Coherent versus incoherent transport in layered doped Mott insulators

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There exists strong experimental evidence for the dimensional crossover from two to three dimensions as $La_{2-x}Sr_xCuO_4$ compounds are overdoped. In this paper we describe the dimensional crossover of the layered correlated metal in the gauge theory framework. In particular, we obtain the anomalous exponent 3/2 for the temperature dependence of resistivity observed in overdoped $La_{2-x}Sr_xCuO_4$.

I. INTRODUCTION

The normal state properties of high-T_c compounds are anomalous. In particular, at optimal doping, in-plane resistivity ρ_{ab} decreases linearly with temperature,[1] while out-of-plane resistivity ρ_c increases with temperature. The difference between in-plane and out-of-plane transport reflects the layered structure of the cuprates and indicates the hopping character of the interlayer transport. It is generally believed that cuprates evolve into Fermi liquids as doping increases.

However, systematic studies[2–5] of transport properties of overdoped $La_{2-x}Sr_xCuO_4$ have shown a deviation from the Fermi liquid as well as from the optimally doped compounds. The temperature dependence of resistivity was found to be T^α with an exponent close to 1.5.[2] Also, in the overdoped regime ($x \geqslant 0.25$), both in-plane and out-of-plane resistivity exhibit similar, although anisotropic, temperature dependence: ρ_c/ρ_{ab}, which is on the order of 50–100, is almost independent of temperature and is not far from the value predicted by the band structure calculation, which is roughly 25.[2,6] This anisotropy of the overdoped $La_{2-x}Sr_xCuO_4$ is to be compared with the anisotropy at optimal doping $x \approx 0.15$, which is on the order of 500–1000 near T_c. The above experimental data force us to conclude that there exists a dimensional crossover from a two dimensional *anomalous (strange) metal* at optimal doping to a three dimensional anisotropic *anomalous metal* in the overdoped case.

The dimensional crossover takes place as the doping varies. In fact, it is also a crossover in temperature. To address the problem of crossover we introduce a temperature scale $T_d(x)$, which depends on doping. At sufficiently low temperatures, $T < T_d(x)$, unless the superconducting transition intervenes, any layered material is essentially three dimensional. This means, in particular, that only one transport time $\tau_{\rm tr}^{-1}$ determines the temperature dependence of in- and out-of-plane resistivity, and hence the ratio ρ_c/ρ_{ab} is independent of temperature. The nearly temperature independent anisotropy ratio ρ_{ab}/ρ_c implies a common scattering mechanism for in-plane and out-of-plane charge transport. In this case, scattering and resistivity generally increase with temperature. We designate this type of transport as *coherent*.

At high temperatures, $T > T_d(x)$, all the relaxation times are shorter than the interlayer hopping time; thus the out-of-plane conductivity is determined by one particle tunneling. When the out-of-plane transport is due to one particle tunneling, we face the question of whether an electron is a quasiparticle or not. If it is, the out-of-plane conductivity is proportional to the one particle relaxation time τ and decreases as temperature increases. In this case, there is no difference between $\tau_{\rm tr}$ and τ and the ratio ρ_c/ρ_{ab} is independent of temperature.

If an electron is not a quasiparticle due to strong interaction, (i.e., its Green function does not possess a pole), a new decay time τ_c to true quasiparticles becomes relevant. We call τ_c the coherence time. If the interlayer hopping time is longer than the coherence time, the electron decays into quasiparticles during the hopping. We refer to this type of transport as *incoherent*. A feature of this incoherent transport is that rising temperature increases the out-of-plane mobility and decreases the resistivity. This incoherent transport, which has no analogue in Fermi liquids is considered in this paper.[7]

Eventually, the temperature scale $T_d(x)$ which determines the dimensional crossover of $La_{2-x}Sr_xCuO_4$ strongly depends on the doping $x \sim (0.15-0.35)$. The experimental data of $La_{2-x}Sr_xCuO_4$ suggest that the overdoped $La_{2-x}Sr_xCuO_4$ most likely lies in the low-temperature coherent regime, $T < T_d(x)$, while the optimally doped $La_{2-x}Sr_xCuO_4$ lies in the high-temperature two dimensional regime, $T > T_d(x)$. We consider the overdoped cuprates as an intermediate metallic state which interpolates between the two dimensional *anomalous strange metal*[8] and the conventional three dimensional metal.

