Exact propagators for a two-dimensional electron in quadratic potentials and a transverse magnetic field

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The propagator for an electron moving in a two-dimensional (2D) saddle-point potential under the influence of a transverse magnetic field is evaluated exactly using the Feynmann path integrals. The applications of this result for the calculation of tunneling through a saddle point are discussed. We also investigate tunneling a 2D electron at finite temperature in the presence of a transverse magnetic field, dissipation, and random scatterers by the use of a nonlocal harmonic oscillator model and find a crossover temperature T_1. The application of the same model at zero temperature for a quantum Hall system is shown to be incorrect.

I. INTRODUCTION

It is well known that a propagator for a quadratic action can be calculated exactly[1] (the WKB result becomes exact). Various calculations using path integrals were done: for systems with a harmonic force, a constant magnetic field, and time-dependent electric field,[2] and for an oscillator with memory (Ref. 3 and references therein). The applications of these results were used to obtain energy eigenvalues and a magnetization of a system,[4] a density of states,[5] and a magnetic susceptibility.[6]

In order to treat systems with random scatterers, Bezak[7] introduced a model for a Boltzmann electron gas in a Gaussian random potential. It allowed the exact calculation (within the model) of the density of states[5] and a magnetic susceptibility.[6] Later the same model was used to calculate exactly the propagator for a two-dimensional (2D) electron moving under the influence of a transverse magnetic field, a time-varying electric field, and a nonlocal harmonic force.[8] This latter result was finally applied to the calculation of a density of states of two-dimensional electrons in high magnetic field at zero temperature.[9]

The tunneling of an electron (vortex in superconductors) was also studied by the path-integral method.[10,11] The influence of magnetic field, potential, and dissipation on tunneling was investigated. The tunneling is also very sensitive to the temperature. This dependence on temperature becomes extremely important in systems exhibiting the quantum Hall effect. It was shown[12] that there is a crossover temperature T_1 above which tunneling acquires the form of thermal activation. The width of the conductance peak between Hall plateaus scales then as T^{κ} with $\kappa=3/7$ at $T<T_1$ and crosses over to $\kappa=6/7$ at $T>T_1$, in good agreement with experimental data.[13,14] Temperature T_1 for a saddle-point potential was calculated.

In this paper we address various aspects of an electron's behavior in the presence of a transverse magnetic field and different types of quadratic potentials. This paper is organized as follows. In Sec. II we present an exact evaluation of a propagator for an electron moving in a two-dimensional saddle-point potential under the influence of a transverse magnetic field and discuss the possibility to apply our result to the calculation of tunneling. In Sec. III with the use of the Bezak model[7] we investigate tunneling of an electron in the presence of random potential, dissipation, and transverse magnetic field at finite temperature and calculate a crossover temperature T_1 for strong magnetic fields. We show in Sec. IV that the Bezak model, while valid for systems with finite temperature (playing a role of imaginary time[6,7]), fails to obtain a correct physical propagator for a 2D electron in a transverse magnetic field and a random potential. We argue that the result obtained in Ref. 8, while absolutely correct mathematically, is inapplicable for such a physical system and consequently for the calculation of a density of states[9] the wrong propagator was used. Conclusions are presented in Sec. V.

II. EXACT CALCULATION OF THE PROPAGATOR

Electrons in a two-dimensional system with a strong magnetic field move along the lines of a constant potential (semiclassical approximation),[15] but when two such lines come close, the quantum tunneling begins to play an important role. This phenomenon is especially important for the description of the transition between plateaus in the quantum Hall effect.[13,14,16,17] The barrier between two equipotentials is well described by a saddle-point potential.[16] We will use the potential $V(x,y)=(m\omega^2/2)(y^2-x^2)$ and direct a magnetic field B along the z axis. The Lagrangian of the corresponding classical system is presented as

$$L=\frac{m}{2}\dot{\mathbf{r}}^2-\frac{m}{2}\omega^2\sigma_3\mathbf{r}^2+\frac{m}{2}\omega_c\dot{\mathbf{r}}J\mathbf{r}, \quad (1)$$

