Screening and energy loss by hot carriers in semiconductors

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Analytical forms are obtained for energy-loss rates due to optical-phonon and plasmon annihilation and creation, and due to quasiparticle scattering. These contain the explicit dependence upon all system parameters and temperature. Screening was obtained through a self-consistent classical calculation, made quantum mechanical by adding the effects of recoil and a quantum treatment of the modes. It is found that if the plasmon and phonon frequencies (ω_p and ω_0, respectively) are much different, the interaction between the hot carrier and optical phonons in compound semiconductors is to a good approximation screened by $\varepsilon(0,\omega_0)=1-\omega_p^2/\omega_0^2$. If the two frequencies are comparable, the usual case, the total rate is well given with *no screening* of either interaction. The interaction producing *quasiparticle* excitations is found to be screened to a good approximation by the quantum dielectric function $\varepsilon(q,\delta\varepsilon/\hbar)\approx 1+(\hbar\omega_p)^2/(2\delta\varepsilon)^2$ with $\delta\varepsilon$ the energy exchanged in the collision, quite different from the static $1+\kappa^2/q^2$ frequently assumed. Losses to plasmons and quasiparticle excitations are seen to be usually comparable and to dominate the phonon rate if the number of carriers per atom exceeds the ratio of the carrier effective mass to the reduced mass of the atoms. This ratio is understandable in terms of classical collisions between carriers and between carriers and atoms. [S0163-1829(96)02619-7]

I. INTRODUCTION

We are concerned with the energy loss by hot carriers in a polar semiconductor, seeking analytic expressions that incorporate the dependence upon all of the system parameters so they can be conveniently used in device modeling. We also address the pragmatic question of how screening is to be included in the calculation. We were unable to extract results such as those given in the abstract from the analysis of energy loss given by Jalabert and Das Sarma[1] nor the more recent analysis by Sanborn,[2] which addressed transport properties. Our analysis would seem to be closer to that given even more recently by Woerner and Elsaesser[3] who considered exactly the same processes we do, for germanium as well as gallium arsenide. Their analysis was directed at holes, and was aimed at numerical solution rather than analytic forms for the result. Obtaining analytic forms requires approximations and *for us* they can best be evaluated on a physical basis by working from the full classical calculation, rather than from a diagrammatic expansion. We make explicit comparisons with the results obtained by Jalabert and Das Sarma,[1] Sanborn,[2] and Woerner and Elsaesser,[3] where they overlap, at the end of Sec. III.

Energy loss arises from three distinguishable mechanisms: excitation of optical-mode vibrations, emission of plasmons, and excitation of quasiparticles. The basic difficulty is that each of these mechanisms also produces a screening of the potential due to the hot carrier, making any kind of exact solution impossible. We seek here at least a good approximation.

The principal approach to this many-body problem has been a quantum-mechanical perturbation-theory expansion in e^2, summing selected terms—or diagrams—to all orders. Such a diagrammatic formulation of screening was given many years ago by Pines,[4] who discussed also the classic problem of energy loss by a fast electron in an electron gas. He gave the full $\varepsilon(q,\omega)$, which screens a classical potential of wave number q and frequency ω in the random phase approximation (RPA), first evaluated by Lindhard.[5] Use of this form, $\varepsilon(q,\omega)$, with the corresponding Fourier components of the potential from the fast electron would constitute a classical treatment of that fast electron, and the neglect of its recoil. This point has been made and discussed by Almbladh and Hedin,[6] who also note that this recoil is included in the full RPA dielectric function reproduced by Pines. Thus in principle it must contain the parts of the electron-electron problem that we discuss here in conjunction with the optical phonons. References 1, 2, and 3, sought numerical solution for the energy loss incorporating these same terms, which we seek to include analytically.

In the treatment of the energy loss of a fast particle by Jackson,[7] an interaction $e^2e^{-\kappa r}/r$ between electrons was used at the outset, which corresponds to a static dielectric function $\varepsilon(q,0)=1+\kappa^2/q^2$. For a degenerate Fermi distribution this is the Fermi-Thomas approximation; for a Maxwell-Boltzmann distribution this is the Debye-Hückel approximation with a different form for κ^2. So also was the static approximation used by Fetter and Walecka[8] in treating energy loss and by Preisel, Mørk, and Haug,[9] in the more recent analysis of carrier capture. We find here that the static approximation is qualitatively correct for quasiparticle scattering, but shows the wrong dependence upon the parameters of the system. For plasmon or phonon excitations a qualitatively different form is needed. A related treatment of the electron gas has been given by Rojas, Godby, and Needs.[10] It provided self-energies but did not answer the specific screening questions addressed here. A major computational treatment of these problems in silicon has been made by Fischetti and Laux,[11] which details earlier analyses and focuses on the quantitative accuracy of predicted transport properties. Where our questions overlap, we agree with their results.

Sotirelis and Hess[12] addressed more specifically the energy loss producing quantum-well capture.

There have been numerous calculations that included exchange and correlation corrections to the static dielectric function, $\varepsilon(q,0)$, generally by adding an exchange and correlation term to Poisson's equation. See Kleinman[13] for one of the earliest such efforts and Glasser[14] for one of the most recent. At small q these terms are completely dominated by the Coulomb term, and at large q the dielectric function is near one in either case so these do not seem essential to the problems motivating this study. We do not include such terms.

It has long been recognized that in semiconductors the plasma frequency and the optical phonon frequencies can be similar and since they are strongly coupled the modes are mixed. The resulting phonon-plasmon modes have been treated by Mooradian and McWharter,[15] more recently by Kuznetsov and Stanton,[16] Jalabert and Das Sarma,[1] and Sanborn,[2] and this coupling is included here.

It is possible to proceed, as an approximation, by treating the field from the hot carrier as a classical potential, expanded in plane waves as $\Sigma_{\mathbf{q},\omega}V^0(\mathbf{q},\omega)\exp(i\mathbf{q}\cdot\mathbf{r}-\omega t)$. Then if one can obtain the linear response to this potential, classically or quantum mechanically, one can obtain a dielectric function $\varepsilon(\mathbf{q},\omega)=\varepsilon_1(\mathbf{q},\omega)+i\varepsilon_2(\mathbf{q},\omega)$ and write the work done by the classical potential as the energy loss. The loss is then proportional to the imaginary part of $1/\varepsilon(\mathbf{q},\omega)$, or $\varepsilon_2/(\varepsilon_1^2+\varepsilon_2^2)$. There are contributions to ε_2 for each of the three loss mechanisms. We treat loss due to phonons and plasmons first, and return at the end to a treatment of quasiparticle excitation.

As mentioned above, the assumption of a classical potential neglects recoil, even for a quantum treatment of $\varepsilon(\mathbf{q},\omega)$. The true quantum energy loss depends upon an energy δ function, the argument of which is missing an $\hbar^2q^2/2m$ term for the hot electron when it is treated as the source of a classical potential in the derivation of ε_2. We shall rectify this by matching our classical calculation with a quantum treatment of the same system, and by using a frequency corrected by the quantum term. This presumably leaves us with an error in the ε_2^2 term in the denominator. Since there are a number of terms in this denominator and the ε_2^2 term is small, this is probably not so serious in any of the cases treated.