We propose that three dimensional anisotropic gauge theory may be a suitable model to describe the charge transport experiments in both the overdoped and the optimally doped cuprates in a unified way.

Among theories for the anomalous normal states of cuprates near optimal doping, the two dimensional (2D) gauge theory[8–10] highly emphasizes the retarded scattering by the

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chirality fluctuations provided by infinitely strong on-site repulsion. In particular it gives T-linear in-plane resistivity. In this paper, we generalize the 2D gauge theory to 3D gauge theory in order to describe the charge transport in the coherent three dimensional regime and the crossover between coherent and incoherent regimes.

We also note that diverse models capture other scattering mechanisms for the peculiar out-of-plane transport of cuprates.[11–18,30]

The main results of this paper can be summarized as follows. In the low-temperature regime, $T<T_d(x)$, the charge transport is coherent and three dimensional, and three dimensional anisotropic gauge theory is employed to describe the coherent charge transport. The resistivities are found to be

$$\rho_{ab}\propto\rho_c\propto T^{3/2}. \tag{1.1}$$

The anomalous exponent 3/2 of resistivities in the three dimensional regime has been experimentally observed in overdoped $La_{2-x}Sr_xCuO_4$.[2,3]

In the high-temperature regime, $T>T_d(x)$, the charge transport is incoherent and two dimensional. This regime can be described by two dimensional gauge theory with an interlayer hopping term. The resistivities behave as

$$\rho_{ab}\propto T,\quad \rho_c\propto\frac{1}{\sqrt{T}}. \tag{1.2}$$

Ironically, gauge theory suggests a different physical mechanism which also gives $\rho_{ab}\propto T^{3/2}$ (but incoherent ρ_c) in the two dimensional regime. At *high* temperature the scattering by the chirality fluctuations becomes inelastic. The inelasticity changes the linear temperature dependence of the resistivity to $T^{3/2}$.

The remainder of the paper is organized in the following way. In Sec. II, we introduce gauge theories for the incoherent and coherent regimes and derive the formulas necessary for the calculation of resistivities. In Sec. III, we calculate resistivities in the coherent regime and the dimensional crossover and experimental data are discussed. In Sec. IV, the out-of-plane conductivity in the two dimensional regime is calculated. In Sec. V, we discuss the inelastic scattering mechanism. We conclude this paper in Sec. VI.

II. THE GAUGE MODELS OF NORMAL STATES

Strong on-site Coulomb repulsion forbids double occupations and imposes the constraint $\Sigma_\alpha c^\dagger_{n,\alpha}(r)c_{n,\alpha}(r)\leqslant 1$ (r are coordinates on a layer and n labels the layers). The gauge field is a tool to deal with this constraint. The constraint can be implemented by representing an electronic operator $c_{n,\alpha}(r)$ by the product of a fictitious spinon $f_{\alpha,n}(r)$ and a holon $b^\dagger_n(r)$ that keeps track of vacant sites: $\Sigma_\alpha f^\dagger_{\alpha,n}(r)f_{\alpha,n}(r)+b^\dagger_n(r)b_n(r)=1$. One of them is a fermion, while the other is a boson. An accepted phenomenological model for each layer that captures the vector character of the interaction has the form[8–10,19–22]

$$H_{2D}=\int d^2r\Big[\sum_\alpha f^\dagger_\alpha(r)\Big(-a_0-\mu_F-\frac{1}{2m^\parallel_F}(\nabla-i\mathbf{a})^2\Big)f_\alpha(r) + b^\dagger(r)\Big(-a_0-\mu_B-\frac{1}{2m^\parallel_B}(\nabla-i\mathbf{a})^2\Big)b(r)\Big]. \tag{2.1}$$

