

Thermal fluctuation effects on the magnetization above and below the superconducting transition in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ crystals in the weak magnetic field limit

J. Mosqueira, E. G. Miramontes, and C. Torrón

Laboratorio de Física de Materiales, Facultad de Física, Universidad de Santiago de Compostela, 15706 Spain

J. A. Campá

Departamento de Cristalografía, Facultad de Ciencias Geológicas, Universidad Complutense de Madrid, Madrid, 28040 Spain

I. Rasines

Instituto de Ciencia de Materiales, Consejo Superior de Investigaciones Científicas, Cantoblanco, Madrid, 28049 Spain

Félix Vidal

Laboratorio de Física de Materiales, Facultad de Física, Universidad de Santiago de Compostela, 15706 Spain

(Received 26 December 1995)

We present detailed experimental data of the magnetization, $M_{ab}(T,H)$, of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ crystals on both sides of the superconducting transition, for magnetic fields, H , applied perpendicularly to the ab (CuO_2) planes and for amplitudes up to $\mu_0 H = 5$ T, which not too close to the superconducting transition correspond to the weak magnetic field amplitude limit. These data are analyzed in terms of thermal fluctuations in this weak H limit: In the reversible mixed state below the transition, by taking into account the fluctuations of the vortex lines positions, as first proposed by Bulaevskii, Ledvij, and Kogan. Above the transition, by taking into account the Cooper pairs created by thermal fluctuations, through a generalization of multilayered superconductors of the Schmidt-like approach. These simultaneous, quantitative and consistent analyses of $M_{ab}(T,H)$ above and below the transition allow us to estimate the effective number of independent fluctuating superconducting CuO_2 planes in the periodicity length $s = c/2$, c being the unit-cell length, and to separate for the first time the in-plane correlation length amplitude, $\xi_{ab}(0)$, and the parameter related to the vortex structure, η . We found $\xi_{ab}(0) = (0.8 \pm 0.1)$ nm and $\eta = 0.15 \pm 0.05$, this last value being well within the one calculated by Fetter by applying the London model to a triangular vortex lattice. For the in-plane magnetic penetration depth, we found a temperature behavior compatible with the clean BCS weak coupling limit, and an amplitude (at $T = 0$ K) of $\lambda_{ab}(0) = (180 \pm 20)$ nm. [S0163-1829(96)02322-3]

I. INTRODUCTION

It is now well established that the high-temperature copper-oxide superconductors (HTSC) present important thermal fluctuation effects on both sides of the superconducting transition.^{1,2} In turn, the study of these fluctuation effects above and below the transition may provide very useful information on various central aspects of these materials, as the different superconducting characteristic lengths or the pair-breaking effects.¹⁻³ One of the observables best adapted to study the thermal fluctuations in high-temperature copper-oxide superconductors (HTSC) is the magnetization, $M_{ab}(T,H)$, for magnetic fields, H , applied perpendicularly to the ab planes (the CuO_2 layers) and in the so-called weak-amplitude limit, characterized by⁴

$$\varepsilon \gg \frac{H}{H_{c2}(0)}, \quad (1)$$

where $H_{c2}(0)$ is the upper critical magnetic field amplitude (at $T = 0$ K) for H perpendicular to the ab planes, $\varepsilon \equiv |T - T_{c0}|/T_{c0}$ is the reduced temperature and T_{c0} is the mean-field transition temperature at $H = 0$. This weak limit will exclude, therefore, two temperature regions, above and below the transition, close to T_{c0} (for $\mu_0 H \leq 5$ T, a few

degrees). These thermal fluctuation effects on $M_{ab}(T,H)$ may be quantified through the so-called excess magnetization, $\Delta M_{ab}(T,H)$, defined as

$$\Delta M_{ab}(T,H) \equiv M_{ab}(T,H) - M_{abB}(T,H), \quad (2)$$

where the background magnetization, $M_{abB}(T,H)$, is the magnetization associated with the normal contributions, i.e., the sample magnetization if the superconducting transition were absent, and which may be approximated by extrapolating through the transition the magnetization measured well above T_{c0} , in a temperature region where the effects of the thermal fluctuations become negligible. Above T_{c0} , $\Delta M_{ab}(T,H)$ is only due to thermal fluctuations, which create Cooper pairs with a finite lifetime.^{1,3-7} In turn, these fluctuation-induced Cooper pairs lead to the appearance of shielding currents near the transition, which round down the $M(T)_H$ curves, i.e., that lead to a negative $\Delta M_{ab}(T,H)$. These effects are explained in the HTSC in terms of generalizations of the Schmidt-like approach which takes into account the layered nature of these materials.⁵⁻⁷ The study of ΔM_{ab} above T_{c0} and in the weak magnetic field limit (or, equivalently, of the so-called excess diamagnetism, $\Delta \chi_{ab}$, see below) may provide direct information on the Ginzburg-Landau superconducting coherence length amplitude (at

$T=0$ K) in the ab plane, $\xi_{ab}(0)$, and on the mean-field transition temperature T_{c0} .⁵⁻¹⁰ In addition, these $\Delta M_{ab}(T,H)$ effects may probe various central and very general aspects of the order-parameter fluctuations (OPF's) above T_{c0} , as the influence or not of the so-called indirect contributions,⁷⁻¹² or the relevance of the multiperiodicity of the superconducting layers in HTSC.^{5-7,10}

Below T_{c0} , in the reversible mixed state, $\Delta M_{ab}(T,H)$ is due to two linearly additive contributions: the London-like diamagnetism and the effects associated with thermal fluctuations.¹³⁻¹⁵ In the weak magnetic field limit, this last contribution is associated with the fluctuations of the vortex lines positions, i.e., with fluctuations of the superconducting order-parameter *phase* (in contrast, in the high magnetic field regime the fluctuations of the order-parameter *amplitude* will give the main contributions).^{13,14} These phase fluctuation effects are particularly important in strongly anisotropic layered HTSC, as the Bi compounds studied here, where the vortex lines appear as weak correlated stacks of two-dimensional (2D) vortices (pancakes). They lead, in particular, to the existence of a temperature, T^* , some degrees below T_{c0} , at which the excess magnetization, $\Delta M_{ab}(H)_{T^*}$, is independent of H [i.e., a crossing point of the $\Delta M_{ab}(T)_H$ curves at T^*].^{13,15} From the analysis of the experimental $\Delta M_{ab}(T,H)$ data below T_{c0} in terms of the Bulaevskii-Ledvij-Kogan (BLK) approach,¹³ it is possible to extract the in-plane magnetic penetration length, $\lambda_{ab}(T)$, and the relationship $\xi_{ab}(0)/\sqrt{\eta}$, where the constant η is related to the vortex lattice. In addition, these data below T_{c0} may probe the applicability to HTSC of other theoretical approaches, as for instance those that take into account the depression of the order parameter in the vortex cores,¹⁶ to the understanding of $\Delta M_{ab}(T,H)$ in strong anisotropic HTSC.

