# ac properties of an anisotropic layered superconductor

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(Received 23 January 1996)

The ac properties of weakly coupled layered thin film superconducting systems are studied for cases of isotropic and anisotropic pairing within a microscopic approach. It is shown that the shape of the ac characteristics crucially depends on the type of symmetry of the anisotropic order parameter because of the sensitivity of the induced ac Josephson current to the order parameter phase. The model suggests a reasonable microscopical explanation for $c$-polarized far-infrared experiments on metal oxides. [S0163-1829(96)05522-1]

## I. INTRODUCTION

A visible attention to the $c$-polarized far-infrared properties of metal oxide superconductors was initialized by observation of a resonance mode with a frequency $\Omega_{ps}$ far below the energy gap value[1–3] $2\Delta$. The phenomenological theory[4] described this phenomenon in terms of the ac Josephson effect taking place between weakly coupled Cu-O layers.[5] However, the simple models[4,6] did not take into account the peculiar dynamic character of the interaction between external electromagnetic fields and the layered superconductors (SC's). The interaction may strongly depend on the low-energy electron excitation spectrum and on the coherent property of the superfluid condensate as well[7,8] similarly as it takes place for $s$-wave homogeneous SC's (see Ref. 7) and for a single Josephson junction.[8] The electrodynamics of a layered SC is closely related also to another issue which is under intensive discussion with respect to metal oxide SC's. Namely the matter is about the unconventional symmetry of the SC order parameter $\Delta(\mathbf{p})$ ($\mathbf{p}$ is the electron momentum) in these materials[9] which was suggested by the recent direct observation of the half-integer flux quantum effect[9] on a tricrystal sample. The mentioned effect is based on the possibility to form a so-called Josephson $\pi$ junction[9] which may take place between weakly coupled unconventional superconducting electrodes having an antiphase orientation of petals of the $d$-wave SC order parameter ($d$ petals). Since the superfluid component of the ac current is sensitive to the phase of $\Delta(\mathbf{p})$ while the ac quasiparticle current depends on the density of electron states in the junction's electrodes, one may expect that the high-frequency properties of layered SC's are consanguineous to the symmetry of $\Delta(\mathbf{p})$ as well.

A separate issue is concerned with the physical origin of the aforementioned mode $\Omega_{ps}$. Since the value of $\Omega_{ps}$ corresponds to $\epsilon_1(\omega=\Omega_{ps})=0$ [$\epsilon_1(\omega)$ is the real part of the dielectric function] it is usually interpreted[4] as a Josephson plasma frequency.[6] However, the postulating of any interaction between $c$-polarized infrared and electron density oscillations[10] (low-frequency plasmons) seems to be artificial due to the following. A simple physical reason is that the electromagnetic wave is a transverse wave while the plasma oscillation (electron density oscillation) is a longitudinal wave. Since the interaction between the purely transverse and longitudinal waves in the limit $\mathbf{k}=0$ is prohibited by selection rules, one has to introduce some special conditions to achieve a sufficient strength of the interaction to be observable. In the normal metal $\omega_{pl}$ is very large, $\sim 1$–10 eV, and thus the skin penetration depth is very small, causing the field to be very inhomogeneous near the metal surface, causing the interaction between the waves to be quite strong. In metal oxides the situation is different because the longitudinal plasma frequency $\omega_{pl}$ (if it exists) is several orders smaller $\sim 0.1$–10 meV. Therefore the field is fairly homogeneous far inside the sample (up to 300 nm in metal oxides even at low temperatures) and thus the $c$-polarized infrared oscillation does not interact with any longitudinal oscillations and the more essential effect is due to the induced transverse screening current oscillations which we shall consider in this paper. Due to Ref. 5 metal-oxide single-crystal samples exhibit properties related to the Josephson effect in the highly capacitive limit. This means that the longitudinal plasma frequency [which in this system is related to the Josephson plasma frequency $\omega_{pl}=(2eI_c/\hbar C)^{1/2}$, where $I_c$ is the interlayer critical current, and $C$ is the interlayer capacitance] should be very small (i.e., $\omega_{pl}\ll 2\Delta$) while in experiments[1–3] $\Omega_{ps}\sim\Delta$. Also $\omega_{pl}$ must depend on the geometry of the sample, which does not follow from experiments.[1–3] Therefore here we shall pay main attention to the transverse ac-current oscillations induced by the external field.

In this paper we propose a microscopic theory of the ac properties of the weakly coupled SC layered system exposed to a far-infrared $c$-polarized electromagnetic field. In the next section we derive the complete set of necessary equations which consist of the equations of the electron spectrum (self-consistency equations) and the expression for the kinetic source. The obtained formulas are to be implemented then to calculate the electric interlayer current. These equations are simplified with the assumption of an explicit form of the matrix interlayer tunneling element in the two limiting cases:

 

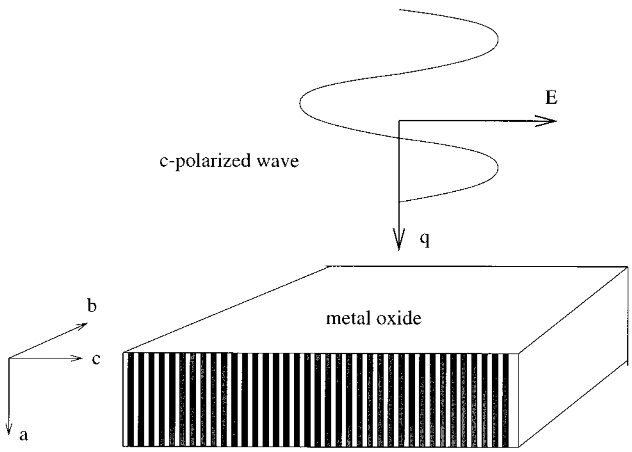


FIG. 1. The system of the stacked Josephson junctions biased by the electromagnetic wave.

(i) The electron momentum **p** is exactly conserved while interlayer tunneling takes place for an ideal ''clean'' periodic layered SC; (ii) **p** is completely not conserved (this happened mostly for a single tunneling junction or in ''dirty'' layered SC's).

Using the obtained general equations for additional simplifying assumptions we derive simple expressions for the ac conductivity, dielectric function, reflectivity, and transmissivity. Such kinds of expressions are used then to calculate the observable characteristics and compare them with known far-infrared experiments on layered metal oxides. We propose to use the measurements of the reflectivity and transmissivity of an external $c$-polarized electromagnetic wave from a layered SC film of a finite thickness to test the $\Delta(\mathbf{p})$ symmetry in metal oxides. We find a pronounced correspondence between the ac properties of the layered SC and the symmetry of the SC order parameter and show that these experiments are unambiguous in distinguishing between different types of order parameter symmetry. Particularly we arrive at a conclusion that the reflectivity and transmissivity characteristics are qualitatively different for $s$-wave, anisotropic $s$-wave, and $d$-wave layered SC's.

## II. INTERACTION BETWEEN ac FIELD AND A LAYERED SC SYSTEM

The calculations of the reflectivity and transmissivity in this article are made for a layered SC film of finite thickness, the geometry of which is sketched in Fig. 1. The electric field vector $\mathbf{E}(t)=(E_\perp(t),0,0)$ is parallel to the $c$ axes and depends on the boundary conditions as well as on the microscopic properties of the layered SC while the electromagnetic wave vector $\mathbf{q}=(0,0,q_z)$ is perpendicular to the surface which consists of strips formed by the superconducting $ab$ planes separated by insulating interplane layers. The dielectric function is

$$\epsilon(\omega)=\epsilon_\infty-\frac{4\pi i\sigma(\omega)}{\omega}, \tag{1}$$

where $\sigma(\omega)$ is the linear response ac conductivity in the $c$ direction which is to be found from the microscopic calculations, $\omega$ is the field frequency, and $\epsilon_\infty$ is the high-frequency dielectric constant. The second term on the right-hand side of this formula corresponds to the contribution of the interlayer ac currents. With the assumption of specular boundary conditions the expressions for the reflectivity $\mathcal{R}$ and for the transmissivity $\mathcal{T}$ are[4]

$$\mathcal{R}(\omega)=\left|\frac{(1-P^2)(1-\epsilon(\omega))}{[1+\sqrt{\epsilon(\omega)}]^2-[1-\sqrt{\epsilon(\omega)}]^2P^2}\right|^2,$$

$$\mathcal{T}(\omega)=\left|\frac{4P\sqrt{\epsilon(\omega)}}{[1+\sqrt{\epsilon(\omega)}]^2-[1-\sqrt{\epsilon(\omega)}]^2P^2}\right|^2, \tag{2}$$

where

$$P=\exp[i(\omega/c)\sqrt{\epsilon(\omega)}d] \tag{3}$$

and where $d$ is the thickness of the film; $c$ is the light velocity.