A small interaction between layers can be represented by adding an interlayer hopping term,

$$H^\perp=t_\perp\sum_n\int d^2r[c_{n,\alpha}(r)c^\dagger_{n+1,\alpha}(r)+\text{ H.c}]. \tag{2.2}$$

We neglect the interlayer magnetic exchange in $La_{2-x}Sr_xCuO_4$ since it is smaller than the intralayer magnetism by a factor of 10^{-5} [see, e.g., (Ref. 23)]. We use the model $H=H_{2D}+H^\perp$ to describe the incoherent out-of-plane transport in the optimally doped two dimensional regime, i.e., at $T>T_d(x)$, where the interlayer hopping $t_\perp$ is the smallest energy scale.

At $T<T_d(x)$, to which regime we believe the overdoped $La_{2-x}Sr_xCuO_4$ belongs, the system is assumed to be three dimensional and we employ a different model which is an anisotropic generalization of the two dimensional gauge theory (2.1):

$$H_{3D}=\int d^3r\Big[f^\dagger_\alpha(r)\Big(\frac{(-i\nabla-\mathbf{a})^2}{2m^\parallel_F}+\frac{(-i\partial_z-a_z)^2}{2m^\perp_F} -\mu_F-a_0\Big)f_\alpha(r)+b^\dagger(r)\Big(\frac{(-i\nabla-\mathbf{a})^2}{2m^\parallel_B}+\frac{(-i\partial_z-a_z)^2}{2m^\perp_B} -\mu_B-a_0\Big)b(r)\Big]. \tag{2.3}$$

In layered materials the interplane masses $m^\perp_B$ and $m^\perp_F$ are much larger than the in-plane masses $m^\parallel_B$ and $m^\parallel_F$.

A few comments are in order before we turn to the perturbative calculation of resistivities. The microscopic basis of the 2D model (2.1) is weak; nevertheless the model has attractive universal features. This model has been derived by different authors[10,21] from different physical assumptions. In Ref. 21 a strong short range magnetic exchange was essential, whereas in Ref. 10 no magnetic exchange was assumed at all. In Ref. 21 $m^\parallel_F$ and $m^\parallel_B$ are determined by the magnetic exchange J and the hopping amplitude t, respectively. In Ref. 10, both $m^\parallel_F$ and $m^\parallel_B$ are determined by the hopping. In both cases, the model (2.1) captures the physics of scattering by chirality fluctuations, namely, by magnetic polarization produced by mobile dopants. In considering the effects of nonlocal retarded processes due to chirality fluctuation on the normal state transport, it is reasonable to treat spinon mass $m^\parallel_F$ and holon mass $m^\parallel_B$ as phenomenological parameters. It is even more so for the 3D model (2.3). The 3D model (2.3) is suggested by transport properties of overdoped $La_{2-x}Sr_xCuO_4$.[2–5] We have failed, however, in justifying this model in a quantitatively microscopic way for the range of parameters known for $La_{2-x}Sr_xCuO_4$.

Another comment is that, although the two models (2.1), (2.2), and (2.3) are different, they equivalently describe the dimensional crossover of the in-plane transport. They are essentially different, however, as far as the out-of-plane

transport is concerned: while both models give the same result for ρ_c in the 3D regime, i.e., at $T<T_d$, they give different ρ_c at $T>T_d$. The reason for this discrepancy at $T>T_d$ is that the 3D model (2.3) neglects fluctuations of the amplitude of effective intra- and interlayer electronic hopping but stresses the fluctuations of their phases. This is a correct approximation at low T. In contrast, the 2D model (2.1) and (2.2) neglects the variations of in-plane hopping amplitude, but takes into account the fluctuations of the interlayer amplitude which become important at high temperature.

