Quantum conductance fluctuations in the large-size-scale regime

C. A. Richter, D. G. Seiler, and J. G. Pellegrino
Semiconductor Electronics Division, National Institute of Standards and Technology, Gaithersburg, Maryland 20899
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We report the results of experimental studies of ''universal'' conductance fluctuations in a variety of millimeter-sized GaAs/Al_xGa_{1-x}As heterostructures. The ability to observe these mesoscopic fluctuations in traditionally macroscopic semiconductor devices is due to the enhanced sensitivity of our magnetic field modulation measurement technique, which allows a coherent interference effect to be observed and studied in the large-size-scale regime where both the sample length and width are much greater than the quantum scattering lengths. [S0163-1829(96)04420-7]

The study of mesoscopic semiconductor devices is a large and exciting area of research[1,2] in condensed matter physics. Coherent quantum effects, such as Aharonov-Bohm oscillations, occur in these devices when an electron maintains phase coherence over the entire device region that is measured. Mesoscopic conductance fluctuations, or ''universal'' conductance fluctuations[1,3–11] (UCF), are perhaps the most robust of these coherent effects. UCF are sample-specific variations in the transport properties of mesoscopic devices, and are an area of active experimental interest that has been intensely studied in small device structures (usually $\cong 1$ μm^2).[6–9] Because some electrons must maintain coherence over the entire device in order for these effects to exist, it has generally been implicitly assumed that, at finite temperatures, coherent quantum effects such as UCF could not be observed in macroscopic or large-area devices (typically $\geqslant 1\times 10^4$ μm^2). Here we report the results of experimental studies of UCF in millimeter-size GaAs/Al_xGa_{1-x}As devices more commonly associated with studies of weak localization or the quantum Hall effect. The observation of a coherent interference effect in this large-size-scale regime indicates that a new perspective may be necessary when studying semiconductor devices. Quantum interference effects cannot be ignored *a priori* in semiconductor devices with large areas (or at moderate temperatures).

UCF have been reported previously in macroscopic devices in nonsemiconductor material systems. Milliken and Ovadyahu[10] reported conduction fluctuations in poorly defined, large-area In_2O_{3-x} films that were in the hopping regime. Smith *et al.*[11] have reported conductance fluctuations in macroscopically long amorphous-metal films; however, these films were 100–200 nm wide. Therefore, the overall area was still mesoscopic. In both instances, the fluctuations were observed at significantly lower temperatures (less than $\cong$150 mK) than those reported here. Furthermore, the signal-to-noise ratio is much better in our work reported here.

Our ability to study UCF in ''macroscopic'' Hall bar devices is due to our use of a measurement technique that is based on ac-magnetic-field modulation and lock-in amplifier techniques. This method allows changes in the resistance as a function of magnetic field to be measured with a greater sensitivity than standard dc magnetotransport techniques.[12] Our application of this technique to the study of mesoscopic effects at moderate magnetic fields allows UCF to be observed and studied in semiconductor devices in a large-size-scale regime where, to the best of our knowledge, coherent interference effects have not been previously observed. Information concerning quantum scattering lengths is derived from proper analysis of these fluctuations. These lengths give insight into the microscopic scattering mechanisms in semiconductor devices, and are necessary design parameters for novel quantum electron devices.[13] UCF can be used to characterize these scattering rates while avoiding the fabrication difficulties associated with small devices.

UCF are a manifestation of the quantum interference of electron waves passing over the weak random potential variations within an entire device. The application of an external magnetic field changes the interference pattern of the various electron paths, leading to reproducible, aperiodic fluctuations in the conductance as a function of magnetic field, $G(B)$. These fluctuations have been referred to as ''universal,'' because, at $T=0$ K, they have a rms amplitude that is of the order e^2/h (where e is the electron charge, and h is Planck's constant), independent of sample size and disorder. A well-developed theory exists that describes the statistical properties of UCF.[3–6] In this theory, a correlation function, $F(\Delta B)$, can be defined that characterizes the range of the observed conductance fluctuations:

$$F(\Delta B)=\langle[G(B_{dc})-\langle G(B_{dc})\rangle][G(B_{dc}+\Delta B) -\langle G(B_{dc}+\Delta B)\rangle]\rangle, \quad (1)$$

