Topological defects in the random-field XY model and the pinned vortex lattice to vortex glass transition in type-II superconductors

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As a simplified model of randomly pinned vortex lattices or charge-density waves, we study the random-field XY model on square ($d=2$) and simple cubic ($d=3$) lattices. We verify in Monte Carlo simulations that the average spacing between topological defects (vortices) diverges more strongly than the Imry-Ma pinning length as the random field strength H is reduced. We suggest that for $d=3$ the simulation data are consistent with a topological phase transition at a nonzero critical field H_c to a pinned phase that is defect free at large length scales. We also discuss the connection between the possible existence of this phase transition in the random-field XY model and the magnetic-field-driven transition from a pinned vortex lattice to a vortex glass in weakly disordered type-II superconductors. [S0163-1829(96)06421-1]

I. INTRODUCTION

There is considerable current interest in the effects of quenched disorder on ordered phases with continuous symmetry. Examples are vortex lattices in type-II superconductors[1,2] and spin- and charge-density-wave systems subject to random pinning,[3] as well as amorphous ferromagnets with random anisotropy[4] and liquid crystals in porous media.[5] The random pinning induces continuous, elastic distortions of the ordered state and it may also induce plastic deformation due to topological defects, such as dislocations, which do not represent continuous distortions of the ideal ordered state. The distortions induced by random pinning in the absence of topological defects can be treated within an elasticity theory, and have received an ever-growing amount of attention over the years.[6–14] Hence, it is important to assess the regime of validity of these approaches that assume that the topological defects are not present. To do so, we focus here specifically on the production of topological defects by the random pinning in equilibrium.[14,15]

The simplest system in which to study these issues appears to be the ferromagnetic XY model with a random field. Here the long-range order in the pure system is ferromagnetism, the pinning is due to the random field, and the topological defects are vortices in the magnetization pattern. The Hamiltonian we consider is

$$H = -\sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_i \mathbf{h}_i \cdot \mathbf{S}_i, \tag{1.1}$$

where the first sum is over all nearest-neighbor pairs of lattice sites, $\mathbf{S}_i$ is a unit-length, two-component (XY) spin at site i, and the static random fields $\mathbf{h}_i$ have rms magnitude H. The ground state of (1.1) evolves from being ferromagnetic and vortex free for $H=0$ to a state with all spins aligned with the random fields, and thus a dense array of vortices, for large H. In the context of vortex lattices, the

$H=0$ limit corresponds to an unpinned Abrikosov lattice, while large H corresponds to a vortex-glass ground state at strong pinning.[1,2,16] The connections between this model and vortex lattices are discussed in more detail in Sec. IV below.

Let us first consider a small random field H, following Imry and Ma.[7] Treating the random field as a perturbation, the long-range-ordered ferromagnetic phase is stable at small H only for spatial dimension $d>4$. For $d<4$ the static elastic relative spin rotations induced by the random field are of order one at the pinning length $\xi_P \sim H^{-2/(4-d)}$. The behavior at longer scales is not accessible by simply perturbing in H. This perturbative treatment only considers continuous elastic distortions of the uniform state, not vortices, which are nonperturbative. Let us ask about the system's stability at small H and length scale $L \lesssim \xi_P$ to static vortex-antivortex pairs in $d=2$ or to vortex loops in $d=3$. The vortices permit the system to align better with the random field on a length scale L. This, naively, lowers the random-field pinning energy by an amount of order $HmL^{d/2}$, where m is the magnetization density at $H=0$. However, the added energy of the elastic strains around the vortices is of order $KL^{(d-2)}|\ln(L)|$, where K is the spin stiffness. Even at $L = \xi_P$, the elastic energy cost is larger than the typical pinning energy gain by a factor of $|\ln(H)|$. Hence, the system appears to be stable against vortices for small H at length scales less than or of order $\xi_P(H)$.[12,17] (Note, however, that there will be a low density of isolated vortex pairs for $d=2$ and loops for $d=3$ that are induced by unusually strong local configurations of the random field.) Thus we expect that at small H, the smallest length scale at which the equilibrium system is unstable to a proliferation of static vortices, ξ_V, is larger than the pinning length ξ_P and *possibly infinite*. Recently, theoretical progress has been made in understanding the essential features of the physics at play in the pinned, vortex-free length scale regime intermediate between ξ_P and ξ_V.[10–13] However, other than the above lower bound predicted for ξ_V, how ξ_V depends on H and when and
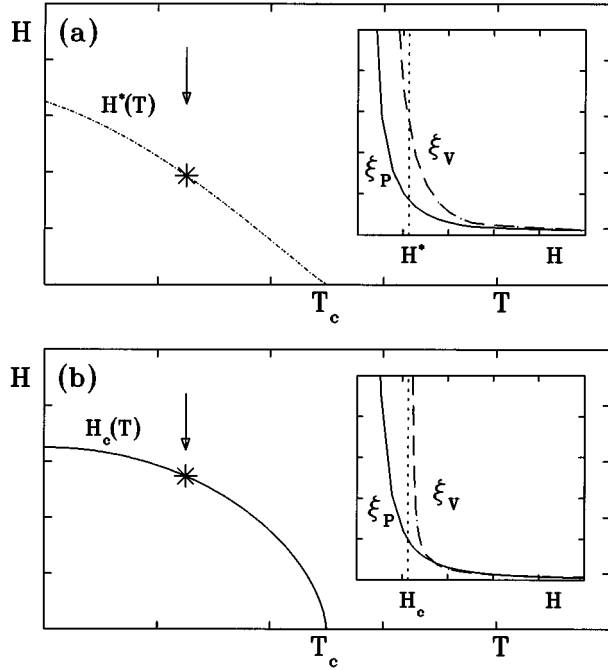
FIG. 1. Schematics of possible phase diagrams for the random-field XY model, and (insets) dependences of the characteristic lengths ξ_P and ξ_V on the random-field strength H along the paths marked by the arrows. In (a) there is no phase transition at nonzero H. The average vortex spacing ξ_V diverges more strongly than the pinning length ξ_P with decreasing random field, but both lengths remain finite as long as $H>0$. At some field H^* that depends on temperature T, the lengths and relaxation times become large enough that equilibrium can no longer be attained, and so $H^*(T)$ is a type of kinetically determined glass transition, whose location depends on the time scales of the experiment or simulation. In (b) there is a true equilibrium thermodynamic phase transition at $H_c(T)$, where ξ_V diverges. The region $0<H<H_c(T)$ is the topologically ordered, pinned phase that is vortex free at large length scales.