We calculate the linear response to the potential from the hot electron of (dispersionless) optical modes and plasma modes classically, without other approximations. This yields a classical absorption remarkably close to a quantum treatment of spontaneous phonon and plasmon emission. We then rectify classical errors in the numerator and in the classical screening represented by the denominator. When we treat also quasiparticle excitation, we proceed in an analogous fashion.

The analysis seems quite intricate and inclusion of the details seems to confuse the central questions. We have therefore tried to trim it to its essential results in this paper. A detailed account of the analysis[17] is available from the author upon request and the specific evaluation of the dielectric function for quasiparticle scattering, for degenerate and nondegenerate carriers, is included here as an Appendix.

II. CLASSICAL ENERGY LOSS WITH SCREENING

The first step is a classical, self-consistent account of the optical-phonon modes and their interaction with the gas of carriers, and the interaction of both with the field arising from a high-energy classical carrier. This defines the parameters of the problem and gives us an energy loss that can be identified with a quantum-mechanical form for this loss. The relative motion of the positive and negative atoms (e.g., Ga and As) in each atomic cell is expanded in normal coordinates representing the longitudinal optical vibrational modes as $\delta\mathbf{r_j}=\Sigma_{\mathbf{q}}\mathbf{u_q}e^{i\mathbf{q}\cdot\mathbf{r}_j}/\sqrt{N_c}$ with N_c the number of primitive cells in the system. This produces a local dipole in each primitive cell of $e_T^*e\,\delta\mathbf{r_j}$ with e_T^* the transverse charge, equal to 2.16 for GaAs.[18] Similarly, for this part of the problem the local displacement of the carriers is expanded in terms of normal coordinates, or plasma coordinates. We further add an ''applied'' potential from the energetic electron of velocity $\mathbf{v}_1$,

$$\frac{e^2}{|\mathbf{r}-\mathbf{v}_1t|}=\sum_{\mathbf{q}}\ \frac{4\pi e^2}{\varepsilon_\infty\Omega q^2}\,e^{i(\mathbf{q}\cdot\mathbf{r}-\mathbf{q}\cdot\mathbf{v}_1t)}\equiv\frac{1}{N_c}\sum_{\mathbf{q}}\ V_q^0e^{i(\mathbf{q}\cdot\mathbf{r}-\mathbf{q}\cdot\mathbf{v}_1t)}. \tag{1}$$

ε_∞ is the optical dielectric constant of the material and Ω is the total volume. Finally, we add a small damping term and, using Poisson's equation, obtain the classical equations of motion for the normal coordinates in terms of the applied field and the fields arising from the phonons (details again are given in Ref. 17 or in the original treatment by Mooradian and McWhorter[15]).

It is helpful to extract some quantities from these equations before proceeding. If there is no applied field, no plasma term, and no damping we obtain ω as the optical-mode frequency ω_0 and

$$\omega_0^2=\omega_T^2+\frac{4\pi e^2e_T^{*2}}{\varepsilon_\infty M_r\Omega_c}\equiv\omega_T^2+\delta\omega_0^2. \tag{2}$$

M_r is the reduced mass $M_+M_-/(M_++M)$ of the two atoms in the cell, of volume $\Omega_c=\Omega/N_c$. ω_T is the transverse optical frequency, equal to the longitudinal frequency in the absence of electric fields, and $\delta\omega_0^2$ is the familiar Lyddane-Sachs-Teller[19] difference between the longitudinal optical-mode frequency and the transverse optical-mode frequency. Similarly we obtain ω_p as the plasma frequency given by

$$\omega_p^2=\frac{4\pi Ne^2}{\varepsilon_\infty m^*}. \tag{3}$$

m^* is of course the effective mass of the carriers making up the plasma. We treat the plasma frequency as independent of q in order to simplify the discussion and to obtain analytic results. We also proceeded numerically, following Jalabert and Das Sarma,[1] and Sanborn,[2] to include the wave-number dependence in the plasma-pole approximation, replacing ω_p everywhere by $\tilde{\omega}\approx\omega_p\sqrt{1+q^2/\kappa^2}$ [Sanborn's Eq. (50)], and shall indicate the resulting modification of the results in Sec. III.

The coupling terms in the classical equations of motion mix the two modes, producing composite phonon-plasma

modes, of the type treated by Mooradian and McWhorter.[15] They are obtained from the two equations of motion, again dropping the applied potential and damping terms, as

$$\omega_{\pm}^2=\frac{\omega_p^2+\omega_0^2}{2}\pm\left[\left(\frac{\omega_p^2-\omega_0^2}{2}\right)^2+\omega_p^2\delta\omega_0^2\right]^{1/2}. \tag{4}$$

Now we may return to the full equations of motion for the atomic coordinates and plasma coordinates, using Poisson's equation to relate the potentials to the normal coordinates and evaluate the ratio of the net potential present to the applied potential [Eq. (1)]. This ratio is $\varepsilon(\omega)=\varepsilon_1(\omega)+i\varepsilon_2(\omega)$, the dielectric function for this classical system. The energy loss can be written in terms of this dielectric function by calculating the work done by the field of the incident electron on the induced charge density. The rate of energy loss is[17]

$$W^{\rm cl}=\frac{\varepsilon_\infty\Omega_c}{4\pi e^2N_c}\sum_{\mathbf{q}}\ \frac{\omega q^2\varepsilon_2(\omega)V_q^{0*}V_q^0}{\varepsilon_1^2+\varepsilon_2^2}, \tag{5}$$

which is the classical loss rate in terms of $\varepsilon(\omega)$. The imaginary part $\varepsilon_2(\omega)$ is obtained by writing $\omega=\mathbf{q}\cdot\mathbf{v}_1$ and changing the sum over all $\mathbf{q}$ to an integral. There are contributions from the poles in this integration that can be written as δ functions. Then returning to the form as a sum over $\mathbf{q}$ the result can be written as

$$W_{\rm pl,ph}^{\rm cl}=\frac{\pi\varepsilon_\infty\Omega_c}{4\pi e^2N_c}\sum_{\mathbf{q}}\left[\frac{\omega_p^2(\omega^2-\omega_T^2)+\delta\omega_0^2\omega^2}{\omega^2-\omega_-^2}\ \delta(\omega-\omega_+)+\frac{\omega_p^2(\omega^2-\omega_T^2)+\delta\omega_0^2\omega^2}{\omega^2-\omega_+^2}\ \delta(\omega-\omega_-)\right]q^2V_q^{0*}V_q^0, \tag{6}$$

where the subscript indicates that we have included the effects of both plasma and phonon coordinates. This classical loss is closely related to the quantum result, Eqs. (15) and (16) of Jalabert and Das Sarma,[1] including the denominator and the matrix elements, but the remaining factors were left in terms of an Im $\chi(q,\omega_\pm)$.