where the angular brackets indicate an ensemble average, B_{dc} is the applied magnetic field, and ΔB is an increment in the applied magnetic field. A correlation field B_C is defined such that the correlation function at $\Delta B=B_C$ is half the value of the function at $\Delta B=0$: mathematically, $F(B_C)=0.5F(0)$. The devices used in this study are large (see Fig. 1); the length (L) and width (w) of the sample are both much larger than the electron scattering lengths ($\leqslant 1$ μm) in the electron gas. Therefore, the two-dimensional limit of the UCF theory applies. In this limit, the correlation field is proportional to the square of the inverse of the quantum coherence breaking length, L_{QC} [$B_C\propto(1/L_{QC})^2$]. We define L_{QC} to be the length scale over which the quantum coherence is maintained. This length is determined by the various scattering lengths in the sample (such as the phase breaking length L_φ, the spin-orbit length L_{SO}, and the thermal length $L_{th}=(hD/2\pi k_BT)$, where

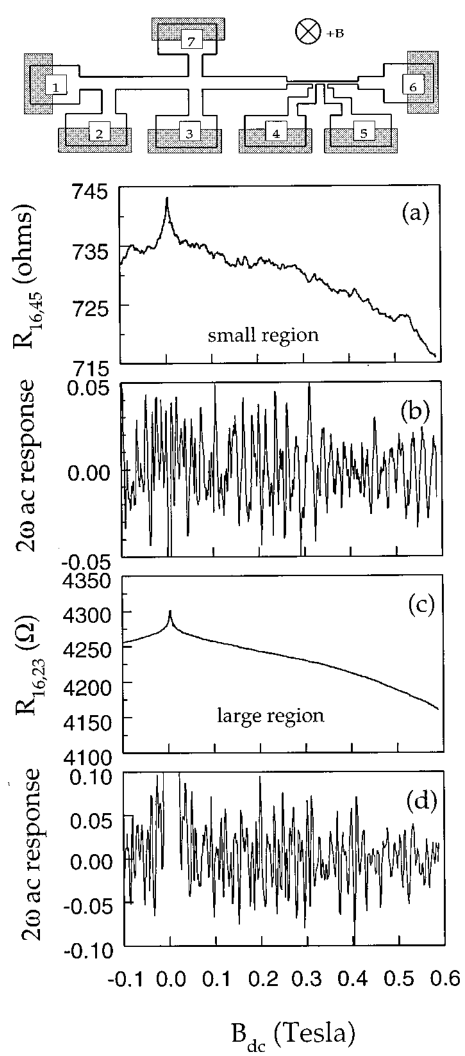

FIG. 1. (a) $R_{16,45}$, the resistance (Ref. 14) of the ''small'' region. $L=16$ μm, $w=8$ μm. (b) 2ω ac response (Ref. 17) between 4 and 5. (c) $R_{16,23}$, the resistance of the ''large'' region. Length, $L=500$ μm, and width, $w=50$ μm. (d) The ac response at 2ω measured between probes 2 and 3; $T=3.0$ K. Top: schematic of the device geometry.

D is diffusivity, k_B is Boltzmann's constant, and T is temperature). $L_{\rm QC}$ is dominated by $L_{\rm th}$ in this work because of the relatively large temperatures used in these experiments. $F(\Delta B=0)$ is the statistical variance of the conductance, $\mathrm{var}(G)$, and describes the amplitude of the fluctuations. For a two-dimensional device,

$$\mathrm{var}(G)/\langle G\rangle^2 \propto (L_{\rm QC}^2)/(Lw). \qquad (2)$$

Both the $\mathrm{var}(G)$ and the falloff of $F(\Delta B)$ (i.e., B_C) depend upon $L_{\rm QC}$. By measuring and analyzing these statistical properties, quantum scattering lengths are obtained.