At a temperature range where the effects of Bose condensation are irrelevant, the chirality fluctuations are small and can be treated perturbatively. The tendency to condense is suppressed by the gauge interaction and strong on-site repulsion: the holons are hard core bosons. These effects are beyond the perturbation theory and have remained obscure. As a result we do not know the low-temperature bound of the perturbation theory. Of course the upper estimate of the bound is given by the mean field value of the Bose condensation temperature $T_0 \sim x/m_B$. For cuprates T_0 is too high (around 1500 K). The Bose condensation effects make a perturbative treatment questionable at $T \leqslant T_0$ unless the interactions decreases the characteristic temperature scale T_0 substantially, and thus drag down the perturbation theory to much lower temperature.[8,22] It is also likely that Bose condensation is completely destroyed due to the retardedness of the gauge interaction and hard core nature of holons. A recent quantum Monte Carlo study also supports the latter point of view.[24]

In three dimensional gauge theory the gauge field fluctuation is less effective than 2D gauge field fluctuation in suppressing Bose condensation. Nevertheless, the retardedness of gauge interaction and the hard-core nature of holons are also valid in the three dimensional case, hence we expect Bose condensation is sufficiently suppressed, or even can be destroyed completely as in the 2D case, to permit the perturbative treatment of 3D gauge theory. We do not address this problem further in this paper.

The strategy of the perturbative calculation of the transport in gauge theory is well known.[8–10,19,21] Let us assign electric charge to, say, the fermions. Then one may find the spinon and holon currents produced by an external electromagnetic field $A_\nu^{\rm ext}$ and the gauge field:

$$j_\mu^F = \Pi_{\mu\nu}^F (a_\nu + A_\nu^{\rm ext}), \quad j_\mu^B = \Pi_{\mu\nu}^B a_\nu ,$$

where $\Pi_{\mu\nu}^{F,B}(\mathbf{k},\omega)$ is a fermionic (bosonic) polarization operator. An infinite on-site repulsion, implemented by the gauge field, causes the spinon current to be opposite to the holon current, $j_\mu^F = -j_\mu^B$. This allowes us to find electromagnetic current as a response to the external electromagnetic field, $j_\mu = j_\mu^F = \Pi_{\mu\nu}^{\rm phys} A_\nu^{\rm ext}$. The physical conductivity $\sigma_{\mu\nu} = \omega^{-1} \Pi_{\mu\nu}^{\rm phys}(\mathbf{k}=0,\omega)$ is given by the combination rule[21]

$$[\Pi_{\mu\nu}^{\rm phys}(\mathbf{k},\omega)]^{-1} = [\Pi_{\mu\nu}^F(\mathbf{k},\omega)]^{-1} + [\Pi_{\mu\nu}^B(\mathbf{k},\omega)]^{-1}.$$

At low temperature the fermionic contribution is smaller than the bosonic one.[8,10,19] This is, roughly, due to the temperature dependence of the number of bosons at a given chemical potential. As a result, at low temperature, $T<T_d(x)$, the conductivity is determined by the bosonic transport relaxation time:

$$\sigma_{ab} \approx \frac{xe^2}{m_B^{\parallel}} \tau_{\rm tr}, \quad \sigma_c \approx \frac{xe^2}{m_B^{\perp}} \tau_{\rm tr}. \tag{2.4}$$

To describe the incoherent out-of-plane transport in the two dimensional regime $T>T_d(x)$, we adopt a different approach. In this case the interlayer tunneling is a perturbation of the 2D model (2.1) and (2.2). In the lowest order of $t_\perp^2$ the Kubo formula gives

$$\sigma_c^{(0)} = 2\, e^2 t_\perp^2 \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \frac{d^2\mathbf{p}}{(2\pi)^2} \left(-\frac{\partial n_F(\epsilon)}{\partial \epsilon} \right) \times \left[-\frac{1}{\pi} \mathrm{Im} G^R(\epsilon,\mathbf{p}) \right]^2, \tag{2.5}$$

where $G^R(x-y)$ is the retarded Green function of two dimensional electrons in a layer.

