functions ψ_k are scattering solutions of the full Hamiltonian H in the region outside the quantum dot and drop smoothly to zero within the dot.[12] Similarly, the states ϕ_j are solutions of the Schrödinger equation involving H within the dot, and are exponentially small outside the dot. As one changes the plunger voltage U_p, one sweeps through a series of resonances each associated with a particular ϕ_j while the states ψ_k are unchanged. Therefore the relative phase of the Φ_0-periodic AB oscillation at a conductance resonance labeled j is determined by the corresponding state ϕ_j on the quantum dot.

It turns out that the sign of the amplitudes V depends on the nature of classical dynamics within the dot. If the equations of motion for the closed dot are integrable, the relative phases are expected to change in a regular way which is specific for the particular system at hand. Consider, for instance, a planar dot of circular shape. Using polar coordinates (r,θ), we can write the eigenfunctions ϕ_j in the form $\phi_{nm}(r,\theta)=R_n(r)\exp(im\theta)$.[21] From Eqs. (7), (8), and (D7) it follows that the amplitudes of the flux-dependent interference terms carry a relative sign $(-1)^{m'-m}$ for resonances with angular momentum m and m', respectively. Therefore resonances with $(m-m')$ even are expected to be in phase while resonances with $(m-m')$ odd should have a phase shift equal to π. For quantum dots with chaotic classical dynamics, on the other hand, it has been shown that the partial width amplitudes and therefore the amplitudes V behave statistically in a manner described by random-matrix theory.[5] We thus expect a statistical behavior for the relative phases of different resonances (with values 0 or π). As a consequence, the amplitude of the Φ_0-periodic oscillations is not expected to have the same sign for all resonances. This statement holds both for integrable and for chaotic dynamics within the quantum dot. Experimentally it was found[6] that all of the observed resonances (the number of which is of order 10) are in phase. Within the framework of our model, this result is surprising. Levy Yeyati, and Büttiker[7] hold the accumulation of additional charge in the AB ring outside the quantum dot responsible for this finding. Their explanation is based on the capacitive coupling between the gate regulating the number of electrons on the dot and the two gates forming the barriers between the dot and the AB ring. Such a coupling is not included in our Hamiltonian and its investigation is beyond the scope of this paper.

IV. SUMMARY

In this paper we have derived an explicit expression for the S matrix of an AB ring with a quantum dot embedded in one of its arms. The transmission amplitude through the ABQD device was shown to exhibit a resonance whenever there is a transmission resonance through the quantum dot. We have also found that the entire dependence of S on the magnetic flux resides in the total widths and the partial width amplitudes of the transmission resonances. These quantities are oscillatory functions of the flux. As a consequence, the excitation spectrum of the dot oscillates with magnetic flux. Such oscillations may be observable in further experiments.

Using the Landauer-Büttiker formalism which expresses the dimensionless conductance $g=(h/e^2)G$ through the ABQD device in terms of the S matrix, we have obtained an analytical expression for g. The AB oscillations with period h/e were shown to vanish close to the peak of each conductance resonance. We find that the higher AB harmonics have maximal amplitude in the vicinity of this node. Our results account for recent experimental[6] as well as numerical[10] findings. However, in the present framework we cannot explain the observation that the AB oscillations with period h/e are in phase for all resonances.

ACKNOWLEDGMENTS

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APPENDIX A

In this Appendix we prove the symmetry properties (21) and (22) of the S matrix. It is useful to write S in the form

$$S^{rs}_{ab}(E,\phi)=\delta^{rs}\delta_{ab}-2\pi i[WD^{-1}W^\dagger]^{rs}_{ab}, \tag{A1}$$

where

$$D_{ik}=(E-\epsilon_i)\delta_{ik}-\sum_j\frac{V^*_{ji}V_{jk}}{E-E_j}+i\pi[W^\dagger W]_{ik} \tag{A2}$$

is the propagator of the full ring. It exhibits resonances of both the dot and the outer part of the ring. The equivalence of Eq. (A1) with Eq. (18) can be shown, e.g., by expanding D^{-1} in powers of V and resumming all terms in the series for S containing V. This yields the third term of Eq. (18) while the V-independent term equals the second term of Eq. (18).