We wish to retain our classical screening, but modify other aspects to match the quantum result. In the quantum development, the δ functions are energy δ functions $\delta(\hbar^2\mathbf{k_1}\cdot\mathbf{q}/m^*+\hbar^2q^2/2m^*-\hbar\omega_\pm)=\delta(\mathbf{v_1}\cdot\mathbf{q}+\hbar q^2/2m^*-\omega_\pm)/\hbar$. In the classical treatment we can see from Eq. (1) that $\omega=\mathbf{v}\cdot\mathbf{k_1}$ but now there is an additional term, which we should include, replacing ω by $\mathbf{v_1}\cdot\mathbf{q}+\hbar q^2/2m^*$. In addition we may treat the plasmon-phonon modes quantum mechanically by writing the V_q^ϕ and V_q^p in terms of the corresponding normal coordinates and write these in terms of annihilation and creation operators $a_\pm$ and $a_\pm^\dagger$. We obtain the quantum loss rate,

$$W_{\rm pl,ph}^{\rm qu}=\frac{\pi\varepsilon_\infty\Omega_c}{4\pi e^2N_c}\sum_{\pm}\ \frac{\omega_p^2(\omega_\pm^2-\omega_T^2)+\delta\omega_0^2\omega_\pm^2}{\omega_\pm^2-\omega_\mp^2}\sum_{\mathbf{q}}\ \hbar q^2\left(\frac{4\pi e^2}{q^2\varepsilon_\infty\Omega_c}\right)^2\left[a_\pm a_\pm^\dagger\delta\left(\frac{\hbar^2\mathbf{k_1}\cdot\mathbf{q}}{m^*}+\frac{\hbar^2q^2}{2m^*}+\hbar\omega_\pm\right)\right.$$
$$\left.-a_\pm^\dagger a_\pm\delta\left(\frac{\hbar^2\mathbf{k_1}\cdot\mathbf{q}}{m^*}+\frac{\hbar^2q^2}{2m^*}-\hbar\omega_\pm\right)\right]. \tag{7}$$

A major simplification has been accomplished by taking the plasma and optical-mode frequencies independent of q so that one complicated factor could be taken out from under the sum. The classical calculation contained no dependence upon the excitation of the mode, so it corresponds only to spontaneous emission, the first term in Eq. (7) with $a_{-\mathbf{q}}a_{-\mathbf{q}}^+$ replaced by 1. Equation (7) should be more closely related to the corresponding result, Eq. (15) of Jalabert and Das Sarma[1] depending upon what is included in their Im $\chi(q,\omega_\pm)$. Our form incorporates the screening explicitly, which was indeed our goal. It is not easy to match up the algebraic expressions with Jalabert and Das Sarma,[1] but ours will lead to appropriate known limits, as must theirs.

This now includes the full classical screening of the modes as well as the formation of composite modes. There will be some error in the use of classical screening, as we shall see in Sec. IV. However, we shall see that the differences are not important in the limits we need and the complex interplay between the frequency shifts, screening, and absorption that is included in our classical calculation would seem to be more essential. Equation (7) does not depend upon taking the electron under consideration to be of high or low energy with respect to thermal electrons nor with respect to $\hbar\omega_0$ or $\hbar\omega_p$.

Equation (7), viewed as an approximate expression of the golden rule for plasmon-phonon scattering of electrons, can be directly modified to treat a number of questions beyond the simple energy-loss rate. We may simply divide inside the sum by the energy $\hbar\omega_\pm$, which is the energy gained or lost, to obtain the transition rate. It is not obvious, but is true, that the factor preceding the square brackets is positive definite, so this will always give a positive rate. We may also obtain the distribution of the wave numbers of final states from such sums over final states, proceeding as we shall in the following section.

The sum over wave number will of course be replaced by an integral over $\mathbf{q}$ in the form

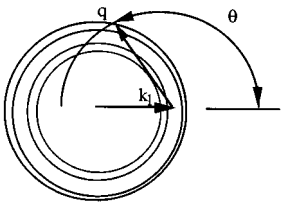

FIG. 1. The initial electron wave number $\mathbf{k_1}$ is shown and the change in electron wave number $\mathbf{q}$, making an angle of θ with $\mathbf{k_1}$. Circles, in order of increasing size, are states of energy $\varepsilon_1-\hbar\omega_+$, $\varepsilon_1-\hbar\omega_-$, $\varepsilon_1+\hbar\omega_-$, and $\varepsilon_1+\hbar\omega_+$. For each case the minimum wave number $q_{\min}$ is the wave-number distance from $\mathbf{k}_1$ to the corresponding circle and $q_{\max}$ the distance to the far side of that circle.

$\Sigma_{\mathbf{q}}=[N_c\Omega_c/(2\pi)^3]\int 2\pi q^2\,dq\int \sin\theta\,d\theta$ using the cylindrical symmetry of the problem around the initial wave number $\mathbf{k_1}$; θ is the angle between $\mathbf{q}$ and $\mathbf{k_1}$. We perform the integral over θ first, holding the magnitude of $\mathbf{q}$ fixed. What this means is seen in Fig. 1. If $\mathbf{q}$ crosses a circle representing an energy-conserving state with the emission or absorption of one of the composite modes (it crosses all four in Fig. 1), the δ function contributes $m^*/(\hbar^2 k_{1q})$ to the integral. We obtain for the energy loss

$$W_{\rm pl,ph}^{\rm qu}=\frac{m^*e^2}{\varepsilon_\infty\hbar k_1}\sum_{\pm}\ \frac{\omega_p^2(\omega_\pm^2-\omega_T^2)+\delta\omega_0^2\omega_\pm^2}{\omega_\pm^2-\omega_\mp^2}\times\left((n_\pm+1)\ln\left|\frac{q_{\max}^{\pm}}{q_{\min}^{\pm}}\right|_{\rm emit}-n_\pm\ln\left|\frac{q_{\max}^{\pm}}{q_{\min}^{\pm}}\right|_{\rm abs.}\right).\quad(8)$$

Here $n_\pm=a_\pm^\dagger a_\pm$ is the number of quanta in the corresponding mode $k_BT/\hbar\omega_\pm$ at high temperatures for thermal equilibrium. $q_{\min}$ and $q_{\max}$ are the minimum and maximum q values for which the crossing occurs for emission or absorption of the corresponding mode, obtained by inspection from Fig. 1. We have dropped the ''quantum'' designation since we no longer need the classical result.

Equation (8) provides an explicit formula for the energy-loss rate due to the phonon-plasmon system. The most important approximation has been the neglect of any dependence of $\omega_\pm$ on q, which allowed us to take the $\omega_\pm$-dependent factor out of the integral and evaluate the integral explicitly. Such formulas allow insight into the nature of the absorption by making explicit the dependence upon parameters such as incident energy, carrier density, temperature, and effective mass. We illustrate this now by considering particular limits.