In a Fermi liquid, where the one particle Green function is characterized by a relaxation time τ, Eq. (2.5) gives $\sigma_c^{(0)} \sim e^2 t_\perp^2 m \tau$; thus ρ_c is proportional to ρ_{ab}. ρ_c appears to be metallic and coherent even though $t_\perp$ is the lowest energy scale.

The situation is very different in the gauge theory as well as in any other theory where the electron is not a quasiparticle, i.e., the electron Green function does not possess a quasiparticle pole. In the gauge theory framework the electron decays into a ''spinon'' and a ''holon'' and does not constitute stable excitations. Spinons and holons themselves are coupled by the gauge field and are not true quasiparticles, either. However, at $T>T_0$, the gauge coupling is weak, so that in the first approximation the electron Green function is simply a product of noninteracting fermion and boson Green functions. The short range decay of the boson Green function in a layer destroys the coherence between electrons on different layers. The calculation of $\sigma_c^{(0)}$ using Eq. (2.5) is presented in Sec. IV. We also note that similar calculations of the out-of-plane conductivity have been attempted in Refs. 18 and 20. Their results do not agree with each other ($\rho_c \propto 1/T$ in Ref. 18, $\rho_c \propto \sqrt{T}$ in Ref. 20) and are also different from ours ($\rho_c \propto 1/\sqrt{T}$).

III. ANISOTROPIC COHERENT TRANSPORT

In this section we calculate the transport time based on anisotropic 3D gauge theory (2.3). The calculation of the conductivity of the system interacting via gauge forces is peculiar. To obtain the conductivity one must sum up the leading corrections to the vertex and Green function of the polarization operator. However, they are connected by the Ward identity. This connection implements the gauge invariance of interaction. Moreover, in two dimensions the corrections to the Green function and to the vertex diverge, although taken all together they give a finite result. Naively it looks as if there exists a difference between transport relaxation time and the one particle relaxation time determined by the decay of the one particle Green function. In fact, in our model, those relaxation times are identical if one chooses a

proper gauge invariant definition of the one particle relaxation time, namely, as the decay of the gauge invariant Green function $G_{\rm inv}(x)=\langle b(x)\exp(i\int_0^x a_i dx^i)b^\dagger(0)\rangle$, calculated on the mass shell. Then the tail factor $\exp(i\int_0^x a_i dx^i)$ takes care of the vertex corrections. For small and smooth gauge fields, the gauge invariant Green function does not depend on the path of the tail.

The first step of the computation of conductivity in the 3D gauge theory (2.3) is to determine the propagators of the gauge fields. Since the gauge field is a Lagrangian multiplier, its dynamics emerges entirely from the polarization produced by bosons and fermions. Perturbatively, it is given by fermionc and bosonic loops:

$$\Pi_{\mu\nu}(\mathbf{k},\omega)=\Pi^F_{\mu\nu}(\mathbf{k},\omega)+\Pi^B_{\mu\nu}(\mathbf{k},\omega). \tag{3.1}$$

The propagators of the gauge fields in the transverse gauge are the inverse of the polarization operator:

$$\langle A_\mu(\mathbf{k},\omega)A_\nu(-\mathbf{k},-\omega)\rangle=D_{\mu\nu}(\mathbf{k},\omega)=(\Pi_{\mu\nu})^{-1}(\mathbf{k},\omega). \tag{3.2}$$