Until now, the fluctuation effects on $M_{ab}(T,H)$ in the weak magnetic field limit in HTSC have been studied separately above^{1,4-12} and below^{2,13,15,17-24} the transition. [Some authors have studied the scaling behavior of $M_{ab}(T,H)$ through the transition, but in an (ε,H) region that will correspond to the high magnetic field limit.^{14,25-27}] However, some of the characteristic parameters arising in the theoretical approaches are the same above and below T_{c0} . This is the case, as we have already stressed, of the in-plane coherence length, or of the (never directly accessible) mean-field critical temperature. Also, some of the possible nonintrinsic effects on $M_{ab}(T,H)$ in real samples are similar on both sides of the transition. This is the case, for instance, of the influence on $M_{ab}(T,H)$ of the presence of stoichiometric inhomogeneities (oxygen content, for instance) at long length scales (at length scales much larger than $\xi_{ab}(T)$, even at temperatures relatively close to T_{c0}). So, the simultaneous analysis of $M_{ab}(T,H)$ on both sides of T_{c0} will reduce the experimental uncertainties and also the number of free parameters arising in the theoretical approaches.

The central aim of this paper is twofold. First, we present detailed experimental data of $M_{ab}(T,H)$ of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ crystals. These data were obtained with H applied perpendicularly to the CuO_2 layers and up to 5 T and they cover, to our knowledge for the first time,²⁸ both sides of the superconducting transition. Then, we analyze these results, simultaneously and consistently above and below T_{c0} , in terms of the available approaches which take

into account the thermal fluctuations in the weak magnetic field limit. These analyses allow us, in particular, to separate for the first time $\xi_{ab}(0)$ and η in these crystals and to estimate the relative strength of the coupling between adjacent superconducting (CuO_2) planes: the magnetic coupling in the case of the fluctuations of the vortex positions below T_{c0} , and the Josephson coupling in the case of the fluctuations of the order parameter amplitude above T_{c0} .

II. EXPERIMENTAL DETAILS AND RESULTS

The $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ (Bi-2212) crystals used in this work were grown from mixtures of analytical grade Bi_2O_3 , $\text{Sr}(\text{OH})_2 \cdot 8\text{H}_2\text{O}$, CaCO_3 , and CuO in stoichiometric amounts. The mixtures were heated to 780 °C in alumina crucibles, ground, heated again to 840 °C for 12 h, reground, tested as pure polycrystalline 2212, transferred to a rotary conical gold crucible (16 and 25 mm diameters, 30 mm high, 60 rpm, and the direction of the rotation changing every 25 s), melt after slowly heated to 980 °C in 6 h 15 min, and maintained at this temperature during 2 h. The melt was cooled to 870 °C at 0.67 °C h⁻¹ and, once crystallized, to 600 °C at a cooling rate of 25 °C h⁻¹, turning then the power off.

A small portion of every batch was finely ground and examined by energy-dispersive x-ray analysis (EDS), and by x-ray powder diffraction (XPD). A detailed structural analysis may be seen in Ref. 29. Two samples of different batches, hereafter named Bi1 and Bi2, of masses 8.31 mg and 19.10 mg, respectively, were chosen for the experiments. These x-ray analyses, as well as measurements of the electrical resistivity performed after the magnetization measurements, show that these two crystals are affected of small compositional or structural inhomogeneities, which may modify the amplitude of the measured magnetization (see below). In addition, SEM analysis showed that sample Bi2 presents large domains with different orientations of the ab planes, which will also affect $M(T,H)$ above and below the transition.

The magnetization measurements were made with a Quantum Design (SQUID) magnetometer. The instrumental resolutions are 10⁻¹¹ A m² for the magnetic moment, 0.1% for the temperature, and 2 × 10⁻⁶ g for the sample mass. The resolution in the magnetization is then better than 0.5%. All the measurements presented here were made with the magnetic fields applied perpendicularly to the ab planes. The samples were wrapped in Teflon tape and situated in the center of the SQUID's pick up coil by means of a plastic straw. The Teflon contribution to the magnetic moment was carefully checked and subtracted to the magnetic moment measured in the samples plus holders. Two series of measurements were made: magnetization versus temperature at a constant field, $M_{ab}(T)_H$, and magnetization versus field at a constant temperature, $M_{ab}(H)_T$. For the $M_{ab}(T)_H$ data, the samples were cooled down to 5 K in the absence of magnetic field [zero-field cooled (ZFC)]. Then the magnetic field was applied and data were taken as the temperature was increased up to 250 K. For the $M_{ab}(H)_T$ measurements, the samples were also zero-field cooled and then the magnetic field was applied and increased in steps of 0.1–0.25 T, up to 5 T, with a 5-min delay between measurements to ensure system stability. To calculate the magnetization, the samples volumes

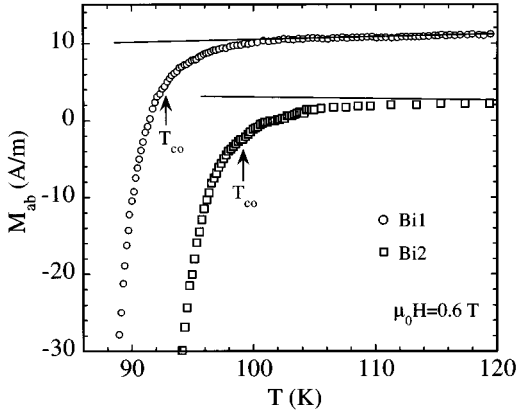


FIG. 1. Magnetization vs temperature around the transition measured with a magnetic field of $\mu_0 H = 0.6$ T applied perpendicularly to the ab planes for samples Bi1 (circles) and Bi2 (squares). The solid lines are the normal-state background magnetizations.

are determined from their masses by using the theoretical density of the Bi-2212 (6.5 g/cm^3). For all the temperatures and magnetic fields studied here, the measured $M_{ab}(T, H)$ are very small compared with H and then the corrections due to demagnetizing effects are negligible. Let us also note here that the ZFC and FC susceptibilities do not show 100% of complete shielding and, respectively, flux expulsion at low temperatures, confirming then the presence of small non-superconducting domains and inclusions in both crystals. In fact, the saturation FC values at *low fields*, already corrected from demagnetization effects and also taking into account the sample misalignments, lead to a magnetic quality factor, C , of the order of 2 and 3 for, respectively, Bi1 and Bi2 samples. We will see in Sec. III that these values are in reasonable agreement with those obtained from the analysis of the thermal fluctuations.

The thermal fluctuation effects on $M_{ab}(T, H)$ will be analyzed above and below T_{c0} through the excess magnetization defined by Eq. (2). In the case of M_{ab} versus T data at constant H , $M_{abB}(T)$ in this equation is approximated by extrapolating through the transition the magnetization measured in the temperature region bounded by $120 \leq T \leq 250$ K, where the influence of the fluctuations is known to be negligible.¹⁰ In this region, the magnetization versus temperature for these samples follows very closely a linear behavior. In Fig. 1 we show the $M_{ab}(T)$ data for sample Bi1 (circles) and sample Bi2 (squares), both measured with $\mu_0 H = 0.6$ T, around the transition. The lines are the extrapolations through the superconducting transitions of the normal-state data of each sample. In the case of the M_{ab} versus H data, we use as background an isotherm taken also at a temperature in which the fluctuation effects are expected to be negligible, about 30 K above the transition. Hereafter, the background magnetization has been subtracted to all the magnetization data presented in this paper.

