Magic numbers and optical-absorption spectrum in vertically coupled quantum dots in the fractional quantum Hall regime

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Exact diagonalization is used to study the quantum states of vertically coupled quantum dots in strong magnetic fields. We find that electron correlation in the double dot leads to angular momentum magic numbers that are distinct from those of a single dot. The double dot magic numbers occur at low angular momentum and the single dot magic numbers reappear at a critical angular momentum determined by the strength of the interdot electron tunneling. We also propose that the magic numbers can be investigated experimentally in vertically coupled dots. Because of the generalized Kohn theorem, the far-infrared optical absorption spectrum of a single dot is unaffected by correlation, but the theorem does not hold for two vertically coupled dots that have different confining potentials. We show that the absorption energy of the double dot should exhibit discontinuities at the magnetic fields where the total angular momentum changes from one magic number to another.

Low-dimensional confined electronic systems, such as quantum wells, wires, and dots, have recently attracted much interest because they exhibit dramatic quantum effects when they are placed in a strong magnetic field. The fractional quantum Hall (FQH) effect,[1] which occurs in a two-dimensional (2D) sheet of electrons, is probably the most spectacular example, although there are many others. Recently, attention has focused on double-layer FQH systems,[2–6] where the additional degree of freedom (a pseudospin that labels the layers) enriches the physics. A central issue in these systems is the interplay of electron correlation and interlayer electron tunneling — because of the competition between these effects, the quantum Hall state evolves continuously from a correlation-dominated (two-component) state down to a tunneling-dominated (single-component) state within the quantum Hall regime.

Another recent development is the study of laterally confined systems, and quantum dots in particular. A dot in the FQH regime contains a few electrons confined on a length scale of the order of the magnetic length. As a function of the total angular momentum, L, the ground-state energy of this system exhibits downward cusps at specific L values which are known as the "magic numbers."[7,8] The magic numbers are caused by electron correlation, and there are general arguments that relate the magic numbers for given numbers of electrons, N_e, to the symmetry of the wave function and the requirement that it satisfies Pauli's principle.[9–11] For instance, the magic numbers, $L = 3, 6, 9, \ldots$, for three spin-polarized electrons correspond to triangular spatial correlation that minimizes the Coulomb repulsion.

It is then intriguing to ask what happens if we laterally confine a double-layer FQH system to form vertically coupled quantum dots. Because fascinating correlation effects are known to occur in double 2D systems (bilayers) and double 1D systems (double quantum wires[12]), we can expect to find interesting phenomena in double 0D systems (double dots), which are the subject of the present work. Specific questions we address are, first, what will happen to the magic numbers as we vary the strength of the tunneling, and second, whether the magic numbers may become observable in double dots. Technically, we believe that the structures considered here are within the scope of current fabrication technology.[13]

The system we study is a double dot containing a total of three spin-polarized electrons. In both dots the electron motion is perfectly two dimensional and the lateral confining potential within each layer is assumed to be parabolic. The dots are separated in the vertical direction with their centers aligned on a common axis. The electrons experience both intralayer and interlayer Coulomb repulsions in the presence of the interlayer tunneling. Related work on multiple dots without tunneling[14,24] has recently been published. We find a new series of magic numbers which are intrinsic to the double dot and correspond to ground states dominated by the interlayer electron correlation. The strength of the interlayer tunneling controls the appearance of these magic numbers. In addition, we propose that the magic numbers can manifest themselves in the far-infrared optical absorption spectrum. In a single dot this is impossible because of the generalized Kohn theorem,[8,15,16] but the theorem does not hold for a double dot with different confining potentials. Consequently, we find that the absorption energy should exhibit discontinuities at the magnetic fields, where the total angular momentum changes from one magic number to another.