### A. Basic equations

The reflectivity and transmissivity of a thin film layered system in the geometry of Fig. 1 are calculated here within a weak-coupling model[8] which was applied in Ref. 11 to examine the electromagnetic properties of granular superconductors. The mentioned model[8,11] implies that only an interaction within nearest neighbor layers is important, and that the field is concentrated inside the interlayer barriers and is homogeneous on the scale $L_g \gg c_\perp$ ($L_g$ is a scale where the field can be considered as homogeneous, $L_g \leqslant \lambda_{\mathrm{field}}$, $c_\perp$ is the lattice constant in the $c$ direction, and $\lambda_{\mathrm{field}}$ is the wavelength of the external field) inside the sample as well. This allows us to calculate the ac current across a single Josephson junction as well as through the stacked layered SC system (Fig. 1) for the unconventional symmetry of the order parameter $\Delta(\mathbf{p})$ and in the presence of frequency dispersion effects. In the case of anisotropic pairing the electron states are strictly separated in momentum space because $\Delta(\mathbf{p})$ forms the petals (see Fig. 2), the phase being the same (e.g., for symmetry $d_{x^2}$) or different (e.g., for symmetry $d_{x^2-y^2}$, Fig. 2). We introduce the $\pi$ factor $\cos\gamma$ to distinguish between the out-of-phase Josephson tunneling ($\gamma=\pi$; see Fig. 2, arrows 1 and 3) and the ordinary in-phase one ($\gamma=0$; see Fig. 2, arrows 2 and 4). The calculations made here are related to the electric current induced in the Josephson junction by the applied external ac field $\mathbf{E}(t)$ of constant amplitude.

### B. Electron spectrum in a layered SC

The ac conductivity $\sigma(\omega)$ for the geometry of Fig. 1 is calculated in a tunneling model which was previously applied in Refs. 11 and 12 to describe the electromagnetic properties of inhomogeneous superconductors. The mentioned model allows one to calculate the ac currents across the stacked system of Josephson junctions (see Fig. 1) for an arbitrary amplitude of the ac field, applied a complex SC order parameter symmetry, and in the presence of frequency dispersion effects.

In this section we formulate the model which describes the periodic multilayered superconductor as a set of weakly coupled $ab$ planes. The electrons move freely in two dimen-

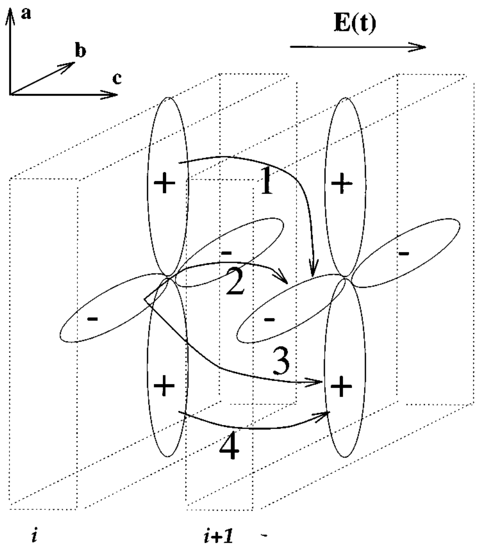


FIG. 2. The order parameter $d_{x^2-y^2}$ in adjacent layers for a $d$-wave SC with petals having different phases.

sions within the $ab$ plane while they tunnel between different planes. The electron part of the total Hamiltonian is

$$H=\sum_i H_i^L+\sum_{ij} H_{ij}^{\mathrm{IL}}+\sum_i H_i^{\mathrm{ext}}. \tag{4}$$

Here

$$H_i^L=\sum_{p\sigma} \epsilon_p^{(i)} a_{p\sigma i}^\dagger a_{p\sigma i}^\dagger+H_i^{\mathrm{int}}, \tag{5}$$

where $\epsilon_p^{(i)}$ is the kinetic energy of electrons with a two-dimensional quasimomentum $\mathbf{p}$ in SC layer $i$, $a_{p\sigma i}^\dagger$ ($a_{p\sigma i}$) is the electron creation (annihilation) operator in layer $i$, and $H_i^{\mathrm{int}}$ includes interactions in the system which cause the SC pairing, mean field, and self-energy effects. The coupling of electrons to bosonlike excitations in the system is included in $H_i^{\mathrm{int}}$ and is also assumed to be responsible for the transfer energy away from the considered structure.

The term $H_{ij}^{\mathrm{IL}}$ describes interlayer (IL) electron transfer:

$$H_{ij}^{\mathrm{IL}}=\sum_{pp'\sigma} \{T_{pp'}^{ij} a_{p\sigma i}^\dagger a_{p'\sigma j}+\text{c.c.}\}, \tag{6}$$

where $T_{pp'}^{ij}$ is the electron IL tunneling matrix element.

With respect to the layered SC for simplicity we assume that the field vector is parallel to the $c$ direction and that the field is homogeneous over the $ab$ planes. Such a situation may be realized in the case of a striplike (shown in Fig. 1) thin film with thickness $d\simeq\min\{\lambda_L,\lambda_\sigma\}$, $\lambda_L$ and $\lambda_\sigma$ being the field penetration depth and skin depth, respectively.

The influence of the external electromagnetic field is taken into account by

$$H_i^{\mathrm{ext}}(t)=\sum_{p\sigma} e[\mu_i(t)-\mu_0]a_{p\sigma i}^\dagger a_{p\sigma i}, \tag{7}$$

where $\mu_0$ is the equilibrium chemical potential, and $\mu_i(t)$ is the electrochemical potential of SC layer $i$ which depends on the time $t$ due to presence of the external field. For the different layers we get the condition $\mu_i(t)-\mu_j(t)=V_{ij}(t)$. The interlayer Josephson phase difference $\phi_{ij}(t)$ is determined as

$$\phi_{ij}(t)=2e\int_{-\infty}^t d\tau V_{ij}(\tau), \tag{8}$$

where the "interlayer voltage" $V_{ij}(t)$ is introduced as

$$V_{ij}(t)=\int_i^j d\mathbf{r}\mathbf{E}(\mathbf{r},t)\approx d_\perp^B\mathbf{E}(t), \tag{9}$$

where $d_\perp^B$ is the interlayer barrier thickness. In principle the above expressions (8) and (9) in combination with the corresponding formulas for the ac interlayer current are to be implemented in the Maxwell equations to determine the gauge-invariant microscopic intergrain field amplitudes $V_{ij}(t)$. Nevertheless, as we shall see for the linear case many important characteristics can be inferred even without any exact knowledge of the field distribution inside the sample and a complicated solution of the whole set of equations is not necessary.

The equations on the electron spectrum in the considered periodic multilayered SC (Fig. 1) are derived from expression for the total electron self-energy:

$$\hat{\Sigma}^{(i)R,A,K}=\hat{\Sigma}_{\mathrm{ph}}^{(i)R,A,K}+\hat{\Sigma}_T^{(i)R,A,K}, \tag{10}$$

where the caret over $\hat{\Sigma}$ implies the matrix structure

$$\hat{\Sigma}=\begin{pmatrix} \Sigma_1 & \Sigma_2 \\ \Sigma_2^+ & \overline{\Sigma_1} \end{pmatrix}. \tag{11}$$

$\hat{\Sigma}_{\mathrm{ph}}^{(i)R,A,K}$ is an ordinary self-energy[15,16] which is responsible for the Cooper pairing and renormalization of the electron spectrum due to electron-phonon interaction in the layer $i$, the indices $R$, $A$, and $K$ correspond to retarded ($R$), advanced ($A$), or correlated ($K$) functions,[14] and $\Sigma_1$ and $\Sigma_2$ are the normal and anomalous components, respectively. The second term in (10) is the self-energy in layer $i$ due to the interlayer electron transfer between different layers[13] $i,j$ which is given by

$$\hat{\Sigma}_T^{(i)R,A,K}(\mathbf{p},t,t')=\sum_{j,p'} |T_{\mathbf{pp}'}^{ij}|^2\hat{S}_{ij}(t)\hat{G}_{\mathbf{p}'}^{(j)R,A,K}(t,t')\hat{S}_{ij}^+(t'), \tag{12}$$

where $T_{\mathbf{pp}'}^{ij}$ is the matrix element of tunneling from layer $j$ to layer $i$ and $G^{(i)R,A,K}(\mathbf{p},t,t')$ denotes the Green function [retarded ($R$), advanced ($A$), or correlated ($K$); see, for instance, Ref. 15]. In (12) the vertex factor[11] is

$$\hat{S}_{ij}(t)=\hat{\tau}_3\exp\left\{i\hat{\tau}_3\int_{-\infty}^t V_{ij}(t')dt'\right\}. \tag{13}$$

