Classical continuum theory of the dipole-forbidden collective excitations in quantum strips

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We investigate the collective mode excitation spectrum of an electron gas in a quantum strip that is subjected to a perpendicular magnetic field, with emphasis on the dipole-forbidden transitions. The quantum strip is assumed to be defined in a two-dimensional electron gas by the application of a parabolic confining potential. A classical continuum theory of the collective modes is developed and solved exactly. These results are used to determine the density-response functions. An experimental method to detect the dipole-forbidden modes, based on the use of an asymmetric planar metal grating above an array of quantum strips, is proposed, and calculations of the expected infrared transmission spectrum of the combined grating-strip system are presented. In a system with reasonable parameters, we find that some of the dipole-forbidden absorption peaks are large enough to be observable.

I. INTRODUCTION

There has been great interest over the past several years in the collective excitations of electrons in semiconductor nanostructures such as quantum wells, quantum wires, and quantum dots.[1] Theoretical studies have focused primarily on the dipole-allowed transitions in these structures, and their dependence on the form of the confining potential, the number of electrons, and the strength and orientation of an applied magnetic field. For example, the long-wavelength optical absorption spectrum in such structures with parabolic confinement is known to be independent of electron-electron interaction, and, in zero field, consists of a single peak at the bare harmonic oscillator frequency, regardless of the number of electrons in the structure.[2] Much theoretical effort has also been devoted to the study of dipole-allowed modes both in parabolic structures with an applied magnetic field and in imperfect parabolic structures.[3–10]

Here we extend this earlier work in a special way, examining the spectrum and external coupling of a subset of the possible collective modes of electrons confined to two-dimensional (2D) strips. For the systems we consider the dynamics are quite different in each of three orthogonal directions. Normal to the strip (along the x direction) we assume complete quantum confinement. Only the lowest subband of such motion is occupied and intersubband transitions out of this subband are ignored. Within the plane of the strip the motion along the z direction is free, while the motion in the y direction is subject to classical confinement; i.e., the motion is confined but a large number of the resulting bound levels lie below the Fermi energy. In this limit it is possible to apply a classical continuum theory for the collective motion of the electrons. Such a classical or semiclassical hydrodynamic approach, where the electrons are treated as a charged fluid, has been extremely useful in other studies.[7,9,11,12] The classical continuum approximation for the 2D motion in the yz plane will be valid if both the equilibrium and induced density vary slowly on the scale of the

average interparticle separation. Its use precludes consideration of quantum size effects on the 2D motion, when only one or a few subbands of the in-plane motion are below the Fermi energy. See Ref. 13 for references to both theoretical and experimental work on this alternative physical limit.

In this paper, we investigate the transverse collective modes in a quantum strip with a parabolic confinement potential, subjected to a perpendicular magnetic field. We study only the modes in which the density is uniform along the z direction. In Sec. II, the classical normal modes and their frequencies are obtained exactly. In Sec. III, we derive expressions for the linear density response due to the collective modes. An efficient way to probe experimentally the dipole-forbidden transitions that are the special focus of this paper is to use the so-called grating-coupler technique.[1,14] Here, a metallic grating near the laterally structured two-dimensional electron system is subjected to far-infrared radiation, resulting in an induced electric field with a wavelength of the order of the grating period. In Sec. IV, we present calculations of the infrared transmission spectrum of an array of quantum strips combined with a nearby planar grating, in zero field, for realistic system parameters. In addition to a large dipole-allowed resonance, we find several observable dipole-forbidden absorption peaks.

II. COLLECTIVE MODES

We consider first a single quantum strip or wire, of infinite length, defined in a two-dimensional electron gas in the yz plane by the application of a parabolic confining potential

$$V = \frac{1}{2} m \Omega^2 y^2, \tag{1}$$

where m is the electron effective mass. The strip is oriented in the z direction, and in this work we assume that the density is always uniform along this direction. The plane in which the strip lies separates two half-spaces with local di-

electric constants ϵ_o and ϵ_s. A magnetic field of strength B is applied perpendicular to the strip in the x direction. We shall be concerned with the classical equations of motion for the velocity field $\vec{v}(y,t)$ of the electron fluid, namely,

$$m\dot{v}_y(y,t) = -m\Omega^2 y - m\omega_c v_z(y,t) + 2\tilde{e}^2 \int dy' \frac{n(y',t)}{y-y'}$$
$$+ eE(y,t) \qquad (2)$$

and

$$m\dot{v}_z = m\omega_c v_y. \qquad (3)$$

Here $e<0$ is the electron charge, $\tilde{e}^2 = 2e^2/(\epsilon_o + \epsilon_s)$ accounts for the screening by the background dielectric, $\omega_c \equiv |e|B/mc$ is the cyclotron frequency, $E(y,t) = E(y)e^{-i\omega t}$ is the y component of an external time-dependent driving field (which varies only in the y direction), and $n(y,t)$ is the two-dimensional number density. In writing these equations of motion we have already discarded terms that are second order in the disturbance, such as $(m\mathbf{v}\cdot\nabla)\mathbf{v}$. We also have neglected the contribution of pressure gradients to the driving terms since these are not of quantitative importance for the long-wavelength response of systems moving in fewer than 3D.[15]