A vertically coupled double dot is characterized by the strength of the parabolic confinement potential of the upper

(lower) layer, $\hbar\omega_+$ ($\hbar\omega_-$), the layer separation, d, and the strength of the interlayer tunneling (measured by Δ_{SAS}, the energy gap between the symmetric and antisymmetric states in the noninteracting system). The Hamiltonian,

$$\mathcal{H}=\mathcal{H}_s+\mathcal{H}_t+\mathcal{H}_C, \tag{1}$$

comprises the single-electron part, $\mathcal{H}_s$, the tunneling term, $\mathcal{H}_t$, and the Coulomb interaction, $\mathcal{H}_C$. We assume that the magnetic field B is so strong that Landau-level mixing is negligible and we write the Hamiltonian in second-quantized form with a Fock-Darwin basis.[17] This gives

$$\mathcal{H}_s=\sum_{\ell}\sum_{\alpha}\varepsilon_{\ell\alpha}c^{\dagger}_{\ell\alpha}c_{\ell\alpha}, \tag{2}$$

$$\mathcal{H}_t=-\frac{\Delta_{SAS}}{2}\sum_{\ell}(c^{\dagger}_{\ell+}c_{\ell-}+c^{\dagger}_{\ell-}c_{\ell+}), \tag{3}$$

$$\mathcal{H}_c=\frac{1}{2}\sum_{\ell_1\sim\ell_4}\sum_{\alpha_1\sim\alpha_4}$$
$$\times\left\langle\ell_1\alpha_1,\ell_2\alpha_2\left|\frac{e^2}{\epsilon\sqrt{|\mathbf{r}_1-\mathbf{r}_2|^2+d^2}}\right|\ell_3\alpha_3,\ell_4\alpha_4\right\rangle$$
$$\times c^{\dagger}_{\ell_1\alpha_1}c^{\dagger}_{\ell_2\alpha_2}c_{\ell_4\alpha_4}c_{\ell_3\alpha_3}. \tag{4}$$

Here, the index α is used to distinguish the two dots, $\alpha=+,-$, $c^{\dagger}_{\ell\alpha}(c_{\ell\alpha})$ are creation (annihilation) operators, and ϵ is the dielectric constant of the host material. The sum over $\alpha_1,\ldots,\alpha_4$ in $\mathcal{H}_c$ guarantees that both intralayer and interlayer interactions are included. The energy of the zeroth Landau-level Fock-Darwin state with angular momentum ℓ (≥ 0) in the αth dot is $\varepsilon_{\ell\alpha}=(1+\ell)\hbar(\omega_c^2/4+\omega_\alpha^2)^{1/2}-\ell\hbar\omega_c/2$, where $\omega_c=eB/m^*c$ is the cyclotron frequency.

We now estimate typical values of the parameters. To obtain approximate values of the confinement energy, we use a simple electrostatic model in which there is a disk of positive charge above and below the two dots, with the entire structure sandwiched between two metallic disks. This is meant to mimic the electrostatic confinement scheme in which a single quantum dot is made by applying a modulated gate electrode to a modulation-doped heterojunction or quantum well.[19] For a single dot, we have found that the model is able to reproduce the confinement energy from an exact solution of the Poisson equation to about 20%. To estimate the confinement energy of the double dot, we take typical device dimensions and dopant densities from the work of Boebinger $et\ al.$[20] and Kumar, Laux, and Stern[19] to find that $\hbar\omega_\alpha$ is about 2–4 meV. The asymmetry in $\hbar\omega_\alpha$ depends on the offset of the two dots from the symmetric configuration and on the positions of the disks. It is typically 5%–10% for disk separations of a few hundred nanometers and dot offsets of a few tens of nanometers. Larger asymmetry could be achieved by making the structure grossly asymmetric. In our calculations we take $\hbar\omega_+=2.0$ meV and $\hbar\omega_-=2.2$ meV. The electrostatic model predicts that the confining potential at the center of each dot will be in general different. We assume that this could be compensated by applying a potential to the entire device. For the dot separation and the symmetric-antisymmetric splitting, we take typical values
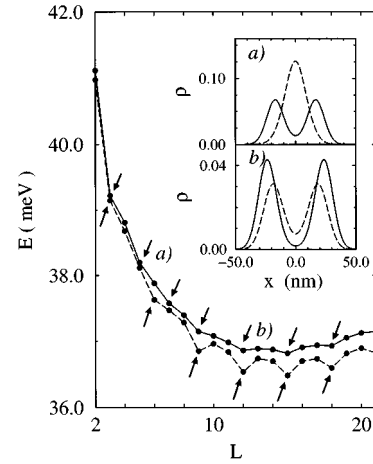