how the static vortices proliferate at length scales $L>\xi_P$ is far from well understood.[12,14] This is what we investigate in this paper.

If vortices are forbidden, then the equilibrium long-distance spin-spin correlation function is expected to decay as a power law.[10–12, 17] This behavior should also apply in the distance range between ξ_P and ξ_V where the system is strongly pinned but still largely vortex free. The above argument says that $\xi_V \gg \xi_P$ for small H, and so this intermediate-distance regime does exist. The true correlation length, beyond which the correlations decay exponentially, should be of order ξ_V. There are two possible behaviors for the vortex spacing $\xi_V(H)$: (a) It may diverge only at $H=0$, but with a stronger divergence than the pinning length ξ_P, or (b) it could diverge at a nonzero critical field H_c. These two possibilities are schematically illustrated in Fig. 1. Our simulation results below appear consistent with $H_c=0$ for $d=2$, but with $H_c>0$ for $d=3$. When this second possibility (b) occurs, there is at low temperatures an intermediate pinned phase for $0<H<H_c(T)$ that is vortex free at the largest length scales, and therefore has topological long-range order.[12] This ordered pinned phase, if it exists, has power-law decay of the long-distance spin-spin correlations, and is separated from the disordered and plastic phases at higher H by a topological phase transition at H_c where large-scale static vortices first appear.

Let us now discuss what are the fixed points governing a hypothetical renormalization-group flow from various portions of the (H,T) phase diagram in a scenario where there is a topologically ordered phase for $0<H<H_c(T)$ at low temperatures. For the ferromagnetic random-field XY model the important energy scales are the temperature T, the spin stiffness J, the random field H, and the core energy per unit length of a vortex line, E. (There is also the uniform field, which we do not consider here). The fixed points are summarized in Table I.

First, let us consider the stability of the pure XY model in the regions $T>T_c$, $T=T_c$, and $T<T_c$ with $H=0$ against

TABLE I. Summary of the various fixed points in the (H,T) phase diagram of the three-dimensional random-field XY model in the scenario where there is a topologically ordered phase in the parameter range $0<H<H_c(T)$ and $T<T_c$. See the text for more discussion.

Portion of (H,T) phase diagram	Governing fixed point	Relevant operators
$H=0$, $T>T_c$	$J=E=H=0$, $T>0$	only H/T is relevant
$H=0$, $T=T_c$	$H=0$, $T\sim J\sim E>0$	H/J and T/J are relevant
$H=0$, $T<T_c$	$H=T=0$, $E\gg J>0$	only H/J is relevant
$H>H_c(T)$	$J=E=0$, $H/T>0$	nothing is relevant
$H=H_c(T)$, $T<T_c$	$T=0$, $H\sim E\gg J$	only H/E is relevant
$0<H<H_c(T)$	$T=0$, $E\gg H\gg J$	nothing is relevant