The collective modes of the system may be found by linearizing the density about the equilibrium distribution $n_0(y)$,

$$n(y,t) = n_0(y) + \delta n(y,t). \qquad (4)$$

In equilibrium (with $E=0$), all velocity components vanish, so $n_0(y)$ satisfies

$$2\tilde{e}^2 \int dy' \frac{n_0(y')}{y-y'} = m\Omega^2 y. \qquad (5)$$

The Hilbert transform of the equilibrium density is therefore equal to $m\Omega^2 y/2\tilde{e}^2$. Hence[16]

$$n_0(y) = \begin{cases} \dfrac{2\lambda}{\pi W}\sqrt{1-\tilde{y}^2}, & |y| \le W \\ 0, & |y| > W, \end{cases} \qquad (6)$$

where $\tilde{y} = y/W$, λ is the number of electrons per unit length in the z direction, and $2W$, defined via

$$\frac{4\lambda\tilde{e}^2}{mW^2} = \Omega^2, \qquad (7)$$

is the width of the electron gas in the strip.

Assuming that the time-dependent quantities vary as $e^{-i\omega t}$ [for example, $\delta n(y,t) = \delta n(y)e^{-i\omega t}$, etc.], (3) leads to $v_z = i\omega_c v_y/\omega$ which may be used to eliminate the v_z in (2) in favor of v_y. Performing the linearization (4) and using the equilibrium condition (5) leads to

$$v_y(y) = \frac{2i\omega\tilde{e}^2}{m(\omega^2 - \omega_c^2)} \int_{-W}^{W} dy' \frac{\delta n(y')}{y-y'} + \frac{ie\omega E(y)}{m(\omega^2 - \omega_c^2)}. \qquad (8)$$

The linearized continuity equation is

$$-i\omega\delta n(y) + \frac{\partial}{\partial y}(n_0 v_y) + \frac{\partial}{\partial z}(n_0 v_z) = 0. \qquad (9)$$

The third term in (9) vanishes because of the assumed translational invariance in the z direction. Then (8) and (9) together lead to

$$\delta n(y) + \frac{2\tilde{e}^2}{m(\omega^2 - \omega_c^2)} \frac{\partial}{\partial y}\left[n_0(y) \int_{-W}^{W} \frac{dy'}{y'-y} \delta n(y') \right]$$
$$= \frac{e}{m(\omega^2 - \omega_c^2)} \frac{\partial}{\partial y}[n_0(y)E(y)]. \qquad (10)$$

The free normal modes of the system are determined by solving the eigenvalue problem (10) with $E=0$. We now present a remarkably simple exact analytic solution of this eigenvalue problem. We shall show that the normal modes may be indexed by positive integers with each one having the form

$$\delta n_j(y) = \eta_j \frac{T_j(\tilde{y})}{\sqrt{1-\tilde{y}^2}} \quad (j=1,2,3,\dots), \qquad (11)$$

where

$$T_j(x) \equiv \frac{(-1)^j}{(2j-1)!!}(1-x^2)^{1/2}\frac{d^j}{dx^j}(1-x^2)^{j-1/2} \qquad (12)$$

are Chebyshev's polynomials of the first kind,[17] and η_j is a normalization constant with the dimensions of density. Furthermore, the frequencies of the normal modes (11) are simply

$$\omega_j \equiv \sqrt{j\Omega^2 + \omega_c^2} \quad (j=1,2,3,\dots). \qquad (13)$$

Note that $\int dy\, \delta n_j = 0$, and also that the $j=0$ Chebyshev polynomial is excluded from (11). The spectrum (13), for the case of no magnetic field, was obtained by Shikin, Demel, and Heitmann.[11] The $j=1$ mode is the well-known center-of-mass mode,

$$\delta n_1(y) = \eta_1 \frac{\tilde{y}}{\sqrt{1-\tilde{y}^2}} \propto -\frac{\partial}{\partial y}n_0(y), \qquad (14)$$

with frequency $\omega_1 = \sqrt{\Omega^2 + \omega_c^2}$. The $j=2$ mode,

$$\delta n_2(y) = \eta_2 \frac{2\tilde{y}^2 - 1}{\sqrt{1-\tilde{y}^2}}, \qquad (15)$$

is a *breathing* mode with frequency $\omega_2 = \sqrt{2\Omega^2 + \omega_c^2}$.

The proof that the normal modes are given by (11) follows from the identities

$$\frac{1}{\pi}\int_{-1}^{1} \frac{d\tilde{y}'}{\tilde{y}'-\tilde{y}} \frac{T_j(\tilde{y}')}{\sqrt{1-\tilde{y}'^2}} = U_{j-1}(\tilde{y}) \qquad (16)$$

and

$$jT_j(\tilde{y}) = \tilde{y}U_{j-1}(\tilde{y}) + (\tilde{y}^2 - 1)U'_{j-1}(\tilde{y}), \qquad (17)$$

where the

$$U_j(x) \equiv \frac{(-1)^j(j+1)}{(2j+1)!!}(1-x^2)^{-1/2}\frac{d^j}{dx^j}(1-x^2)^{j+1/2}$$

(18)

are Chebyshev's polynomials of the second kind.[17] Using these relations we find that (11) is indeed a solution of Eq. (10) when $\omega = \omega_j$.