where $\omega_c=eB/mc$ is the cyclotron frequency. We introduced a 2×2 matrix $J=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ as in Ref. 2, σ_3 is a Pauli matrix, and a vector $r=\begin{pmatrix}x\\y\end{pmatrix}$. Since the Lagrangian given by Eq. (1) is quadratic the path integral can be evaluated exactly as

$$K(\mathbf{r}_T T|\mathbf{r}_0)=\frac{m}{2\pi i\hbar(\det D(T))^{1/2}}\exp\left(\frac{i}{\hbar}S(\mathbf{r}_T T|\mathbf{r}_0)\right), \quad (2)$$

where S is the classical action and the matrix $D(T)$ is defined as $mD^{-1}(T)=-\partial^2 S/\partial bfr_T \partial \mathbf{r}_0$. The action is calculated along the trajectory evolving via the equations of motion,

$$\ddot{\mathbf{r}}+\omega_c J\dot{\mathbf{r}}-\omega^2\sigma_3\mathbf{r}=0. \tag{3}$$

According to Ref. 4, we will look for the solution of Eq. (3) in the form $\mathbf{r}\sim\exp(Rt)$, where the matrix R has the structure $R=\begin{pmatrix}0&\beta\\ \alpha&0\end{pmatrix}$. After this substitution, Eq. (3) looks like

$$R^2+\omega_c JR+\omega^2\sigma_3=0, \tag{4}$$

which has two solutions, $R_1=\begin{pmatrix}0&-\omega_2\\ \omega_1&0\end{pmatrix}$ and $R_2=-\begin{pmatrix}0&\omega_4\\ \omega_3&0\end{pmatrix}$, where $\omega_{1,3}=\pm(\omega_c^2-2\omega^2\pm\sqrt{\omega_c^4+4\omega^4})/2\omega_c$, $\omega_{2,4}=2\omega_c\omega^2/(\omega_c^2+2\omega^2\pm\sqrt{\omega_c^4+4\omega^4})$. One can show that exponents can be presented as matrices

$$\exp(R_1t)=\begin{pmatrix}\cos\omega' t & \dfrac{\omega_1}{\omega'}\sin\omega' t\\ -\dfrac{\omega_2}{\omega'}\sin\omega' t & \cos\omega' t\end{pmatrix}, \tag{5}$$

$$\exp(R_2t)=\begin{pmatrix}\cosh\omega'' t & \dfrac{\omega_3}{\omega''}\sinh\omega'' t\\ -\dfrac{\omega_4}{\omega''}\sinh\omega'' t & \cosh\omega'' t\end{pmatrix},$$

where $\omega'=\sqrt{\omega_1\omega_2}$, $\omega''=\sqrt{\omega_3\omega_4}$.

Therefore the general solution of Eq. (3) is $\mathbf{r}(t)=\exp(R_1t)\mathbf{a}+\exp(R_2t)\mathbf{b}$. Vectors **a** and **b** are determined from the boundary conditions $\mathbf{r}(T)=\mathbf{r}_T$, $\mathbf{r}(0)=\mathbf{r}_0$. In the case of homogeneous quadratic Lagrangians, the classical action between the points $(\mathbf{r}_T T)$ and $(\mathbf{r}_0 0)$ becomes

$$S(\mathbf{r}_T T|\mathbf{r}0)=\frac{m}{2}(\mathbf{r}_T\dot{\mathbf{r}}_T-\mathbf{r}_0\dot{\mathbf{r}}_0). \tag{6}$$

After simple but long enough calculations one finally obtains the following formula:

$$\begin{aligned}S(\mathbf{r}_T T|\mathbf{r}0)=\frac{m}{2}\frac{1}{\Delta}\{&(x_T^2+x_0^2)[(\omega_2\omega_3/\omega'+\omega')\sin(\omega' T)\cosh(\omega'' T)-(\omega_1\omega_4/\omega''+\omega'')\cos(\omega' T)\sinh(\omega'' T)]\\&+2x_Tx_0[-(\omega_2\omega_3/\omega'+\omega')\sin(\omega' T)+(\omega_1\omega_4/\omega''+\omega'')\sinh(\omega'' T)]\\&+(x_Ty_T-x_0y_0)[(\omega_1-\omega_2-\omega_3-\omega_4)(1-\cos\omega' T)\cosh(\omega'' T)+(\omega_1-\omega_2)\omega''/\omega'\\&+(\omega_3-\omega_4)\omega'/\omega''\sin(\omega' T)\sinh(\omega'' T)]\\&+(x_Ty_0-y_Tx_0)(\omega_1+\omega_2+\omega_3-\omega_4)(\cos\omega' T)-\cosh(\omega'' T)\\&+(y_T^2+y_0^2)[(\omega'-\omega_1\omega_4/\omega')\sin(\omega' T)\cosh(\omega'' T)\\&+(\omega_2\omega_3/\omega''-\omega'')\cos(\omega' T)\sinh(\omega'' T)]+2y_Ty_0[(\omega_1\omega_4/\omega'-\omega')\sin(\omega' T)\\&+(\omega''-\omega_2\omega_3/\omega'')\sinh(\omega'' T)]\},\end{aligned} \tag{7}$$

where $\mathbf{r}_T=\begin{pmatrix}x_T\\ y_T\end{pmatrix}$, $\mathbf{r}_0=\begin{pmatrix}x_0\\ y_0\end{pmatrix}$, and $\Delta=2-2\cos(\omega' T)\cosh(\omega'' T)+[(\omega_2\omega_3-\omega_1\omega_4)/\omega'\omega'']\sin(\omega' T)\sinh(\omega'' T)$. For the determinant of the matrix D^{-1} we obtain

$$\begin{aligned}\det(D^{-1})=&4[\omega_1\omega_4+(\omega'')^2-(\omega')^2-\omega_2\omega_3]\sin^2(\omega' T)+4[2\omega'\omega''+(\omega_2\omega_3-\omega_1\omega_4)(\omega''/\omega'-\omega'/\omega'')\\&+(\omega_1^2\omega_4^2+\omega_2^2\omega_3^2)/(\omega'\omega'')]\sin(\omega' T)\sinh(\omega'' T)+4[(\omega'')^2+\omega_1\omega_4-(\omega')^2-\omega_2\omega_3]\sinh^2(\omega'' T)\\&+(\omega_1+\omega_2+\omega_3-\omega_4)^2[\cosh(\omega'' T)-\cos(\omega' T)]^2.\end{aligned} \tag{8}$$

One can easily check that if we substitute $\omega=0$ in the initial Hamiltonian then we obtain the well-known propagator for an electron in the perpendicular magnetic field. In the other case $\omega_c=0$ we get the answer for an electron in the saddle-point potential, which is represented by the product of two independent one-dimensional propagators, for the harmonic oscillator (along the y axis) and for the inverse parabola potential (along the x axis). The x-axis propagator is obtained from the y-axis one by substitution of an imaginary frequency.

The presented result can be used to consider the behavior of a tunneling electron. In this sense it is related to the transmission coefficient through the saddle point calculated exactly in Ref. 18. The initial Hamiltonian was transformed into the sum of two commuting Hamiltonians with new coordinates X and s and momenta conjugates P and p, which are the inverse parabola and harmonic oscillator Hamiltonians, respectively. The relation between x,y,p_x,p_y and new variables is given by

$$x=\sqrt{\frac{2}{m\omega_c}}(\alpha_1 X-\beta_2 s),\quad y=\sqrt{\frac{2}{m\omega_c}}(\beta_1 P+\alpha_2 p),$$

$$p_x=\sqrt{\frac{m\omega_c}{2}}(\alpha_3 P-\beta_4 p),\quad p_y=\sqrt{\frac{m\omega_c}{2}}(\beta_3 X+\alpha_4 s),\tag{9}$$

where α_i,β_i are coefficients dependent on m, ω_c, and ω (Ref. 18). The propagator for new coordinates is a product of two independent propagators, K_1 for the inverse parabola and K_2 for the harmonic oscillator, as was mentioned above. These propagators can be found in any book on path integrals (see, for example, Ref. 1). The relation between the propagators in ‘‘old’’ coordinates Eqs. (7,8) and in coordinates from Ref. 18 is given by the following integral:

$$K(\mathbf{r}_T T|\mathbf{r}_0)=\int\int\int\int dX'ds'dX''ds''K_1(X''T|X')K_2(s''T|s')\,\delta\left(x_T-\sqrt{\frac{2}{m\omega}}(\alpha_1 X''-\beta_2 s'')\right)\delta\left(x_0-\sqrt{\frac{2}{m\omega}}(\alpha_1 X'-\beta_2 s')\right)$$
$$\times\exp\left[\frac{i}{\hbar}\sqrt{\frac{m\omega}{2}}(\beta_3 X''+\alpha_4 s'')y_T\right]\exp\left[\frac{i}{\hbar}\sqrt{\frac{m\omega}{2}}(\beta_3 X''+\alpha_4 s')y_0\right].\tag{10}$$

