Electronic contributions to spin-wave characteristics in antiferromagnetic metals

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Magnetic properties of three-dimensional (3D) and two-dimensional (2D) metallic antiferromagnets in the spin-wave temperature region are investigated within the s-$d(f)$ exchange model. The spin-wave damping owing to one- and two-magnon scattering processes is calculated. The electronic contributions to the sublattice magnetization are obtained. At $T \ll T^*$ and $T \gg T^*$ (T^* is the threshold energy for one-magnon processes) the leading nonanalytical terms are proportional for $D=3$ to $T^4 \ln T$ and $\ln T$, respectively. Peculiar logarithmic contributions in the 2D case are derived. Nonanalytical corrections to the magnon spectrum and local moment on a site are analyzed. A comparison with the case of a ferromagnet is performed. [S0163-1829(96)04418-9]

The problem of magnon-magnon interactions and of the corresponding contributions to thermodynamic and magnetic properties in the Heisenberg model has been investigated for a long time (see, e.g., Refs. 1–4). Recently interest in quantum magnets, especially in two-dimensional (2D) and frustrated systems, has grown in connection with discovery of high-T_c superconductivity. At the same time, a number of anomalous metallic magnets (e.g., Kondo lattices, heavy-fermion compounds) has been studied experimentally.

Unlike insulating ''Heisenberg'' magnets, magnetic properties of metals are determined to a considerable extent by conduction electrons. There is a number of difficult problems and interesting peculiarities for weak itinerant electron magnets where spin waves possess a strong damping practically in the whole Brillouin zone, and paramagnonlike excitations play a dominant role.[5] On the other hand, for metals with localized magnetic moments (e.g., for rare earths and some d and f compounds) the situation, although being simpler, is not investigated in detail. It is usually accepted that these substances may be described by the s-$d(f)$ exchange model. To second order in the s-f parameter I the latter model is reduced to an effective Heisenberg model, so that spin waves are well defined unlike the case of itinerant magnets. However, there exist peculiar electronic contributions to spin-wave spectrum characteristics of local-moment metallic magnets. In particular, magnons possess finite damping at zero temperature, which is owing to electron-magnon scattering processes. Corresponding effects in thermodynamic properties can lead to drastic differences in comparison with insulating magnets.

For a ferromagnet (FM) higher-order corrections to the magnon spectrum were treated in Refs. 6–8. In the present work we consider the case of an antiferromagnet (AFM). We calculate the magnon damping, nonanalytical corrections to the spin-wave frequency and related contributions to the sublattice magnetization and local moment.

We start from the Hamiltonian of the s-$d(f)$ exchange model,

$$H=\sum_{\mathbf{k}\sigma} t_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} J_{\mathbf{q}} \mathbf{S}_{-\mathbf{q}} \mathbf{S}_{\mathbf{q}} - I \sum_{i\alpha\beta} \mathbf{S}_i \boldsymbol{\sigma}_{\alpha\beta} c^\dagger_{i\alpha} c_{i\beta}, \quad (1)$$

where $t_{\mathbf{k}}$ is the band energy, $\mathbf{S}_i$ and $\mathbf{S}_{\mathbf{q}}$ are spin-density operators and their Fourier transforms, and σ are the Pauli matrices. For the sake of a convenient construction of perturbation theory, we explicitly include the Heisenberg exchange interaction with the parameters $J_{\mathbf{q}}$ in the Hamiltonian. It should be noted that similar results may be reproduced for a Hubbard antiferromagnet provided that local moments are well defined (cf. Refs. 8,9).