In Figs. 2(a) and 2(b) we present the $\Delta M_{ab}(T)_H$ data for sample Bi1 and sample Bi2, respectively, in the reversible mixed state, for several applied magnetic fields, where the crossovers of the different curves (at $(T^*, \Delta M_{ab}^*)$) are clearly seen. As an example, the $\Delta M_{ab}(H)_T$ data of sample Bi1 for several temperatures are shown in Fig. 3. The data of this last figure have already been corrected by a C factor to take into

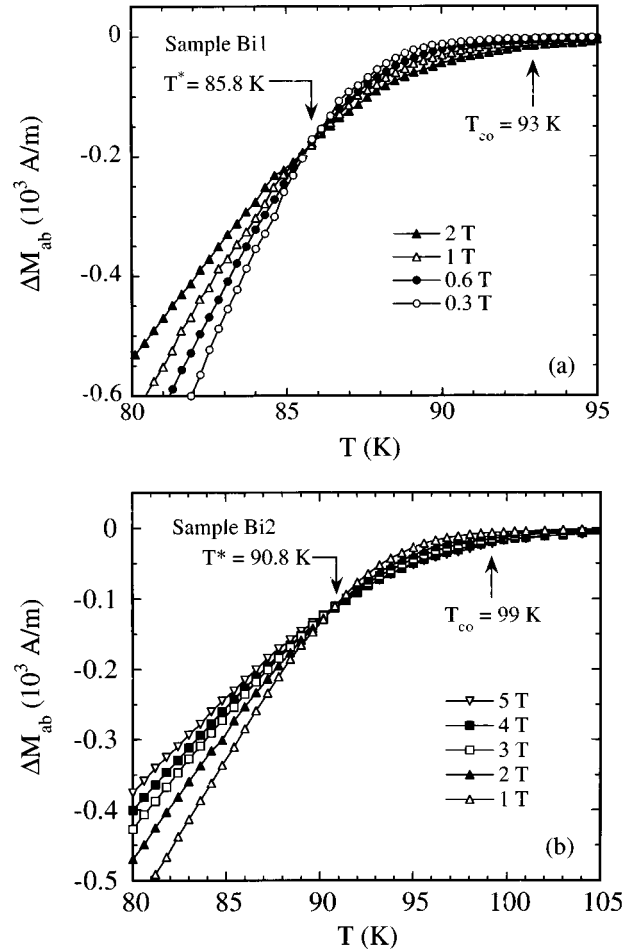


FIG. 2. (a) Excess magnetization vs temperature for sample Bi1, measured with magnetic fields of 0.3, 0.6, 1, and 2 T applied perpendicularly to the ab planes, around T^* . (b) Excess magnetization vs temperature for sample Bi2, measured with magnetic fields of 1, 2, 3, 4, and 5 T applied perpendicularly to the ab planes, around T^* . The solid lines are guides for the eyes.

account the possible presence of nonsuperconducting domains in the sample (see Sec. III). Note that ΔM_{ab} shows a linear behavior versus $\log H$ but presenting a slight deviation from this linear behavior for temperatures below about 83 K. This effect may be due to the contribution of the quantum fluctuations of vortices,³⁰ or due to the influence of pinning on the distribution of vortices in the material,³¹ but it has no relevance on the analysis presented in this paper.

III. DATA ANALYSIS

A. Theoretical background

1. Above T_{c0}

For H perpendicular to the ab planes (the CuO_2 layers) and in the weak magnetic field limit, both the measured and the background magnetization arising in ΔM_{ab} [Eq. (2)] depend linearly, above T_{c0} , on the magnetic field amplitude. Therefore, in this case, to characterize the thermal fluctuation effects on $M_{ab}(T, H)$ it will be useful to use instead of ΔM_{ab} the so-called in-plane excess diamagnetism (also called in-plane fluctuation induced diamagnetism), $\Delta \chi_{ab}(\varepsilon)$, defined as

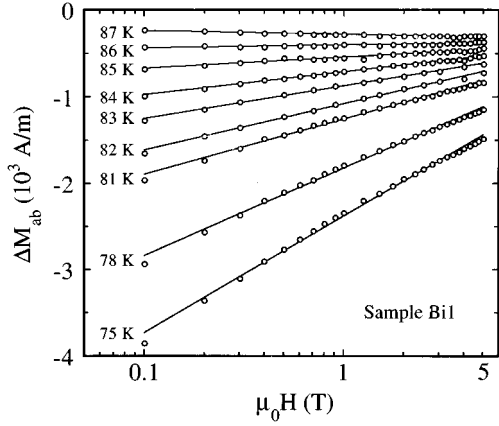


FIG. 3. Excess magnetization for sample Bi1 vs external magnetic field (applied perpendicularly to the ab planes) for several temperatures in the reversible mixed state. These data are already corrected of the C factor which accounts for the possible existence of nonsuperconducting domains in the sample. The lines are fittings of the BLK model for the thermal fluctuations of the vortex positions [Eq. (6)] to each isotherm, with λ_{ab} and ηH_{c2} as free parameters.

$$\Delta\chi_{ab}(\varepsilon) \equiv \chi_{ab}(\varepsilon) - \chi_{abB}(\varepsilon), \quad (3)$$

where $\chi_{ab}(\varepsilon) = M_{ab}(\varepsilon)_H/H$ and $\chi_{abB}(\varepsilon) = M_{abB}(\varepsilon)_H/H$ are, respectively, the measured and the background susceptibilities in the weak magnetic field limit. In layered superconductors with two superconducting layers per periodicity length and with two different Josephson coupling strengths, γ_1 and γ_2 , between adjacent layers, which in principle is the case well adapted to the Bi-2212 compounds studied here, $\Delta\chi_{ab}(\varepsilon)$ in the mean-field-like temperature region and in the weak magnetic field limit is given by⁷

$$\frac{\Delta\chi_{ab}}{T}(\varepsilon) = -N_e(\varepsilon) \frac{A_S}{\varepsilon} \left(1 + \frac{B_{LD}}{\varepsilon} \right)^{-1/2}, \quad (4)$$

where $N_e(\varepsilon)$ is an effective number of independent fluctuating planes per periodicity length, $A_S \equiv \pi\mu_0 k_B \xi_{ab}^2(0)/3\phi_0^2 s$ is the Schmidt diamagnetism amplitude (we will use through this paper MKSA units) and $B_{LD} = [2\xi_c(0)/s]^2$ is the Lawrence-Doniach parameter governing the dimensionality of the thermal fluctuations of the order parameter (OPF). In these expressions k_B is the Boltzmann constant, μ_0 is the vacuum permeability, ϕ_0 is the flux quantum, $\xi_{ab}(0)$ and $\xi_c(0)$ are the superconducting coherence length amplitudes (at $T=0$ K) in the ab planes and, respectively, in the c direction, and s is the periodicity length of the superconducting CuO_2 layers, which in the case of the Bi-2212 compounds is one-half the crystallographic unit-cell length in the c direction, i.e., $s = c/2 = 1.54$ nm (as noted before, there are two superconducting layers in s). Mainly due to the presence in Eq. (4) of $N_e(\varepsilon)$, which in fact depends also on γ_1/γ_2 , this expression for $\Delta\chi_{ab}(\varepsilon)$ could be quite complicated.⁷ However, earlier paraconductivity measurements showed that in all the mean-field region (MFR) above T_{c0} the OPF effects in these Bi-based HTSC are essentially two dimensional (2D), i.e., $\xi_c(\varepsilon) \ll s$ (and then, $B_{LD} \ll \varepsilon$) in that ε region.³² Such a 2D behavior was latter confirmed by different studies of the OPF effects above the superconducting

transition.^{1,7-9,27,33} The analysis in Refs. 7,8, and 33 also show that in Bi-2212 compounds $N_e \approx 2$, i.e., that the two superconducting CuO_2 planes in s are, in what concerns the OPF effects in the MFR above T_{c0} , completely uncorrelated. So, for these HTSC, Eq. (4) reduces to

$$\frac{\Delta\chi_{ab}}{T}(\varepsilon) = \frac{-2A_S}{\varepsilon}, \quad (5)$$

an expression which, as could be expected, coincides with the conventional $\Delta\chi_{ab}(\varepsilon)$ result for layered superconductors with only one layer per periodicity length, but with $s/2$ as effective interlayer distance.⁷