FIG. 1. Ground-state energy against the total angular momentum, L, in vertically coupled dots with three electrons for the strength of the interlayer tunneling $\Delta_{SAS}=0.2$ meV (solid line) or $\Delta_{SAS}=0.5$ meV (broken line). Strength of the confinement potential is $\hbar\omega=2.0$ (2.2) meV for the upper (lower) layer, and the layer separation is $d=20$ nm. Arrows indicate the positions of the cusps. The inset shows a cross section of the charge density in the upper (solid lines) and lower layer (broken lines) against the lateral distance from the center of each dot for $L=5$ (a) and $L=12$ (b).

from double layer studies of Boebinger $et\ al.$[20] and Eisenstein $et\ al.$,[4] leading to $d=20$ nm and Δ_{SAS} in the range 0.2–0.5 meV.

The ground-state energy (Fig. 1) is calculated as a function of the total angular momentum, L, by diagonalizing the Hamiltonian in a Slater determinant basis at $B=10$ T for $\Delta_{SAS}=0.2$ meV (solid line) and $\Delta_{SAS}=0.5$ meV (broken line). Qualitatively, the behavior shown in the figure is typical for a large range of B values, although the value of L at which the minimum energy occurs depends strongly on B. The magic numbers can be identified from the positions of downward cusps. For $\Delta_{SAS}=0.2$ meV we have a new period of two up to $L=3,5,7,9$, followed by a period of three, $L=9,12,\ldots$, while for a larger $\Delta_{SAS}=0.5$ meV the period is three throughout, $L=3,6,9,12,\ldots$, as in the case of a single dot containing three electrons.

To identify the mechanism for the change of period in the magic number for smaller Δ_{SAS}, we show the charge density (the inset to Fig. 1) and the pair correlation function (Fig. 2) before ($L=5$) and after ($L=12$) the change in the period sets in. For $L=5$ [inset (a)] the density against the lateral distance from the center has a peak at the center in the lower layer while the density is double peaked in the upper layer. For $L=12$ [inset (b)] the densities in both layers are double peaked. We investigate further by looking at the pair correlation function $P(\mathbf{r},\mathbf{r}_0)$ (Fig. 2), which is defined as the conditional probability of finding an electron at position $\mathbf{r}$ given that there is one at position $\mathbf{r}_0$. The fixed electron is at $r_0=16.9$ nm [for $L=5$ (a)] or 23.5 nm [for $L=12$ (c)] in the upper layer where the charge density is at maximum. From the figure we can immediately see that the ground-state electron configuration changes from one dominated by interlayer correlation to one dominated by intralayer correlation. For $L=5$ the form of the correlation corresponds to a triangular ``electron molecule'' developing $across$ the two layers, with one electron at the center of the lower layer while the other two are in the upper layer. In contrast, the triangular form
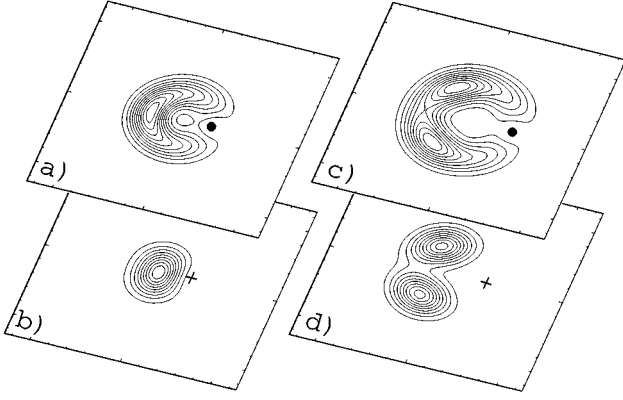
FIG. 2. Intralayer (upper panels) and interlayer (lower panels) pair correlation functions, $P(\mathbf{r}, \mathbf{r}_0)$, for $L=5$, (a,b) or $L=12$ (c,d). One electron (solid circle) is fixed in the upper layer at $r_0=16.9$ nm ($L=5$) or 23.5 nm ($L=12$) where the charge density has a maximum. The symbol $+$ denotes the projection of the solid circle onto the lower layer. An area with the linear dimension of 128 nm is displayed. The confinement energy is $\hbar\omega=2.0$ (2.2) meV for the upper (lower) layer, the layer separation $d=20$ nm, and $\Delta_{SAS}=0.2$ meV.