To investigate the magnon spectrum we pass in the local coordinate system from spin operators to the Bose spin deviation operators $b^\dagger_i, b_i$ and calculate the corresponding retarded Green's function. Writing down the sequence of equations of motion and performing decouplings to accuracy of I^2 [cf. the calculations for FM's (Ref. 6) and AFM's (Ref. 10) we represent this in the form

$$\langle\langle b_{\mathbf{q}} | b^\dagger_{\mathbf{q}} \rangle\rangle_\omega = \frac{\omega + C_{\mathbf{q}-\omega}}{(\omega - C_{\mathbf{q}\omega})(\omega + C_{\mathbf{q}-\omega}) + D^2_{\mathbf{q}\omega}}, \quad (2)$$

$$\begin{aligned} C_{\mathbf{q}\omega} = {} & S(J^{\text{tot}}_{\mathbf{Q}+\mathbf{q},\omega} + J^{\text{tot}}_{\mathbf{q}\omega} - 2J^{\text{tot}}_{\mathbf{Q}0}) \\ & + \sum_{\mathbf{p}} [C_{\mathbf{p}} \Phi_{\mathbf{pq}\omega} - (C_{\mathbf{p}} - D_{\mathbf{p}}) \Phi_{\mathbf{p}00} + \phi^+_{\mathbf{pq}\omega} + \phi^-_{\mathbf{pq}\omega}] \\ & + \sum_{\mathbf{p}} [(2J_{\mathbf{Q}} + 2J_{\mathbf{q}-\mathbf{p}} - 2J_{\mathbf{p}} - J_{\mathbf{Q}+\mathbf{q}} - J_{\mathbf{q}}) \langle b^\dagger_{\mathbf{p}} b_{\mathbf{p}} \rangle \\ & - 2J_{\mathbf{p}} \langle b_{-\mathbf{p}} b_{\mathbf{p}} \rangle], \end{aligned} \quad (3)$$

$$\begin{aligned} D_{\mathbf{q}\omega} = D_{\mathbf{q}-\omega} = {} & S(J^{\text{tot}}_{\mathbf{q}\omega} - J^{\text{tot}}_{\mathbf{Q}+\mathbf{q},\omega}) \\ & + \sum_{\mathbf{p}} D_{\mathbf{p}} \Phi_{\mathbf{pq}\omega} + \sum_{\mathbf{p}} [(J_{\mathbf{Q}+\mathbf{q}} - J_{\mathbf{q}}) \langle b^\dagger_{\mathbf{p}} b_{\mathbf{p}} \rangle \\ & - 2J_{\mathbf{q}-\mathbf{p}} \langle b_{-\mathbf{p}} b_{\mathbf{p}} \rangle]. \end{aligned}$$

Here $\mathbf{Q}$ is the wave vector of the AFM structure (for simplicity $2\mathbf{Q}$ is supposed to be equal to a reciprocal lattice vector) and S is the spin value. The terms that contain the spin deviation correlation functions describe the magnon-magnon interaction. The s-d exchange contributions of first order in the quasiclassical small parameter $1/2S$ correspond to the RKKY approximation where

$$J^{\rm tot}_{\mathbf{q}\omega}=J_{\mathbf{q}}+I^2\sum_{\mathbf{k}}\frac{n_{\mathbf{k}}-n_{\mathbf{k}-\mathbf{q}}}{\omega+t_{\mathbf{k}}-t_{\mathbf{k}-\mathbf{q}}} \tag{4}$$

[$n_{\mathbf{k}}=n(t_{\mathbf{k}})$ is the Fermi function], the second summand in (4) being the ω-dependent RKKY indirect exchange interaction. Unlike the standard canonical transformation method, our approach permits one to calculate the spin-wave damping. The function Φ, which determines the second-order corrections, is given by

$$\Phi_{\mathbf{pq}\omega}=(\phi^+_{\mathbf{pq}\omega}-\phi^-_{\mathbf{pq}\omega})/\omega_{\mathbf{p}}, \tag{5}$$

$$\phi^{\pm}_{\mathbf{pq}\omega}=I^2\sum_{\mathbf{k}}\frac{n_{\mathbf{k}}(1-n_{\mathbf{k}+\mathbf{p}-\mathbf{q}})+N(\pm\omega_{\mathbf{p}})(n_{\mathbf{k}}-n_{\mathbf{k}+\mathbf{p}-\mathbf{q}})}{\omega+t_{\mathbf{k}}-t_{\mathbf{k}+\mathbf{p}-\mathbf{q}}\mp\omega_{\mathbf{p}}},$$