infinitesimally small H. In the fully disordered paramagnetic phase the random field is in a sense marginal, since the frozen-in magnetization induced by the random field and the rms thermally fluctuating magnetization are both of the order of the square root of the number of sites, $L^{d/2}$ (where L is the length scale considered), with a ratio which varies continuously as one varies H/T. Thus it would appear that the fully disordered phase is governed by a trivial fixed line of which the $H/T=0$ paramagnetic fixed point is a special, higher-symmetry point. For the $(H=0,T=0)$ fixed point that governs the low-temperature $H=0$ ferromagnetic phase, one has $H/T=0$, but J/T and E/T are both infinite, with E/T larger than J/T by a factor of order $\ln(L)$, due to the logarithmically divergent energy of a vortex line. Here H/J is relevant for $d<4$, as argued by Imry and Ma. For the $(H=0,T=T_c)$ fixed point, $H/T=0$, and J/T and E/T are of order one. For $H=0$, T/J is obviously a relevant operator at the (unstable) nontrivial $T=T_c$ critical fixed point of the pure model. H/J is also relevant at $T=T_c$. One can obtain the scaling of the correlation length at $T=T_c$ and $H>0$ by comparing the random-field free energy scale $Hm(L)L^{d/2}$ with the spin stiffness on length scale L, which is L independent at $T=T_c$. Since at criticality $m(L)\sim L^{-(d-2+\eta)/2}$, we find, taking $\xi=L$, that $\xi(H,T=T_c)\sim H^{-2/(2-\eta)}$, where η is the usual critical exponent for the spatial decay of the correlations of the pure XY model at $T=T_c$ and $H=0$.

Now, if an intermediate topologically ordered phase exists, it is governed by a zero-temperature fixed point (as in other random-field problems) where H/T is infinite, but there E/T is also infinite and much larger than H/T (since large-scale vortex loops must be absent to ensure topological order). J/T might well be zero at that fixed point (or at least much smaller than H/T). At the fixed point governing the $H>0$ critical line, $H=H_c(T)>0$, H/T and E/T would be comparable (probably both infinite) and again J/T much smaller or even zero. For the fixed point governing the disordered phase, $H>H_c(T)$, one has $J=E=0$ and only $H/T>0$, and none of the scaling fields are relevant, although, as discussed above, H/T is in some sense marginal here.

We have performed Monte Carlo simulations of the random-field XY model (1.1) on simple cubic ($d=3$) and square ($d=2$) lattices. In both cases we find that, as expected by the above heuristic arguments and also suggested by Giamarchi and Le Doussal,[12] the spacing between vortices diverges more strongly with decreasing H than the pinning length ξ_P obtained from the Imry-Ma argument. For $d=3$, the data appear consistent with a transition to a topologically ordered phase at a nonzero critical field $H_c>0$, with the correlation length and vortex spacing diverging as a power of $(H-H_c)$. For $d=2$, on the other hand, the vortex density is better fit by a power law in H itself, indicating that there is no intermediate phase (i.e., $H_c=0$). For $d=3$ we have tried to more precisely locate the proposed phase transition at H_c using various forms of finite-size scaling involving moments of the magnetization distribution and the derivative of the magnetization with respect to H, but have been unsuccessful in obtaining a convincing one-parameter scaling $L/\xi(H)$, where L is the linear size of the samples. We suspect that this lack of simple finite-size scaling may be

due to the presence of two distinct diverging length scales in this problem, namely, ξ_P and ξ_V, which complicates the finite-size scaling.[18]

The rest of the paper is organized as follows: The details and results of our Monte Carlo simulations are presented in the next section. Section III contains a discussion of the possible nature of the phase transition in $d=3$ and of the underlying topologically ordered phase at small random field. The connections between the random-field XY model and vortex lattices and the possible existence of two thermodynamically distinct superconducting ''glassy'' vortex phases in type-II superconductors are discussed in Sec. IV. Section V contains a brief conclusion.

II. SIMULATIONS AND RESULTS

Here we report results from simulations of the random-field XY model on large lattices (up to 10^6 spins) in the temperature and field range where we could obtain true thermodynamic equilibrium. We simulated two copies (replicas) of each sample, one with a ferromagnetic initial condition and one with the spins initially aligned with the random field at each site. We took care to only use late-time results where both replicas give the same time-independent averages. At the lowest fields studied, this required up to 10^5 Monte Carlo steps per spin (single-spin rotations, Metropolis algorithm). We studied the model with independently Gaussian-distributed random fields at each site $[\mathbf{h}_i]=0$, $[\mathbf{h}_i\cdot\mathbf{h}_j]=H^2\delta_{ij}$, where the square brackets represent an average over the distribution of random fields. We measured the time-averaged magnetization $\mathbf{m}_i=\langle\mathbf{S}_i\rangle$ at each site. The angle between the magnetization vectors $\mathbf{m}_i$ on each nearest-neighbor pair of sites was obtained, with the convention that it lies between $-\pi$ and π. For each elementary square plaquette, these angles were added to obtain the total rotation of the magnetization on moving around the plaquette. This sum is a multiple of 2π; if it is nonzero, then there is a vortex in that plaquette in the equilibrium (static) magnetization pattern. We also measured the correlation function $g(r)=[\mathbf{m}_i\cdot\mathbf{m}_j]$, for pairs of sites i and j that are separated by a distance r along a lattice axis.