develops within each layer for $L=12$. Similar physics should occur when the two dots have the same confining potential ($\omega_+=\omega_-$).

The change in correlation can be understood by considering the energy. As L is decreased the lateral spatial extent of the wave function becomes comparable with the vertical separation of the layers. When the total angular momentum is small enough, the intralayer Coulomb interaction dominates the interlayer Coulomb interaction, so electrons try to avoid each other by developing an interlayer correlation. Although this has to involve mixing of states in the two dots and costs an energy Δ_{SAS}, the electron correlation still dominates as long as Δ_{SAS} is small enough. We believe this is why the new magic numbers $L=5,7$ appear for smaller Δ_{SAS}. The global minimum energy and the angular momentum of the absolute ground state depends on the magnetic field. By scanning a range of magnetic fields we have found that the new magic number states at $L=5$ and $L=7$ become the absolute ground state when $B\sim4$ T and $B\sim6$ T, respectively.

A comparison of our results with the phase diagram[18] for the bulk double-layer FQH system is not straightforward. The latter phase diagram is drawn against two dimensionless quantities, d/ℓ_B and $\Delta_{SAS}/(e^2/\epsilon\ell_B)$, where $\ell_B = (c\hbar/eB)^{1/2}$ is the magnetic length. Because of the confining potential the relevant length scale for dots becomes the effective magnetic length λ with $\lambda^2=\hbar/m^*(\omega_c^2+4\omega_0^2)^{1/2}$. With the parameters we have used $\lambda=(0.91-0.97)\ell_B$ for $B=(5-10)$ T. This yields $e^2/\epsilon\lambda=14.5$ meV for $B=10$ T, so that $\Delta_{SAS}/(e^2/\epsilon\lambda)=0.01-0.03$ for the double dots considered here, while the Landau-level filling, ν, which is usually defined as $\nu=N_e(N_e-1)/2L$ for dots, ranges from $\nu=3/5$ for $L=5$ to $\nu=1/4$ for $L=12$. It is an interesting problem to see how the intradot to interdot crossover in double dots may be related to the one- to two-component crossover in the double layers.

Now we move on to the far-infrared (FIR) optical absorption spectrum. In a single dot with a parabolic confinement potential, the electron-electron interaction does not affect the FIR absorption. This follows from the generalized Kohn theorem: long-wavelength electromagnetic radiations with electric vector $\mathbf{E}$ couple to the dot via the perturbation Hamiltonian

$$\mathscr{H}' = \sum_{i=1}^{N} e\mathbf{E}\cdot\mathbf{r}_i, \tag{5}$$

which depends only on the center-of-mass coordinate. In a single dot with parabolic confinement the Hamiltonian separates into the center-of-mass and relative (interaction) parts and the latter is irrelevant to optical transitions. In contrast, the separation does not occur in vertically coupled dots having different confinement energies even if both dots have parabolic confinement. This means the Coulomb interaction should affect FIR absorption spectra.