where $N(\omega)$ is the Bose function; note that $\phi^+_{\mathbf{pq}\omega}=-\phi^-_{\mathbf{pq}-\omega}$. The quantities $C_{\mathbf{p}},D_{\mathbf{p}}$ in the right-hand side of (3) should be substituted to zeroth order in I and $1/2S$, $(C_{\mathbf{p}}-D_{\mathbf{p}})\to 0$ at $p\to 0$, $(C_{\mathbf{p}}+D_{\mathbf{p}})\to 0$ at $\mathbf{p}\to\mathbf{Q}$, and

$$\omega_{\mathbf{p}}=(C^2_{\mathbf{p}}-D^2_{\mathbf{p}})^{1/2}=2S(J_{\mathbf{p}}-J_{\mathbf{Q}})^{1/2}(J_{\mathbf{Q}+\mathbf{p}}-J_{\mathbf{Q}})^{1/2}$$

is the magnon frequency. We have taken into account in (3) the expressions for the static correlation functions that occurred in the equations of motion,

$$I\sum_{\mathbf{k}}\langle c^\dagger_{\mathbf{k}-\mathbf{Q}\downarrow}c_{\mathbf{k}\uparrow}\rangle=-S(J^{\rm tot}_{\mathbf{Q}0}-J_{\mathbf{Q}}), \tag{6}$$

$$I\sum_{\mathbf{k}}\langle b^\dagger_{\mathbf{p}}(c^\dagger_{\mathbf{k}-\mathbf{p}\uparrow}c_{\mathbf{k}\uparrow}-c^\dagger_{\mathbf{k}-\mathbf{p}\downarrow}c_{\mathbf{k}\downarrow})\rangle=-(2S)^{1/2}(C_{\mathbf{p}}-D_{\mathbf{p}})\Phi_{\mathbf{p}00}. \tag{7}$$

These are obtained by calculating the corresponding retarded Green's functions and using the spectral representation.

Although the energy denominators in (4), (5) do not take into account the splitting of the band by AFM ordering, we have to separate the contributions of the transitions within the AFM subbands and between them (cf. Ref. 9). Generally speaking, the latter contributions are more singular, but in fact they are cut off owing to the AFM splitting $\Delta=2|I|\bar{S}$, $\bar{S}$ being the sublattice magnetization. The corresponding threshold value of the magnon quasimomentum transfer is estimated as $\min|\mathbf{p}-\mathbf{Q}|=q_0\sim\Delta/v_F$ (v_F is the electron velocity at the Fermi level). This quantity determines a characteristic temperature and energy scale

$$T^*=\omega(q_0)=cq_0\sim(\Delta/v_F)T_N, \tag{8}$$

with c being the magnon velocity. Note that for FM's with the quadratic dispersion law of spin waves one has $T^*\sim(\Delta/v_F)^2T_C$.[8]

Despite the absence of long-range magnetic ordering at finite temperatures, the result (2) is valid also in the two-dimensional (2D) case up to $T\sim J$ (i.e., $T_N\to JS^2$) owing to the strong short-range order. The quantity $\bar{S}$ is replaced by square root of the Ornstein-Cernike peak in the spin correlation function and has in the spin-wave region a linear T dependence (cf. Ref. 11).

The spin-wave damping owing to one-magnon decay processes, determined by the imaginary part of (4), reads at small q

$$\gamma^{(1)}_{\mathbf{q}}=\pi S\left[\frac{A}{L}\omega_{\mathbf{q}}+B\psi(q)\right], \tag{9}$$

where $L=2S(J_0-J_{\mathbf{Q}})$, the function ψ describes entering the "Stoner continuum," $\psi(q<q_0)=0$, $\psi(q\geqslant q_0)=1$, and

$$A=cI^2\lim_{\mathbf{q}\to 0}\mathbf{q}\sum_{\mathbf{k}}\delta(t_{\mathbf{k}})\delta(t_{\mathbf{k}-\mathbf{q}}), \tag{10}$$

$$B=LI^2\sum_{\mathbf{k}}\delta(t_{\mathbf{k}})\delta(t_{\mathbf{k}-\mathbf{Q}}), \tag{11}$$