III. LINEAR RESPONSE

The response of the electron gas to a weak driving field $E(y,t) = E(y)e^{-i\omega t}$ may be obtained by expanding the charge density as

$$e\,\delta n(y) = \sum_{j=1}^{\infty} c_j \frac{T_j(\widetilde{y})}{\sqrt{1-\widetilde{y}^2}}.$$

(19)

Then (10) and the orthonormality relation[18]

$$\int_{-1}^{1} \frac{dx}{\sqrt{1-x^2}} T_i(x)T_j(x) = \frac{\pi}{2}\delta_{ij},$$

(20)

valid for nonzero i and j, lead to the response amplitudes

$$c_j = \frac{\alpha_j \kappa \Omega^2}{\omega^2 - \omega_j^2}$$

(21)

where $\kappa = (\epsilon_o + \epsilon_s)/2$ and

$$\alpha_j \equiv -\frac{1}{\pi^2}\int_{-1}^{1} d\widetilde{y}\, T_j'(\widetilde{y})\sqrt{1-\widetilde{y}^2}\,E(y).$$

(22)

If we describe $E(y)$ with a scalar potential $\phi(y)$, the coupling coefficient α_j becomes

$$\alpha_j = \frac{1}{W\pi^2}\int_{-1}^{1} d\widetilde{y}\, T_j'(\widetilde{y})\sqrt{1-\widetilde{y}^2}\,\phi'(\widetilde{y})$$

$$= -\frac{1}{W\pi^2}\int_{-1}^{1} d\widetilde{y}\, [T_j'(\widetilde{y})\sqrt{1-\widetilde{y}^2}]'\,\phi(\widetilde{y})$$

$$= \frac{j^2}{W\pi^2}\int_{-1}^{1} d\widetilde{y}\, \frac{T_j(\widetilde{y})}{\sqrt{1-\widetilde{y}^2}}\,\phi(\widetilde{y}),$$

(23)

where the last step follows from another formal identity.[18] Thus

$$e\,\delta n(y) = \int_{-W}^{W} dy'\,\chi(y,y')\phi^{\mathrm{ext}}(y'),$$

(24)

where the susceptibility is

$$\chi(y,y') = \sum_j \left(\frac{j}{\pi W}\frac{T_j(\widetilde{y})}{\sqrt{1-\widetilde{y}^2}}\right)\frac{\kappa\Omega^2}{\omega^2-\omega_j^2}\left(\frac{j}{\pi W}\frac{T_j(\widetilde{y}')}{\sqrt{1-\widetilde{y}'^2}}\right).$$

(25)

We have added a superscript of "ext" to the scalar potential in (24) to emphasize that this equation describes the density response due to an external potential acting on a single strip. It is also useful to consider the response to the total potential or field. Then the two terms on the right hand side of (8) are combined into a single term involving $E^{\mathrm{tot}}(y,t)$ and (10) becomes

$$\delta n(y) = \frac{e}{m(\omega^2-\omega_c^2)}\frac{\partial}{\partial y}[n_0(y)E^{\mathrm{tot}}(y)].$$

(26)

If we again assume that $\delta n(y)$ can be expanded as in (19), we find by a similar reduction that

$$e\,\delta n(y) = \int_{-W}^{W} dy'\,\chi^{(0)}(y,y')\phi^{\mathrm{tot}}(y'),$$

(27)

where the bare susceptibility is

$$\chi^{(0)}(y,y') = \sum_j \left(\frac{j}{\pi W}\frac{T_j(\widetilde{y})}{\sqrt{1-\widetilde{y}^2}}\right)\frac{\kappa\Omega^2}{\omega^2-\omega_c^2}\left(\frac{j}{\pi W}\frac{T_j(\widetilde{y}')}{\sqrt{1-\widetilde{y}'^2}}\right).$$

(28)

The only difference between $\chi^{(0)}$ and χ is in the energy denominators. As an independent-particle response, $\chi^{(0)}$ does not depend on the collective mode frequencies ω_j.

The induced density due to various functional forms of ϕ is readily determined from (25) and (28). For instance, if one has many parallel wires spaced by $d > 2W$ and if

$$\phi^{\mathrm{tot}}(y) = \sum_{n>0} \phi_n^{\mathrm{tot}}\sin(G_n y)$$

(29)

with $G_n = 2\pi n/d$, then $e\,\delta n(y) = \delta\sigma(y)$ is also given by a sum of sines with

$$\delta\sigma_n = \sum_{m>0} \chi_{n,m}^{(0,e)}\phi_m^{\mathrm{tot}},$$

(30)

where

$$\chi_{n,m}^{(0,e)} = \frac{2}{d}\sum_j \frac{\kappa\Omega^2}{\omega^2-\omega_c^2}M_n^{(e,j)}M_m^{(e,j)}.$$

(31)

The "matrix elements,"

$$M_n^{(e,j)} = \frac{j}{\pi}\int_{-1}^{1} d\widetilde{y}\,\frac{T_j(\widetilde{y})}{\sqrt{1-\widetilde{y}^2}}\sin(G_n W\widetilde{y}),$$