The samples used in these studies were fabricated from conventional, single-interface, modulation-doped GaAs/$Al_{0.3}Ga_{0.7}$As heterostructures. The device geometry (shown in Fig. 1) allows the longitudinal resistance of both a ''large'' region ($R_{16,23}$) and a ''small'' region ($R_{16,45}$) to be simultaneously measured, where $R_{ij,kl}$ is the four-terminal resistance when the current is passed from probes i to j and the voltage is measured between k and l. The small region is still bigger than most devices typically used to study UCF. A heterostructure containing a two-dimensional electron gas (2DEG) with a carrier density, $n_s \cong 3.4\times10^{15}$ m^{-2}, and a zero magnetic field mobility, $\mu\cong4.4$ m^2/V s at $T=4.2$ K, was chosen for these experiments. The short scattering lengths in this low-mobility 2DEG ensure that the device is in the diffusive, two-dimensional regime and in the low-magnetic field regime, allowing a straightforward comparison with conventional UCF theory; also, Shubnikov–de Haas (SdH) oscillations are not observed at 3.0 K until above 0.6 T, thus allowing UCF to be observed over a large magnetic field range which results in good statistics.

The devices are measured by an ac-magnetic-field modulation and lock-in amplifier technique,[12] which is extremely sensitive to changes in the resistance as a function of magnetic field. Equation (2) predicts that while the amplitude of UCF is extremely small in large-area devices, it is still present. The enhanced sensitivity of this measurement technique allows us to observe UCF in the large-device-area regime, confirming the theoretical predictions of Eq. (2). Experimentally, in addition to $B_{\rm dc}$, we apply an ac magnetic field of frequency ω with a constant amplitude B_0. (For this work, $B_0=5$ mT, and ω corresponds to a frequency of 12.5 Hz. Both $B_{\rm dc}$ and B_0 are applied perpendicular to the 2DEG.) As a dc current is applied to a device, the device response at twice the modulation frequency, 2ω, is measured by using lock-in amplifier techniques. This ac response at 2ω is related to the second derivative of the device resistance with respect to magnetic field. The dc current magnetoresistance is measured simultaneously with the 2ω response. In order to show a comparison between the 2ω response and ''typical'' low-noise resistance measurements, the magnetoresistance curves in Figs. 1(a) and 1(c) were taken using standard ac lock-in techniques and low ac currents (100 nA at 18 Hz) with the modulated magnetic field off ($B_0=0$).[14]

The modulated-magnetic-field technique is shown here to be an extremely powerful tool for studying small variations in the device resistance. First, this technique allows the observation of small changes in resistance that cannot be measured when more conventional dc magnetic field techniques are used. We have measured fluctuations roughly corresponding to changes in device resistance as small as 1 part in 10^7. Another strength of this measurement technique is that the fluctuations in the 2ω response are centered around zero signal and can be studied directly. In general, when studying UCF with more conventional methods, the measured conductance has a slowly varying background, which must be fit and subtracted from the total conductance in order to obtain the variance. This numerical manipulation of the data adds to the uncertainty in the analysis of the fluctuations.

Referring to Eq. (2), it is expected that UCF can be observed at low temperatures in the smaller region of the device. Experimentally, in addition to the weak localization peak[1,15] centered at $B_0=0$ T and a slowly changing background,[1,16] the resistance $R_{16,45}$ [Fig. 1(a)] clearly shows the reproducible fluctuations, or magnetofingerprint, due to the UCF. The resistance of the large device segment $R_{16,23}$ [Fig. 1(c)] shows no indication of similar fluctuations because they are smaller than the resolution capabilities of the standard ac lock-in measurement. Reproducible structure due

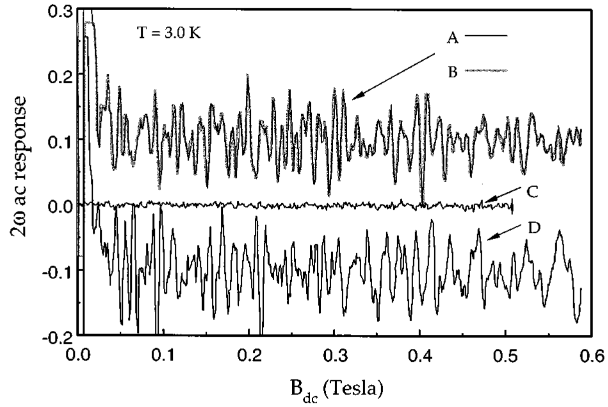

FIG. 2. A and B are two different measurements of the 2ω response (Ref. 17) between probes 2 and 3 at $T=3.0$ K. Note, on this scale, the two traces coincide. C: 2ω ac response with $B_0=0$ illustrating the size of the system noise. D: 2ω ac response between probes 2 and 3 at $T=3.0$ K after warming the sample above 200 K.