III. THE EVALUATION FOR SPECIFIC LIMITS

We make the evaluation first for optical-phonon emission when ω_0 and ω_p are very different and the incident energy ε_1 is much larger than the phonon energy $\hbar\omega_0$. When ω_0^2 and ω_p^2 are very different we may expand Eq. (5) around the root near ω_0 as $\omega^2\approx\omega_0^2-\omega_p^2\delta\omega_0^2/(\omega_p^2-\omega_0^2)$, to first order in $\delta\omega_0^2\omega_p^2/(\omega_p^2-\omega_0^2)$. The $\omega^2-\omega_T^2$ in the numerator of the factor preceding the integral in Eq. (8) becomes $\omega_-^2-\omega_T^2=\delta\omega_0^2-\omega_p^2\delta\omega_0^2/(\omega_p^2-\omega_0^2)=-\delta\omega_0^2\omega_0^2/(\omega_p^2-\omega_0^2)$ and the entire factor becomes $\delta\omega_0^2\omega_0^4/(\omega_p^2-\omega_0^2)^2=\delta\omega_0^2/\varepsilon(0,\omega_0)^2$, where the dielectric function

$$\varepsilon(0,\omega)=1-\frac{\omega_p^2}{\omega^2}\quad(9)$$

is the long-wavelength dielectric function for a free-carrier gas, with Boltzmann or Fermi distributions (obtained, for example, from the second of Eq. (2) and Poisson's equation, with V_q^ϕ dropped or from the $q=0$ limit of the quantum dielectric function in the random phase approximation as we shall see in the Appendix). The same result applies for $\omega_0^2\gg\omega_p^2$ and for $\omega_0^2\ll\omega_p^2$. Note that in the first case it is a small antiscreening $[1/\varepsilon(0,\omega)^2\gtrsim 1]$ and in the second case it is a strong screening $[1/\varepsilon(0,\omega)^2\approx\omega_0^4/\omega_p^4\ll 1]$. [Similarly, we can obtain the screening of the *plasma like* mode by the phonons when the two frequencies are much different. When the $\omega_p^2\ll\omega_0^2$, the frequency-dependent factor in Eq. (8) becomes ω_p^2/ε^2 with the dielectric function $\varepsilon\approx 1+\delta\omega_0^2/\omega_0^2$ and when $\omega_p^2\gg\omega_0^2$ there is a weak antiscreening.]

We may look at the other factors in Eq. (8) for phonon emission and absorption. From Fig. 1 we may see that for large k_1 the lower limit $q_{\min}\approx\hbar\omega_0 m^*/\hbar^2 k_1$ obtained from $q_{\min}\partial\varepsilon_k/\partial k\approx\hbar\omega_0$, and the upper limit is $q_{\max}=2k_1\pm q_{\min}$ with the plus for absorption and the minus for emission. The terms in $n_\pm$, which we write here as n_0 and which could be a thermal occupation $k_BT/\hbar\omega_0$, cancel except over the range $2q_{\min}$ near $q=2k_1$. The remaining term contributes over the entire range. This leads to

$$W_{\rm pl,ph}=\frac{4\pi e^4 e_T^{*2}m^*}{\varepsilon_\infty^2\varepsilon(0,\omega_0)^2\Omega_c M_r\hbar k_1}\left[-\frac{n_0\hbar\omega_0}{2\varepsilon_1}+\ln\frac{4\varepsilon_1}{\hbar\omega_0}\right]\quad(10)$$

for ω_p very different from ω_0. The first term comes from the difference between absorption and stimulated emission and the second term arises from spontaneous emission of phonons, with the screening of each matrix element by $1/\varepsilon(0,\omega_0)$.

The conditions leading to Eq. (10) apply for a hot electron in a metal, $\hbar\omega_0<\varepsilon_1-\varepsilon_F\ll\hbar\omega_p$, but they will ordinarily not apply in semiconductor devices. In GaAs with n-type doping of 8×10^{17} electrons/cm^3, the optical-phonon frequency, the plasma frequency, and $k_BT/\hbar$ at room temperature are all comparable.

It will be informative therefore to make an approximate evaluation when again the incident energy is high compared to $\hbar\omega_\pm$, but ω_+ and ω_- are similar. The determination of limits on q for each case is the same as above. Also, the integrals themselves are similar enough for ω_+ and ω_- that we take them to be equal [at a common value, which we write as $\langle\omega_\pm\rangle=\sqrt{(\omega_0^2+\omega_p^2)/2}$, suggested by Eq. (4)] and combine the terms that precede them in Eq. (8) to obtain

$$W_{\rm pl,ph}=\frac{m^*e^2}{\varepsilon_\infty\hbar k_1}(\omega_p^2+\delta\omega_0^2)\left[-\frac{n_+\hbar\omega_+ + n_-\hbar\omega_-}{4\varepsilon_1}+\ln\left|\frac{4\varepsilon_1}{\hbar\langle\omega_\pm\rangle}\right|\right],\quad(11)$$

to lowest order in $(\omega_+-\omega_-)/\langle\omega_\pm\rangle$. In thermal equilibrium at high temperatures, $n_+\hbar\omega_+=n_-\hbar\omega_-\approx k_BT$.

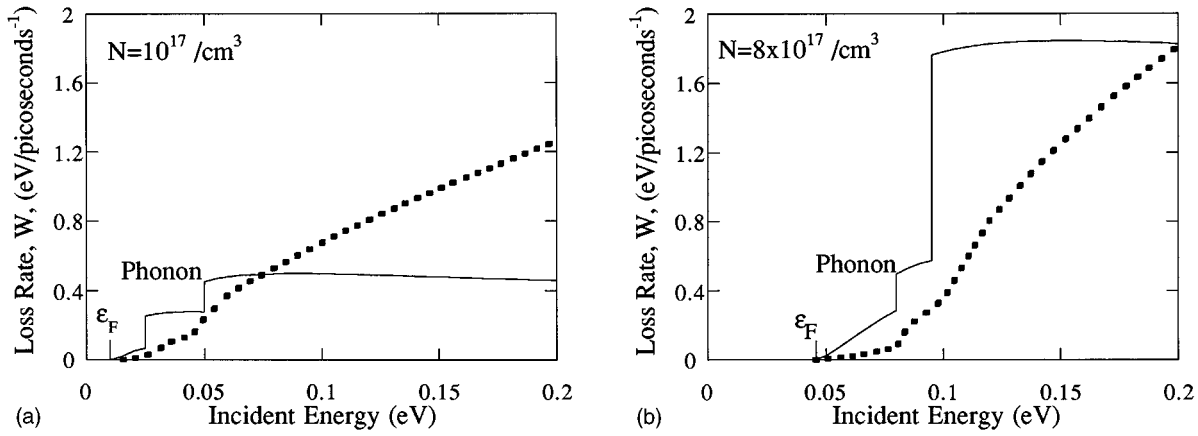

FIG. 2. The total absorption rate W as a function of incident energy ε_1 for two concentrations N of electrons in GaAs. The dotted line is the corresponding calculation from Ref. 1.