(32)

are easily computed using $T_j[\cos(\theta)] = \cos(j\theta)$.[18] Since $T_j(\widetilde{y})$ has the parity of $\widetilde{y}^j$, the $M_n^{(e,j)}$ are nonzero only for odd j. The extra e superscript stands for even, which is the parity in y of the associated fields and currents for these excitations. There are also odd parity excitations, for which ϕ and $\delta\sigma$ have a cosine expansion, and the analogue of (31) is

$$\chi_{n,m}^{(0,o)} = \frac{2}{d}\sum_j \frac{\kappa\Omega^2}{\omega^2-\omega_c^2}M_n^{(o,j)}M_m^{(o,j)}$$

(33)

with

$$M_n^{(o,j)} = \frac{j}{\pi}\int_{-1}^{1} d\widetilde{y}\,\frac{T_j(\widetilde{y})}{\sqrt{1-\widetilde{y}^2}}\cos(G_n W\widetilde{y}),$$

(34)

which are nonzero only for even j. In Fig. 1 we show how the dimensionless $M^{(j)}$ vary with GW. They all vanish at
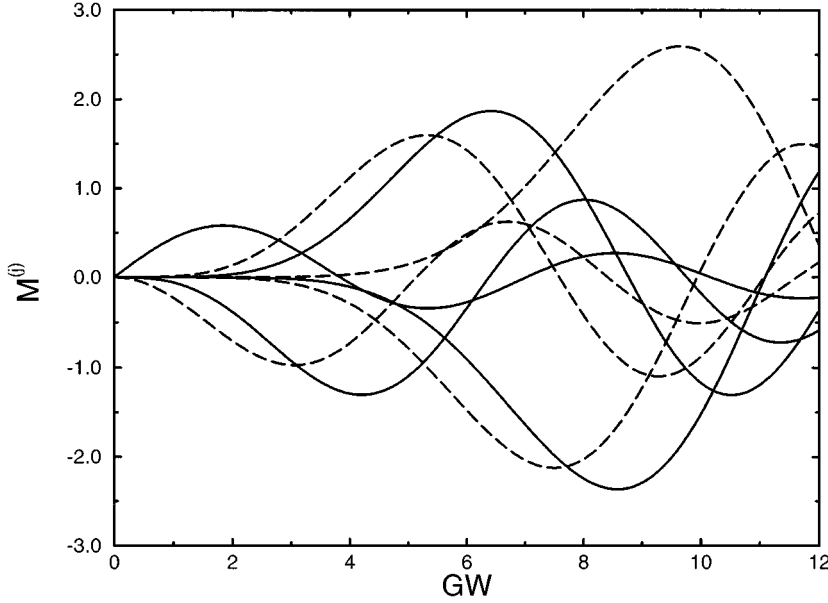
FIG. 1. Matrix elements for coupling of collective modes to sinusoidal fields. The solid (dashed) curves are for odd (even) values of j. Results for $j = 1, \ldots, 8$ are shown. The larger j is, the smaller $|M^{(j)}|$ is near the origin.

$GW = 0$. For increasing j the maximum in $|M^{(j)}|$ for larger j is higher and occurs at a larger value of GW.

It is also of interest to consider the response to a uniform, external field $E^{\text{ext}}(y) = \bar{E}$. For this case (20) and (22) imply that only the $j = 1$ center-of-mass mode will be excited:

$$\delta\sigma(y) = -\frac{\bar{E}}{2\pi} \frac{\kappa\Omega^2}{\omega^2 - \omega_1^2} \frac{T_1(\widetilde{y})}{\sqrt{1 - \widetilde{y}^2}}, \quad |\widetilde{y}| < 1. \quad (35)$$

This is the dipole-allowed excitation.

For an external electric field that is linear in y,

$$\delta\sigma(y) \sim \frac{\kappa\Omega^2}{\omega^2 - \omega_2^2} \frac{T_2(\widetilde{y})}{\sqrt{1 - \widetilde{y}^2}}, \quad |\widetilde{y}| < 1, \quad (36)$$

which is resonant, in zero magnetic field, only at the breathing mode frequency $\sqrt{2}\Omega$. This result is in contrast to those in a recent paper that considered the response of electrons in a quantum dot.[19] These authors found that a quadrupolar perturbation leads to energy absorption at twice the Kohn mode frequencies as well as at the frequencies for modes of relative motion (such are our $j > 1$ modes). Because our strip contains $N \to \infty$ number of electrons, the mass associated with its center-of-mass motion, $M = Nm$, is divergent, which suppresses the possibility of multiple excitation of this mode.

IV. TRANSMISSION SPECTRUM OF THE GRATING-WIRE SYSTEM

In this section we explore the feasibility of detecting with a grating coupler some of the rich spectrum our model predicts. The grating coupler we have in mind is a conducting sheet that lies parallel to and separated by h from the plane containing the quantum wires. Although it is common (but not necessary) practice[1,20–22] to use the grating's shape to produce the static potentials that confine the electrons, we shall ignore how the ground state configuration is actually produced and instead concentrate on how the system responds to infrared fields. Then it is sufficient to describe the grating as a flat 2D conductor whose (local) resistivity varies periodically in the y direction. To enhance the signal strength and simplify the analysis, we assume that the single wire studied before has been periodically repeated in the y direction with the same period $d > 2W$ that the grating has.