We wanted to study the ordered-phase (low-temperature) behavior, but be at a high enough temperature that we can equilibrate in not too much computer time. For $d=3$, where the critical point in the absence of the random field ($H=0$) is at $T_c^{3D}\cong2.2$,[19] we examined $T=1.5$, which is sufficiently far below T_c^{3D} to avoid the $H=0$ critical regime. For $d=2$, where $T_c^{2D}\cong0.9$,[20] we worked at $T=0.7$. The quantities we have measured are all self-averaging, and so they can be accurately determined from a single sample, provided it is large compared to the correlation lengths. We generally simulated more than one sample and the sample-to-sample differences were small, as expected. We have also varied the sample size to check that any finite-size effects in the data reported are smaller than the statistical errors.

The fraction f_V of elementary square plaquettes occupied by static vortices is shown vs H in Fig. 2. The solid ($d=2$) and dashed ($d=3$) lines indicate what the slopes would be if the intervortex spacing was the pinning length ξ_P given by the Imry-Ma argument, and so $f_V\sim\xi_P^{-2}$. The argument given in the Introduction says that f_V should van-
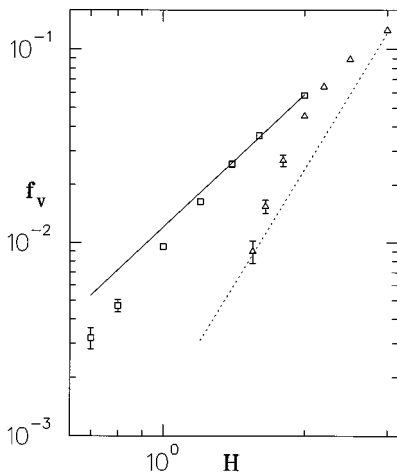
FIG. 2. The fraction of plaquettes occupied by vortices vs the rms random-field strength for the random-field XY model. Triangles are $d=3$, $T=1.5$; squares are $d=2$, $T=0.7$. The slopes of the lines on this log-log plot are 4 (for $d=3$) and 16/7 (for $d=2$), as given by naive extensions of the Imry-Ma argument (see text). These data are, in most cases, from simulations of two large samples (10^5–10^6 spins), and so the statistical errors, indicated by the error bars, are only roughly estimated.

ish more rapidly than this with decreasing H, and the data clearly support this. For $d=3$, the vortex density is not well approximated by a power of H over any substantial field range in $H > 1.5$, the range we can equilibrate.

In Fig. 3(a) we show the correlation function for $d=3$, which is very well fit by a simple exponential $g(r) \sim \exp(-r/\xi)$. The measured correlation length ξ also diverges substantially faster than the Imry-Ma estimate of the pinning length, $\xi_P \sim H^{-2}$, as H is decreased. We find that the vortex density f_V equilibrates much earlier in the simulation than the long-distance correlation function; hence we have results that we trust for f_V to slightly lower field than for $g(r)$ and ξ.

One naively expects that if there is a transition to a topologically ordered phase for $d=3$ at a nonzero H_c (which depends on T), the correlation length ξ diverges as a power of $(H-H_c)$. In Fig. 4 we show that our data for $T=1.5$ are consistent with such a critical behavior with $H_c \cong 1.35$. In fact, the vanishing of the vortex density, f_V, is also consistent with such a power law. However, such a power law for f_V should not hold all the way to H_c because a small density of small isolated vortex loops should be present even in the ordered phase, due to rare, strong local random-field configurations. The fact that f_V is vanishing almost as fast as ξ^{-2} suggests that these small loops represent only a small fraction of the full vortex density over the field range studied here. We did not attempt to separate the population of vortices into small and large loops for $d=3$ (but see below for $d=2$). The apparent exponents with $H_c=1.35$, indicated by the solid lines in Fig. 4, are $\xi \sim (H-H_c)^{-\nu}$ with $\nu \cong 0.85$, and $f_V \sim (H-H_c)^\rho$ with $\rho \cong 1.4$. Note that the range of the scaling fits for all the quantities in Fig. 4 is less than one decade, and so this apparent scaling should not be taken too seriously. But we can definitely say that the data for ξ and f_V in this field range are very different from a power-law critical point with $H_c=0$ and are consistent with a power-
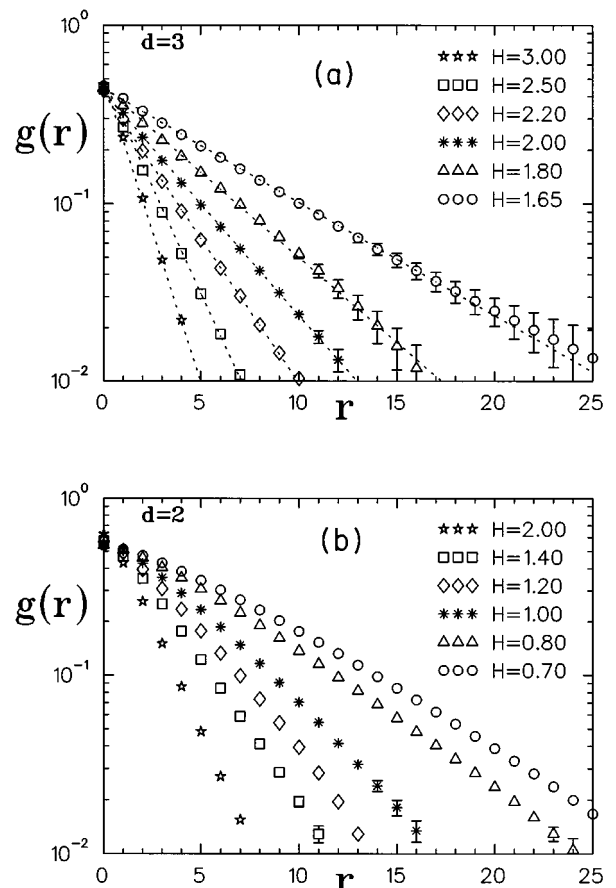