$t_{\mathbf{k}}$ being referred to the Fermi level. Generally speaking, A depends on the direction of the vector $\mathbf{q}$ (see, e.g., Refs. 5,12). For an isotropic electron spectrum one has at $D=3$

$$A=cI^2\Omega_0\{4\pi^2|k^{-1}\partial t_{\mathbf{k}}/\partial k|^2_{k=k_F}\}^{-1}, \tag{12}$$

where Ω_0 is the lattice cell volume.

One can see that the one-magnon damping (9) is finite at arbitrarily small q(in contrast with the FM case), but becomes considerably larger when intersubband transitions begin to work ($q>q_0$). Unlike the FM situation ($\mathbf{Q}=0$), the intersubband damping does not contain the factor of q^{-1}. However, such a dependence occurs in some $\mathbf{q}$ region provided that the electron spectrum approximately satisfies the "nesting" condition $t_{\mathbf{k}+\mathbf{Q}}=-t_{\mathbf{k}}$ in a large part of the Fermi surface.

The damping owing to two-magnon scattering processes is determined by the imaginary part of the function (5). The intersubband transitions contribute at $\max(T,\omega_{\mathbf{q}})>T^*$. Using the identity

$$n(\epsilon)[1-n(\epsilon')]=N(\epsilon-\epsilon')[n(\epsilon')-n(\epsilon)], \tag{13}$$

we obtain from (2) to leading order in $\omega=\omega_{\mathbf{q}}$

$$\gamma^{(2)}_{\mathbf{q}}=\frac{\pi}{2}I^2\sum_{\mathbf{k,p}}\sum_{\alpha,\beta=\pm}\left(\frac{C_{\mathbf{q}}-\alpha D_{\mathbf{q}}}{\omega}\frac{C_{\mathbf{p}}+\alpha D_{\mathbf{p}}}{\omega_{\mathbf{p}}}-\beta\right)(\omega-\beta\omega_{\mathbf{p}})$$
$$\times[N(\omega_{\mathbf{p}})-N(\omega_{\mathbf{p}}-\beta\omega)]\delta(t_{\mathbf{k}})\delta(t_{\mathbf{k}+\mathbf{p}-\mathbf{q}}). \tag{14}$$

Integration at $T\ll\omega$ gives

$$\gamma^{(2)}_{\mathbf{q}}=\frac{\Omega_0}{24\pi c^3}\left[\left(\frac{9}{5}A\omega+\tilde{B}\right)\omega^2+4\pi^2(2A\omega+\tilde{B})T^2\right],$$
$$D=3, \tag{15}$$

$$\gamma^{(2)}_{\mathbf{q}}=\frac{\Omega_0}{8c^2}\left[\left(\frac{5}{3}A\omega+\tilde{B}\right)\omega+4(A\omega+\tilde{B})T\ln\frac{\omega}{T}\right],$$
$$D=2, \tag{16}$$

with $\tilde{B}(\omega\gg T^*)=B$ and $\tilde{B}(\omega\ll T^*)=0$. At $\omega\ll T$ we find

$$\gamma^{(2)}_{\mathbf{q}}=\frac{\Omega_0}{2\pi c^3}\left[6\zeta(3)AT+\frac{\pi^2}{3}\tilde{B}\right]T^2,\quad D=3, \tag{17}$$

$$\gamma^{(2)}_{\mathbf{q}}=\frac{\Omega_0}{2c^2}\left[\frac{\pi^2}{12}AT+\tilde{B}\ln\frac{T}{\omega}\right]T,\quad D=2, \tag{18}$$

with $\widetilde{B}(T \geqslant T^*) = B$, $\widetilde{B}(T \ll T^*) = 0$ and $\zeta(z)$ the Riemann function, $\zeta(3) \simeq 1.2$. As well as for the one-magnon damping, the intersubband contributions (terms with $\widetilde{B}$) become modified and increase considerably in the ''nesting'' situation. Although the two-magnon damping is smaller than the ''Landau'' damping (9), the former can be picked up owing to its strong temperature dependence.