The experiment we have in mind is for unpolarized infrared light to be incident along the normal. The corrugation in both the grating coupler and quantum wire systems will cause local fields varying on the scale of d to be produced, which in turn can excite various collective modes and reduce the net transmission of the radiation. Since the collective mode frequencies have a common dependence on normal magnetic field (with no avoided crossings or splittings) we will only analyze the case of $B = 0$. This allows the simplification that the responses to incident fields polarized along the y or z direction may be treated separately. We do not expect that the coupling strengths we find will be significantly changed for $B \neq 0$, at least within our classical theory. Let us also emphasize that we are proposing the use of the spatial variation of the infrared exciting fields to get around the restrictions of the generalized Kohn theorem[2,19] rather than modifying the parabolic form of the bare confining potential.[3–7,9,21–23] An alternative path to the same goal is provided by Raman scattering.[24,25]

We assume that the grating period d is on the order of micrometers and hence much smaller than the vacuum wavelength of the infrared radiation. This plus the requirement that $h \approx d$ allows us to separate the calculation into macroscopic and microscopic stages.[26] For the former the whole system appears as a single, homogeneous, conducting layer separating vacuum from the substrate whose dielectric constant is ϵ_s. The transmission coefficient through this layer is

$$T_i = \sqrt{\epsilon_s} |t_i|^2 \quad (37)$$

where the transmission amplitude is

$$t_i = 2 \left/ \left[1 + \sqrt{\epsilon_s} + \frac{4\pi}{c} \Sigma_{ii} \right] \right. \quad (38)$$

Here the subscript i refers to a Cartesian index and for un-polarized light we need $T = \frac{1}{2}(T_y + T_z)$. The Σ_{ii} in (38) is the diagonal element of the macroscopic surface conductance, which must be calculated microscopically.

It is easy to find Σ_{zz} because we have assumed that there is no spatial variation in the z direction. There are independent contributions from the grating coupler and the quantum wire system, $\Sigma_{zz} = \Sigma_{zz}^{(g)} + \Sigma_{zz}^{(s)}$, where

$$\Sigma_{zz}^{(g)} = \frac{1}{d} \int_0^d dy\, \sigma(y) \equiv \bar{\sigma} \qquad (39)$$

and at frequency ω

$$\Sigma_{zz}^{(s)} = \frac{i\lambda e^2}{dm(\omega + i/\tau)} = \frac{c}{4\pi}\left[2\pi^2 i\nu d \left(\frac{W}{d}\right)^2 \frac{\kappa\Omega^2}{\tilde{\omega}^2} \right]. \qquad (40)$$

Here $\sigma(y) = 1/\rho(y)$ is the local, frequency-independent, 2D

conductivity of the grating coupler and τ is the Drude relaxation time for the electrons in a strip. The final expression in (40), which derives from (7), uses $\tilde{\omega}^2 = \omega(\omega + i/\tau)$ and $\nu = \omega/(2\pi c)$ for the frequency in wave numbers. The dimensionless product νd is small.

The calculation of Σ_{yy} is much more involved because we must self-consistently account for the mutual influence of density fluctuations in the quantum wires and grating coupler. We do this by adapting earlier work,[27] called hereafter SPM. We work in the electrostatic limit, separating the y component of the total electric field into $E^{tot}(y) = E_0 - \partial\phi/\partial y$. The constant term of E_0 represents the transverse field, which on the scale of d and h scarcely varies. In contrast, the longitudinal electric field does have significant microscopic variations and we represent it with a scalar potential that is expanded in a series of sines and cosines. With the grating plane at $x = 0$ and the quantum wire system in the $x = h$ plane, we write

$$\phi^{tot} = \begin{cases} \sum_{n>0} \left\{ e^{G_n x} \left[-r_n^{(e)} \frac{\sin G_n y}{G_n} + r_n^{(o)} \frac{\cos G_n y}{G_n} \right] \right\}, & x < 0 \\[2ex] \sum_{n>0} \left\{ e^{-G_n x} \left[-\lambda_n^{(-,e)} \frac{\sin G_n y}{G_n} + \lambda_n^{(-,o)} \frac{\cos G_n y}{G_n} \right] + e^{-G_n(h-x)} \left[-\lambda_n^{(+,e)} \frac{\sin G_n y}{G_n} + \lambda_n^{(+,o)} \frac{\cos G_n y}{G_n} \right] \right\}, & 0 < x < h \\[2ex] \sum_{n>0} \left\{ e^{-G_n(x-h)} \left[-t_n^{(e)} \frac{\sin G_n y}{G_n} + t_n^{(o)} \frac{\cos G_n y}{G_n} \right] \right\}, & h < x, \end{cases}$$

$$(41)$$

where $G_n = 2\pi n/d$ with n a positive integer. The amplitudes r, t, and $\lambda^{(\pm)}$ all have the units of electric field and are the same symbols used in SPM. We have added e/o superscripts to denote whether the associated y component of electric field has even or odd parity in y. The latter possibility was not allowed by SPM.