FIG. 3. The magnetization correlation function versus distance for (a) $d=3$, $T=1.5$, and (b) $d=2$, $T=0.7$, for the indicated random-field strengths. The dotted lines in (a) are fits to simple exponentials. The error bars, where shown, indicate variations between two and three large samples.

law critical point with H_c near 1.3.

For $d=2$ the Imry-Ma argument gives $\xi_P \sim 1/H$ at $T=0$. At finite temperature, the magnetization at scale L, $m(L)$, is renormalized by thermal fluctuations in the critical phase be-
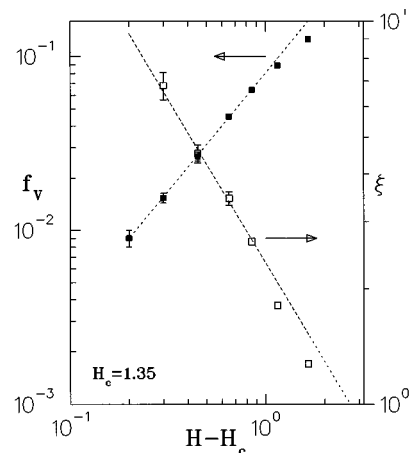

FIG. 4. For $d=3$, $T=1.5$, and $H_c=1.35$ the vortex density f_V (solid symbols) and correlation length ξ (open symbols) vs $(H-H_c)$ on a log-log plot. ξ is obtained from the simple exponential fits in Fig. 2(a).

low the Kosterlitz-Thouless temperature T_{KT} [$m(L)$ $\sim L^{-(d-2+\eta)/2}$ with $d=2$]. When the Imry-Ma argument is modified to take this into account one obtains $\xi_P \sim H^{-2/(2-\eta)}$. If we then take the largest value $\eta = 1/4$ at the Kosterlitz-Thouless transition temperature,[21] we obtain the maximum possibly Imry-Ma slope 16/7 ($f_V \sim \xi_p^{-2}$), indicated by the solid line near the $d=2$ data in Fig. 2. As expected, the vortex density for $d=2$ is found to vanish even faster than this with decreasing H. This again indicates that, as found in $d=3$, the vortex spacing diverges more strongly than the pinning length given by the Imry-Ma argument. However, for $d=2$ the low-field behavior is quite consistent with a power law in H, just with a larger exponent (roughly $f_V \sim H^3$), rather than a phase transition at nonzero H_c. We have checked that even for the lowest H we could equilibrate, the majority of the vortices are well separated; less than half are in closely spaced vortex-antivortex pairs. (Note that thermally excited vortices are not present in our measurements because we time average to obtain the *static* magnetization pattern.) The correlation function for $d=2$ [Fig. 3(b)] and low field does not fit a simple exponential as found in $d=3$, and so it is not obvious how one should define the correlation length.