Equation (11) is remarkable. The final factor is a generalization of Eq. (10), but otherwise the term in $\delta\omega_0^2$ is exactly the result we obtained in Eq. (10) for spontaneous emission of phonons, but *without the screening factor* $1/\varepsilon(0,\omega_0)^2$. Similarly the term in ω_p^2 is that corresponding to spontaneous emission of plasmons without screening, or other effects, from the phonons. We find that the total energy loss is correctly computed by treating the phonons and plasmons independently and without screening (other than the $1/\varepsilon_\infty$ factors).

This depended upon our approximation that $\omega_\pm$ were similar, but in that case the mixing of the two levels and the mutual screening of each other cancel out completely. Though in detail there is screening, or antiscreening, of the phonon field by the plasmons, the modification in the intensity of phonon emission is compensated by a change in the loss rate to plasmons. The actual energy-loss events occur at $\omega_\pm$, not ω_0 and ω_p, which would lead to some modification of the dependence upon temperature and incident energy. There is no difficulty in keeping the two terms, $\omega_\pm$, separately as in Eq. (11) if one so chooses, and we shall do that in Sec. V.

An additional interesting conclusion can be drawn. The ratio of spontaneous loss rate due to plasmon creation to loss rate due to optical phonons is estimated from Eq. (11) to be simply

$$\frac{W_{\rm pl}}{W_{\rm ph}}=\frac{\omega_p^2}{\delta\omega_0^2}=\frac{N\Omega_c M_r}{m^* e_T^{*2}}. \tag{12}$$

This ratio may be rationalized by thinking of the emission of a plasmon as a collision with a carrier and emission of a phonon as a collision with an atom. One is less likely to collide with a carrier by a factor of the number of carriers per atom, $N\Omega_c/2$ for two atoms per cell, hence the factor $N\Omega_c$. However, in a direct collision with a stationary carrier, the incident carrier loses all of its energy. In a direct collision with a stationary atom (of mass $2M_r$ if the two atoms in the cell have about the same mass) one loses only $4m^*/(2M_r)$ of the initial energy. Finally, the atoms behave as if having a charge e_T^* times the carrier charge and the scattering cross section goes as the square of the charge. This crude argument suggests a ratio $N\Omega_c M_r/(4m^* e_T^{*2})$, giving all of the dependences correctly, with too small a numerical factor, partly because of the assumed head-on collision.

IV. COMPARISON WITH MORE COMPLETE CALCULATIONS

For comparison with numerical calculations we must substitute numbers for a particular system. We do this for n-type gallium arsenide. The carrier effective mass is $m^*=0.067m$. The experimental transverse-optical-mode frequency is[18] 0.509×10^{14} rad/sec and with a transverse charge of[18] $e_T^*=2.16$ for GaAs, $\delta\omega_0^2=0.028\times10^{28}/\text{sec}^2$, and from Eq. (2) $\omega_0=0.536\times10^{14}$/sec. Thus we have $\hbar\omega_0=0.035$ eV, in comparison to the plasmon energy of $\hbar\omega_p=0.014$ eV for n-type doping of $N=10^{17}/\text{cm}^3$ or 0.039 eV for $8\times10^{17}/\text{cm}^3$, two of the cases treated by Jalabert and Das Sarma.[1] It is quite informative to make a direct comparison with their calculations by returning to Eq. (8) and calculating the energy loss as a function of incident energy ε_1 for these two concentrations. This clarifies the weaknesses and strengths of our analysis. The two terms in Eq. (8) are given by

$$W_\pm=\frac{m^*e^2}{\varepsilon_\infty\hbar k_1}\,\frac{\omega_p^2(\omega_\pm^2-\omega_T^2)+\delta\omega_0^2\omega_\pm^2}{\omega_\pm^2-\omega_\mp^2}\,\ln\left|\frac{1+\sqrt{1-\hbar\omega_\pm/\varepsilon_1}}{1-\sqrt{1-\hbar\omega_\pm/\varepsilon_1}}\right|. \tag{13}$$

The comparison is given in Fig. 2, where we have added the two terms $W_\pm$, and included also quasiparticle excitations, discussed in the following section.

Jalabert and Das Sarma[1] considered the zero-temperature case, so that the electron gas was degenerate with a Fermi energy shown in the figure. This affected the results by excluding events for which the final electron energy would be below that Fermi energy. It also sets the $n_\pm$ equal to zero in Eq. (8). The basic assumptions are the same in both calculations, but we have made additional simplifying approximations in order to obtain analytic forms so it may be reasonable to assume that the differences are deficiencies in our approximations.

The small initial absorption, at energies, just above ε_F, is from quasiparticle excitations, to which we return. We see two additional abrupt rises, at $\varepsilon_F+\hbar\omega_-$ and $\varepsilon_F+\hbar\omega_+$, which are identifiable with shoulders in the curves given by

Jalabert and Das Sarma,[1] also shown. They are abrupt in our calculation because we did not include dispersion in the phonon or plasmon modes; emission became possible at all wave numbers at the same energy ε_1. We redid this including plasmon dispersion, which indeed rounded the rises, but not to the extent indicated by Jalabert and Das Sarma, and it required numerical integration rather than the explicit form given in Eq. (13).

The coupling between phonon and plasmon, represented by $\delta\omega_0^2$, is weak enough that we can associate the rises with the uncoupled mode to which it is closest, and we have so labeled the optical phonon mode in the figure. We see that the total rise associated with that mode is given rather well relative to Ref. 1, and is quite similar at both densities, as expected, and it occurs at the same energy above ε_F for both densities. The rise in absorption due to plasmons of course shifts with electron density, as we find, but our calculation gives considerably less growth with energy than does the full calculation. Perhaps the finite lifetime of the plasmons, which would enter somewhat like a broadening of the mode, might account for some of the difference. The figure may well be a fair representation of the validity of our approximate calculation.

It is not quite so simple to make the comparison with the numerical results given by Woerner and Elsaesser,[3] who included only contributions due to phonons. We evaluate the second term in Eq. (11), that due to optical phonons, taking $\omega_\pm$ equal to the longitudinal optical-mode frequency and evaluate it for heavy holes of mass of 0.66 (a weighted average over direction) using parameters that they listed. We note that the $1/k_1$ factor is related to $\varepsilon_1=\hbar^2k_1^2/(2m_{\rm HH})$ and obtain a rate $W_Q=0.93\ \ln|4\varepsilon_1/\hbar\omega_0)|/\sqrt{4\varepsilon_1/\hbar\omega_0}$ in eV per picosecond. This form rises from zero to a peak of 0.7 eV/picosecond at $\varepsilon_1=0.065$ eV. The corresponding curve in their Fig. 5 rises to a peak of 0.45 eV/picosecond at perhaps 0.1 eV. Woerner and Elsaesser have included a more complete description of the heavy-hole states and anisotropies, but we suspect that the differences arising from such refinements are quite small. Our result is higher, presumably largely because separating out the phonon term in Eq. (11) leaves out any screening of the interaction, since the effects of that screening are then part of the plasma term. It is difficult to make a comparison of the exact values from the two theories, but there is no reason to believe that they are in serious disagreement.