To determine the coefficients in (41) we impose boundary conditions on parallel E and normal D across the $x = 0$ and $x = h$ interfaces. The background dielectric constant is 1 in $x < 0$, ϵ_o in $0 < x < h$, and ϵ_s in $h < x$. Since the different parity terms separate, we suppress the e/o superscripts to write the matching conditions just once. Continuity of parallel E leads to

$$r_n = \lambda_n^{(-)} + e^{-G_n h}\lambda_n^{(+)}, \qquad (42)$$

$$t_n = \lambda_n^{(-)}e^{-G_n h} + \lambda_n^{(+)}. \qquad (43)$$

The discontinuity in normal D is set by the induced charge

$$\epsilon_s t_n - \epsilon_o[\lambda_n^{(-)}e^{-G_n h} - \lambda_n^{(+)}] = 4\pi\delta\sigma_n^{(s)}, \qquad (44)$$

$$\epsilon_o[\lambda_n^{(-)} - \lambda_n^{(+)}e^{-G_n h}] + r_n = 4\pi\delta\sigma_n^{(g)}. \qquad (45)$$

Here the transforms of the interface charge densities, $\delta\sigma$, are with respect to $-\sin G_n y$ ($\cos G_n y$) for the e (o) cases. For the grating we use the equation of continuity to replace

$$\delta\sigma_n^{(g)} = \frac{n}{i\nu dc} J_n^{(g)}, \qquad (46)$$

where the y component of the grating current is transformed with respect to $\cos G_n y$ ($\sin G_n y$) for the e (o) cases. The $J_n^{(g)}$ can be expressed in terms of transforms of the grating resistivity profile and the fields E_0 and $-(\partial\phi/\partial y)|_{x=0}$.[27] Rather than proceeding similarly for the quantum strip $\delta\sigma_n^{(s)}$, we make use instead of the susceptibility (27) and (28) so

$$\delta\sigma_n^{(s)} = \sum_{m>0} \chi_{n,m}^{(0)} t_m / G_m + \gamma_n^{(0)} E_0 \qquad (47)$$

where

$$\gamma_n^{(0)} = \frac{W}{d} \frac{\kappa\Omega^2}{\omega^2} M_n^{(1)} \qquad (48)$$

and is nonzero only for the even-n terms.

It is straightforward to algebraically eliminate the $\lambda_n^{(\pm)}$ and t_n, yielding for $n > 0$

$$\sum_{m>0} F(n,m) r_m = \frac{4\pi}{c} J_n^{(g)} + \Gamma_n E_0 \qquad (49)$$

where

$$\Gamma_n = i\frac{\nu d}{n} \frac{\epsilon_o}{\sinh G_n h} \sum_{\ell>0} D_e^{-1}(n,\ell) 4\pi\gamma_\ell^{(0)}, \qquad (50)$$

$$F(n,m)=i\frac{\nu d}{n}\left\{[1+\epsilon_o\coth G_n h]\delta_{n,m}\right.$$

$$\left.-\frac{\epsilon_o}{\sinh G_n h}D^{-1}(n,m)\frac{\epsilon_o}{\sinh G_m h}\right\}, \quad (51)$$

with

$$D(n,m)=(\epsilon_s+\epsilon_o\coth G_n h)\delta_{n,m}-4\pi\chi_{n,m}^{(0)}/G_m. \quad (52)$$

Note that the Γ_n (like the $\gamma_n^{(0)}$) are nonzero only for the even terms. However, the matrices F and D have both even and odd contributions, but no cross couplings.

The quantity we need, Σ_{yy}, can be split into contributions from the grating, $\Sigma_{yy}^{(g)}$, and from the quantum wire system, $\Sigma_{yy}^{(s)}$. The latter may be calculated from $(1/d)\int_0^d dy J^{(s)}(y)/E_0$, where by the equation of continuity

$$\int_0^d dy J^{(s)}(y)=-i\omega W^2\int_{-1}^1 d\tilde{y}\,\tilde{y}\delta\sigma^{(s)}(y). \quad (53)$$

Since $\tilde{y}=T_1(\tilde{y})$, only the $j=1$ term in (19) will contribute. We find using (20) and the first line of (23)

$$\frac{4\pi}{c}\Sigma_{yy}^{(s)}=4\pi i\nu d\left(\frac{W}{d}\right)^2\frac{\kappa\Omega^2}{\omega^2}\int_{-1}^1 d\tilde{y}\sqrt{1-\tilde{y}^2}E^{\text{tot}}(y)/E_0$$

$$=2\pi^2 i\nu d\left(\frac{W}{d}\right)^2\frac{\kappa\Omega^2}{\omega^2}\left[1+2\sum_n\frac{M_n^{(e,1)}t_n^{(e)}/E_0}{G_n W}\right]$$

$$(54)$$

where

$$t_n^{(e)}=\sum_{m>0}D_e^{-1}(n,m)\left[\frac{\epsilon_o}{\sinh G_m h}r_m^{(e)}+4\pi\gamma_m^{(0,e)}E_0\right]. \quad (55)$$