We take into account that

$$e^{i\hat{\alpha}\cos\omega t}\equiv\sum_n J_n(\hat{\alpha})e^{in\omega t},$$

where $J_n(\alpha)$ is the Bessel function of order $n$, $\hat{\alpha}=\hat{\tau}_3\cdot\alpha$, $\alpha=eV_{ij}/\omega$, and $\tau_i$, $i=\overline{1,\ldots,3}$, are the Pauli matrices. Then in case of harmonic alternation $V_{ij}(t)=V_{ij}\cos\omega t$ one can reduce (12) to

$$\hat{\Sigma}_T^{(i)R,A,K}(\mathbf{p},\epsilon,\epsilon')=\sum_{j,p} |T_{\mathbf{pp}'}^{ij}|^2\sum_{n,m} J_n(\hat{\alpha})\hat{G}^{(j)R,A,K} \times(\mathbf{p}',\epsilon-n\omega,\epsilon'-m\omega)J_m(\hat{\alpha}), \tag{14}$$

where $\hat{G}$ and $J_n(\hat{\alpha})$ mean

$$\hat{G}=\begin{pmatrix} G & F \\ F^{+} & \overline{G} \end{pmatrix}, \quad J_n(\hat{\alpha})=\begin{pmatrix} J_n(\alpha) & 0 \\ 0 & J_n(-\alpha) \end{pmatrix},$$

where $G$ (or $\overline{G}$) denotes an electron (or hole) normal Green function while $F$ (or $F^{+}$) is related to an anomalous function.

In order to simplify the following consideration we shall restrict ourselves to second-order terms over the interlayer interaction. Thus we assume that

$$\overline{\Gamma}=\langle\Gamma_i^{(j)}\rangle_{\text{layers}}\ll\overline{E}, \tag{15}$$

where $\langle\cdots\rangle_{\text{layers}}$ means averaging over the different layers and a typical electron energy is $E\sim\max\{\Delta,\omega,T\}$. In this limit all the terms that are nondiagonal over the energy variable like $G_{\epsilon,\epsilon-n\omega}$, $\Sigma_{\epsilon,\epsilon-n\omega}$, or $\Delta_{\epsilon,\epsilon-n\omega}$ ($\Delta_{\epsilon,\epsilon-n\omega}$ is a nonstationary contribution to the energy gap) have the additional small parameter $\overline{\Gamma}/\overline{E}\ll 1$ in comparison with diagonal stationary terms like $G_\epsilon$, $\Sigma_\epsilon$, or $\Delta_\epsilon$. This means that the main contribution to the electron kinetic characteristics is coming from the electron energies $\sim\overline{E}$; thus the terms containing the above-mentioned nonstationary quantities are negligible in comparison with stationary ones. Therefore when calculating the kinetic characteristics one can for instance replace

$$\int d\epsilon_1\hat{\Sigma}(\mathbf{p},\epsilon,\epsilon_1)\hat{G}(\mathbf{p},\epsilon_1,\epsilon)\rightarrow\hat{\Sigma}(\epsilon)\hat{G}(\mathbf{p},\epsilon)+O(\overline{\Gamma^2}/\overline{E^2}), \tag{16}$$

where $O(\eta)$ means negligible terms to be small over parameter $\eta$. This allows us to express the final results through time-averaged quantities like the energy gap $\Delta(\epsilon)=\langle\overline{\Delta(\epsilon,t)}\rangle^t$ or the electron energy $\xi_p(\epsilon)=\langle\overline{\xi_p(\epsilon,t)}\rangle^t$.

Expanding the retarded self-energy over the Pauli matrices as

$$\hat{\Sigma}^{(i)R}(\epsilon)=[1-Z_i(\epsilon)]\epsilon\hat{1}+\chi_i(\epsilon)\hat{\tau}_3+\Phi_i(\epsilon)\hat{\tau}_1 \tag{17}$$

[here $Z_i(\epsilon)$ is the renormalization function, $\chi_i(\epsilon)$ is the shift of the electrochemical potential, and $\Phi_i(\epsilon)$ is the Cooper pairing potential] and using the equation for $G^R(\epsilon)$ (see Ref. 14),

$$[\hat{G}^R(\epsilon)]^{-1}=[\hat{G}_0^R(\epsilon)]^{-1}-\hat{\Sigma}^R(\epsilon), \tag{18}$$

we get the following analytic expression for the averaged over time (diagonal over the energy variable $\epsilon$) retarded Green function:[16]

$$\hat{G}_i^R(\mathbf{p},\epsilon)=\frac{Z_i(\epsilon)\epsilon\hat{1}+\xi_i(\mathbf{p})\hat{\tau}_3-\Phi_i(\epsilon)\hat{\tau}_1}{[Z_i(\epsilon)\epsilon]^2-\xi_i^2(\mathbf{p})-\Phi_i^2(\epsilon)}. \tag{19}$$

Acting here in spite of Ref. 13 and using the explicit expression for $\hat{\Sigma}^{(i)R}$ [formula (14)] and expansion (17) one gets the closed system of equations for $Z_i(\epsilon)$ and for the averaged over the time energy gap function $\Delta_i(\epsilon)=Z_i^{-1}(\epsilon)\cdot\Phi_i(\epsilon)$,

$$[1-Z_i(\epsilon)]\epsilon=\sum_{j,p}|T_{\mathbf{pp}'}^{ij}|^2\sum_n J_n^2(\alpha)G^{(j)R}(\mathbf{p}',\epsilon-n\omega), \tag{20}$$

$$Z_i(\epsilon)\Delta_i(\epsilon)=\Delta_i^c+\sum_{j,p}|T_{\mathbf{pp}'}^{ij}|^2\sum_n(-1)^nJ_n^2(\alpha)\times F^{(j)R}(\mathbf{p}',\epsilon-n\omega), \tag{21}$$

where $\Delta_i^c$ depends on the pairing mechanism and in the case of Cooper pairing satisfies the BCS equation

$$\Delta_i^c=-\Upsilon_i^c\sum_p\int\frac{d\epsilon}{\pi i}\text{Tr}\{\hat{G}_i^K(\mathbf{p},\epsilon)\}, \tag{22}$$

where $\hat{G}^K=(\hat{G}^R-\hat{G}^A)[1-2f_\epsilon]$, $\Upsilon_i^c$ is the intralayer Cooper pairing potential in the layer $i$, and $f_\epsilon$ is the electron distribution function which has to be determined from the corresponding kinetic equation.

The above general self-consistency equations are simplified with the assumption of an explicit form of the interlayer tunneling matrix element. Below we consider two limiting cases: (i) The electron momentum is conserved for interlayer tunneling and we assume that $|T_{\mathbf{pp}'}^{ij}|^2=|T|^2\delta(\mathbf{p}-\mathbf{p}')$ which takes place for a ''clean'' periodic layered SC; (ii) there is no conservation of $\mathbf{p}$ and $|T_{\mathbf{pp}'}^{ij}|^2=|T|^2$ (this makes sense in the case of a single tunnel junction or a ''dirty'' layered SC).

In case (i) one has to take into account the periodicity of the layered system. The electron spectrum of the double-layer periodic normal/superconductor system was calculated in Refs. 17–20. The calculations performed in the mentioned articles were based on the matching of boundary conditions for the electron wave functions in the neighboring layers within the mean field approximation for superconductivity. They predicted the appearance of subbands in the electron spectrum due to Andreev reflection processes in such a periodic system. The authors of Ref. 21 implemented a model of the interlayer tunneling interaction[22–24] and calculated the electron spectrum of the multilayered superconducting system with five different layers. Nevertheless, the direct contribution of the interlayer interaction to the energy gap which is responsible for the proximity effect[13] as well as the dynamic renormalization of the electron spectrum due to retardation effects and translation invariance of the system in the simple model of Ref. 21 was neglected.

In this section we formulate the system of equations describing the electron spectrum of a layered periodic SC under the influence of an external electromagnetic field. Since the layers are assumed to be weakly coupled, we do not expect any essential contribution to the electron spectrum from the Andreev reflection processes as took place in Refs. 17–19. Instead the role of the interlayer interaction in the development of subbands in the quasiparticle excitation spectrum[4] as well as in the interlayer superconducting proximity effect[13] seems to be important.