Inserting Eq. (A1) into Eqs. (21) one observes that the latter immediately follow from the equation

$$D - D^\dagger = 2i\pi W^\dagger W, \tag{A3}$$

which in turn follows from Eq. (A2). Time-reversal invariance implies that the coupling matrix elements obey $W(\phi) = W^*(-\phi)$ and $V(\phi) = V^*(-\phi)$. This implies $D(\phi) = D^T(-\phi)$. With this relation, Eqs. (22) also reduce to Eq. (A3).

APPENDIX B

In the following, we derive the relationship $\Gamma_j = \Sigma_{c,t} |\gamma_{cj}^t|^2 = \Sigma_{c,t} |\overline{\gamma_{cj}^t}|^2$ between the total resonance width and the partial width amplitudes. This relation follows from the unitarity of the scattering matrix.

In keeping with the main text, we assume the resonances of the quantum dot to be isolated. In the vicinity of a particular resonance j we write the S matrix (18) in the form

$$S = S^{(0)} - i \frac{\gamma \times \overline{\gamma^*}}{E - E' + i\Gamma}, \tag{B1}$$

where we have suppressed the index j (labeling the resonance energy, the resonance width, and the partial width amplitudes). The matrix $S^{(0)}$ is unitary and symmetric by construction and can be diagonalized by an orthogonal matrix U,

$$S^{(0)} = 1 - 2\pi i W D_{\rm ring}^{-1} W^\dagger \tag{B2}$$

$$= U \exp(2i\delta) U^\dagger, \tag{B3}$$

where δ is diagonal and real. Now, consider

$$\hat{S} = \exp(-i\delta) U^\dagger S U \exp(-i\delta) \tag{B4}$$

$$= 1 - i \frac{\alpha \times \overline{\alpha^*}}{E - E' + i\Gamma/2}, \tag{B5}$$

where

$$\alpha = \exp(-i\delta) U^\dagger \gamma, \tag{B6}$$

$$\overline{\alpha^*} = \exp(-i\delta) U^T \overline{\gamma^*}. \tag{B7}$$

We have $\hat{S}\hat{S}^\dagger = \hat{S}^\dagger \hat{S} = 1$ which yields

$$\alpha \times \overline{\alpha^*} = \overline{\alpha} \times \alpha^*, \tag{B8}$$

$$\left(\sum_{c,t} |\overline{\alpha_c^t}|^2 \right) \alpha \times \alpha^* = \left(\sum_{c,t} |\alpha_c^t|^2 \right) \overline{\alpha} \times \overline{\alpha^*}, \tag{B9}$$

$$\frac{1}{2}\Gamma(\alpha \times \overline{\alpha^*} + \overline{\alpha} \times \alpha^*) = \left(\sum_{c,t} |\overline{\alpha_c^t}|^2 \right) \alpha \times \alpha^*. \tag{B10}$$

In Appendix C we show that $\alpha = \overline{\alpha}$. This implies Eqs. (B8) and (B9), and Eq. (B10) reduces to

$$\Gamma = \sum_{c,t} |\alpha_c^t|^2 = \sum_{c,t} |\overline{\alpha_c^t}|^2. \tag{B11}$$

Finally, using Eqs. (B6) and (B7) to replace α and $\overline{\alpha}$ by γ and $\overline{\gamma}$, using the unitarity of U, and including the index j for completeness we find

$$\Gamma_j = \sum_{c,t} |\gamma_{cj}^t|^2 = \sum_{c,t} |\overline{\gamma_{cj}^t}|^2. \tag{B12}$$

As a corollary, we present a relation between the partial widths γ and $\overline{\gamma}$. This relation could be useful for showing explicitly the symmetry of the conductance with respect to flux. We recall that the entire flux dependence resides in the matrix elements V, and we work within the single-level approximation. We use Eqs. (B6) and (B7) for the partial widths α, $\overline{\alpha^*}$ and the relation $\alpha(\phi) = \overline{\alpha(\phi)}$ derived in Appendix C. We have $\gamma(\phi) = U \exp(i\delta)\alpha(\phi) = U \exp(i\delta)\overline{\alpha(\phi)} = U \exp(2i\delta) U^T \overline{\gamma(\phi)} = S^{(0)} \overline{\gamma(\phi)}$. Hence $\gamma(\phi) = S^{(0)} \overline{\gamma(\phi)}$.