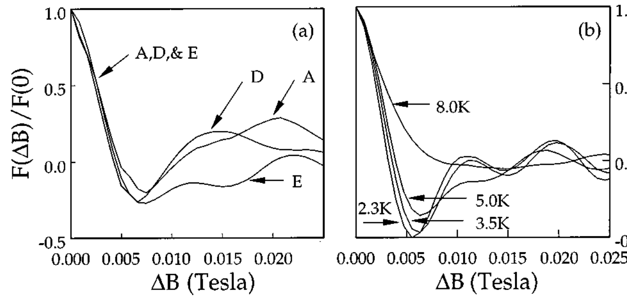

FIG. 3. Normalized correlation functions, $F(\Delta B)/F(\Delta B=0)$, of the 2ω ac response. (a) A and D correspond to A and D in Fig. 2. E is the correlation of the 2ω ac response of the small region [Fig. 1(d)]. (b) $F(\Delta B)/F(\Delta B=0)$, of the 2ω ac response of the large region of a second device for various temperatures.

to fluctuations in the magnetoresistance is observed in the 2ω response that is measured across both the small region (probes 4 and 5), and across the large region (probes 2 and 3) [Figs. 1(b) and 1(d)].[17] The 2ω response measured across the small region can be directly correlated with the magnetoresistance fluctuations observed in $R_{16,45}$; thus, the conductance fluctuations observed by using the modulated-magnetic-field technique are due to UCF.

A series of measurements was performed to ensure that these fluctuations are not a measurement artifact, or do not arise from some mechanism other than UCF. A and B, shown in Fig. 2, are the 2ω responses measured across the large region (probes 2 and 3) in two different experimental sweeps separated in time by a few hours, but obtained while the sample was still maintained at low temperatures; these data illustrate the reproducibility of the fluctuations. D (Fig. 2) is the experimental data after the device has been warmed to temperatures above 200 K and then cooled to 3 K. The exact structure observed is different from that in A and B; however, the magnitude and the characteristic field of the fluctuations are the same as before the device was thermally cycled. This is the expected behavior for UCF. At the higher temperatures, the impurities and dopants change their electronic configurations, so that when the device is again cooled, the random potential is different. This new random potential leads to a new magnetofingerprint. As an illustration that the amplitude of the instrumental noise in the system is significantly less than the observed fluctuations, C (Fig. 2) is the ac response measured when $B_0=0$ T. We have observed similar fluctuations in a variety of device geometries and heterostructure materials in addition to those described here.

The sensitivity of the modulated-magnetic-field measurement technique allows these small fluctuations to be studied, but the derivative nature of this method makes a direct, quantitative comparison with the statistical UCF theory for the conductance difficult. Furthermore, we know of no theoretical results concerning the effects of modulated magnetic fields on UCF to compare directly with our data. However, the trends predicted for the conductance fluctuations will also be observed in the 2ω response. The correlation function [Eq. (1)] of the 2ω response has been analyzed, and a great deal of qualitative information can be derived. A normalized correlation function, $F(\Delta B)/F(0)$, can be defined for the 2ω response, and the trends observed will be the same as for the correlation of the overall conductance. This normalized correlation function, $F(\Delta B)/F(0)$, is shown in Fig. 3 for various data sets. There is a negative drop in these normalized correlation functions that arises from the derivative nature of the measurement technique. This drop would not be observed in the correlation functions of the direct conductance fluctuations.

Recall that $B_C \propto (1/L_{QC})^2$; therefore, the falloff of $F(\Delta B)$ is determined by the phase coherence length. $F(\Delta B)/F(0)$ is the same for both the large and the small segment under the same measurement conditions [Fig. 3(a)]. This is a strong indication that the physical mechanism giving rise to the fluctuations is the same in both device segments. Although the exact pattern of the conductance fluctuations is different before and after thermal cycling, the weighted correlation is also the same in these cases. While the exact configuration of the scattering centers is changed by warming the sample, the ensemble average that determines L_{QC} remains the same. Another device, with a slightly larger electron mobility ($\mu \cong 7.7$ m^2/V s), but otherwise identical, was also measured. Due to the larger scattering lengths in this sample as indicated by the higher mobility, UCF theory predicts that the correlation function (not shown) for this device would have a faster falloff than the previously discussed device. The data indicate that this is true. [The effective B_C, where $F(B_C)/F(0)=0.5$, is $\cong 1.7$ mT compared with $\cong 2.5$ mT.]