2. Below T_{c0}

The $M_{ab}(T, H)$ data below T_{c0} and in the weak magnetic field amplitude limit will be analyzed in terms of the BLK approach.¹³ In this model, which is based on the London theory but by taking also into account the contributions associated with the thermal fluctuations of the vortex positions, the excess magnetization, $\Delta M_{ab}(T, H)$, for H perpendicular to the ab planes, may be approximated as^{13,15}

$$\Delta M_{ab}(T, H) = -\frac{\phi_0}{8\pi\mu_0\lambda_{ab}^2(T)} \ln\left(\frac{\eta H_{c2}(T)}{H}\right) + \frac{k_B T}{\phi_0 s} \ln\left(\frac{8\pi\mu_0 k_B T \lambda_{ab}^2(T)}{s\phi_0^2 e} \frac{\eta H_{c2}(T)}{H}\right), \quad (6)$$

where the first term on the right corresponds to the usual London magnetization, whereas the second one is the correction due to the fluctuations of the vortex positions. [Some authors denote the excess magnetization below T_{c0} as $M_{ab}(T, H)$. However, our notation is not only consistent above and below T_{c0} , but also avoids possible confusions between the excess magnetization and the magnetization itself.] In Eq. (6), $e \approx 2.718$ is the Euler constant, λ_{ab} is the penetration depth in the ab plane, η is a constant related to the vortex lattice structure and s is again the periodicity length of the superconducting CuO_2 layers. The above equation assumes one superconducting layer in s . In the case of the Bi-2212 compounds this would be a good approximation below T_{c0} if the magnetic coupling between pancakes in the nearest layers were strong enough to make them fluctuate jointly. In this case, each bilayer in s may be considered, in what concerns the fluctuations of the vortex positions, as a single superconducting layer without internal structure. We will see here that this point of view, already proposed in the original BLK approach^{13,15} seems to be the adequate one for Bi-2212. However, for completeness let us note that Eq. (6) could easily be adapted to the opposite scenario, in which the pancakes in each CuO_2 layer fluctuate independently and, therefore, there are two completely uncorrelated superconducting planes in s , by just using as an effective interlayer periodicity $s/2$ instead of s . We will examine both scenarios in this work. Finally, from Eq. (6) the magnetization at the crossing point of the $\Delta M_{ab}(T)_H$ curves may be easily obtained as^{13,15}

$$-\Delta M_{ab}^* = \frac{k_B T^*}{\phi_0 s}, \quad (7)$$

an expression that again may be also applied to the case of uncorrelated interlayer pancakes by just replacing s by $s/2$.

B. Excess magnetization in the reversible mixed state below T_{c0}

A first direct comparison between our experimental results below T_{c0} and the BLK approach may be done through Eq. (7) relating the excess magnetization, ΔM_{ab}^* , to the temperature, T^* , of the crossing point of the $\Delta M_{ab}(T)_H$ curves. For sample Bi1, the results of Fig. 2(a) lead to $\Delta M_{ab}^* = -176$ A/m and $T^* = 85.8$ K. By using in Eq. (7) this T^* and $s = 1.54$ nm, one obtains $\Delta M_{ab}^* = -372$ A/m. As the value of T^* is very reliable, such a strong discrepancy between the measured and the calculated ΔM_{ab}^* could be due, on the grounds of the BLK approach, to the value of the CuO_2 layer periodicity length or to the measured ΔM_{ab}^* . However, $s = c/2$, which corresponds to the case of strong correlation of the pancakes in the two closest CuO_2 superconducting planes, is the biggest possible effective interlayer periodicity for the fluctuating vortex positions. Therefore, the origin of this discrepancy must be attributed to extrinsic effects on the measured ΔM_{ab}^* . For instance, the measured ΔM_{ab}^* may be affected by the presence of nonsuperconducting domains in the sample (associated, in turn, with small stoichiometric inhomogeneities) or by slight misalignments of different sample domains with respect to the applied field, both effects reducing the absolute value of the measured M_{ab} , and then of ΔM_{ab}^* . This simple explanation is supported by the fact that, as stressed in Sec. II, the ZFC susceptibility does not show complete magnetic shielding at low temperatures. So, these extrinsic effects may be easily taken into account by correcting the absolute values of the total measured magnetization by the constant factor C introduced in Sec. II. For the sample under study, the agreement between ΔM_{ab}^* calculated as indicated above and the measured one is obtained for $C = 2.1$. So, in the remaining part of this paper all the magnetization data (including those above T_{c0}) of sample Bi1 will be corrected by this C factor. This same analysis performed on sample Bi2 leads to $C = 3.5$. A so important correction factor is, in fact, a clear indication of the relatively low quality of this crystal, as already stressed in Sec. II, and introduces some additional uncertainty on the analyses based on the absolute amplitude of $M(T, H)$ in this sample. Note, however, that these C factors are, for the two samples, in good qualitative agreement with the values found before from the analysis of the FC low-field susceptibility versus temperature curves, the relatively small differences being probably due to sample misalignments (see also Sec. III C).

To further analyze $\Delta M_{ab}(T, H)$ below T_{c0} , the measured $\Delta M_{ab}(H)_T$ of sample Bi1 (already corrected from its corresponding C factor) is compared in Fig. 3 with the BLK theoretical approach. The solid lines in this figure are the best fits of Eq. (6) to the different isotherms, with λ_{ab} and ηH_{c2} as free parameters for each isotherm. The fittings were done in the range $0.1 \text{ T} \leq \mu_0 H \leq 5 \text{ T}$ that, even for the isotherm taken at the highest temperature (87 K), is well within the weak magnetic field amplitude limit, for which the BLK approach is valid. The rms of all the fittings are at about 2%. The circles in Fig. 4 are the resulting ηH_{c2} for each temperature.

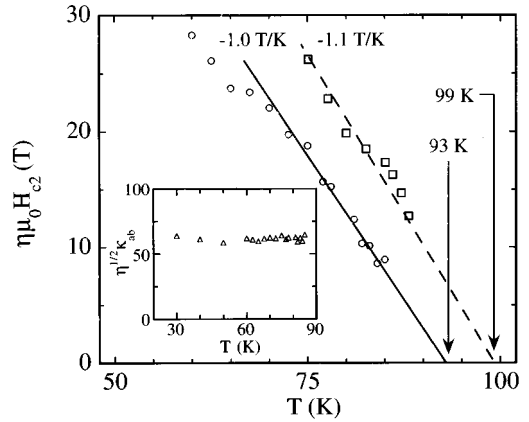


FIG. 4. Upper critical magnetic field (multiplied by η) vs temperature for samples Bi1 (circles) and Bi2 (squares), obtained by fitting the model for thermal fluctuations of vortices [Eq. (6)] to the $\Delta M_{ab}(T, H)$ data in the reversible mixed state (already corrected of the C factor). The solid and dashed lines are linear fittings for temperatures above 75 K. Inset: ab -plane Ginzburg-Landau parameter (multiplied by $\sqrt{\eta}$) vs temperature, calculated from the $\eta \mu_0 H_{c2}(T)$ data shown in the main figure and the $\lambda_{ab}(T)$ data of Fig. 5.