III. TUNNELING OF A 2D ELECTRON IN THE PRESENCE OF MAGNETIC FIELD, DISSIPATION, AND RANDOM POTENTIAL AT FINITE TEMPERATURE

We consider the Euclidean action studied in Ref. 10, but instead of a harmonic pinning potential we introduce a random potential $U(x,y)$,

$$S=\int_0^{\hbar\beta}d\tau\left\{\frac{1}{2}M\dot{\mathbf{r}}^2+ieB\dot{x}y+V_1(y)+U(x,y)+\sum_j\left[\frac{1}{2}m_j\dot{\mathbf{q}}_j^2+\frac{1}{2}m_j\omega_j^2\left(\mathbf{q}_j-\frac{c_j}{m_j\omega_j^2}\right)\mathbf{r}\right]\right\},\tag{11}$$

where coordinate $\mathbf{q}_j$ describes environmental degrees of freedom. Assuming $U(x,y)$ to be a random Gaussian function, the average over all realizations of U of the exponential functional $\exp(-S/\hbar)$ can be calculated exactly[19] adding the following term to the sum in the exponent: $(1/2)\int_0^{\hbar\beta}\int_0^{\hbar\beta}d\tau d\tau W(\mathbf{r}(t)-\mathbf{r}(t'))$, where W is the autocorrelation function. If W is a Gaussian function, $W(\mathbf{r}-\mathbf{r}')=W(0)\exp[-(\mathbf{r}-\mathbf{r}')^2/L^2]$ (L is the correlation length), then one can use the idea of Bezak,[7] for large correlation L length, to expand the Gaussian in the series and to keep only the first two terms. This is a correct procedure if for all relevant trajectories $|\mathbf{r}(t)-\mathbf{r}(t')|<L$. We will discuss the conditions of applicability of the method later in this section.

Summing up over the final states and integrating over the q_j coordinate we obtain the following action:

$$S=\int_0^{\hbar\beta}d\tau\left\{\frac{1}{2}M\dot{\mathbf{r}}^2+ieB\dot{x}y+V_1(y)+\frac{1}{2}\int_0^{\hbar\beta}d\tau'\left[k(|\tau-\tau'|)+\frac{M\Omega^2}{2\hbar\beta}\right][\mathbf{r}(\tau)-\mathbf{r}(\tau')]^2\right\},\tag{12}$$

where $k(\tau)=(1/2\pi)\int_0^\infty d\omega J(\omega)\cosh[\omega(\hbar\beta/2-\tau)]/\sinh[\omega\hbar\beta]$, $M\Omega^2/2\hbar\beta=W(0)/L^2$. The effect of the dissipative environment is described by the spectral function $J(\omega)$. Now following Ref. 10 we write down a classical equation for coordinate x and solve with boundary conditions $x(0)=x(\hbar\beta)$ considering coordinate y as an external force:

$$M\ddot{x}+i\frac{e}{c}B\dot{y}-\int_0\hbar\beta d\tau'\left[k(|\tau-\tau'|)+\frac{M\Omega^2}{\hbar\beta}\right][x(\tau)-x(\tau')]=0.\tag{13}$$