In calculations of the average quantities in higher orders of perturbation theory translation invariance is taken into account in the model using the Bloch theorem

$$a_j^\dagger|\rangle=\exp\{ikm\}a_{j+m}^\dagger|\rangle, \tag{23}$$

where $m$ is period, $k$ is the electron quasimomentum in the $c$ direction, and the ground state is denoted as $|\rangle$. Thus splitting the quantum mechanical and statistical averages $\langle\cdots\rangle$ at second order over the interlayer interaction for instance one gets

$$\langle M\rangle=\left\langle \sum_{i_2 j_2} T_{i_2 j_2} a^{\dagger}_{i_2} a_{j_2} a_i a^{\dagger}_i \sum_{i_1 j_1} T^{*}_{i_1 j_1} a^{\dagger}_{i_1} a_{j_1}\right\rangle = \pm \sum_{i_1 j_1 i_2 j_2} \langle a_{j_2} a^{\dagger}_i\rangle\langle a^{\dagger}_{i_2} a_{j_1}\rangle\langle a_i a^{\dagger}_{i_1}\rangle T_{i_2 j_2} T^{*}_{i_1 j_1}$$

$$= \pm \sum_{l_1 l_2 l_3 l_4} \langle a_{i+ml_2} a^{\dagger}_i\rangle\langle a^{\dagger}_{j+ml_1} a_{j+ml_4}\rangle\langle a_i a^{\dagger}_{i+ml_3}\rangle T_{j+ml_1,i+ml_2} T^{*}_{i+ml_3,j+ml_4}, \tag{24}$$

where $\{l_1 l_2 l_3 l_4\}$ are arbitrary integers. The last equality was written taking into account the fact that only correlators on equivalent layers are not equal to zero, i.e., $\langle a^{\dagger}_{j+ml_1} a_j\rangle = \exp\{ikml_1\}\langle a^{\dagger}_j a_j\rangle \neq 0$ while $\langle a^{\dagger}_{j+ml_1} a_i\rangle = \exp\{ikml_1\}\langle a^{\dagger}_j a_i\rangle_{i\neq j}=0$. Thus in (24) we have used that

$$i_2 = j+ml_1,\ \ j_2 = i+ml_2,\ \ i_1 = i+ml_3,\ \ j_1 = j+ml_4. \tag{25}$$

When making calculations of averages as in (24) a phase multiplier of the kind $\exp\{ikml\}\neq 1$ does not appear if one assumes that only interlayer tunneling matrix elements between neighbor layers are nonzero while all the others are equal to zero, i.e., $T_{j,i}=T_{j+ml,i+ml}\neq 0 (j=i\pm 1)$ while $T_{j+ml_1,i+ml_2}\approx 0 (l_1\neq l_2)$. This means that no electron quasimomentum appears in the $c$ direction in this limit. The last situation is similar to the case of a single tunnel junction formed by a couple of two-dimensional electrodes. Another situation takes place if $T_{j+ml_1,i+ml_2}\neq 0 (l_1\neq l_2)$. Then the averages (24) gain the phase multiplier

$$\langle M\rangle = \pm \sum_{l_1 l_2 l_3 l_4} \exp\{-ikm(l_2-l_1-l_3+l_4)\}\langle a_i a^{\dagger}_i\rangle\langle a^{\dagger}_j a_j\rangle\langle a_i a^{\dagger}_i\rangle \cdot T_{j+ml_1,i+ml_2} T^{*}_{i+ml_3,j+ml_4}, \tag{26}$$

which provides the final electron momentum along the $c$ direction. In the simplified case $T_{j+ml_1,i+ml_2}=T_{j,i}$ for arbitrary $l_1$ and $l_2$. Summarizing in (26) over the all layers we get for the phase multiplier $\beta_k$ the geometrical progression

$$\beta_k = \frac{1-\exp\{ikmN\}}{1-\exp\{ikm\}}, \tag{27}$$

where $N$ is the number of periods in the sample. The external field causes an additional phase multiplier of the kind

$$\sum_n J_n\left(\frac{ml_1 eV_{ij}}{\omega}\right) e^{in\omega t}, \tag{28}$$

which yields a small contribution for $l_1\neq 0$.

Thus in the above-mentioned limiting case (i) the self-consistency equations are overwritten then as

$$[1-Z_i(k,\mathbf{p},\epsilon)]\epsilon = \beta_k \sum_{j,n} J_n^2(\alpha)\frac{|T_p^{ij}|^2}{Z_j(\epsilon-n\omega)}\frac{\epsilon-n\omega}{(\epsilon-n\omega)^2-\tilde{\xi}_j^2(\mathbf{p},\epsilon-n\omega)-\Delta_j^2(\epsilon-n\omega)}, \tag{29}$$

$$\chi_i(k,\mathbf{p},\epsilon) = \beta_k \sum_{j,n} J_n^2(\alpha)\frac{|T_p^{ij}|^2}{Z_j(\epsilon-n\omega)}\frac{\tilde{\xi}_j(\mathbf{p},\epsilon-n\omega)}{(\epsilon-n\omega)^2-\tilde{\xi}_j^2(\mathbf{p},\epsilon-n\omega)-\Delta_j^2(\epsilon-n\omega)}, \tag{30}$$

$$Z_i(\epsilon)\Delta_i(k,\mathbf{p},\epsilon) = \Delta_i^{\mathrm{par}} + \beta_k \sum_{j,n} (-1)^n J_n^2(\alpha)\frac{|T_p^{ij}|^2}{Z_j(\epsilon-n\omega)}\frac{\Delta_j(\epsilon-n\omega)}{(\epsilon-n\omega)^2-\tilde{\xi}_j^2(\mathbf{p},\epsilon-n\omega)-\Delta_j^2(\epsilon-n\omega)}, \tag{31}$$

where $\tilde{\xi}_i(k,\mathbf{p},\epsilon) = Z_i^{-1}(k,\mathbf{p},\epsilon)[\xi_i(k,\mathbf{p},\epsilon)+\chi_i(k,\mathbf{p},\epsilon)]$ is the renormalized electron energy. In the other case (ii) the self-consistency equations are written as

$$[1-Z_i(\epsilon)]\epsilon = \sum_{j,n} J_n^2(\alpha)\frac{\Gamma_i^{(j)}(\epsilon-n\omega)}{\sqrt{\Delta_j^2(\epsilon-n\omega)-(\epsilon-n\omega)^2}}, \tag{32}$$

$$Z_i(\epsilon)\Delta_i(\epsilon) = \Delta_i^{\mathrm{ph}} + \sum_{j,n} (-1)^n J_n^2(\alpha)\frac{\Gamma_i^{(j)}\Delta_j(\epsilon-n\omega)}{\sqrt{\Delta_j^2(\epsilon-n\omega)-(\epsilon-n\omega)^2}}, \tag{33}$$

where $\Gamma_i^{(j)}=\mathcal{A}|T|^2 N_j(0)$ is the tunneling rate from layer $j$ to layer $i$, $\mathcal{A}$ is the cross section of the sample, and $N_j(0)$ is the electron density of states on the Fermi level in a layer $j$.

### C. ac currents and anisotropic pairing

Here we derive also the necessary expressions for the components of the ac current and then take the limit of a weak field. The general solution for a limit of the intensive field is quite complicated and some special cases will be considered elsewhere. We calculate the interlayer tunneling current on the space scale of $d_\perp^G$ which satisfies $c_\perp \ll d_\perp^G \ll \lambda_{\rm field}$. Since the interlayer tunneling current $J(\tau,\mathcal{T})$ is expressed through the correlators of the electron creation and annihilation operators at different time moments $t$ and $t'$, it depends on the ''difference'' time $\tau=(t-t')/2$ which describes the adiabatic switching of the interactions in the system as well as the ''summarized'' time $\mathcal{T}=(t+t')/2$ characterizing the alternation of the correlators in the real time (see Ref. 23).