We see in that figure that near T_{c0} , ηH_{c2} is linear versus temperature. To calculate the slope, $(dH_{c2}/dT)_{T_{c0}}$, we made a linear fitting to the experimental data for temperatures above 75 K, which leads to $\eta \mu_0 (dH_{c2}/dT)_{T_{c0}} = (-1.0 \pm 0.1)$ T/K, and $T_{c0} = (93 \pm 1)$ K. Let us stress already here that this value of T_{c0} is close to the one that we are going to obtain later by analyzing the OPF above the transition. From this result one may easily obtain $\xi_{ab}(0)/\sqrt{\eta}$ by using

$$\xi_{ab}(0) = \sqrt{\frac{\phi_0}{-2\pi T_{c0} \mu_0 (dH_{c2}/dT)_{T_{c0}}}}, \quad (8)$$

which may be obtained by just combining, $\xi_{ab}(T) = \xi_{ab}(0) \varepsilon^{-1/2}$ and $\mu_0 H_{c2}(T) = \phi_0 / 2\pi \xi_{ab}^2(T)$. The resulting value is $\xi_{ab}(0)/\sqrt{\eta} = (1.9 \pm 0.3)$ nm, the error being mainly due to the background choice and to the fitting uncertainties. The squares in Fig. 4 represent the results obtained for sample Bi2, following the same procedure as for sample Bi1. The resulting parameters are $\eta \mu_0 (dH_{c2}/dT)_{T_{c0}} = (-1.1 \pm 0.2)$ T/K, which is close to the one obtained for sample Bi1, and $T_{c0} = (99 \pm 2)$ K, which is quite higher than the one obtained for sample Bi1, but is close to the transition temperatures observed by other authors in other Bi-2212 samples.^{13,20,24} From these data, we have obtained, through Eq. (8), $\xi_{ab}(0)/\sqrt{\eta} = (1.7 \pm 0.2)$ nm. These values for $\xi_{ab}(0)/\sqrt{\eta}$ are in good agreement with those obtained for crystal and polycrystalline Bi-2212 samples in Refs. 13,20,21, and 24. We will see in the next subsection that this common value of $\xi_{ab}(0)/\sqrt{\eta}$ leads, however, to very different proposals for the value of $\xi_{ab}(0)$ and η .

The λ_{ab} values resulting from the fits of Eq. (6) to the $\Delta M_{ab}(H)_T$ data are plotted in Figs. 5(a) and 5(b) for samples Bi1 and Bi2, respectively. We also show in those figures the fittings to the $\lambda_{ab}(T)$ predicted by the BCS theory

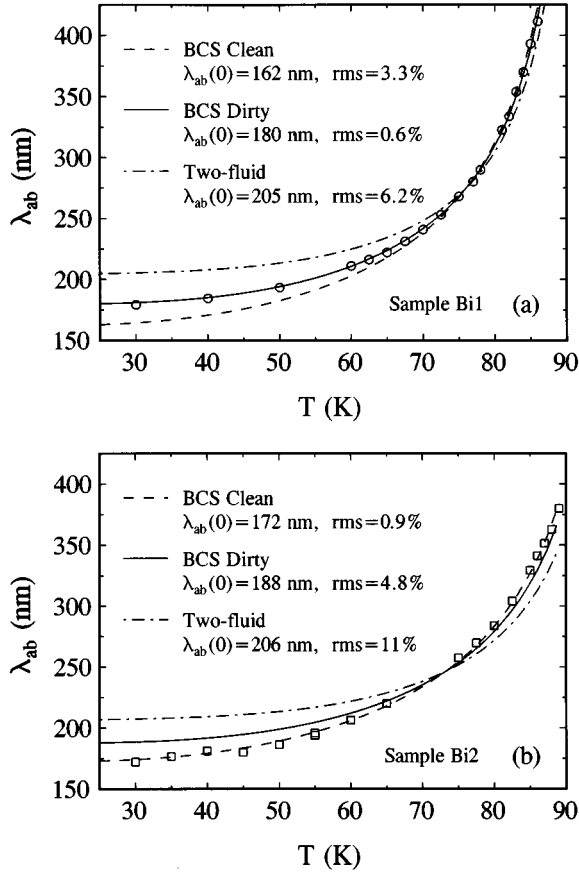


FIG. 5. In-plane magnetic penetration depth vs temperature of (a) sample Bi1 and (b) sample Bi2, obtained by fitting the model for thermal fluctuations of vortices [Eq. (6)] to the $\Delta M_{ab}(T, H)$ data in the reversible mixed state (again corrected of the C factor). The dashed and solid lines are fittings to the BCS functional forms in the clean and dirty limits, respectively. The dot-dashed line is the fitting to the two-fluid model. The fitting free parameter is $\lambda_{ab}(T=0)$ K. The mean-field critical temperature, T_{c0} , is the one obtained from the analysis of $\eta\mu_0 H_{c2}(T)$ near the transition.

in the clean limit (dashed line) and in the dirty limit (solid line), as tabulated in Ref. 34, and also by the two-fluid model:³⁵

$$\lambda_{ab}(T) = \frac{\lambda_{ab}(0)}{\sqrt{1 - (T/T_{c0})^4}} \quad (9)$$

(dot-dashed line). In these fittings we have set $\lambda_{ab}(0)$ as free parameter and imposed $T_{c0}=93$ K for sample Bi1 and $T_{c0}=99$ K for sample Bi2, as determined from the analyses of dH_{c2}/dT done before. The resulting values are summarized in Fig. 5, where one may see that both BCS weak-coupling approaches are in good agreement with the experimental data. These results seem to exclude the validity of the functional form predicted by the two-fluid model. Moreover, the good agreement for both the dirty and clean BCS weak-coupling approaches just suggests that these data cannot discriminate the small differences between these two limits. In fact, as the Bi-based compounds are expected to be in the clean limit, the main conclusion here is, therefore, that our results for $\lambda_{ab}(T)$ are well compatible with the clean BCS

weak-coupling limit. In addition, the corresponding $\lambda_{ab}(0)$ are close to the ones proposed in Refs. 13,15,20, and 21 for the Bi-2212 compounds.

Finally, by combining the value obtained above for $\lambda_{ab}(T)$ and $\xi_{ab}(T)/\sqrt{\eta}$ one may obtain $\sqrt{\eta}\kappa_{ab}(T)$ versus T , where $\kappa_{ab} \equiv \lambda_{ab}/\xi_{ab}$ is the in-plane Ginzburg-Landau parameter. As an example, it can be seen in the inset of Fig. 4 that $\sqrt{\eta}\kappa_{ab}$ for sample Bi1 is temperature independent, even up to T^* . To obtain such a temperature-independent behavior it is crucial to take into account the thermal fluctuations of the vortex positions. As stressed already by other authors,^{13,15} this result provides another confirmation of the adequacy of the BLK approach to explain the magnetization behavior below T_{c0} of the Bi-based HTSC.