The scalar component of the gauge field is short ranged due to Debye screening, while the unscreened transverse part of the vector potential produces anomalously strong scattering. As in the 2D case, the fermionic contribution is the larger one, so that only the transverse components of $\Pi^F_{\mu\nu}(\mathbf{k},\omega)$ are needed. Owing to the uniaxial symmetry, the matrices D_{ij} and Π_{ij} $(i,j=x,y,z)$ can be parametrized by two elements $(D_\parallel,D_\perp)$ and $(\Pi_\parallel,\Pi_\perp)$, respectively,

$$\begin{aligned} D_{xx}&=\hat{k}_y^2 D_\parallel+\hat{k}_z^2 D_\perp, \quad D_{yy}=\hat{k}_x^2 D_\parallel+\hat{k}_z^2 D_\perp,\\ D_{zz}&=(\hat{k}_x^2+\hat{k}_y^2)D_\perp, \quad D_{xy}=-\hat{k}_x\hat{k}_y D_\parallel,\\ D_{xz}&=-\hat{k}_x\hat{k}_z D_\perp, \quad D_{yz}=-\hat{k}_y\hat{k}_z D_\perp, \end{aligned} \tag{3.3}$$

where

$$D_\parallel(\omega,\mathbf{k})=\frac{\Pi_\perp k^2+\Pi_\perp k_z^2-\Pi_\parallel k_z^2}{\Pi_\perp(\Pi_\parallel k_\parallel^2+\Pi_\perp k_z^2)}, \quad D_\perp(\omega,\mathbf{k})=\Pi_\perp^{-1}, \tag{3.4}$$

$\hat{\mathbf{k}}=\mathbf{k}/|\mathbf{k}|$ is a unit wave vector along $\mathbf{k}$, and $k_\parallel^2$ is an in-plane momentum. We assume that the condition $(m_F^\perp d)^{-1}k_z \ll v_F k_\parallel$ holds for typical momentum transfer $k_z\sim(m_B^\perp T)^{1/2}$, $k_\parallel\sim(m_B^\parallel T)^{1/2}$, where d is the interlayer distance. Then

$$\begin{aligned} \Pi^R_\parallel(\omega,\mathbf{k})&=k^2\left(\chi_\parallel-i\frac{p_F}{\pi d}\frac{\omega}{k_\parallel^3}\right),\\ \Pi^R_\perp(\omega,\mathbf{k})&=k^2\left[\chi_\perp-i\left(\frac{m_F^\parallel}{2dp_F m_F^\perp}\right)^2\frac{p_F}{\pi d}\frac{\omega}{k_\parallel^3}\right], \end{aligned} \tag{3.5}$$

where $\chi_\parallel\propto 1/m_F^\parallel$ and $\chi_\perp\propto 1/m_F^\perp$ are the components of the diamagnetic susceptibilities, and p_F and v_F are the Fermi momentum and velocity of the two dimensional Fermi surface. The imaginary parts of the fermion loop are given by the Landau damping:

$$\begin{aligned} \mathrm{Im}\Pi^R_{ij}(\omega,\mathbf{k})&=-2\pi\omega\int\frac{d^3p}{(2\pi)^3}v_i(\mathbf{p}+\mathbf{k})v_j(\mathbf{p})\\ &\quad\times\left(-\frac{\partial n_F(\xi_\mathbf{p})}{\partial\xi_\mathbf{p}}\right)\delta(\omega+\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{k}})\\ &\approx-2\pi m_F^\parallel\omega\int\frac{d\theta}{2\pi}\frac{dp_z}{2\pi}v_{Fi}v_{Fj}\delta(\xi_{\mathbf{p}_F+\mathbf{k}}), \end{aligned} \tag{3.6}$$

where $v_i(\mathbf{p})=\partial\epsilon_\mathbf{p}/\partial p_i$, $\xi_\mathbf{p}=\epsilon_\mathbf{p}-\mu_f$, $\mathbf{v}_F=\mathbf{v}(\mathbf{p}_F)$ is the Fermi velocity, θ is the angle between $\mathbf{v}_F$ and $\mathbf{k}$, $p_F\equiv m_F^\parallel v_F$, and the integration over p_z is limited by the inverse interlayer distance π/d. Using $(m_F^\perp d)^{-1}k_z\ll v_F k_\parallel$ we find that $\mathbf{v}_F$ is almost perpendicular to $\mathbf{k}_\parallel$. Under these conditions the Landau damping is similar to the 2D one. At low $\omega<v_F k_\parallel$ we have