APPENDIX C

Here, we prove the relation $\alpha = \overline{\alpha}$ which was used in Appendix B. We assume

$$\alpha = c\,\overline{\alpha} \tag{C1}$$

and show that $c = 1$. To that end, Eq. (C1) is multiplied by $U \exp(i\delta)$. Using the definitions of both α, $\overline{\alpha}$ and γ, $\overline{\gamma}$, one obtains

$$W D_{\rm ring}^{-1} V^\dagger = c U \exp(2i\delta) U^\dagger W (D_{\rm ring}^\dagger)^{-1} V^\dagger \tag{C2}$$

$$= c[1 - 2i\pi W (D_{\rm ring})^{-1} W^\dagger] W (D_{\rm ring}^\dagger)^{-1} V^\dagger, \tag{C3}$$

where we have inserted $S^{(0)}$ from Eq. (B2). Exploiting the symmetry of $D_{\rm ring}$ and inserting the relation

$$D_{\rm ring} - D_{\rm ring}^\dagger = 2i\pi W^\dagger W, \tag{C4}$$

we find for the right-hand side of Eq. (C3)

$$c[W (D_{\rm ring}^\dagger)^{-1} V^\dagger - 2\pi i W (D_{\rm ring})^{-1} W^\dagger W (D_{\rm ring}^\dagger)^{-1} V^\dagger] \tag{C5}$$

$$= c[W (D_{\rm ring}^\dagger)^{-1} V^\dagger + W (D_{\rm ring})^{-1} V^\dagger - W (D_{\rm ring}^\dagger)^{-1} V^\dagger] \tag{C6}$$

$$= c W (D_{\rm ring})^{-1} V^\dagger, \tag{C7}$$

which proves $c = 1$ and hence Eq. (C1).

APPENDIX D

In this appendix we derive the flux dependence of the various terms contributing to the conductance (35). First, the flux dependence of the total width Γ_j is calculated starting from Eqs. (25),

$$\Gamma_j = -2\,{\rm Im}[V D_{\rm ring}^{-1} V^\dagger]_{jj} \tag{D1}$$

$$= -2\,{\rm Im} \sum_{k,l} \{[(v_{kj}^L + v_{kj}^R \exp(-i\phi)] \times D_{\rm ring}^{-1} [v_{lj}^L + v_{lj}^R \exp(i\phi)]\}, \tag{D2}$$

where we have inserted the parametrizations (7) and (8). The term proportional to $\sin\phi$ vanishes since $D_{\rm ring}$ is symmetric. Hence one obtains

$$\Gamma_j = \overline{\Gamma}_j(1 + x_0\cos\phi), \tag{D3}$$

where both $\overline{\Gamma}_j$ and x_0 are flux independent.

To derive Eqs. (37) and (38) we recall that the couplings V and W obey the symmetries $V(\phi) = V^*(-\phi)$ and $W = W^*$. Therefore the propagator $D_{\rm ring}$ defined in Eq. (16) is symmetric and the partial width amplitudes γ, $\overline{\gamma}$ given in Eqs. (19) and (20) are related by

$$\gamma(\phi) = \overline{\gamma^*}(-\phi). \tag{D4}$$

Hence they can be written in the form

$$\gamma(\phi) = \mu + \nu\ \exp(-i\phi), \tag{D5}$$

$$\overline{\gamma^*}(\phi) = \mu + \nu\ \exp(+i\phi), \tag{D6}$$

where μ and ν are complex and independent of ϕ. Using this, one finds that the term $\sim\sin\phi$ in the expression

$$\gamma^{1*}_{aj}\overline{\gamma^2_{bj}} = (\mu^1_a\mu^2_b + \nu^1_a\nu^2_b) + (\mu^1_a\nu^2_b + \nu^1_a\mu^2_b)\cos\phi + i(\mu^1_a\nu^2_b - \nu^1_a\mu^2_b)\sin\phi \tag{D7}$$

is antisymmetric with respect to the interchange $1,a \leftrightarrow 2,b$. The transmission amplitude $t_{{\rm ring},ab}$, by contrast, is symmetric under the same interchange. As a result, the sum $2\Sigma_{a,b}t_{{\rm ring},ab}\gamma^{1*}_{aj}\overline{\gamma^2_{bj}}$ has the form given in Eq. (37) with positive amplitudes y_0, y_1 and real phase shifts ξ_0, ξ_1. The form of Eq. (38) follows from the parametrizations (D5) and (D6) since the conductance is a symmetric function of ϕ.

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[21] Here, we assumed that both the boundary and the potential outside the dot have rotational symmetry.