Taking the current at a certain $\tau$ (for instance at $\tau=0$) and using the causality principle one arrives

$$J(\tau=0,\mathcal{T})=e\,\mathrm{Tr}\int d\epsilon \langle \mathrm{Re}\{\hat{I}_T(\epsilon,\mathcal{T})\}\rangle_p, \qquad (34)$$

where we denoted the electron charge as $e$, $\langle\cdots\rangle_p=N(0)\int d\xi_p (d\Omega_p/4\pi)$ means the averaging over the angles of the electron momentum and integration over the kinetic electron energy $\xi_p$, $N(0)$ is the electron density of states, ''Tr'' means the trace operation, and $\hat{I}_T(\epsilon,\mathcal{T})$ is the nonequilibrium source,

$$\hat{I}_T^{(i)}(\epsilon,\mathcal{T})=\int d\tau e^{-i\epsilon\tau}\hat{I}_T(\tau,\mathcal{T}), \qquad (35)$$

which describes the electron tunneling into layer $i$ from the nonequivalent layers (for details see Refs. 11 and 15) in the quantum kinetic equation for the electron distribution function $f_\epsilon$,

$$N_i(\epsilon)\frac{\partial f_\epsilon}{\partial \mathcal{T}}=\langle \mathrm{Tr}\{\hat{I}_{\rm ph}^{(i)}(\epsilon,\mathcal{T})+\hat{I}_{\rm int}^{(i)}(\epsilon,\mathcal{T})+\hat{I}_T^{(i)}(\epsilon,\mathcal{T})\}\rangle_p. \qquad (36)$$

In the last formula $\hat{I}_{\rm ph}^{(i)}$ denotes the electron-phonon collision integral responsible for the electron energy and momentum relaxation and $\hat{I}_{\rm int}^{(i)}$ takes into account some other possible interactions in the system; the tunnel density of electron states is

$$N_i(\varepsilon)=\mathrm{Re}\left\{\frac{\varepsilon}{\sqrt{\varepsilon^2-\Delta_i^2(\varepsilon)}}\right\}. \qquad (37)$$

The source $\hat{I}_T^{(i)}$ is determined for the two time moments $t$ and $t'$ as

$$\hat{I}_T^{(i)}(t,t')=\{\hat{G}^K\hat{\Sigma}_T^A-\hat{\Sigma}_T^R\hat{G}^K+\hat{G}^R\hat{\Sigma}_T^K-\hat{\Sigma}_T^K\hat{G}^A\}_{t,t'}. \qquad (38)$$

The ''product'' in (38) means

$$\{AB\}_{t,t'}=\int_\infty^t dt_1 A(t,t_1)B(t_1,t'). \qquad (39)$$

We emphasize that the upper limit of integration here is different from infinity, which is the consequence of the causality principle when calculating the total current and is an important point to avoid confusion, when calculating the Josephson part of the current.

After taking the trace operation from expression (36) in the absence of the charge imbalance[24] one gets terms of the two kinds $I_T^{(i)}=I_{\rm qp}^{(i)}+I_{\rm Jos}^{(i)}$:

$$\begin{aligned}I_{\rm qp}^{(i)}(t,t')&=[\hat{I}_{\rm qp}^{(i)}(t,t')]_{11}=\{G^K\Sigma_T^A-\Sigma_T^R G^K+G^R\Sigma_T^K\\&\quad-\Sigma_T^K G^A\}_{t,t'},\\ I_{\rm Jos}^{(i)}(t,t')&=[\hat{I}_{\rm Jos}^{(i)}(t,t')]_{11}=\{F^K\Sigma_{2T}^{+A}-\Sigma_{2T}^R F^{+K}+F^R\Sigma_{2T}^{+K}\\&\quad-\Sigma_{2T}^K F^{+A}\}_{t,t'}.\end{aligned} \qquad (40)$$

From the last equations it is seen that for layer $i$ the source $I_{\rm qp}^{(i)}$ is proportional to the product of the normal Green functions and normal self-energies and is attributed to the photon-assisted quasiparticle IL tunneling. The term $I_{\rm Jos}^{(i)}$ which is proportional to the products of anomalous functions is related to the IL Cooper pair tunneling that we shall consider below. This source $I_{\rm Jos}^{(i)}$ takes into account the influence of the external field on the Josephson current via the superfluid channel. Besides, if one could check if the dc voltage were included, the obtained source would yield an ordinary expression for the Shapiro steps in the $I$-$V$ Josephson curves.[24]

Here we also restrict ourselves to second-order terms over the interlayer interaction. For the harmonically alternating external field $E_\perp(z_i,t)=E_{0\perp}^{(i)}\cos(\omega t)$ ($E_{0\perp}^{(i)}$ is the field amplitude in the $i$th layer) the quasiparticle (qp) part of the tunneling source $\hat{I}_T^{(i)}$ is written down in explicit form.[12] For example in the aforementioned limiting case (i) $|T_{\mathbf{pp}'}^{ij}|^2=|T|^2\delta(\mathbf{p}-\mathbf{p}')$ and the quasiparticle tunneling source becomes

$$\begin{aligned}I_{\rm qp}^{(i)}(\epsilon,\mathcal{T})=\beta_k\sum_{l,m}\sum_j |T_p^{ij}|^2 J_{l+m}(\alpha)J_{m-l}(\alpha)\int d\epsilon_1\Bigg\{&\mathrm{Im}G_p^{(i)R}(\epsilon_1)G_p^{(j)A}(\epsilon-m\omega)\frac{(1-2f_{\epsilon_1})e^{-il\omega\mathcal{T}}}{\epsilon-\epsilon_1-l\omega-i\delta}\\&-\mathrm{Im}G_p^{(i)R}(\epsilon-l\omega)G_p^{(j)R}(\epsilon_1)\frac{(1-2f_{\epsilon-l\omega})e^{il\omega\mathcal{T}}}{\epsilon-\epsilon_1-m\omega-i\delta}+G_p^{(i)R}(\epsilon_1)\mathrm{Im}G_p^{(j)A}(\epsilon-m\omega)\frac{(1-2f_{\epsilon-m\omega})e^{-il\omega\mathcal{T}}}{\epsilon-\epsilon_1-l\omega-i\delta}\\&-G_p^{(i)A}(\epsilon-l\omega)\mathrm{Im}G_p^{(j)R}(\epsilon_1)\frac{(1-2f_{\epsilon_1})e^{il\omega\mathcal{T}}}{\epsilon-\epsilon_1-m\omega-i\delta}\Bigg\},\end{aligned} \qquad (41)$$

where $\delta\to 0$ and is taking into account the causality requirements in the ac Josephson effect. In this case one can find also the expressions for the quasiparticle and for Josephson components of the ac current. Using the dispersion relations[16] for the functions $G_p^{R(A)}(k,\mathbf{p},\epsilon)$ the quasiparticle component of the ac current is written as

$$J_{\rm qp}^{(i)}(\mathcal{T})=e\sum_{j,k,p}\beta_k|T_p^{ij}|^2\sum_{l,m}J_{l+m}(\alpha)J_{m-l}(\alpha)\int d\epsilon' d\epsilon\langle A_i(k,\mathbf{p},\epsilon')A_j(k,\mathbf{p},\epsilon)\rangle_p[f_{\epsilon'}-f_\epsilon]\frac{e^{-il\omega\mathcal{T}}}{\epsilon-\epsilon'-(m-l)\omega-i\delta}, \tag{42}$$

where

$$A_i(k,\mathbf{p},\epsilon)=\mathrm{Im}\{G_i^R(k,\mathbf{p},\epsilon)\}, \tag{43}$$

while for the Josephson (superfluid) part one finds

$$J_{\rm Jos}^{(i)}(\mathcal{T})=e\sum_{j,k,p}\beta_k|T_p^{ij}|^2\sum_{l,m}J_{l+m}(\alpha)J_{l-m}(\alpha)\int d\epsilon' d\epsilon\langle B_i(k,\mathbf{p},\epsilon')B_j(k,\mathbf{p},\epsilon)\rangle_p[f_{\epsilon'}-f_\epsilon]\frac{e^{-im\omega\mathcal{T}}}{\epsilon-\epsilon'+(m-l)\omega-i\delta}, \tag{44}$$

where

$$B_i(k,\mathbf{p},\epsilon)=\mathrm{Im}\{F_i^R(k,\mathbf{p},\epsilon)\}. \tag{45}$$