Up to this point the algebraic reduction has been exact. But, since we expect the dipole-forbidden absorption to be weak, we now introduce the approximation [see Eq. (A10) of SPM for the exact relation]

$$r_n\approx\rho_n j_0, \quad (56)$$

where $j_0=J_{n=0}^{(g)}=\Sigma_{yy}^{(g)}E_0$. Then from Ohm's law for the grating,

$$E_0=\bar{\rho}j_0+\frac{1}{2}\sum_{n>0}[\rho_n^{(e)}J_n^{(e,g)}+\rho_n^{(o)}J_n^{(o,g)}], \quad (57)$$

and Eq. (49), we obtain

$$\frac{4\pi}{c}\Sigma_{yy}^{(g)}\approx\left[1+\frac{1}{2}\sum_n\left(\frac{c\rho_n^{(e)}}{4\pi}\right)\Gamma_n^{(e)}\right]\Bigg/\left\{\left(\frac{c\bar{\rho}}{4\pi}\right)\right.$$

$$+\frac{1}{2}\sum_{n,m}\left[\left(\frac{c\rho_n^{(e)}}{4\pi}\right)F^{(e)}(n,m)\left(\frac{c\rho_m^{(e)}}{4\pi}\right)+\left(\frac{c\rho_n^{(o)}}{4\pi}\right)F^{(o)}\right.$$

$$\left.\left.\times(n,m)\left(\frac{c\rho_m^{(o)}}{4\pi}\right)\right]\right\}, \quad (58)$$

where we have restored all the e/o superscripts. The ρ_n are transforms of the grating's resistivity profile $\rho(y)$ with respect to $\cos G_n y$ ($\sin G_n y$) for the e (o) cases and $\bar{\rho}$ is the spatial average of $\rho(y)$.

Sharp structure in the transmission coefficient will arise from rapid variations in the D^{-1} matrices, whose singularities define the collective modes. The frequencies of these excitations will differ from those of an isolated strip for several reasons. The nearby grating changes the effective dielectric background in which a wire sits. This is represented by the fact that the diagonal term in (52) is not the constant $(\epsilon_s+\epsilon_o)$. However, deviations due to this ''screening'' by the grating are only important for small n. Another factor influencing mode locations is the interwire coupling. The Coulomb potential of each strip will affect the response of its neighbors. One can separate out this effect by writing $v=v^{(s)}+\delta v$ where $v^{(s)}(y,y')=-(2/\kappa)\ln|y-y'|$ only if both y and y' are inside the same strip, otherwise $v^{(s)}$ vanishes. It is $v^{(s)}$ that enters the relation between $\chi^{(0)}$ and the susceptibility for an independent set of strips: $\chi^{(s)}=\chi^{(0)}+\chi^{(0)}v^{(s)}\chi^{(s)}$. The solution of this equation gives $\chi^{(s)}(y,y')$ in the form of (25), with the constraint that y and y' must be inside the same strip. The corrections due to δv can then be added back in a rapidly converging series of multipolar couplings, leading to small depolarization shifts away from the ω_j. This two-step resummation of the infinite series expansion of $2\kappa D^{-1}\sim(1-\chi^{(0)}v)^{-1}$ is important since it allows one to avoid a direct evaluation of $\chi_{n,m}^{(0)}$, which is numerically difficult to treat.

We have now determined all the formal ingredients, so we turn to the choice of model parameters. With systems based on GaAs/ $Al_x Ga_{1-x}$ As in mind, we set $m=0.069m_e$ and $\epsilon_o=\epsilon_s=13$. The Drude relaxation time is chosen to be $\tau=10$ ps, which corresponds to a mobility of 255 000 $cm^2 V^{-1} s^{-1}$. The factor of ω^2 in (25) and (48) is replaced with $\tilde{\omega}^2=\omega(\omega+i/\tau)$ to broaden out the resonances and Ω is set initially at 10 cm^{-1}. For simplicity we ignore the possible dependence of τ on frequency. The resistivity profile for the grating is represented with steps and ramps: $\rho(y)=\rho^{(e)}(y)+\rho^{(o)}(y)$ where

$$\rho^{(e)}(y)=\begin{cases}\rho_a, & |y|<a/2 \\ \rho_b, & |y-d/2|<b/2,\end{cases} \quad (59)$$

$$\rho^{(o)}(y)=\begin{cases}\rho_c\dfrac{y-d/2}{b/2}, & |y-d/2|<b/2 \\ 0, & |y|<a/2.\end{cases} \quad (60)$$

Here the grating period $d=a+b$ and the above definitions are to be periodically extended. The Fourier transforms are readily found. Note that $\bar{\rho}=(a\rho_a+b\rho_b)/d$ while $\bar{\sigma}=\{a/\rho_a+0.5b\ \ln[(\rho_b+\rho_c)/(\rho_b-\rho_c)]/\rho_c\}/d$. We assume $\rho_a\geq\rho_b\geq\rho_c$. If $\rho_c=0$, then $\rho(y)$ has even parity while if $\rho_a=\rho_b$ there is odd parity for $\rho(y)-\bar{\rho}$ about $y=d/2$. We imagine that the quantum wires lie either under the ρ_a region or the ρ_b region, with $2W<a$ or $2W<b$, respectively. Note that if $W=1\,\mu m$, then from (7) $\lambda=3.14\times10^7$ cm.