C. Excess diamagnetism above T_{c0} :

Comparison with the magnetization below T_{c0}

The analysis of $\Delta\chi_{ab}(\varepsilon)$ above T_{c0} in terms of thermal fluctuations of the order parameter amplitude will provide a direct estimation of $\xi_{ab}(0)$ which, when combined with the $\xi_{ab}(0)/\sqrt{\eta}$ resulting from the analysis of $\Delta M_{ab}(T, H)$ below T_{c0} , allows the experimental determination of η , the vortex structure parameter. As a first check of the applicability of Eq. (5), which corresponds to OPF in the two-dimensional (2D) limit, to the $\Delta\chi_{ab}(\varepsilon)$ data above T_{c0} measured in the sample Bi1, in the inset of Fig. 6(a) we have plotted $T/\Delta\chi_{ab}(T)$ versus T , for $\mu_0 H = 0.6$ T. These data were obtained from the data points (circles) of Fig. 1, already corrected of the C factor estimated in the preceding subsection. This curve displays good linearity, as predicted by Eq. (5), between 95 and 101.5 K. This temperature interval will correspond, therefore, to the MFR above T_{c0} .⁵⁻⁸ In addition, the mean-field critical temperature may be estimated by just extrapolating to the temperature axis a linear fit to $T/\Delta\chi_{ab}$ in this region (solid line). The resulting value, $T_{c0} \approx 93$ K, is in excellent agreement with the mean-field critical temperature found before from the analysis of $\Delta M_{ab}(T, H)$ below the transition. This is, therefore, a first indication of the consistency of both analyses. Note that for $\mu_0 H = 0.6$ T, the weak magnetic field limit bounded by Eq. (1) will extend well beyond this lower limit of the MFR used here. The comparison between Eq. (5) and the measured $\Delta\chi_{ab}(\varepsilon)$ is presented in Fig. 6(a). Here, the solid line is the best fit of Eq. (5) to the data in the same reduced temperature as before, i.e., $2 \times 10^{-2} \leq \varepsilon \leq 10^{-1}$, with $T_{c0} = 93$ K and $s = 1.54$ nm as fixed parameters and with $\xi_{ab}(0)$ as the only free parameter. The fit rms is 5%, and the resulting value is $\xi_{ab}(0) = (0.8 \pm 0.1)$ nm, where the uncertainty is again mainly due to the background used to extract the experimental $\Delta\chi_{ab}(\varepsilon)$. This value of $\xi_{ab}(0)$ agrees, within the experimental uncertainties, with the ones found for Bi-2212 crystals from paraconductivity and fluctuation induced magnetoconductivity measurements,³³ and it is also consistent with the $\xi_{ab}(0)$ proposed in Ref. 8 for a Bi-2212 single crystal, and obtained from $\Delta\chi_{ab}(\varepsilon)$ measurements above T_{c0} . When combined with $\xi_{ab}(0)/\sqrt{\eta} = (1.9 \pm 0.2)$ nm resulting from the analysis of $\Delta M_{ab}(T, H)$ below T_{c0} , this value of $\xi_{ab}(0)$ leads to $\eta = 0.15 \pm 0.05$, which is in excellent agreement with $\eta = 0.16$ calculated by Fetter by applying the London theory to a triangular vortex lattice.³⁶

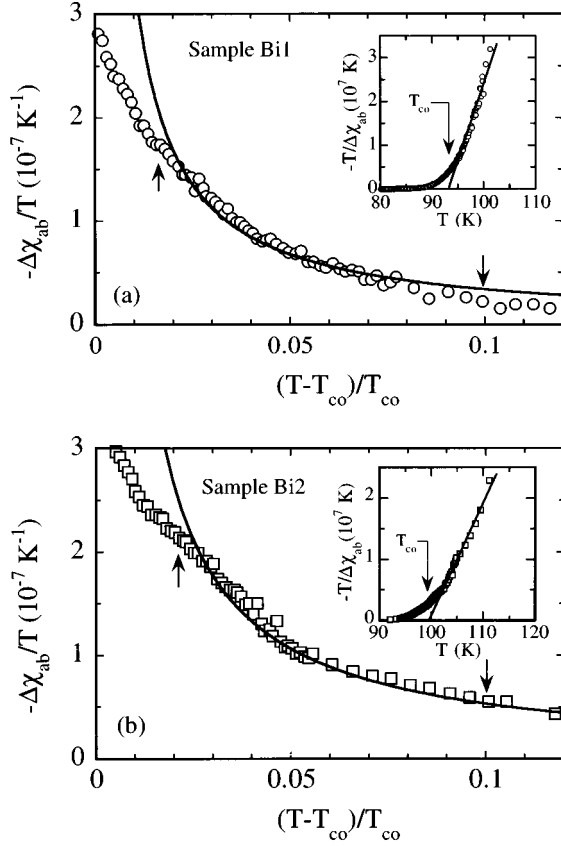


FIG. 6. Excess diamagnetism (over T) vs reduced temperature of (a) sample Bi1 and (b) sample Bi2, measured with $\mu_0 H = 0.6$ T applied perpendicularly to the ab planes. These data above T_{c0} have been corrected by the same C factor as we have used in the analysis below T_{c0} to take into account the possible existence of nonsuperconducting domains in the sample (see the text for details). The solid line is the best fit of the theoretical approach [Eq. (5)] to the experimental data in the reduced temperature interval marked between arrows $2 \times 10^{-2} \leq \varepsilon \leq 10^{-1}$, with $\xi_{ab}(0)$ as a free parameter. The inset shows the same data in a $-T/\Delta\chi_{ab}$ vs T plot. The mean-field critical temperature, T_{c0} , has been estimated by extrapolating to the temperature axis a linear fit to these data in the mean-field-like region (solid line in the inset).

In Fig. 6(b), we present $\Delta\chi_{ab}(T)/T$ for sample Bi2. These results were obtained from the data points of Fig. 1 (squares) corrected of the corresponding C factor obtained in Sec. III B by analyzing the data below T_{c0} . Note that a so high C factor may introduce additional uncertainties in the absolute values of $\Delta\chi_{ab}(T)/T$, but it does not affect its temperature behavior. In the inset, the linear extrapolation of $T/\Delta\chi_{ab}$ to the temperature axis is shown. This leads to $T_{c0} \approx 99.5$ K, in good agreement with the critical temperature found for this sample by analyzing $dH_{c2}(T)/dT$. The solid line in Fig. 6(b) is the best fit of Eq. (5) to the measured $\Delta\chi_{ab}(T)/T$ also in the reduced temperature interval $2 \times 10^{-2} \leq \varepsilon \leq 10^{-1}$ and with $\xi_{ab}(0)$ as the only free parameter. The good quality of this fitting (the rms is 5%) is again a clear indication of the 2D behavior predicted in Eq. (5) for the order-parameter fluctuations in this type of compounds. The resulting value is $\xi_{ab}(0) = (1.0 \pm 0.3)$ nm which, when combined with the value obtained for $\xi_{ab}(T)/\sqrt{\eta}$ from the analysis below T_{c0} , leads to $\eta = 0.30 \pm 0.12$.