III. DISCUSSION

We now give a heuristic argument for why rare, strong-pinning regions should prevent the proposed topologically ordered thermodynamic phase from being stable at $H>0$ for $d=2$. In $d=2$ the vortices are point defects. The elastic energy cost of a vortex is finite, being proportional to $\ln(\xi_P)$, due to integrating the elastic energy from the lattice spacing out to the pinning length ξ_P. Beyond ξ_P the system is strongly pinned and no longer behaves as an elastic medium. The local random-field pattern over any given area of order ξ_P^2 or larger has a nonzero probability of favoring the presence of a vortex by enough to compensate this finite elastic energy cost of the vortex, thus forcing in a vortex there in the ground state. For example, consider the extreme random-field configuration that has the random fields in a vortex pattern out to distance ξ_P. This random-field pattern, which occurs with a nonzero probability for finite ξ_P, favors a spin pattern that contains a vortex over one without a vortex by an energy proportional to ξ_P, for $T=0$. In an infinite sample, the density of occurrences of rare, special random field patterns that induce vortices in the ground state will be nonzero as long as $H>0$. Therefore, for $d=2$ there cannot be a vortex-free equilibrium phase in the thermodynamic limit of an infinite sample, except at $H=0$. Our data are quite consistent with this conclusion.

A similar conclusion does not apply for $d=3$, where the vortices are lines so that the elastic energy cost of a vortex loop is proportional to the length (perimeter) of the loop. For such a vortex loop to be present at equilibrium, this elastic energy cost must be compensated by a larger pinning energy that favors the presence of the vortex loop. Naively, for small random field H, the probability of such a large pinning energy occurring falls off exponentially with the length of the loop, and thus vanishes in the limit of a large loop, because it would require a rare, strong-pinning configuration

favoring the vortex that extends along the entire length of the loop. (In a recent paper, Kierfeld *et al.*[14] use a scaling argument and a variational calculation to argue that the algebraic translational order of the vortex lattice predicted by Giamarchi and Le Doussal[12] should be stable at small disorder against the proliferation of large dislocation loops.) Thus we see that the vortex-free phase may be stable against rare, strong-pinning configurations for $d=3$. However, this is just a heuristic argument, and there remains the possibility that some sort of random-field configuration does destabilize the vortex-free phase even at arbitrarily small $H>0$. One scenario is that the $T=0$ correlation length ξ could vary faster than any power of H with a form $\xi \sim \exp(1/H^\mu)$, with some exponent μ. Our data certainly do not rule out this possibility.

How should one think about the physics at length scales beyond ξ_P? One proposal is the following: Consider first $d=2$, for concreteness. First, forbid vortices and find the lowest-energy state satisfying this no-vortex constraint. This state is pinned with some particular nontrivial spin pattern resulting from the competition between the random-field energy at each site and the ''elastic'' exchange energy. Now consider a patch of linear size L (area L^2), with $L \gg \xi_P$. Introduce a vortex and an antivortex separated by a distance of order L and, within this patch, choose their locations and the spins' orientations to minimize the energy with this pair present. For $L < \xi_V$ this new energy is presumably typically higher than the lowest-energy vortex-free state, but for $L > \xi_V$ it is typically lower, and so the true ground state has typical vortex spacing ξ_V.

What does the unconstrained ground state, with vortex separation ξ_V, look like? The *relative* spin orientation between it and the lowest-energy vortex-free state must rotate by 2π on encircling any vortex. But the system (for $\xi_V \gg \xi_P$) is strongly pinned, and so it will typically cost a lot of energy to locally rotate the spins away from their local ground state. Thus we expect that the relative spin rotation will be concentrated in line defects (like sine-Gordon solitons or domain walls) that each extend from a vortex to an antivortex. Thus to find the ground state one must optimize not only over the positions of the vortices but also of these line defects (these defects are present only in the *relative* spin orientations and are presumably of width of order ξ_P). Since the line defects are not permitted in the vortex-free state, they can have negative energy relative to the lowest-energy vortex-free state once their positions are optimized. Generally, it must require a length of defect line with end-to-end distance at least of order ξ_V for its negative energy to be enough to ''pay for'' the positive energy of the vortex cores. This picture provides an energy-balance mechanism that can set the density of topological defects.