In the opposite limiting case (ii) $|T_{\mathbf{pp}'}^{ij}|^2=|T|^2$ one obtains

$$\begin{aligned}\langle I_{\rm qp}^{(i)}(\epsilon,\mathcal{T})\rangle_p=&\sum_j\Gamma_i^{(j)}\sum_{l,m}J_{l+m}(\alpha_{ij})J_{m-l}(\alpha_{ij})\int d\epsilon_1\Bigg[\mathrm{Im}\,\{g^{(i)R}(\epsilon_1)\}g^{(j)A}(\epsilon-m\omega)\frac{(1-2f_{\epsilon_1})e^{-il\omega\mathcal{T}}}{\epsilon-\epsilon_1+l\omega-i\delta}\\&-\mathrm{Im}\,\{g^{(i)R}(\epsilon-l\omega)\}g^{(j)R}(\epsilon_1)\frac{(1-2f_{\epsilon-l\omega})e^{il\omega\mathcal{T}}}{\epsilon-\epsilon_1-m\omega-i\delta}+g^{(i)R}(\epsilon_1)\mathrm{Im}\,\{g^{(j)A}(\epsilon-m\omega)\}\frac{(1-2f_{\epsilon-m\omega})e^{-il\omega\mathcal{T}}}{\epsilon-\epsilon_1+l\omega-i\delta}\\&-g^{(i)A}(\epsilon-l\omega)\mathrm{Im}\,\{g^{(j)R}(\epsilon_1)\}\frac{(1-2f_{\epsilon_1})e^{il\omega\mathcal{T}}}{\epsilon-\epsilon_1-m\omega-i\delta}\Bigg],\end{aligned} \tag{46}$$

where $g^{(i)R(A)}(\epsilon)=\int d\xi_p\langle G_i^{R(A)}(\mathbf{p},\epsilon)\rangle_p$, $\xi_p$ is the electron kinetic energy, $\Gamma_i$ is the virtual tunneling rate,[13] and $\alpha_{ij}=eE_{0\perp}^{(i)}d_\perp^B|i-j|/\omega$. In many applications, when strong anisotropy of the electron spectra is present the electron states are strictly separated in momentum space. For instance in the case of anisotropic pairing the SC order parameter forms petals (see Fig. 2) having the same (i.e., for symmetry $d_{x^2}$) or different (i.e., for symmetry $d_{x^2-y^2}$; Fig. 2) superfluid phase. Thus we distinguish the cases when the Josephson tunneling is happening either between the out-of-phase petals (see Fig. 2, arrows 1 and 3 correspondly) or between the in-phase petals (see Fig. 2, arrows 2 and 4) introducing the $\pi$ factor $\exp(i\gamma)$. Then the integration over the angles of electron momentum in formulas (36), (42), and (44) as well as in the corresponding expression for the self-energy $\hat{\Sigma}$ (see Refs. 11 and 12) is restricted within the petals having the same phase. The Josephson term gets the $\pi$ factor $\cos\gamma=\pm1$ for in-phase (upper sign) or out-of-phase (lower sign) tunneling processes. The expression (46) for the qp source as well as an analogous expression for the Josephson source allow us to find formulas for the quasiparticle and for the Josephson components of the interlayer ac current which are naturally derived from the quantum kinetic equation (36). Below we restrict ourselves to consider the dirty case (ii) leaving the clean case for a detailed description elsewhere. Using the dispersion relation

$$g^{R(A)}(\varepsilon)=\frac{1}{\pi}\int d\zeta\frac{\mathrm{Im}\,\{g^R(\zeta)\}}{\zeta-\varepsilon\pm i\delta}, \tag{47}$$

where the upper sign is attributed to index $R$ while the lower sign to index $A$ for the quasiparticle component of the ac current, Eq. (42), one obtains

$$J_{\rm qp}^{(i)}(\mathcal{T})=\sum_j\frac{1}{\pi eR_{ij}}\sum_{l,m}J_{l+m}(\alpha_{ij})J_m(\alpha_{ij})\times\int d\epsilon' d\epsilon L_{\epsilon',\epsilon}^{ij}\mathrm{Re}\left\{\frac{e^{-il\omega\mathcal{T}}}{\epsilon-\epsilon'-(m+l)\omega-i\delta}\right\}, \tag{48}$$

with

$$L_{\epsilon',\epsilon}^{ij}=\mathrm{Im}\,\{g^{(i)R}(\epsilon')\}\mathrm{Im}\,\{g^{(j)R}(\epsilon)\}(f_\epsilon-f_{\epsilon'}), \tag{49}$$

while for the Josephson (superfluid) part, Eq. (44), one finds

$$J_{\rm Jos}^{(i)}(\mathcal{T})=\sum_j\frac{\cos\gamma}{\pi eR_{ij}}\sum_{l,m}(-1)^mJ_{l+m}(\alpha_{ij})J_m(\alpha_{ij})\times\int d\epsilon' d\epsilon M_{\epsilon',\epsilon}^{ij}\mathrm{Re}\left\{\frac{e^{-il\omega\mathcal{T}}}{\epsilon-\epsilon'-(m+l)\omega-i\delta}\right\}, \tag{50}$$

with

$$M_{\epsilon',\epsilon}^{ij}=\mathrm{Im}\,\{\mathcal{F}^{(i)R}(\epsilon')\}\mathrm{Im}\,\{\mathcal{F}^{(j)R+}(\epsilon)\}(f_\epsilon-f_{\epsilon'}). \tag{51}$$

In formula (50), $\mathcal{F}^{(i)R(A)}(\epsilon)=\int d\xi_p\langle F_i^{R(A)}(\mathbf{p},\epsilon)\rangle_p$ and $R_{ij}$ is the interlayer resistivity. The above formulas obtained describe the ac current induced in the layered SC system by the applied external stationary ac field with the assumption that the interlayer voltage is alternating harmonically. These formulas also are appropriate to take into account many-body or depairing effects as well as the different symmetry of the SC order parameter. Because $J^{(i)}_{\rm Jos}$ is sensitive to the phase of the order parameter, this allows us to indicate the contribution of the $\pi$ junctions which distinguish the $d$-wave case from the anisotropic $s$-wave case.

Since we are interested in the characteristics for which mostly the energy range $0<\epsilon<\overline{E}\simeq\max(\Delta,T,\omega)$ is relevant, we include the superconducting correlations using the ordinary stationary expressions for the electron Green's function[16] entering (49)–(51). In addition we assume that the superconductivity is described within the simplest model with a three-dimensional energy gap. The different symmetry of the SC order parameter $\Delta(\theta,\phi)$ (where $\theta$ and $\phi$ are the angles between the electron momentum and the quantization axes) is considered assuming the BCS pairing to be of $s$-wave or of $d$-wave like. For the case of $s$-wave pairing $\Delta$ is isotropic while the $d$-wave case for example for the orthorhombic symmetry of the crystal lattice (e.g., $YBa_2Cu_3O_{7-\delta}$) is described by the point symmetry group $D_{2h}$ (see for instance Refs. 25 and 26).

Thus the Green functions calculated in the pole approximation are written as

$$g_i^R(\varepsilon)=i\pi\left\langle \frac{\varepsilon}{\sqrt{\varepsilon^2-\Delta_i^2(\theta,\phi)}}\right\rangle_{\theta,\phi},$$

$$g_i^R(\varepsilon)=[g_i^A(\varepsilon)]^*,$$

$$g_i^R(\varepsilon)-g_i^A(\varepsilon)=2\pi i\mathrm{Re}\{g_i^R(\varepsilon)\},$$

$$\mathcal{F}_i^{R(A)}=i\pi\left\langle \frac{\Delta_i(\theta,\phi)}{\sqrt{\varepsilon^2-\Delta_i^2(\theta,\phi)}}\right\rangle_{\theta,\phi}, \tag{52}$$

where $\langle\cdots\rangle_{\theta,\phi}$ means the averaging over the angles of the electron momentum within the same $d$ petal. Besides, since the double integrals incorporated in formulas (42)–(44) and (48)–(50) are expressed through the retarded [advanced] functions $g^{R(A)}(\varepsilon)$ $[f^{R(A)}(\varepsilon)]$, they are reduced to single integrals using their analytic properties[8] in the following way:

$$\mathcal{I}^{ij}(\zeta)=\mathcal{P}\int d\varepsilon_1 d\varepsilon_2 \mathrm{Im}\,\{g^{(i)R}_{\varepsilon_1}\}\mathrm{Im}\,\{g^{(j)R}_{\varepsilon_2}\}\frac{f_{\varepsilon_1}-f_{\varepsilon_2}}{\varepsilon_1-\varepsilon_2-\zeta}\Rightarrow\int d\varepsilon_1\mathcal{K}^{ij}_{\varepsilon_1,\zeta}(1-2f_{|\varepsilon_1|}),$$

$$\mathcal{K}^{ij}_{\varepsilon_1,\zeta}=\mathrm{Re}\{g^{(i)R}_{\varepsilon_1-\zeta}\}\mathrm{Im}\,\{g^{(j)R}_{\varepsilon_1}\}+\mathrm{Im}\,\{g^{(i)R}_{\varepsilon_1}\}\mathrm{Re}\,\{g^{(j)R}_{\varepsilon_1+\zeta}\}. \tag{53}$$