We look for a solution in the form $x(\tau)=\Sigma_{n=-\infty}^{\infty}c_n\exp(i\nu_n\tau)$, where $\nu_n=2\pi n/\hbar\beta$. Substituting this form into Eq. (13) and using periodic boundary conditions $y(0)=y(\hbar\beta)$, we obtain

$$c_n=-\frac{\frac{e}{c}B\nu_n}{M\nu_n^2+M\Omega^2+\xi_n}\int_0\hbar\beta d\tau y(\tau)\exp(-i\nu_n t),\tag{14}$$

where $\xi_n=1/\pi\int_0^\infty d\omega[J(\omega)/\omega]2\nu_n^2/(\omega^2+\nu_n^2)$. Substituting the solution obtained from Eq. (13) into Eq. (12) we get the effective Euclidean action,

$$S_{\text{eff}}=\int_0^{\hbar\beta}d\tau\left\{\frac{1}{2}M\dot{y}^2+V_1(y)+\frac{1}{2}\int_0^{\hbar\beta}d\tau'\left[k(|\tau-\tau'|)+g(\tau-\tau')+\frac{M\Omega^2}{2\hbar\beta}\right][y(\tau)-y(\tau')]^2\right\},\tag{15}$$

where

$$g(\tau)=(1/2M\hbar\beta)(eB/c)^2\sum_{n=-\infty}^{\infty}\exp(i\nu_n\tau)\times(M\Omega^2+\xi_n)/(M\nu_n^2+M\Omega^2+\xi_n).$$

This type of tunneling problem in one dimension has been extensively studied.[20,21] Let us now consider the characteristic temperature T_1, which separates the region wherein the main contribution to the partition function comes from periodic solutions near the extremal points (above T_1) from that one wherein the bounce contributes (below T_1). According to Grabert and Weiss,[21] it corresponds to the transition between thermal hopping and quantum tunneling. T_1 is determined in the following way. We consider the action near the local maximum of the potential V_1, say at point $y=0$; i.e., we can present $V_1(y)=-K_y y^2/2$ near it (obviously the curvature of the barrier should be proportional on average to the inverse square of the correlation length, $K_y\sim 1/L^2$). Representing the periodic path near $y=0$ as

$$y(t)=\sum_{n=-\infty}^{\infty} Y_n \exp(i\nu_n t), \tag{16}$$

we obtain the second-order action,

$$S[y]=\frac{1}{2}M\hbar\beta\sum_{n=-\infty}^{\infty}\lambda_n Y_n Y_{-n}, \tag{17}$$

where eigenvalues λ_n are

$$\lambda_n=\nu_n^2-K_y/M+\Omega^2+\frac{\xi_n}{M}+\omega_c^2\frac{\nu_n^2}{\nu_n^2+\Omega^2+(\xi_n/M)}. \tag{18}$$

If the zero λ_0 eigenvalue is negative, then the integral over Y_0 becomes divergent. If all other λ_n are positive, then this divergence is overcome by distorting integrals into the imaginary plane, which leads to the imaginary part of the partition function (decay rate). This immediately puts a natural condition on the applicability of the Bezak method. For temperatures below $W(0)/K_yL^2$ the eigenvalue $\lambda_0>0$, meaning that the partition function has no imaginary part (decay rate), which is obviously impossible. Therefore, $T>W(0)/K_yL^2$ is a lower boundary for the Bezak method. The condition that all characteristic trajectories should obey $|\mathbf{r}(t)-\mathbf{r}(t')|<L$ is obviously satisfied: we consider small oscillations near the local maximum, which are much smaller than the correlation length. The semiclassical approach [the representation of the solution in the form of Eq. (18)] breaks down when the first eigenvalue λ_1 vanishes at some temperature T_1. In this case the integral over Y_1 and Y_{-1} becomes divergent. Grabert and Weiss[21] state that the vanishing eigenvalue points to the fact that below T_1 the classical equation of motion for $y(t)$ admits a new oscillatory solution (a nonlinear one called bounce). So we have the equation $\lambda_1(T_1)=0$. Because we are interested in T_1, we are still in the limits of the applicability of the Bezak method. Indeed, the bounce trajectory cannot exceed the distance between the local maximum and minimum, which is of order or less than the correlation length. For the limits of applicability in the x direction in the limit $\hbar\beta\to\infty$, we can change the summation in $x(\tau)=\Sigma_{n=-\infty}^{\infty}c_n\exp(i\nu_n\tau)$ by integration and obtain

$$x(\tau)=(\omega_c/\pi)\int_0^{\hbar\beta}d\tau' \times\int_0^{\infty}y(\tau')\nu\sin[\nu(\tau-\tau')]d\nu/(\nu^2+\Omega^2).$$