For $d=3$, instead of vortex-antivortex pairs one has vortex loops, and instead of defect lines one has defect surfaces that span the vortex loops or extend from loop to loop. Again an optimally positioned defect surface spanning a vortex loop can have a negative energy whose magnitude increases as the loop grows, but now the positive core energy of the vortex loop also grows in proportion to its perimeter. The vortex-free phase will occur if the negative defect energy typically increases in magnitude more slowly with L than the loop's core energy, and so the defect surface is generally not
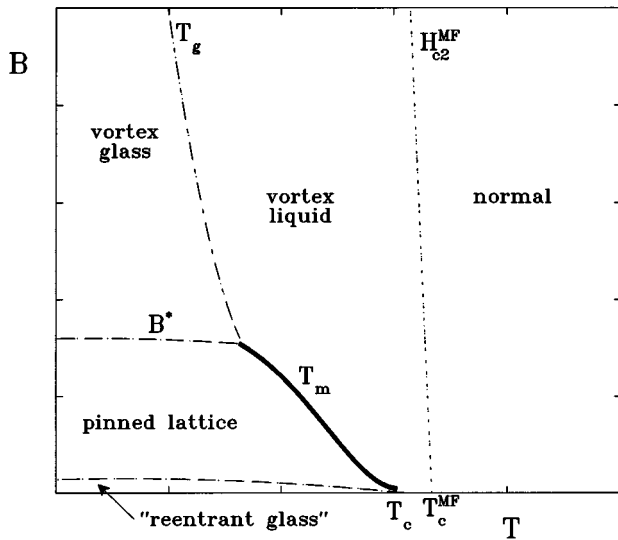
FIG. 5. Internal magnetic field B vs temperature T schematic phase diagram for a layered type-II superconductor with strong thermal fluctuations and weak random pinning. For an applied field less than the lower critical field $\mathscr{H}_{c1}$, the system is in the Meissner phase with $B=0$. The true zero-field critical temperature T_c is depressed from the mean-field estimate T_c^{MF} by thermal fluctuations. For large B and T, the system is in the normal (nonsuperconducting) state. Below $\mathscr{H}_{c2}^{MF}$, the system is in the so-called vortex-liquid state. Here it exhibits some local pairing and an increased conductivity due to superconducting fluctuations, but no long-range off-diagonal order. The vortex liquid has a nonzero Ohmic resistivity. $\mathscr{H}_{c2}^{MF}$ is not a thermodynamic phase transition, and so the vortex-fluid phase is not a distinct phase from the normal state. In absence of any disorder, the vortex fluid freezes into an Abrikosov vortex lattice via a first-order transition at T_m. For weak disorder and small field this first-order melting transition remains and the system enters the superconducting, pinned vortex-lattice phase, which we propose is devoid of large-scale lattice dislocations at equilibrium. For low temperatures at $B>B^*(T)$, the random pinning induces dislocations and the system instead enters the amorphous vortex-glass phase that we argue may be thermodynamically distinct from the pinned vortex lattice. By increasing the microscopic disorder, the vortex glass to pinned lattice phase boundary $B^*(T)$ moves to smaller fields, reducing the range of B where the pinned lattice exists and eventually eliminating this phase altogether for strong enough pinning. At very low fields where the vortices are far apart compared to the magnetic penetration length there is also an amorphous glass phase due to the vortex-vortex interactions becoming small compared to the random pinning; this is indicated as ''reentrant glass.'' Here, for simplicity, we have shown the lines B^*, T_g, and T_m, all meeting at a multicritical point. Other topologies of the phase diagram, including possibly the proposed ''vortex slush'' regime (Ref. 26), are also possible (Ref. 24).

able to ''pay for'' the vortex loop (again, except for a low density of small loops that are present due to anomalously strong local pinning configurations). Similar scenarios (for $d=2$ and 3) may also apply to random-*anisotropy XY* models.[4,22]

IV. VORTEX LATTICES

We now comment on the connection between the $d=3$ random-field XY model and the Abrikosov vortex lattice in type-II superconductors with uncorrelated random pinning.

There are at least two aspects of the vortex lattice that are not captured by the XY model. The single continuous degree of freedom in the XY model, the spin orientation, plays the role of the vector displacement of the vortex lattice. One consequence of this simplification is that the topological charge of a vortex in the XY model is a scalar (how much the spin orientation winds upon encircling the vortex), while that of a dislocation in the vortex lattice is the two-component Burgers vector. The other simplification is that the superconductor has the additional U(1) symmetry of the complex scalar Ginzburg-Landau order parameter ψ under rotations in the complex plane; this symmetry is absent in the random-field XY model. This U(1) symmetry can be spontaneously broken in the absence of any vortex-lattice order in the superconductor, yielding the vortex glass phase.[1,2,16] The random-field XY model has no analogous ordered phase. Another consequence of the U(1) symmetry is that in the vortex lattice an interstitial (or vacancy), being a change in the vortex number, is itself a topological defect; the random-field XY model has no analogous defect.

The topologically ordered pinned phase at intermediate disorder H that we discuss in this paper for the $d=3$ random-field XY model would correspond for a superconductor to a pinned, Abrikosov vortex-lattice phase that is dislocation free at large length scales.[16] In the latter system, the structure factor $S(\mathbf{q})$ would have power-law singularities at the basic reciprocal lattice vectors $\mathbf{Q}$ of the form $S(\mathbf{q}) \sim |\mathbf{q}-\mathbf{Q}|^{-(2-\eta)}$.[11,12] By increasing the disorder H in the random-field XY model, we drive the system into the fully disordered phase. In that case, the disordered phase is not thermodynamically distinct from the high-temperature paramagnetic phase. For the superconductor, on the other hand, there are at least two noncrystalline phases: the superconducting vortex-glass phase at zero and/or low T and the resistive vortex-liquid phase at higher T. The higher-temperature transition directly from the vortex-lattice phase to the vortex liquid involves loss of both crystalline and superconducting order, and so is certainly not fully modeled by the random-field XY model. In the superconductor, for weak pinning this melting transition is first order;[23–25] the XY model shows no such transition. For example, the XY model would not display an analogue of the ''vortex-slush'' phase that has been discussed based on some transport measurements.[26] However, for the zero-temperature transition from the pinned vortex lattice to the vortex glass, both phases have off-diagonal long-range (superconducting) order,[27] and so the superconducting order could be effectively just a bystander, and the random-field XY model, which ignores this order, might capture the essential physics of this transition.