To quantify the effect we have calculated the FIR absorption spectrum of vertically coupled dots from the matrix element of the perturbation Hamiltonian, $\langle\mathscr{H}'\rangle$, between the ground state and all the excited states. Before discussing the results, we comment on the applicability of this approach to real systems. One important question is the nature of the electric field $\mathbf{E}$. Several authors have questioned the relation between the applied electric field and the internal electric field in mesoscopic systems[21–23] with the general conclusion that depolarization effects are important. Therefore we would have to calculate the internal electric field to obtain the absolute value of the absorption coefficient. In addition, precise calculation of the absorption spectrum would require us to take account of other device properties that affect absorption, such as finite thickness of the individual dots and deviations from a parabolic potential, about which scant information is available. We therefore make the reasonable assumption that
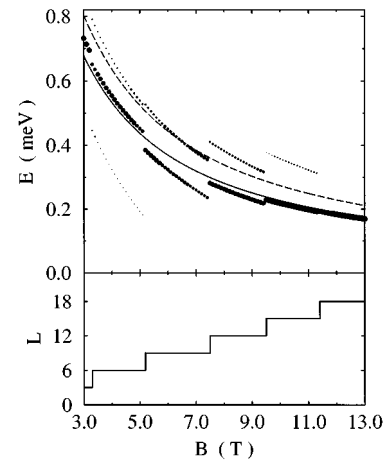

FIG. 3. FIR absorption spectrum (upper panel) and total angular momentum (lower panel) of vertically coupled dots for $\hbar\omega_+=2.0$ meV and $\hbar\omega_-=2.2$ meV, with the layer separation $d=20$ nm and $\Delta_{SAS}=0.5$ meV. The position of each filled circle gives the energy of the transition, while the size of the circle represents the absorption intensity. The solid (broken) line corresponds to the single-electron absorption spectrum for $\hbar\omega_+=2.0$ (2.2) meV.

the internal electric field is uniform and discuss only the absorption energy and the relative intensities of various transitions. This should be sufficient for our purpose of demonstrating that the FIR absorption of vertically coupled dots is affected by electron correlation.

The results of our calculations (Fig. 3) for $\Delta_{SAS}=0.5$ meV show that the spectrum indeed exhibits a series of jumps. In a single dot the FIR absorption has two branches: the upper branch for inter-Landau-level transitions and the lower one for intra-Landau-level transitions. Because we consider only the lowest Landau level here we have only calculated the lower branch, but we anticipate that the upper branch will exhibit similar jumps. For comparison, the energy of the lower branch for noninteracting electrons given by

$$\hbar\omega_{single}=\frac{\hbar}{2}(\omega_c^2+4\omega_0^2)^{1/2}-\frac{1}{2}\hbar\omega_c \tag{6}$$

is also shown in the figure for $\hbar\omega_+=2.0$ meV (solid line) and $\hbar\omega_-=2.2$ meV (broken line). It is clear that the coupled dot absorption spectrum is not simple such as that of a single dot and is split into pieces. This means that in the weak magnetic-field region, or in a small total angular momentum region, the center-of-mass and relative motions are strongly mixed.

In particular, the jumps in the absorption energy occur at the magnetic fields at which the total angular momentum changes from one magic number to another. Thus the ground-state transitions should be directly observable in the FIR absorption spectrum. The figure also shows that the absorption intensity ($\propto$ square of the matrix element) is not monotonic. For $\Delta_{SAS}=0.2$ meV the FIR absorption spectrum is similar, although the jumps in energy are smaller than for $\Delta_{SAS}=0.5$ meV.

In conclusion, we have found magic numbers intrinsic to vertically coupled quantum dots and have shown that they could be probed experimentally.

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