Calculation of the two-magnon damping in a disordered AFM (the one-magnon damping is treated in Ref. 10, $\gamma^{(1)}_{\mathbf{q}\to 0} \propto \omega^2_{\mathbf{q}}$) can be performed by analogy with the case of the ferromagnet.[13] After averaging the δ function over the Fermi surface [see (14)] we have for $D=3$ to replace as compared to the ''clean'' limit

$$|\mathbf{p}-\mathbf{q}|^{-1} \to (2/\pi) b(|\mathbf{p}-\mathbf{q}|l)|\mathbf{p}-\mathbf{q}|^{-1}, \tag{19}$$

$$b(z) = (1+1/z^2)\arctan z - 1/z,$$

with l being the electron mean free path. Then we obtain in the hydrodynamic limit $ql \ll 1$, $T \ll c/l$ instead of (15), (17),

$$\gamma^{(2)}_{\mathbf{q}} = \frac{\Omega_0}{c^4} Al \times \begin{cases} \omega^4/10\pi^2, & T \ll \omega, \\ 8\pi^2 T^4/45, & \omega \ll T \end{cases} \tag{20}$$

(at small T, ω only intrasubband transitions yield a contribution). Thus the disorder in the electron subsystem leads to a decrease of the two-magnon damping.

Electronic corrections to the sublattice magnetization which are due to the one-magnon damping (9) are calculated in terms of the averages $\langle b^\dagger_{\mathbf{q}} b_{\mathbf{q}} \rangle$ with the use of the spectral representation for the Green's function (2),

$$\delta \overline{S}_{\mathrm{el}} = \frac{1}{\pi} \sum_{\mathbf{p}} \int_{-\infty}^{\infty} d\omega\ N(\omega) \left(\frac{\mathrm{Im} C_{\mathbf{p}\omega}}{\omega^2 - \omega^2_{\mathbf{p}}} + \frac{2 C_{\mathbf{p}} \omega \gamma^{(1)}_{\mathbf{p}}}{(\omega^2 - \omega^2_{\mathbf{p}})^2} \right). \tag{21}$$

The zero-point contributions owing to the first term in the brackets yield a Kondo-type logarithmic correction to the sublattice magnetization,[14,15]

$$[\delta \overline{S}_{\mathrm{el}}(0)]_1 = -2(I\rho)^2 S\ \ln(W/\overline{\omega}), \tag{22}$$

with ρ being the electron density of states at E_F, $\overline{\omega} = \omega(2k_F)$, and W of order of the bandwidth. The temperature correction to (22) for $D=2$ is

$$(\delta \overline{S}_{\mathrm{el}})_1 = -\frac{\Omega_0}{3c^2 L} \pi S B T^2 \ln \frac{\overline{\omega}}{\max(T, T^*)} \tag{23}$$

[the sense of the dependence $\overline{S}(T)$ in the 2D case is discussed above]. For $D=3$ the intrasubband $T^2 \ln T$ terms in the staggered magnetization are canceled, as well as for the paramagnetic susceptibility in the Fermi-liquid theory,[17] and the leading nonanalytical corrections owing to intrasubband transitions are proportional to $T^4 \ln T$. At $T \ll T^*$ the correction owing intersubband transitions from the second term in (21) reads

$$(\delta \overline{S}_{\mathrm{el}})_2 = -\frac{\Omega_0}{6c^2} SBL \times \begin{cases} 4\pi^2 T^4/15(T^*)^3 c, & D=3, \\ \pi(T/T^*)^2, & D=2, \end{cases} \tag{24}$$

and nonanalytical terms are absent. For $D=3$, $T > T^*$ the intersubband contribution takes the form

$$(\delta \overline{S}_{\mathrm{el}})_2 = -\frac{\Omega_0}{\pi^2 c^3} SLBT^* \ln \frac{T}{T^*}. \tag{25}$$