Two comments on the above results are in order. Note first that until now all the different groups that have studied $\Delta M_{ab}(T, H)$ in the weak magnetic field limit below T_{c0} in Bi-2212 HTSC did not determine $\xi_{ab}(0)$ and η separately.^{13,15,20,21,24} Therefore, to obtain $\xi_{ab}(0)$, these authors impose in their $\xi_{ab}(T)/\sqrt{\eta}$ results (which in general are, as noted in the preceding subsection, close to ours) a value of η . Most of the authors have used $\eta \approx 1$, which is not too far from $\eta = 1.4$ calculated by Hao and co-workers¹⁶ by taking into account the depression of the order parameter in the vortex cores. This leads, then, to $\xi_{ab}(0) \approx 2$ nm, a value discarded, however, from our analysis of $\Delta\chi_{ab}(\varepsilon)$ above T_{c0} . These results on η and $\xi_{ab}(0)$ suggest, therefore, that the possible effects of the order-parameter depression in the vortex core play a much less important role in the vortex lattice than the one assumed in Ref. 16. In addition, our results strongly suggest that η does not depend on H , at least in the weak H limit, in contrast with the proposals of other authors.³¹

The other aspect of our results to be commented on here concerns the effective number, N_e , of fluctuating planes in the periodicity length, s . Until now, we have assumed strong magnetic coupling but weak Josephson coupling between the two closest CuO_2 layers per periodicity length, s , in our Bi-2212 crystals. In the case of the fluctuations of the vortex positions below T_{c0} , this leads to $N_e = 1$, in agreement with the BLK approach,^{13,15} whereas in what concerns the OPF above T_{c0} this leads to $N_e = 2$. As this last result is now well established from different measurements above T_{c0} in Bi-2212 crystals,^{6-8,33} an alternative scenario could be to suppose also $N_e = 2$ for the vortex fluctuations below T_{c0} (i.e., weak magnetic field coupling between adjacent CuO_2 planes and, therefore, completely independent fluctuating pancakes). As already indicated in Sec. III A, this scenario is equivalent to use $s/2 (= c/4)$ instead of s in the BLK equations. In the case of sample Bi1, the analysis of the measured T^* and ΔM_{ab}^* in terms of this last scenario [i.e., through Eq. (7), but with $s/2$ instead of s] will lead to $C \approx 4$ which will imply a too low (at about 25%) superconducting fraction for this crystal. Moreover, this scenario will also lead to $\xi_{ab}(0) \approx 1.1$ nm and $\eta = 0.3$, this last value still being very different from that calculated in Refs. 16 and 37. In the case of sample Bi2, this scenario with $N_e = 2$ below T_{c0} will lead to $C \approx 7$, which even for this low-quality sample seems to be too high. So, we must conclude here that the scenario we have used all along the paper, that is summarized in Table I, is the most plausible one for the thermal fluctuation effects on $M_{ab}(T, H)$ in the Bi-based HTSC. The in-plane paraconductivity and the magnetoconductivity measured above T_{c0} in Bi-2212 crystals may also be explained on the grounds of this scenario.³³ However, the confirmation of these conclusions through measurements in other HTSC systems and in single crystals with lower C factors will be very useful.

Let us, finally, briefly summarize here how our conclusions would be modified if $C = 1$ is imposed, i.e., if the corrections mainly associated with the presence of nonsuperconducting sample domains are neglected. In that case, to explain the behavior of the reversible magnetization below T_{c0} in the framework of the BLK approach for thermal fluc-

TABLE I. Characteristic parameters for the thermal fluctuation effects on the magnetization above and below the superconducting transition in the two Bi-2212 crystals studied here. N_e is the effective number of fluctuating superconducting planes in the periodicity length, s (which is equal to one-half the unit-cell length in the c direction): Above T_{c0} , N_e is related, through the *Josephson coupling* between adjacent layers, with the fluctuations of the order parameter amplitude (with the number of fluctuating Cooper pairs). Below T_{c0} , N_e is related, through the *magnetic coupling* between adjacent CuO_2 planes, with the fluctuations of the order-parameter phase (with the number of independent fluctuating magnetic pancakes in s). The N_e values suggest weak Josephson coupling but strong magnetic coupling between the two closest CuO_2 planes in s . $\lambda_{ab}(0)$ is the magnetic penetration length amplitude for H perpendicular to the CuO_2 layers, $\xi_{ab}(0)$ is the in-plane coherence length amplitude and η is the vortex lattice parameter.

Sample	T_{c0} (K)	s (nm)	N_e		$\lambda_{ab}(0)$ (nm)	$\xi_{ab}(0)$ (nm)	η
			above T_{c0}	below T_{c0}			
Bi1	93	1.54	2	1	180 ± 20	0.8 ± 0.1	0.15 ± 0.05
Bi2	99	1.54	2	1	170 ± 25	1.0 ± 0.3	0.30 ± 0.12

tuations of vortices (with $N_e = 1$), it would be necessary to assume an *effective* periodicity length for these fluctuations of 3.2 and 5.4 nm for, respectively, samples Bi1 and Bi2. These effective values are not only very different from each other but also they are unreasonably much bigger than the superconducting layer periodicity length (1.5 nm for Bi-2212). Above T_{c0} , the fluctuation-induced diamagnetism will still follow the 2D reduced temperature behavior but, with $s = 1.5$ nm and $N_e = 2$, the $\xi_{ab}(0)$ resulting values would be underestimated by a factor \sqrt{C} . This would lead to $\xi_{ab}(0) = 0.55$ nm and 0.58 nm for, respectively, samples Bi1 and Bi2. These new coherence length amplitudes are lower than any value proposed until now in the literature for Bi-2212 compounds. We may conclude, therefore, that the use for our samples of $C = 1$ would lead to unphysical results.

IV. CONCLUSIONS

We have presented in this paper detailed experimental data of the in-plane magnetization, for H perpendicular to the superconducting layers and in the weak amplitude limit, of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ crystals. These data were analyzed simultaneously and consistently, about and below the superconducting transition, in terms of thermal fluctuations. This analysis strongly suggests that in the case of the fluctuations of the vortex positions below T_{c0} , the two nearest planes per periodicity length have a strong magnetic coupling and, therefore, the pancakes in these planes will fluctuate together, as first proposed by Bulaevskii and co-workers.^{13,15}

In this case, the effective number of independent fluctuating superconducting CuO_2 layers is $N_e = 1$. In contrast, above T_{c0} , our results suggest a weak Josephson coupling between adjacent planes which, therefore, will lead to independent fluctuations of the superconducting order parameter, with $N_e = 2$, as first stressed by Klemm⁶ and Johnston and co-workers,⁸ and with a strong 2D behavior, as first proposed by Vidal and co-workers.³² In addition, by combining our results above and below T_{c0} , we obtained $\xi_{ab}(0) = (0.8 \pm 0.2)$ nm and $\eta = 0.15 \pm 0.05$. This last value is well within the one calculated in Ref. 36 by applying the London approach to a triangular vortex lattice. Although these results may be affected by the presence of some stoichiometric inhomogeneities in the Bi-2212 crystals measured here, our analysis seems to discard the values $\xi_{ab}(0) \approx 2.0$ nm and $\eta \approx 1$ proposed by other authors that have also analyzed the fluctuation effects on $M_{ab}(T, H)$ below the superconducting transition.^{15,20,21,24} For the in-plane magnetic penetration length, we found a temperature behavior compatible with the clean BCS weak-coupling limit, and an amplitude of $\lambda_{ab}(0) = (180 \pm 20)$ nm.