In the above formula $\mathcal{P}$ means the Cauchy principal value of the integral. In the case of a weak external field one may linearize the expressions over the field amplitude $E_\perp(t)$ using the Josephson relation $\varphi(t)=2ed_\perp^B\int dtE_\perp(t)$. For the quasiparticle $J^{(i)}_{\rm qp}\times(t)$ and Josephson $J^{(i)}_{\rm Jos}(t)$ current then one finds

$$J^{(i)}_{\rm qp}(t)=\sum_j\frac{1}{eR_{ij}}\frac{\alpha_{ij}}{2}[\cos\omega t\mathcal{I}^{ij}_{\rm qp}(\omega)-\sin\omega t\mathcal{I}^{ij}_{\rm qp,1}(\omega)],$$

$$J^{(i)}_{\rm Jos}(t)=\sum_j\frac{1}{eR_{ij}}\cos\gamma\frac{\alpha_{ij}}{2}[\cos\omega t\mathcal{I}^{ij}_{\rm Jos,2}(\omega)-\sin\omega t\mathcal{I}^{ij}_{\rm Jos,1}(\omega)], \tag{54}$$

where $\gamma=0$ for the ordinary junctions while $\gamma=\pi$ for $\pi$ junctions, $i,j$ are the indices of the adjacent layers, $R_{ij}$ is the interlayer normal state resistance. The functions in (54) are

$$\mathcal{I}^{ij}_{\rm qp}(\omega)=\int d\epsilon\mathrm{Im}\,\{g^{(i)R}(\epsilon-\omega)\}\mathrm{Im}\,\{g^{(j)A}(\epsilon)\}(f_{\epsilon-\omega}-f_\epsilon),$$

$$\mathcal{I}^{ij}_{\rm qp,1}(\omega)=\int d\epsilon[\mathrm{Re}\{g^{(i)R}_{\varepsilon-\omega}\}\mathrm{Im}\,\{g^{(j)R}_{\varepsilon}\}+\mathrm{Im}\,\{g^{(i)R}_{\varepsilon}\}\mathrm{Re}\,\{g^{(j)R}_{\varepsilon+\omega}\}](1-2f_{|\varepsilon|}),$$

$$\mathcal{I}^{ij}_{\rm Jos,1}(\omega)=\int d\epsilon[\mathrm{Re}\{\mathcal{F}^{(i)R}_{\varepsilon-\omega}\}\mathrm{Im}\,\{\mathcal{F}^{+(j)R}_{\varepsilon}\}+\mathrm{Im}\,\{\mathcal{F}^{(i)R}_{\varepsilon}\}\mathrm{Re}\,\{\mathcal{F}^{+(j)R}_{\varepsilon+\omega}\}](1-2f_{|\varepsilon|}),$$

$$\mathcal{I}^{ij}_{\rm Jos,2}(\omega)=\int d\epsilon\mathrm{Im}\,\{\mathcal{F}^{(i)R}(\epsilon-\omega)\}\mathrm{Im}\,\{\mathcal{F}^{+(j)A}(\epsilon)\}(f_{\epsilon-\omega}-f_\epsilon). \tag{55}$$

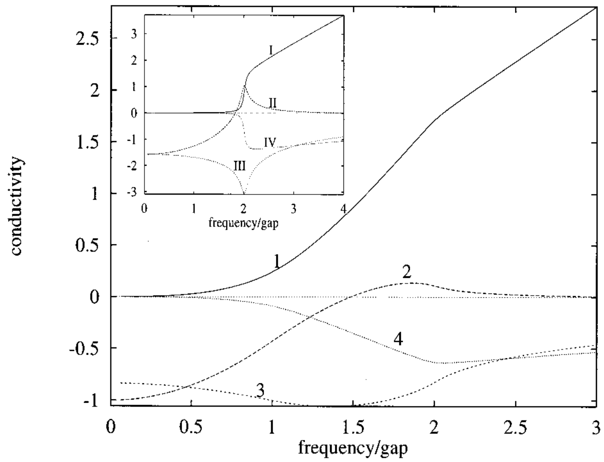


FIG. 3. The different components of the quasiparticle and Josephson currents for the anisotropic case.

For the real and imaginary components of the ac conductivity from (54) one obtains

$$\sigma_1(\omega)=\frac{\sigma_{N\perp}}{\omega}[\mathcal{I}^{ij}_{\rm qp}(\omega)+\cos\gamma\mathcal{I}^{ij}_{\rm Jos,2}(\omega)],$$

$$\sigma_2(\omega)=-\frac{\sigma_{N\perp}}{\omega}[\mathcal{I}^{ij}_{\rm qp,1}(\omega)+\cos\gamma\mathcal{I}^{ij}_{\rm Jos,1}(\omega)], \tag{56}$$

where $\sigma_{N\perp}$ is the normal state conductivity in the $c$ direction.

### III. RESULTS OF THE NUMERICAL CALCULATION

We calculate numerically the dielectric function, the reflectivity, and transmissivity for the $s$-wave (isotropic and anisotropic) and $d$-wave junctions using the above formulas.

In order to make an analogy to the case of a single Josephson junction[8] we calculate the different components of the quasiparticle and Josephson currents for the anisotropic case. In Fig. 3 one can compare the mentioned components $\mathcal{I}^{ij}_{\rm qp}(\omega)$ (see curve 1), $\mathcal{I}^{ij}_{\rm qp,1}(\omega)$ (curve 2), $\mathcal{I}^{ij}_{\rm Jos,1}(\omega)$ (curve 3), and $\mathcal{I}^{ij}_{\rm Jos,2}(\omega)$ (curve 4) computed for the case of in-phase tunneling and $d$-wave symmetry of the order parameter with the classic $s$-wave case presented in the inset to Fig. 3 where curves I, II, III, and IV correspond to the analogous components $\mathcal{I}^{ij}_{\rm qp}(\omega)$, $\mathcal{I}^{ij}_{\rm qp,1}(\omega)$, $\mathcal{I}^{ij}_{\rm Jos,1}(\omega)$, and $\mathcal{I}^{ij}_{\rm Jos,2}(\omega)$, respectively. From these figures one can infer that in the former case no distinguished gap features are exhibited at $\omega\simeq 2\Delta$ contrary to what takes place for the last case. Instead the amplitudes of the quasiparticle $\mathcal{I}^{ij}_{\rm qp}$, Josephson $\mathcal{I}^{ij}_{\rm Jos,1}$, and interference $\mathcal{I}^{ij}_{\rm Jos,2}$ currents become of the same order of value already at $\omega\simeq\Delta$, causing a visible modification of all the ac characteristics.

In Fig. 4 we show the frequency dependence of the imaginary part of the dielectric function $\epsilon_2(\omega)$ calculated in accordance with (1), (56) for the $s$-wave $o$ junction's system[8] (curve 1), for the $d$-wave $g$ junction's system (curve 2) with in-phase tunneling and for the $d$-wave $\pi$ junction's system (curve 3) with out-of-phase tunneling at $T=0.12$ [all temperature and energy units here are expressed in values of the order parameter amplitude $\Delta(0)$ at $T=0$; particularly, in these units $\sigma_{N\perp}=0.7$ which corresponds to the value 1.6 $\Omega^{-1}$ cm$^{-1}$] and $\epsilon_\infty=23$. The contribution of the possible depairing effects as well as inelastic collisions was taken into account by introducing an imaginary addition to the energy gap $\eta(T)=0.02\Delta(T)$. From Fig. 4 (curve 1) one can see that for the chosen parameters a visible threshold takes place at $\omega\leqslant 2\Delta$, being only a few percent less than the predicted Mattis-Bardeen theory[7] value at $\omega=2\Delta$ for a homogeneous isotropic SC. This threshold is caused by breaking off the Cooper pairs and by a contribution of the excited quasiparticles. Curve 2 in the same Fig. 4 shows $\epsilon_2(\omega)$ calculated for a $g$ junction. One can see that in this case $\epsilon_2(\omega)\neq 0$ already at low frequencies $\omega\to 0$ because of a contribution of the quasiparticles located along the lines and points of nodes in the anisotropic gap $\Delta(\mathbf{p})$. In contradiction to the previous curve 1 the energy gap value at $\omega=2\Delta_0$ ($\Delta_0$ is the amplitude of the gap) is not pronounced in this case. Curve 3 of the same figure shows $\epsilon_2(\omega)$ computed for a Josephson $\pi$ junction at the same parameters as before.

Figure 5 shows the frequency dependences $\epsilon_1(\omega)$ for the same $\Delta(\mathbf{p})$ symmetries and the same parameters as before. One can see that the value of $\Omega_{\rm ps}(T)$ is determined by a

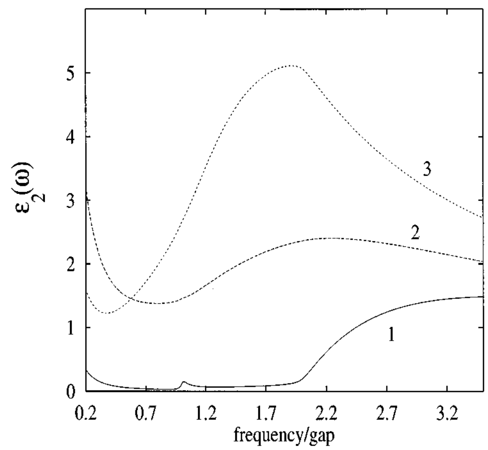


FIG. 4. The frequency dependence of the imaginary part of the dielectric function $\epsilon_2(\omega)$.