The correction of the form (25) can be obtained also for FM [but at low T, $\delta \overline{S}_{\mathrm{el}} \sim -(T/T_C)^2$ (Ref. 8)]. Note the difference in comparison with the case of weak itinerant magnets where the damping in the denominators of (21) may not be neglected. Then $\delta \overline{S}$ is proportional to $T^{4/3}$ and $T^{3/2}$ at not too low T (above T^*) for FM's and AFM's, respectively.[5,8]

In the 2D case the second term in (21) yields the logarithmic contribution

$$(\delta \overline{S}_{\mathrm{el}})_2 = \frac{\Omega_0}{2\pi c^2} SLB\ \ln \frac{\overline{\omega}}{\max(T, T^*)}. \tag{26}$$

For $T=0$ this expression yields an increase of $\overline{S}$ and describes the suppression of zero-point magnon oscillations owing to the spin-wave damping. Such corrections are specific for 2D antiferromagnets (and for 3D ''nested'' ones) and are absent in the FM case. Thus we have a new class of logarithmic divergences that depend essentially on the space dimensionality, unlike the Kondo terms.[15]

The electronic contributions to $\overline{S}$ owing to the two-magnon damping yield only small corrections to the usual spin-wave term (cf. the case of a ferromagnet[7]).

Consider the square of the total momentum on a site,

$$\langle \mathbf{S}^2_{i\mathrm{tot}} \rangle = \left\langle \left(\mathbf{S}_i + \frac{1}{2} \sum_{\alpha\beta} \boldsymbol{\sigma}_{\alpha\beta} c^\dagger_{i\alpha} c_{i\beta} \right)^2 \right\rangle. \tag{27}$$

This quantity is an analog of square of local moment in a Hubbard magnet. The spin-wave correction to (27) is given by

$$\delta \langle \mathbf{S}^2_{i\mathrm{tot}} \rangle_{\mathrm{SW}} = -\frac{\partial}{\partial I} \delta F_{\mathrm{SW}} \sim I\rho (T/T_N)^4, \tag{28}$$

where δF is the free energy of magnons,

$$\delta F_{\mathrm{SW}} = T \sum_{\mathbf{q}} \ln[1 + N(\omega_{\mathbf{q}})], \tag{29}$$

and the I dependence comes from the RKKY interaction (cf. Ref. 9). As well as standard spin-wave T^2 corrections to magnetization, corrections of the type (24)–(26) are canceled in (27) by transverse fluctuation contributions. For $D=3$ the leading nonanalytical electronic corrections are proportional to $T^4 \ln T$. Using (7) we obtain for $D=2$

$$\delta \langle \mathbf{S}^2_{i\ \mathrm{tot}} \rangle_{\mathrm{el}} = \frac{4\Omega_0}{3c^2} \frac{B}{I} \pi S T^2 \ln \frac{\overline{\omega}}{\max(T, T^*)} = -\frac{\partial}{\partial I} \delta F_{\mathrm{el}} \tag{30}$$

(cf. Ref. 16). The $T^2\ln T$ corrections occur also for a 3D FM (see Ref. 8). The ''Kondo'' correction to δF is of the order of $(I\rho)^2\overline{\omega}\ln(W/\overline{\omega})$.[14]

To calculate singular corrections to the spin-wave frequency, i.e., to the pole of (2), we substitute s-d contributions to the averages $\langle b^\dagger_{\mathbf{q}} b_{\mathbf{q}}\rangle, \langle b_{-\mathbf{q}} b_{\mathbf{q}}\rangle$, expand $\phi^{\pm}_{\mathbf{pq}\omega}$ in $\omega=\omega_{\mathbf{q}}$, and use (13). Integrating by parts we obtain for the spin-wave velocity