V. LOSS TO QUASIPARTICLE EXCITATIONS

Equation (8), or the more approximate Eq. (11), can be used to obtain the energy loss for hot carriers due to optical phonons and plasmons. There are also quasiparticle excitations that take energy from the incident hot electron. These require the introduction of a much larger class of coordinates, those of the individual electrons rather than the local center of gravity utilized above.

We again matched a classical self-consistent solution with a Born-approximation expression for electron-electron scattering to obtain an energy loss rate due to quasiparticle excitation,[17]

$$W_{\rm qp}=\frac{2\pi}{\hbar}\sum_{\mathbf{q},\mathbf{k}} f_0(\varepsilon_{\mathbf{k}})\left(\frac{4\pi e^2}{q^2\varepsilon_\infty\Omega}\right)^2\frac{\hbar\omega}{\varepsilon(\mathbf{q},\omega)^*\varepsilon(\mathbf{q},\omega)}\times\delta\left(\frac{\hbar^2}{2m^*}(2\mathbf{k}\cdot\mathbf{q}+q^2)-\hbar\omega\right),\tag{14}$$

with $\hbar\omega$ found to be the energy exchanged between the colliding electrons,

$$\hbar\omega=\frac{\hbar^2(2\mathbf{k_1}\cdot\mathbf{q}-q^2)}{2m^*}.\tag{15}$$

This plausible, but not completely obvious, identification follows from the classical analysis. With this identification, Eq. (14) could be written down immediately from the golden rule. Equation (15) was contained in Sanborn's Eq. (46). The unscreened matrix element for transfer of momentum $\hbar\mathbf{q}$ is $4\pi e^2/(q^2\varepsilon_\infty\Omega)$.

For a high-energy incoming electron we might neglect $\mathbf{k}$ in the δ function of Eq. (14) relative to $\mathbf{k_1}$. Then $\hbar\omega$ becomes $\hbar^2q^2/2m^*$ and with no remaining dependence upon $\mathbf{k}$ in the integral so we may perform the $\Sigma_{\mathbf{k}}\mathbf{f_0}(\varepsilon_{\mathbf{k}})=N\Omega$ with N the carrier density. Then writing $\varepsilon(\mathbf{q},\omega)^*\varepsilon(\mathbf{q},\omega)=\varepsilon_1^2+\varepsilon_2^2$ Eq. (14) reduces to

$$W_{\rm qp}=\frac{2\pi}{\hbar}\sum_{\mathbf{q}}\left(\frac{4\pi e^2}{q^2\varepsilon_\infty\Omega}\right)^2\frac{N\Omega}{\varepsilon_1^2+\varepsilon_2^2}\frac{\hbar^2q^2}{2m^*}\times\delta\left(\frac{\hbar^2}{m^*}(-\mathbf{k_1}\cdot\mathbf{q}+q^2)\right),\tag{16}$$

again for $k_1\gg k$, the wave number of the struck electron.

We write the sum over $\mathbf{q}$ as an integral $[\Omega/(2\pi)^3]\int 2\pi q^2dq\int\sin\theta\,d\theta$ and the δ function becomes $\delta(\hbar^2(q^2-k_1q\cos\theta)/m^*)$. We perform the angular integral first, in analogy with Fig. 1, obtaining a contribution as long as q is less than k_1. We have,[17] again for large k_1,

$$W_{\rm qp}=\frac{Nm^*}{2\pi\hbar^2k_1}\int_{0,k_1}q\,dq\left(\frac{4\pi e^2}{q^2\varepsilon_\infty}\right)^2\frac{1}{\hbar}\frac{\hbar^2q^2}{2m^*}\frac{1}{\varepsilon_1^2+\varepsilon_2^2}=\frac{4\pi Ne^4}{\varepsilon_\infty^2\hbar k_1}\int_{0,k_1}\frac{dq}{q}\frac{1}{\varepsilon_1^2+\varepsilon_2^2}.\tag{17}$$

It will turn out that $\varepsilon_1^2+\varepsilon_2^2$ is near one over most of the range of integration, but if it were equal to one at small q the integral would diverge. $\varepsilon_1^2+\varepsilon_2^2$ provides the cutoff at small q and is thus essential. One might ask if our taking of $\mathbf{k}$ negligible compared to $\mathbf{k_1}$ in obtaining Eq. (16) might have caused the divergence, but that is not the case. Even if we keep the distribution $f_0(\varepsilon_{\mathbf{k}})$ there remains a $\ln|k_1/q|$ term in the integral at small q.

In this case the quantum derivation of the dielectric function in terms of a density matrix and the random phase approximation is essentially as easy as a classical treatment based, for example, upon the Boltzmann equation. We therefore obtain the quantum $\varepsilon(q,\omega)$. This will include plasma contributions to this screening, but we drop any smaller effects of screening by optical phonons. Sanborn[2] has in fact argued that these phonon corrections vanish, a point that may well be similar to our finding that when both plasmons and optical phonons contribute, the screening of one compen-

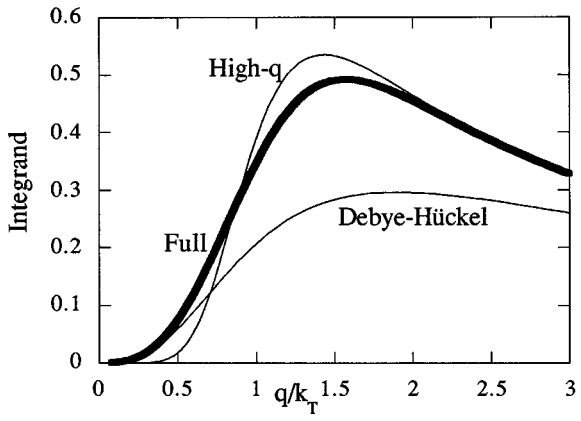

FIG. 3. The integrand of Eq. (17), $(k_T/q)(1/\varepsilon^2)$ with ε the full dielectric function, $\sqrt{\varepsilon_1[q,\hbar q^2/(2m^*)]^2+\varepsilon_2[q,\hbar q^2/(2m^*)]^2}$ [from Eqs. (A7) and (A8)]. The heavy line is based upon a Boltzmann distribution with $N=8\times10^{17}/\mathrm{cm}^3$, $k_BT=0.025$ eV, $\varepsilon_\infty=11$, $m^*=0.066m$, for which $\kappa_{\mathrm{DH}}^2=1.20k_T^2$. Also shown is the integrand based upon the Debye-Hückel (low-q) approximation to the dielectric function, Eq. (A5), and the high-q approximation, Eq. (A9), for the same parameters.

sates the antiscreening of the other. The losses due to the optical modes have already been included in the preceding sections without screening by the quasiparticles and here we should treat the quasiparticle loss without screening by the phonons. The ε_∞ that we have used throughout is the optical dielectric constant, arising from polarization of the bonds, and this will *not* be changed to the static dielectric constant ε_0, which includes the effect of atomic displacements.