As a first illustration of a transmission spectrum we show in Fig. 2 results for an even-parity grating, with $\rho_a=1000\ \Omega$/sq, $\rho_b=30\ \Omega$/sq, and $\rho_c=0$. We choose
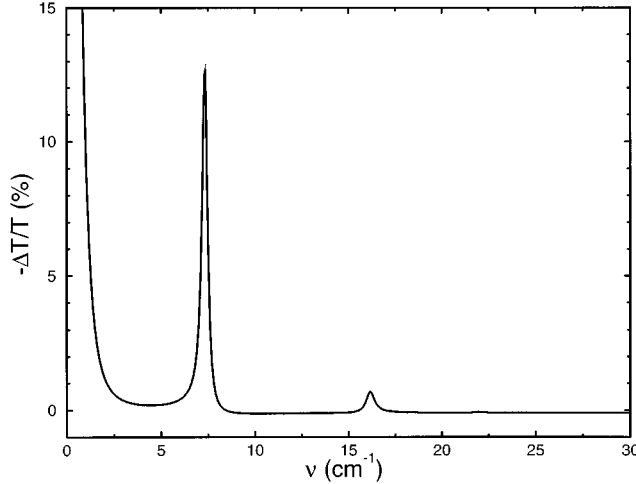
FIG. 2. Relative change in transmission between when the quantum strips are absent or present versus infrared radiation frequency ν. See text for specific parameter values of the grating coupler and the quantum wire system. There is no coupling to modes of even j in (19).
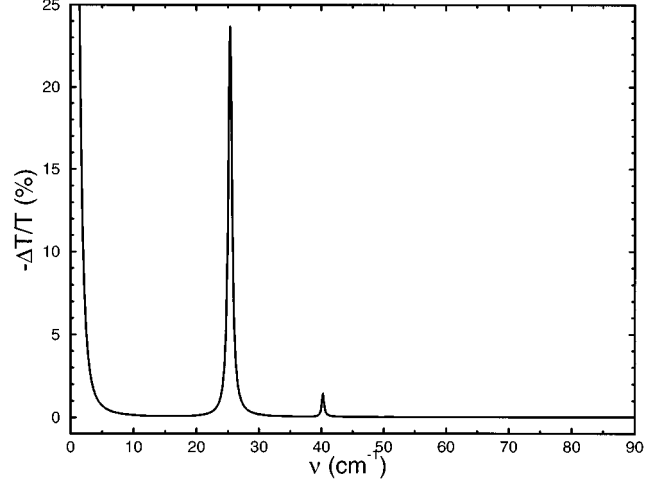

FIG. 3. Relative change in transmission between when the quantum strips are absent or present versus infrared radiation frequency ν. See text for specific parameter values of the grating coupler and the quantum wire system. There is coupling to modes of all j in (19).

$d=4$ μm, $h=0.3$ μm, and $a/d=0.2$. The quantum strips are centered under the ρ_b regions with $W/b=0.4$ so $W=1.28$ μm. If W is chosen to be smaller the assumption of parabolic confinement is better satisfied, but the coupling strength is weaker. For good convergence of peak positions and heights, one needs to keep about 30 reciprocal lattice vectors. The strong peak near $\nu=7$ cm^{-1} is due to the $j=1$ mode. Its position has been shifted downwards due to the Coulomb interactions with the grating and between adjacent strips. The next visible peak near 16 cm^{-1} arises from the $j=3$ resonance (at $\sqrt{3}\Omega=17.3$ cm^{-1} for an isolated strip). The excitation of the resonances for even j is forbidden by symmetry and the coupling to higher odd-j modes is too weak to be seen. The Drude peak as $\nu \to 0$ is due to conduction along the length of the wires.

In Fig. 3 we show what happens at the other symmetry extreme. Here we have chosen $\rho_a=\rho_b=1000$ Ω/sq and $\rho_c=900$ Ω/sq so all $\rho_{n>0}^{(e)}$ vanish. The strips are still centered under the ρ_b regions but now with $W/b=0.3$ and $b/d=0.6$. We have decreased d to 4/3 μm and h to 0.1 μm while Ω has been increased to 30 cm^{-1} so λ is roughly the same as for Fig. 2. The dipole-allowed mode at 25 cm^{-1} is the dominant structure. Again, screening by the grating and interwire interactions are responsible for its depolarization shift below the value 30 cm^{-1} for a single strip. The peak near $\nu=40$ cm^{-1} is due to the first odd-parity collective mode, with $j=2$. The next even-j peak is near 60 cm^{-1}, but it is not visible. By symmetry, direct coupling from the grating to odd-j peaks, except $j=1$, is forbidden. The quantum wire array can itself provide the necessary momentum, but for our choice of parameters this mechanism of excitation does not appear to be efficient.

We have examined other cases in the large parameter space of variable ρ_a, ρ_b, ρ_c, d, h, a, b, W, τ, and Ω. The relative strengths of the dipole-forbidden peaks are comparable to or smaller than those shown here (unless one increases τ), so it will be a definite experimental challenge to observe them. We hope that someone will take on this task.

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