It has recently been suggested that there could be (at least) *two* types of glass phases caused by point impurities in type-II superconductors,[12,14] i.e., two types of vortex-glass phases. Indeed, there might already exist indirect experimental evidence for two distinct superconducting phases in clean crystals of Y-Ba-Cu-O and Bi-Sr-Ca-Cu-O high-T_c superconductors. At low applied magnetic fields, experiments see the first-order melting transition of the vortex lattice.[23–25, 28] At higher fields, the effective random pinning appears stronger,[29] and the superconducting transition is continuous,

as expected for the melting of a vortex glass. For Bi-Sr-Ca-Cu-O, a neutron-scattering study saw the lattice Bragg peaks in the low-field regime but not in the high-field regime.[30] A similar field-driven transition has been observed in muon spin resonance (μSR) experiments on Bi-Sr-Ca-Cu-O.[31] This is all consistent with these samples having the pinned vortex-lattice phase in the low-field regime and the amorphous vortex-glass phase in the high-field regime. Also, similar to what has been found in Bi-Sr-Ca-Cu-O,[31] recent μSR results on YBa$_2$Cu$_3$O$_{6.6}$ show a rapid modification of the μSR line shape above a critical field.[32] Some signs of a transition between these two superconducting phases have recently been seen in the nonlinear transport properties Y-Ba-Ca-O,[33] and in DyBa$_2$Cu$_3$O$_7$/(Y$_{1-x}$Pr$_x$)Ba$_2$Cu$_3$O$_7$ superlattices.[34] The neutron[30] and μSR (Refs. 31,32) results have been interpreted in terms of a field-driven $d=3$ to $d=2$ crossover as discussed by Glazman and Koshelev.[2,35] However, in presence of disorder, this crossover makes the effective pinning potential (relative to the vortex-vortex interactions) change rapidly around the crossover field value,[29] which may transform it into a true thermodynamic phase transition. This transition, we argue, is characterized by the proliferation of dislocations in the lattice, reducing it to the amorphous vortex glass (see Fig. 5). It is this transition that is mimicked by the random-field XY model we have studied here. In the ideal pure disorder-free system, there is no true structural phase transition in the vortex lattice at the dimensional crossover at zero temperature. So, in this sense, in the presence of the disorder, the vortex-lattice to vortex-glass transition observed experimentally[23(b),30–34] is more than ''just'' the $d=3$ to $d=2$ crossover.[35] Finally, it has recently been reported that the first-order melting in Y-Ba-Ca-O is destroyed by point defects caused by electron irradiation, giving rise to a second-order transition.[36] Interestingly, the critical exponents measured in these electron-irradiated samples differ largely from those measured for the vortex-glass transition.[2] This may indicate that a sufficiently large density of point defects have destroyed the first-order vortex-lattice melting transition, and converted it into a second-order transition from a vortex-liquid to pinned vortex-lattice, as opposed to a vortex liquid to vortex glass transition that would occur at even larger density of point defects (or at larger applied magnetic fields).

V. CONCLUSION

In conclusion, we have performed extensive Monte Carlo simulations of the $d=2$ and $d=3$ random-field XY model. In both dimensions, the spacing between static vortices grows faster than the pinning length obtained from the Imry-Ma argument as the random-field amplitude is decreased. In $d=3$, our results appear to be consistent with a phase transition at nonzero critical random field into a topologically ordered (vortex-free) phase with power-law decay of the spin-spin correlation function. We have also discussed the relationship between this possible phase transition and the pinned vortex-lattice to vortex-glass transition in type-II superconductors.

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ing the transition could be done by varying the aspect ratio of the simulation cell. In the UF XY model, there exists the possibility of two very close phase transitions preventing a unique (independent of system size) crossing of Binder ratios of the staggered magnetization M_s (our results for the size and field dependence of various moments of the static uniform magnetization in the random-field XY model are qualitatively very similar to those for M_s in the UF XY model). In the UF XY model, some progress may have been made because of the theoretical analytical input used in analyzing the scaling behavior of various extracted thermodynamic quantities. In our case, we do not dispose of a well-established theoretical framework to allow us to perform a one-parameter scaling in terms of a ''renormalized'' coupling. (We thank E.S. Sørensen for suggesting that we clarify this issue.)

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