The temperature dependence of the fluctuations has been studied in a second device nominally the same as the one previously discussed in detail (Fig. 4) and is in qualitative agreement with the theory of UCF. Notice that, at a given temperature, the conductance fluctuations are uniform in amplitude across the entire span of magnetic field, and the fluctuations are largest at our base temperature and decrease with increasing temperature until they cannot be experimentally resolved at temperatures above ≈ 12 K. The variance of the 2ω response, var(2ω), depends upon L_{QC} [Eq. (2)]. L_{QC} decreases with increasing temperature, and this leads to the observed decrease in the amplitude of the fluctuations. This decrease in L_{QC} can be investigated more quantitatively by studying the normalized correlation function. As predicted

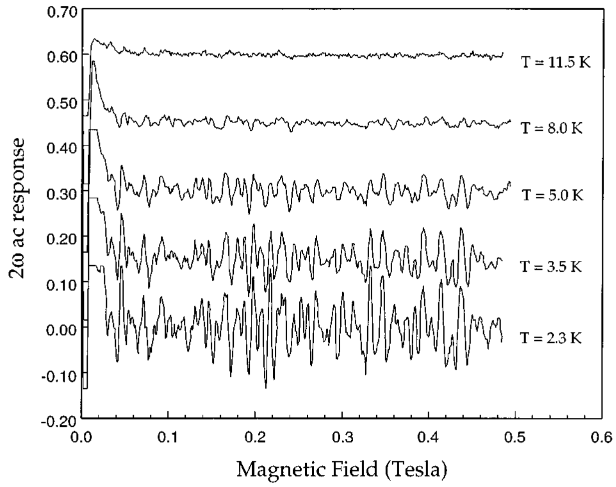

FIG. 4. Temperature dependence of the 2ω ac response between probes 2 and 3 for the second device. The curves are offset for viewing purposes. Note that the reproducibility of the fluctuations is again shown by these data.

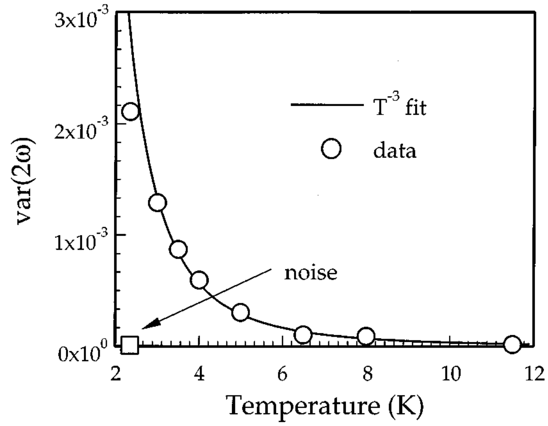

FIG. 5. Temperature dependence of the variance of the 2ω ac response shown in Fig. 4. The open circles represent the experimentally obtained data. The solid line is an empirical T^{-3} fit. The experimental noise level is indicated by the open square.

by UCF theory, the decrease in L_{QC} with increasing the temperature shifts B_C and subsequently the dropoff of $F(\Delta B)$ to larger magnetic fields. This trend is illustrated in Fig. 3(b). Experimentally, the var(2ω) is observed to have a T^{-3} functional dependence (Fig. 5) in the temperature regime of this work. This is dramatically different from the T^{-1} dependence that is expected from the results of standard measurements of the variance of the conductance in this relatively high temperature regime.[6] It is most probable that this difference arises because the 2ω response is related to the second derivative, not the conductance itself. It may also be possible that the observed temperature dependence in these large samples is due to a different physical mechanism than that described by conventional UCF theory. Because there is no quantitative theory for the effects of a modulated-magnetic-field on UCF, it is difficult to resolve this issue conclusively.