In the Appendix we evaluate the quantum dielectric function $\varepsilon(q,\omega)$ both for a Boltzmann distribution and a degenerate Fermi distribution. Evaluating it for a high-energy incident electron, which we saw in the derivation of Eq. (16) corresponded to $\hbar\omega=\hbar^2q^2/(2m^*)\equiv\delta\varepsilon$ and a proportionality of the response to the density of carriers, we find what we call the ''dynamic dielectric function for quasiparticle scattering'' given by

$$\varepsilon_{\mathrm{qp}}(q)\approx1+\frac{(\hbar\omega_p)^2}{4\delta\varepsilon^2} \tag{18}$$

for both the Boltzmann and degenerate distributions.

We find this result remarkable in several ways. First, written in this form it is identical for a Boltzmann and a degenerate Fermi distribution, and therefore for intermediate degeneracy. Second we see that it has similarity to the dielectric function for phonon scattering, Eq. (9), which may be written as $1-(\hbar^2\omega_p^2)/\delta\varepsilon^2$. However, the sign of the final term is changed, and there is a new factor of $\frac{1}{4}$. Third, the second term in either form of Eq. (A9) is one-quarter of (and the same sign as) the corresponding term in the large-q limit of the quantum $\varepsilon_1(q,0)$, so the inclusion of the frequency dependence through $\hbar\omega=\delta\varepsilon$ was of some importance. Fourth, the energy-loss rate for a carrier of wave number k_1, the integral $\int_{0,k_1}dq/(q\varepsilon_{qp}^2)$ from Eq. (17), even for a ''warm electron'' with $k_1\approx1.25k_T$, is within 3% of the value obtained with the full dielectric function and beyond that is quite accurately given. This is seen in Fig. 3, where the integrand is plotted for the full form, as well as using the $\varepsilon_{\mathrm{qp}}$ and the Debye-Hückel form. We may use Eq. (18) for screening of quasiparticle scattering quite generally and with some confidence.

Substituting $\varepsilon_{\mathrm{qp}}(q)^2$ from Eq. (18) for $\varepsilon_1^2+\varepsilon_2^2$ in Eq. (17) and integrating gives the loss rate as

$$W_{\mathrm{qp}}=\frac{\pi Ne^4}{\varepsilon_\infty^2\hbar k_1}\left(\ln\left|1+\frac{4\varepsilon_1^2}{(\hbar\omega_p)^2}\right|-\frac{4\varepsilon_1^2}{4\varepsilon_1^2+(\hbar\omega_p)^2}\right). \tag{19}$$

It is applicable independent of the degeneracy of the electron gas, but applies to ε_1 large compared to the energy of the carriers of density N. If ε_1 is also large compared to $\hbar\omega_p$ Eq. (19) could be approximated by

$$W_{\mathrm{qp}}\approx\frac{2\pi Ne^4}{\varepsilon_\infty^2\hbar k_1}\left(\ln\left|\frac{2\varepsilon_1}{\hbar\omega_p}\right|-\frac{1}{2}\right). \tag{20}$$

We may correct Eq. (19) for one aspect of treating ε_1 as large compared to the Fermi energy of the carriers for the degenerate Fermi distribution that becomes serious at lower energies. We should exclude any scattering events in which the final electron state lies below the Fermi energy, as we did for phonons and plasmons for Fig. 2. We approximate this by noting that we included scattering by all electrons, leading to the factor N in Eq. (16). If the electron energy is $\varepsilon_1-\varepsilon_F$ above the Fermi energy, it cannot excite an electron below a cutoff energy ε_c for which $\varepsilon_F-\varepsilon_c$ is greater than $\varepsilon_1-\varepsilon_F$. This eliminates a fraction $(\varepsilon_c/\varepsilon_F)^{3/2}$ of the electrons, reducing the loss rate by a factor

$$\begin{aligned} f_{\mathrm{eff}}&=0 \quad \text{if } \varepsilon_1<\varepsilon_F,\\ f_{\mathrm{eff}}&=1-\left(\frac{2\varepsilon_F-\varepsilon_1}{\varepsilon_F}\right)^{3/2} \quad \text{if } \varepsilon_F<\varepsilon_1<2\varepsilon_F,\\ f_{\mathrm{eff}}&=1 \quad \text{if } 2\varepsilon_F<\varepsilon_1, \end{aligned} \tag{21}$$

in either Eq. (19) or (20).

Equation (19) with the correction Eq. (21), is the form that was used in Fig. 2. This only partly compensated for the error of the high-ε_1 approximation, which may account for our predicted rate at low energies, in the curve of Fig. 2 for $N=8\times10^{18}$ carriers, being significantly higher than that given by Jalabert and Das Sarma.[1] At large k_1 Eq. (19), or more clearly Eq. (20), drops with energy because the $1/k_1$ factor dominates the logarithmic term. This $1/k_1$ factor arose from a factor $1/(\partial\varepsilon/\partial k)$, which converted the integration over wave number to an integration over energy for use with the energy δ function in deriving Eq. (17). It arose also in the loss to phonons and plasmons, causing the rates to saturate and drop at large energy in Fig. 2. We do not understand the origin of the continued increase shown by the Jalabert and Das Sarma result though, as we indicated in our discussion of Fig. 2, it might be related to a broadening of the plasma modes from interaction with quasiparticle excitations.

Finally, we may compare the energy loss due to quasiparticle excitations with that due to plasma emission. We estimate the latter from the ω_p^2 term in Eq. (11), and substituting from Eq. (3) for ω_p^2 we see that the factor preceding the square brackets becomes $4\pi Ne^2/(\varepsilon_\infty^2\hbar k_1)$, the same as the leading factor in Eqs. (19) and (20) except for a factor $\frac{1}{4}$,

which came from the form we chose for the integral in those equations. For the parameters we have discussed the remaining factors are of order one and the two rates are comparable. From the detailed forms, we could see what conditions would make one dominate the other. Kleinman[20] also found the two mechanisms gave comparable contributions for hot carriers in aluminum, assuming the electron energy was large enough to allow plasmon emission.

VI. CONCLUSION

Indeed it has been possible to obtain analytic forms for each contribution to the energy loss by an energetic carrier, Eq. (8) for loss to the phonon-plasmon system and Eqs. (19) and (20) for excitation of quasiparticles. We have determined the form of the screening that entered these expressions, and have evaluated them for some interesting cases. The advantages of such expressions, which contain explicitly the dependence upon system parameters, may be sometimes sufficient to outweigh the extra accuracy, which can and has been obtained by detailed numerical calculation for individual systems.