ACKNOWLEDGMENTS

This work has been supported by the Comisión Interministerial de Ciencia y Tecnología (MAT95-0279) and by the European Economic Community Grant No. CHRX-CT93-0325. One of us, J.M., acknowledges financial support from the Xunta de Galicia.

¹For earlier references on the thermal fluctuation effects above the superconducting transition in HTSC see, e.g., M. Akinaga, in *Studies of High Temperature Superconductors*, edited by A. Narlikar (Nova, New York, 1991), Vol. 8, p. 297; M. Ausloos, S. K. Patapis, and P. Clippe, in *Physics and Materials Science of High Temperature Superconductors II*, edited by R. Kossowsky, B. Raveau, D. Wohlleben, and S. K. Patapis (Kluwer, Dordrecht, 1992), p. 775.

²Some general aspects and early references on the effects of thermal fluctuations below the transition in HTSC, in the presence of vortex lines, may be seen in, e.g., G. Blatter, M. V.

Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, *Rev. Mod. Phys.* **66**, 1125 (1994); D. Feinberg, *J. Phys. III (France)* **4**, 169 (1994).

³See, e.g., W. J. Skocpol and M. Tinkham, *Rep. Prog. Phys.* **38**, 1049 (1975), and references therein.

⁴See, e.g., S. Ullah and A. T. Dorsey, *Phys. Rev. B* **44**, 262 (1991).

⁵W. C. Lee, R. A. Klemm, and D. C. Johnston, *Phys. Rev. Lett.* **63**, 1012 (1989).

⁶R. A. Klemm, *Phys. Rev. B* **41**, 2073 (1990).

⁷M. V. Ramallo, C. Torrón, and F. Vidal, *Physica C* **230**, 97 (1994).

- ⁸D. C. Johnston and J. H. Cho, Phys. Rev. B **42**, 8710 (1990); W. C. Lee, J. H. Cho, and D. C. Johnston, *ibid.* **43**, 457 (1991).
- ⁹C. Torrón, O. Cabeza, J. A. Veira, J. Maza, and F. Vidal, J. Phys. Condens. Matter **4**, 4273 (1992).
- ¹⁰C. Torrón, A. Díaz, A. Pomar, J. A. Veira, and F. Vidal, Phys. Rev. B **49**, 13 143 (1994).
- ¹¹M. Randeria and A. A. Varlamov, Phys. Rev. B **50**, 10 401 (1994).
- ¹²M. V. Ramallo *et al.* (unpublished).
- ¹³L. N. Bulaevskii, M. Ledvij, and V. G. Kogan, Phys. Rev. Lett. **68**, 3773 (1992).
- ¹⁴Z. Tešanović, L. Xing, L. N. Bulaevskii, Q. Li, and M. Suenaga, Phys. Rev. Lett. **69**, 3563 (1992).
- ¹⁵V. G. Kogan, M. Ledvij, A. Yu. Simonov, J. H. Cho, and D. C. Johnston, Phys. Rev. Lett. **70**, 1870 (1993).
- ¹⁶Z. Hao, J. R. Clem, M. W. McElfresh, L. Civale, A. P. Malozemoff, and F. Holtzberg, Phys. Rev. B **43**, 2844 (1991); Z. Hao and J. R. Clem, Phys. Rev. Lett. **67**, 2371 (1991).
- ¹⁷F. Zuo, D. Vacaru, H. M. Duan, and A. M. Hermann, Phys. Rev. B **47**, 8327 (1993).
- ¹⁸J. H. Cho, D. C. Johnston, M. Ledvij, and V. G. Kogan, Physica C **212**, 419 (1993).
- ¹⁹J. R. Thompson, J. G. Ossandon, D. K. Christen, B. C. Chakoumakos, Y. R. Sun, M. Paranthaman, and J. Brynstad, Phys. Rev. B **48**, 14 031 (1993).
- ²⁰G. Triscone, A. F. Khoder, C. Opagiste, J.-Y. Genoud, T. Graf, E. Janod, T. Tsukamoto, M. Couach, A. Junod, and J. Muller, Physica C **224**, 263 (1994).
- ²¹A. Schilling, R. Jin, J. D. Guo, and H. R. Ott, Physica B **194-196**, 2185 (1994).
- ²²Z. J. Huang, Y. Y. Xue, R. L. Meng, X. D. Qui, Z. D. Hao, and C. W. Chu, Physica C **228**, 211 (1994).
- ²³J. R. Thompson, D. K. Christen, and J. G. Ossandon, Physica B **194-196**, 1557 (1994).
- ²⁴G. Triscone, A. Junod, J.-Y. Genoud, C. Opagiste, T. Tsukamoto, and J. Muller, J. Phys. Condens. Matter **6**, L399 (1994).
- ²⁵Q. Li, K. Shibusaki, M. Suenaga, I. Shigaki, and R. Ogawa, Phys. Rev. B **48**, 9877 (1993).
- ²⁶U. Welp, S. Fleshler, W. K. Kwok, R. A. Klemm, V. M. Vinokur, J. Downey, B. Veal, and G. W. Crabtree, Phys. Rev. Lett. **67**, 3180 (1991).
- ²⁷Q. Li, M. Suenaga, T. Hikata, and K. Sato, Phys. Rev. B **46**, 5857 (1992).
- ²⁸Some of our preliminary results on the thermal fluctuation effects on $M(T,H)$ on both sides of the transition were presented in the M^2 HTSC IV Conference: E. G. Miramontes, J. A. Campá, A. Pomar, I. Rasines, C. Torrón, J. A. Veira, and F. Vidal, Physica C **235-240**, 2931 (1994).
- ²⁹J. A. Campá, E. Gutierrez-Puebla, M. A. Monge, I. Rasines, and C. Ruiz-Valero, J. Cryst. Growth **127**, 17 (1992).
- ³⁰J. C. Martinez, P. J. E. M. van der Linden, L. N. Bulaevskii, S. Brongersma, A. Koshelev, J. A. A. J. Perenboom, A. A. Menovsky, and P. H. Kes, Phys. Rev. Lett. **72**, 3614 (1994); L. N. Bulaevskii, J. H. Cho, M. P. Maley, P. H. Kes, Q. Li, M. Suenaga, and M. Ledvig, Phys. Rev. B **50**, 3507 (1994); L. N. Bulaevskii, M. P. Maley, and J. H. Cho, Physica C **235-240**, 87 (1994).
- ³¹R. Puźniac, J. Ricketts, J. Schützmann, G. D. Gu, and N. Koshizuka, Phys. Rev. B **52**, R7042 (1995).
- ³²F. Vidal, J. A. Veira, J. Maza, F. Garcia-Alvarado, E. Morán, J. Amador, C. Cascales, A. Castro, M. T. Casais, and I. Rasines, Physica C **156**, 807 (1988); J. A. Veira and F. Vidal, Phys. Rev. B **42**, 8748 (1990).
- ³³A. Pomar *et al.* (unpublished).
- ³⁴B. Mühlshlegel, Z. Phys. **155**, 313 (1959).
- ³⁵M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1980), p. 80.
- ³⁶A. L. Fetter, Phys. Rev. **147**, 153 (1966); see also, A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988), p. 425. (See also Ref. 12 of Ref. 4.)
- ³⁷See, e.g., P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966), p. 71.