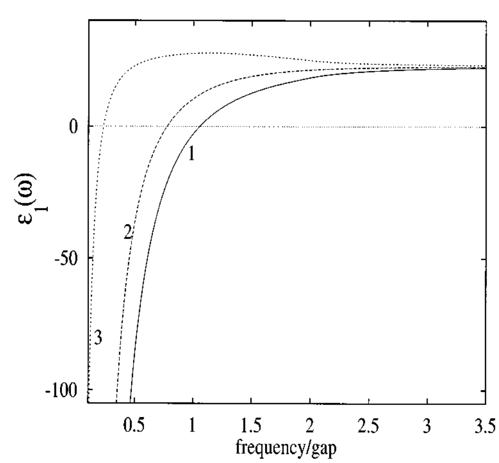


FIG. 5. The frequency dependences $\epsilon_1(\omega)$ for the same $\Delta(\mathbf{p})$ symmetries and the same parameters as before.

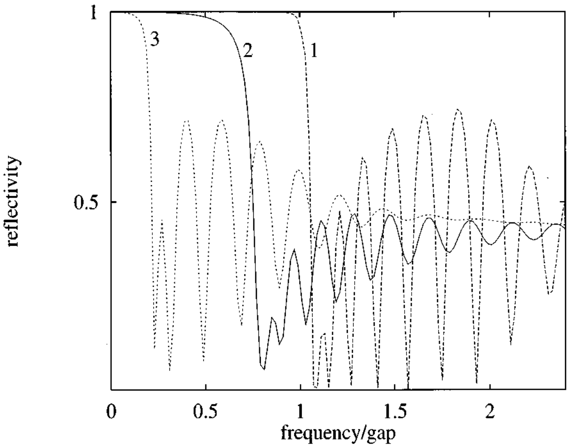


FIG. 6. The results of computation of the reflectivity $\mathcal{R}(\omega)$ for the $s$-wave (curve 1) and $d$-wave (curves 2, 3) order parameter's symmetry. Curve 2 corresponds to the in-phase tunneling while the curve 3 is for the out-of-phase tunneling.

crossing of $\epsilon_1(\omega)$ with the $\omega$ axes (similarly as takes place for the definition of the ordinary plasma frequency). Nevertheless, the resonance frequency $\Omega_{ps}(T)$ is established here by a balance between the quasiparticle and Josephson currents. Thus in our case $\Omega_{ps}(T)$ corresponds to transverse oscillations while the Josephson plasma frequency is related rather to longitudinal density oscillations.

Since the measurements of the reflectivity and transmissivity are more spectacular, we present the computation of these values of $\mathcal{R}(\omega)$ and $\mathcal{T}(\omega)$ in accordance with formulas (2) in Fig. 6 and Fig. 7. The calculations are performed for a thin film of thickness $d$. Curve 1 in Fig. 6 is the reflectivity $\mathcal{R}(\omega)$ calculated at $d=0.7c\hbar/\Delta_0$ for the classic $s$-wave case and coincides with the result obtained in Ref. 4 while curve 2 corresponds to the $d_{x^2-y^2}$ symmetry of $\Delta(\mathbf{p})$ for the case of in-phase IL tunneling. Curve 3 is attributed to $d_{x^2-y^2}$ symmetry as well but is for the out-of-phase Josephson tunneling between adjacent layers (see Fig. 2). The curves for the transmissivity $\mathcal{T}(\omega)$ (see Fig. 7) were obtained for the same parameters as before and the curves with the same numbers as in Fig. 6 correspond to the same order parameter's symmetry and type of tunneling as in that figure. Comparing curves 1, 2, and 3 in both Fig. 6 and Fig. 7 one can infer that the $\mathcal{R}(\omega)$ and $\mathcal{T}(\omega)$ curves are quite sensitive to the order parameter symmetry and to the type of tunneling as well.

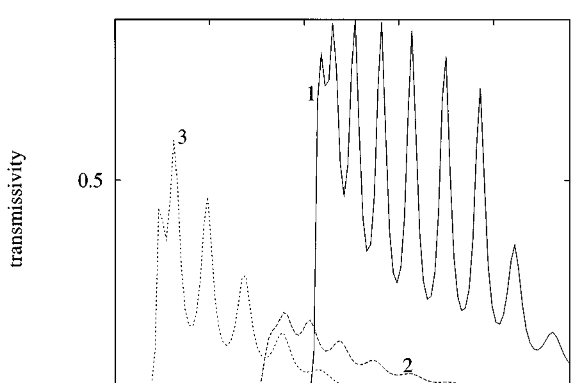


FIG. 7. The transmissivity $\mathcal{T}(\omega)$ for the same order parameter symmetries and types of tunneling as in the previous figure.

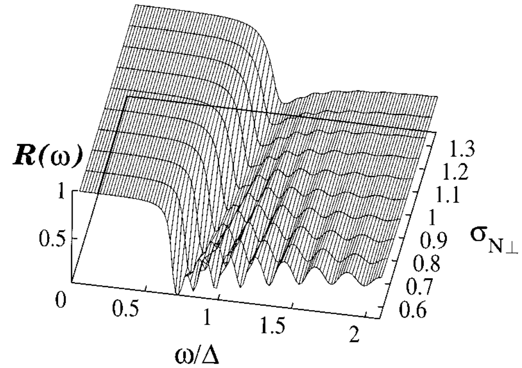


FIG. 8. Three-dimensional plot of the reflectivity $\mathcal{R}$ versus the field frequency $\omega$ and the $c$-axis normal state conductivity $\sigma_{N\perp}$.

In order to see how the above effect depends on the normal interlayer conductivity $\sigma_{N\perp}$ we present a three-dimensional plot of the reflectivity $\mathcal{R}(\omega)$ for the $d_{x^2-y^2}$ symmetry of $\Delta(\mathbf{p})$ and for the case of in-phase IL tunneling in Fig. 8. This figure demonstrates that the reflectivity $\mathcal{R}(\omega)$ depends on the value of $\sigma_{N\perp}$ in quantitative way: As $\sigma_{N\perp}$ decreases the reflectivity threshold $\Omega_{ps}$ is diminished and the oscillatory behavior of $\mathcal{R}(\omega)$ at $\omega>\Omega_{ps}$ becomes more pronounced.

From the above results one can see that the ac properties of the layered SC are quite sensitive to the order parameter symmetry because (i) the energy gap has lines and points of nodes along the Fermi surface; therefore the quasiparticle excitations exist until $T\rightarrow 0$; this causes a visible quasiparticle contribution to the total ac current already at small $T$ and $\omega$ as takes place in the homogeneous anisotropic superconductor;[27] (ii) due to the gap anisotropy effect the interference component $\mathcal{I}^{ij}_{\mathrm{Jos},2}(\omega)$ becomes comparable to the value with the quasiparticle component $\mathcal{I}^{ij}_{\mathrm{qp}}(\omega)$ already at $\omega\sim\Delta$; (iii) the Josephson current is sensitive to the order parameter phase and this may contribute to the ac properties as well; (iv) the reflectivity and transmissivity characteristics are very sensitive to the shape of the $\sigma(\omega)$ curves. They show striking undamped oscillatory behavior for the case of anisotropic pairing (in-phase tunneling) which itself may serve as an independent test for the unconventional superconductivity in the layered SC.

## IV. CONCLUSIONS

Finally one can conclude that the large value $\epsilon_\infty=23$ as well as the small $c$-axis normal state conductivity $\sigma_{N\perp}=1.6\ \Omega^{-1}\ \mathrm{cm}^{-1}$ used above are not sufficient to explain the low value of $\Omega_{ps}$ in metal oxides within an isotropic pairing model. The consistency is improved if one assumes that the order parameter is anisotropic. It causes the quasiparticle contribution to the total current even at low temperatures and frequencies which itself may reduce the value of $\Omega_{ps}$. However, the analysis made here shows that fairly closer agreement with known experiments may be achieved if one accepts the $d$-wave symmetry of the order parameter. In the last case the reflectivity and transmissivity characteristics (which in general are qualitatively different for the $s$-wave, anisotropic $s$-wave, and $d$-wave layered SC's) show an undamped oscillatory behavior which may serve as an independent test for this kind of pairing in the layered SC.

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