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APPENDIX

The quantum dielectric function $\varepsilon(q,\omega)$ can be obtained using a density-matrix formulation to determine the electron response, and solving together with Poisson's equation, as, for example, in Ref. 21, p. 290, to obtain

$$\varepsilon(q,\omega)=1+\frac{4\pi e^2}{\varepsilon_\infty q^2\Omega}\sum_{\mathbf{k}} f_0(\varepsilon_{\mathbf{k}})\left(\frac{1}{\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}+\hbar\omega+i\hbar\alpha}+\frac{1}{\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}-\hbar\omega-i\hbar\alpha}\right). \tag{A1}$$

This was first evaluated for a degenerate electron gas by Lindhard[5] and the real and imaginary parts are given for that case by Pines,[4] for example. It is less familiar for a Boltzmann distribution so we treat that case in detail here. In either case the sum over wave numbers is replaced by an integral. The angular integral can be performed analytically to obtain both the real and imaginary parts as integrals over the magnitude k. The real part with a Boltzmann distribution is given by

$$\varepsilon_1(q,\omega)=1+\frac{\kappa_{\rm DH}^2}{k_T^2}\frac{1}{\sqrt{\pi}Q^3}\int dx\ x\ \exp(-x^2)\left[\ln\left(\frac{1+2x/B_+}{1-2x/B_+}\right)+\ln\left(\frac{1+2x/B_-}{1-2x/B_-}\right)\right]. \tag{A2}$$

Here we have introduced the renormalized frequency $w=\hbar\omega/(k_BT)$ and renormalized wave numbers as $x=k/k_T$ and $Q=q/k_T$ with again $\hbar^2k_T^2/(2m^*)\equiv k_BT$. Also $B_\pm=Q\pm w/Q$. The Debye-Hückel screening parameter $\kappa_{\rm DH}$ is given by

$$\kappa_{\rm DH}^2=\frac{4\pi Ne^2}{\varepsilon_\infty k_BT}. \tag{A3}$$

The imaginary part can be obtained analytically as

$$\varepsilon_2(q,\omega)=\frac{\kappa_{\rm DH}^2}{\sqrt{\pi}k_T^2Q^3}\left[\exp\left(-\frac{1}{4}\left|\frac{w}{Q}-Q\right|^2\right)-\exp\left(-\frac{1}{4}\left|\frac{w}{Q}+Q\right|^2\right)\right]. \tag{A4}$$

We might note two interesting limits that we have mentioned before, in both of which ε_2 vanishes. First is $\varepsilon(0,\omega)=\varepsilon_1(0,\omega)$, which we gave already in Eq. (9) and second is the static dielectric constant at long wavelengths,

$$\varepsilon(q,0)=\varepsilon_1(q,0)\approx 1+\frac{\kappa_{\rm DH}^2}{q^2}\quad(\text{small } q). \tag{A5}$$

Both forms are valid both for the degenerate electron gas or for the Boltzmann distribution. However, for the degenerate electron gas $\kappa_{\rm DH}$ is replaced by the Fermi-Thomas screening parameter $\kappa_{\rm FT}$ given by

$$\kappa_{\rm FT}^2=\frac{6\pi Ne^2}{\varepsilon_\infty\varepsilon_F}, \tag{A6}$$

with ε_F the Fermi energy.

The dielectric function we need for this case, $k\ll k_1$, is $\varepsilon(q,\omega)$ for $\hbar\omega=\hbar^2q^2/2m^*$. Then $w=Q^2$ so $B_+=2Q$ and $B_-=0$. Thus Eq. (A2) for a Boltzmann distribution becomes

$$\varepsilon_1\left(q,\frac{\hbar q^2}{2m^*}\right)=1+\frac{\kappa^2}{k_T^2}\frac{1}{Q^3\sqrt{\pi}}\int_{0,\infty}dx\ x\times\exp(-x^2)\ln\left(\frac{Q+x}{Q-x}\right), \tag{A7}$$

and Eq. (A4) becomes

$$\varepsilon_2[q,\hbar q^2/(2m^*)]=\frac{\kappa^2}{k_T^2}\frac{1}{\sqrt{\pi}Q^3}(1-e^{-Q^2}). \tag{A8}$$

We have evaluated the integrand of Eq. (17) for the energy-loss rate due to quasiparticle excitations (times k_T so

that it is dimensionless) numerically using Eqs. (A7) and (A8) with the result plotted as the heavy line in Fig. 3.

$\varepsilon_2(q)$ itself is small and makes only a small contribution to $\varepsilon_1^2+\varepsilon_2^2$. (Note that, at small q, ε_2^2 grows as $1/q^2$ but ε_1^2 contains a term in $1/q^4$.) The dominant term at small q is the Hückel approximation, Eq. (A5), which gives an integrand labeled as Debye-Hückel in Fig. 3. At large q the imaginary part ε_2 drops as $1/q^6$, while ε_1^2 is equal to one plus a term proportional to $1/q^4$. The combination approaches the high-q expression for $\varepsilon_1(q,\hbar q^2/2m^*)$, which is given by

$$\varepsilon_{\rm qp}(q)\approx 1+\frac{\kappa_{\rm DH}^2 k_T^2}{2q^4}=1+\frac{(\hbar\omega_p)^2}{4\delta\varepsilon^2}, \tag{A9}$$

the *dynamic dielectric function for quasiparticle scattering* by a Boltzmann distribution at large q. In the final form we have written the energy transfer $\delta\varepsilon=\hbar^2q^2/2m^*$. In obtaining the final form we used Eqs. (3) and (A3). Equation (A9) leads to the integrand labeled ''high-q'' in Fig. 3.

This result is interesting in a number of ways that we discussed following Eq. (18). In particular, in the final form it has no dependence upon the parameters $\kappa_{\rm DH}$ and k_T associated with the Boltzmann distribution and we may confirm that the final form is valid also for a degenerate distribution.

The Lindhard form for $\varepsilon(q,\omega)$ for a degenerate Fermi distribution, with $\hbar\omega=\hbar^2q^2/(2m^*)$, is, analogous to Eq. (A7),

$$\varepsilon_1\left(q,\frac{\hbar q^2}{2m^*}\right)=1+\frac{\kappa_{\rm FT}^2 k_F}{2q^3}\left(\frac{1-(q/k_F)^2}{2}\ln\left|\frac{q+k_F}{q-k_F}\right|+\frac{q}{k_F}\right). \tag{A10}$$

The high-q form is obtained directly by expanding the logarithm as

$$\varepsilon_{\rm qp}(q)\approx 1+\frac{\kappa_{\rm FT}^2 k_F^2}{3q^4}=1+\frac{(\hbar\omega_p)^2}{4\delta\varepsilon^2}, \tag{A11}$$

where in the final step we used Eqs. (3) and (A6). As in Eq. (A9), the final term is one-fourth the high-q form for a degenerate electron gas of $\varepsilon(q,0)\approx 1+4\kappa_{\rm FT}^2 k_F^2/(3q^4)$. We have again dropped the small contribution of $\varepsilon_2[q,\hbar q^2/(2m^*)]$.

These forms enter the energy-loss calculation through the integrand shown in Fig. 3, the integration being carried up to q equal to the wave number k_1 of the incident particle. That integral, and therefore the loss, can be seen to be roughly equal for the full result and the high-q result if the upper limit k_1 is around $2k_T$ or larger.

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