$$\frac{\delta c}{c}=-I^2\sum_{\mathbf{kp}}\delta(t_{\mathbf{k}})\delta(t_{\mathbf{k-p}})\int_{-\infty}^{\infty}\omega\, d\omega\left\{2\left[1-\left(1-\frac{J_{\mathbf{p}}}{J_0}\right)\omega_{\mathbf{p}}\frac{\partial}{\partial\omega_{\mathbf{p}}}\right]\frac{N(\omega)}{\omega^2-\omega^2_{\mathbf{p}}}+\left(1-T\frac{\partial}{\partial T}\right)\frac{L(C_{\mathbf{p}}-D_{\mathbf{p}})}{(\omega^2-\omega^2_{\mathbf{p}})^2}N(\omega)\right\}, \qquad (31)$$

where we have accepted for simplicity the nearest-neighbor approximation $J_{\mathbf{q+Q}}=-J_{\mathbf{q}}$. The first term in square brackets yields at $T=0$ the ''Kondo'' correction to the spin-wave frequency, $\delta\omega_{\mathbf{q}}/\omega_{\mathbf{q}}=-2(I\rho)^2\ln(W/\overline{\omega})$ for arbitrary $\mathbf{q}$ (cf. Ref. 15 where only magnon-magnon anharmonicity terms were treated). The intrasubband $(p\rightarrow 0)$ temperature correction reads for $D=3$

$$(\delta c/c)_1=-\frac{\Omega_0}{12c^3}AT^2\ln\frac{\overline{\omega}}{T}. \qquad (32)$$

Note that in the 3D FM case the correction to the spin-wave stiffness has the form $\delta\mathscr{D}/\mathscr{D}=-K(T/\overline{\omega})^2\ln(T/\overline{\omega})$ with $K(T\ll T^*)\sim 1$, but $K(T>T^*)\sim(I\rho)^2$ because of the strong compensation of intra- and intersubband contributions.[8] For $D=2$ the intrasubband nonanalytical corrections for AFM's are absent.

The intersubband transition ($\mathbf{p}\rightarrow\mathbf{Q}$) temperature correction to δc arising from the same term in (31) is singular for $D=2$. We have $(\delta c/c)_1=5(\delta S_{\rm el})_1/S$ with $(\delta S_{\rm el})_1$ given by (23). More singular intrasubband contributions for AFM's are absent.

$$(\delta c/c)_2=(1-T\partial/\partial T)(\overline{\delta S_{\rm el}})_2/2S, \qquad (33)$$

with $(\overline{\delta S_{\rm el}})_2$ given by (24)–(26). For finite q we have at $D=2$

$$\frac{\delta\omega_{\mathbf{q}}}{\omega_{\mathbf{q}}}=\frac{\Omega_0}{4\pi c^2}LB\ \ln\frac{\overline{\omega}}{\max(T,T^*,\omega_{\mathbf{q}})}, \qquad (34)$$

so that $\delta\omega_{\mathbf{q}}\propto q\ \ln q$ at $q>q_0$.

The obtained corrections to $\omega_{\mathbf{q}}$ result in nonanalytical terms in the Dyson expansion[1] of the sublattice magnetization,

$$\delta\overline{S}_{\rm mag}=-\sum_{\mathbf{p}}(C_{\mathbf{p}}/\omega_{\mathbf{p}})[\partial N(\omega_{\mathbf{p}})/\partial\omega_{\mathbf{p}}]\delta\omega_{\mathbf{p}}. \qquad (35)$$

At $T\ll T^*$ we have $\delta\overline{S}_{\rm mag}\propto T^4\ln T$ (for a ferromagnet the corresponding result has the form $\delta S_{\rm mag}\propto T^{7/2}\ln T$).

To conclude, we have presented a detailed study of the temperature and wave vector dependences of spin-wave characteristics in AFM metals, especially of the nonanalytical terms. The electronic contributions lead to strong effects which are absent in the insulating ''Heisenberg'' AFM (where effects of magnon-magnon interaction are rather weak[2,3]). This difference is usually disregarded in experimental investigations of magnetic and thermodynamic properties of metallic substances. Thus the search for the temperature dependences predicted is of great interest.

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