In the absence of a theory of the modulated-magnetic-field response, we have simply integrated twice to obtain conductance fluctuations that can be compared directly with the existing theory. (The necessary scaling factors are obtained by a comparison with the dc resistance that is measured simultaneously.) From the fluctuations in this derived device conductance, L_{QC} can be obtained from both the variance and B_C. The values of L_{QC} derived by this process are in qualitative agreement with the thermal lengths ($L_{\mathrm{th}}\approx 0.42$ μm at $T=2.3$ K and $L_{\mathrm{th}}\approx 0.23$ μm at 8.0 K), and the range of these values is supported by values of the phase coherence length independently obtained from fits to the weak localization peak. (The $L_\varphi \approx 0.7$ μm at $T=2.3$ K and ≈ 0.4 μm at $T=8.0$ K from weak localization techniques.)

The conductance fluctuations that we have studied in these large samples exhibit the traits of UCF. By using modulated-magnetic-field measurement techniques, we have observed and studied in semiconductor devices, a coherent interference effect in a large-size-scale regime. Because a quantum interference effect is observed in these devices, they can no longer be considered macroscopic, as seems most intuitive, but must be thought of as mesoscopic device structures. The enhanced measurement sensitivity demonstrated in these measurements should allow other known quantum interference effects to be studied in new temperature or size regimes, and even make possible the observation of novel, small-amplitude, quantum effects that have not yet been discovered when using more conventional experimental techniques.

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[1] C. W. J. Beenakker and H. van Houten, in *Solid State Physics*, edited by H. Ehrenreich and D. Turnbull (Academic, New York, 1991), Vol. 44, p. 1.
[2] *Nanostructured Systems*, edited by R. K. Willardson, A. C. Beers, and E. R. Weber, Semiconductors and Semimetals Vol. 35 (Academic, New York, 1992).
[3] P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985); B. L. Al'tshuler, Pis'ma Zh. Éksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)].
[4] P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).
[5] A. D. Stone, in *Physics of Nanostructures*, edited by J. H. Davies and A. R. Long (IOP Publishing, Bristol, 1992), p. 65.
[6] O. Millo *et al.*, Phys. Rev. Lett, **65**, 1494 (1990).
[7] A. K. Geim *et al.*, Phys. Rev. Lett. **69**, 1248 (1992).
[8] A. Morgan *et al.*, Phys. Rev. B **50**, 12 187 (1994).
[9] Y. Ochiai *et al.*, Physica B **201**, 357 (1994).
[10] F. P. Milliken and Z. Ovadyahu, Phys. Rev. Lett. **65**, 911 (1990).
[11] W. F. Smith *et al.*, Phys. Rev. B **43**, 12 267 (1991).
[12] A. Goldstein, S. J. Williamson, and S. Foner, Rev. Sci. Instrum. **36**, 1356 (1965). See discussion of D. G. Seiler and A. E. Stephens, in *Landau Level Spectroscopy*, edited by G. Landwehr and E. I. Rashba (North-Holland, Amsterdam, 1991), p. 1031.
[13] F. Capasso and S. Datta, Phys. Today **43** (2), 74 (1990).

[14]The magnetoresistance (MR) curves acquired using strictly dc techniques simultaneously with the 2ω response have a larger electronic noise level than the ac current curves. The 2ω response is strictly correlated with these simultaneously acquired dc MR curves. The fluctuations in the ac current MR curves are different than those in the dc MR curves and therefore, do not correspond to the 2ω response. This is as expected, because the Fermi energy will be at a different location when a 3-μA dc current is applied than when a 100-nA ac current is applied. (A change in the Fermi level leads to a change in the UCF patterns.)

[15]B. L. Altshuler, A. G. Aronov, D. E. Khmelnitskii, and A. I. Larkin, in *Quantum Theory of Solids*, edited by I. M. Lifshits (MIR, Moscow, 1982).

[16]K. K. Choi *et al.*, Phys. Rev. B **33**, 8216 (1986).

[17]Because of the qualitative nature of the arguments in this paper, the units of the 2ω response are left as arbitrary units in the figures. The actual units shown are an ohm: the (rms ac voltage at 2ω) divided by the (dc current in the sample). This value remains constant over a range of dc current, indicating that the samples are in the linear response regime, and not heating. The sample current for Figs. 1(b), 1(c), 2, and 3(a) is 3 μA, and the sample current for a second device used in Figs. 3(b), 4, and 5 is 2 μA. These sample currents gave reasonable signal-to-noise ratios without showing signs of sample heating.