Taking into account that the effective width of the bounce is ω_b^{-1} (where $\omega_b\approx[K_y/M+\Omega^2+\omega_c^2]^{1/2}$ is the characteristic frequency of bounces) and using the condition for the y direction we obtain that $\max(x(\tau))<L$ if $\omega_c/\omega\leqslant 1$, which is obviously satisfied for all temperatures. For the case of Ohmic damping $J(\omega)=\eta\omega$ the term $\xi_n=\eta|\nu_n|$ and the equation looks like

$$\frac{4\pi^2}{(\hbar\beta)^2}-\frac{K_y}{M}+\frac{2W(0)\beta}{ML^2}+\frac{2\pi\eta}{M\hbar\beta}+\omega_c^2\frac{4\pi^2}{4\pi^2+\dfrac{2W(0)\hbar^2\beta^3}{ML^2}+\dfrac{2\pi\eta\hbar\beta}{M}}=0. \tag{19}$$

For comparison the equation for T_1 for the potential considered in Ref. 10 is

$$\frac{4\pi^2}{(\hbar\beta)^2}-\frac{K_y}{M}+\frac{2\pi\eta}{M\hbar\beta}+\omega_c^2\frac{4\pi^2}{4\pi^2+\dfrac{K_x}{M}(\hbar\beta)^2+\dfrac{2\pi\eta\hbar\beta}{M}}=0, \tag{20}$$

where K_x is a curvature in the X direction. It is clear from Eqs. (19) and (20) that in both cases the temperature T_1 is decreased by a switching on of the magnetic field. If $\omega_c=0$, then one can immediately find that T_1 in the case of the random potential is lower than for the case of pinning.

In the absence of a random potential (pinning) in the limit of high magnetic fields $\omega_c\gg(K_y/M)^{1/2},(2\eta/M)^{1/2}$ we obtain

$$T_1\approx\frac{\hbar K_y\eta}{2\pi M^2\omega_c^2}, \tag{21}$$

which is consistent with our general statement that the magnetic field decreases the temperature T_1 and shows that Ohmic dissipation leads to a small [order of ℓ Ref. 4, where $\ell=(\hbar c/eB)^{1/2}$ is a magnetic length] effect.[12]

In the absence of dissipation, Eq. (20) can be solved exactly and yields

$$T_1=\frac{\hbar}{\pi}\left(\frac{K_y}{2M}\right)^{1/2}\left[1+\frac{M\omega_c^2}{K_x}-\frac{K_y}{K_x}+\sqrt{\left(1+\frac{M\omega_c^2}{K_x}-\frac{K_y}{K_x}\right)^2+\frac{4K_y}{K_x}}\right]^{-1/2}. \tag{22}$$

In the limit $\omega_c\gg(K_{y,x}/M)^{1/2}$ we have

$$T_1\approx(1/2\pi)\ell^2\sqrt{K_xK_y}\sim\frac{\ell^2}{L^2} \tag{23}$$

as was found in Ref. 12.

Solving Eq. (19) in the absence of dissipation and in the strong magnetic fields $\omega_c\gg(K_y/M)^{1/2}$ yields the transition temperature

$$T_1 \approx \left(\frac{K_y W(0) \hbar^2}{2\pi^2 \omega_c^2 M^2 L^2} \right)^{1/3} = \left(\frac{K_y W(0) \ell^4}{2\pi^2 L^2} \right)^{1/3} \sim \left(\frac{\ell^2}{L^2} \right)^{2/3}. \tag{24}$$

The influence of the temperature on conductance peak σ_{xx} width was discussed in detail in Ref. 12. If T_1 is in the range of temperatures studied in the experiment then an intermediate value of the critical exponent κ between 3/7 and 6/7 could be measured. We have found that T_1 calculated in the case of the random potential exhibits similar behavior to T_1 calculated for the regular potential[12] (the dependence on the parameters of the system is weaker because of the power 2/3). It increases with increasing Landau level (decreasing magnetic field), predicting smaller κ, and decreases with increasing correlation length of the random potential in agreement with experiment,[13] indicating larger κ.

IV. APPLICABILITY OF THE BEZAK MODEL TO THE CALCULATION OF THE DENSITY OF STATES FOR A 2D ELECTRON IN HIGH MAGNETIC FIELDS AT ZERO TEMPERATURE

As we mentioned in the Introduction any propagator with a quadratic action can be calculated exactly. Sa-yakanit, Choosiri, and Robkob[8] considered the action presented in the following form:

$$S(\mathbf{r}_T T|\mathbf{r_0}) = \int_0^T \frac{m}{2} (\dot{\mathbf{r}}^2 + \omega_c \dot{\mathbf{r}} J \mathbf{r}) dt - \frac{m\nu^2}{4T} \int_0^T \int_0^T |\mathbf{r} - \mathbf{r}'|^2 d\tau dt, \tag{25}$$

where ν is the frequency of the nonlocal harmonic oscillator and we omitted an electric field term from Ref. 8. This is an analog of the action from Eq. (11) but without a barrier and dissipation and presented in real time. They have calculated the propagator explicitly. Then Nithisoontorn, Lassing, and Gornik[9] investigated a two-dimensional electron system in a high magnetic field with random scatterers. After averaging over all possible configurations, assuming high density and weak scatterers and applying the Bezak method, they obtained the effective action in the form of the Eq. (25) form of Eq. (10), with $\nu^2 = 2i\rho\eta^2 T/m\hbar L^2$, where ρ is the density and ϵ is the strength of scatterers. Then by taking the trace and performing a Fourier transform to the energy representation, the density of states was obtained.[9]

We repeated such a substitution ν in the exact propagator expression obtained in Ref. 8, but instead of taking the trace, we investigated the behavior of the propagator for time $T \to \infty$. It turns out that *the propagator diverges in this limit as*

$$K(\mathbf{r}_T T \to \infty | \mathbf{r}0) \propto \exp\left[\frac{5m\tilde{\omega}}{32} (\mathbf{r}_T - \mathbf{r}_0)^2 \right], \tag{26}$$

i.e., *it does not correspond to the real physical picture* $(\tilde{\omega}^2 = \omega_c^2/4 + \nu^2)$. The consequence of this divergence can also be found in Ref. 9. Before taking the Fourier transform, the authors neglected a small (but finite) positive imaginary part $\nu^2 T/\omega_c$ in the argument of the sine. Then they performed the commonly used expansion,

$$\frac{1}{\sin(\omega_c T/2)} = 2i \sum_{n=0}^{\infty} \exp[(-i\omega_c T/2)(n+1/2)]. \tag{27}$$

But this expansion is correct only if one adds an appropriate small negative imaginary part to frequencies.[1] Therefore if one neglects a small imaginary part of the frequency in Ref. 9 then the expansion would look like

$$\frac{1}{\sin(\omega T/2 + \nu^2 T/\omega_c)} = -2i \sum_{n=0}^{\infty} \exp[(+i\omega T/2)(n+1/2) - (|\nu^2| T/\omega_c)(n+1/2)] \tag{28}$$

and after performing a Fourier transformation one would get Landau levels with *negative* energies, which obviously does not correspond to the physical picture.

V. CONCLUSIONS

We have presented the exact evaluation of the propagator for a 2D electron in a saddle-point potential and a transverse magnetic field by the use of path integrals and have shown its relation to the tunneling probability calculated in Ref. 18. We considered the tunneling of the electron in the presence of the random potential, dissipation, and magnetic field at finite temperature by the Bezak method. We obtained a crossover temperature T_1 at which tunneling changes from the zero temperature behavior to an activated one and found its dependence on the magnetic field and correlation length of a random potential to be in agreement with experimental data. We discussed the applicability of the Bezak model to the quantum Hall system at zero temperature. The original idea was proposed for a Boltzmann gas of electrons where the characteristic ''diffusion length'' L_{dif} can be defined.[7] The method is justified if $L_{\mathrm{dif}} < L$. In the system under consideration, such a characteristic length cannot be defined. The negative result (divergent propagator) means that by using only the first two terms of the expansion of the Gaussian correlation function, one neglects trajectories which give a crucial contribution to the path integral. We therefore conclude that at zero temperature this model is inapplicable to the quantum Hall system.

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