$$\begin{aligned} \mathrm{Im}\Pi^R_{yy}(\omega,\mathbf{k})&=-\frac{p_F}{\pi d}\frac{\omega}{k_\parallel},\\ \mathrm{Im}\Pi^R_{zz}(\omega,\mathbf{k})&=-\left(\frac{m_F^\parallel}{2dp_F m_F^\perp}\right)^2\frac{p_F}{\pi d}\frac{\omega}{k_\parallel}. \end{aligned} \tag{3.7}$$

Assuming that the gauge field is in equilibrium, the relaxation time of bosons scattered by the gauge field in the second order of the gauge field is[8,10]

$$\begin{aligned} \tau_{\rm tr}^{-1}(\mathbf{p})\sim\int\frac{d^3\mathbf{k}}{(2\pi)^3}\int_0^\infty\frac{d\omega}{\pi}\mathrm{Im}\langle[\mathbf{v}_\mathbf{p}\times\hat{\mathbf{k}}\cdot\mathbf{B}(\omega,k)]\\ \times[\mathbf{v}_\mathbf{p}\times\hat{\mathbf{k}}\cdot\mathbf{B}(\omega,k)]\rangle[1+n_B(\omega)]\\ \times[1+n_B(\xi_{\mathbf{p}+\mathbf{k}})]\delta(\xi_\mathbf{p}-\xi_{\mathbf{p}+\mathbf{k}}-\omega)|\mathbf{p}|^{-2} \end{aligned} \tag{3.8}$$

where $\xi_\mathbf{p}=\epsilon_\mathbf{p}-\mu_B$. The ''magnetic field'' $\mathbf{B}=\nabla\times\mathbf{A}$ is a chirality:

$$\langle[\mathbf{v}_\mathbf{p}\times\hat{\mathbf{k}}\cdot\mathbf{B}(\omega,k)][\mathbf{v}_\mathbf{p}\times\hat{\mathbf{k}}\cdot\mathbf{B}(\omega,k)]\rangle=|\mathbf{k}|^2 v_\mathbf{p}^i v_\mathbf{p}^j D_{ij}(\omega,\mathbf{k}). \tag{3.9}$$

The factor k^2 in the above expression comes from the tail and guarantees the convergence of the scattering by soft chirality fluctuations.

The perturbation theory is valid only at high temperature where the effects of Bose condensation are negligible. Therefore the factor $n_B(\xi_{\mathbf{p}+\mathbf{k}})$ is less than unity and can be neglected. For the scattering of fermions, Eq. (3.8) remains the same, except that the factor $1+n_B(\xi_{\mathbf{p}+\mathbf{k}})$ is replaced by $1-n_F(\xi_{\mathbf{p}+\mathbf{k}})$. According to Sec. II, the transport relaxation time of bosons (2.4) dominates over the fermionic one and determines conductivities.

At low temperature, the scattering is elastic. This means that the gauge fluctuations are damped if the frequency $\omega^*\sim\chi\gamma^{-1}(k_\parallel)^3$ [see $\Pi_\parallel(\mathbf{k},\omega)$ in (3.5), $\gamma=p_F/(\pi d)$] exceeds temperature. This happens at $T<T_{\rm in}\equiv(\gamma/\chi)^2(m_B^\parallel)^{-3}$ (the opposite, inelastic case is discussed in Sec. V). The out-of-plane component of the gauge field is damped at even higher frequency, $(m_B^\perp/m_B^\parallel)\omega^*$. Therefore one may take into

account only the *static* chirality fluctuations. In static approximation Eq. (3.8) takes the form

$$\tau_{\rm tr}^{-1}(\mathbf{p}) \sim T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{|\mathbf{k}|^2}{|\mathbf{p}|^2} v_{\mathbf{p}}^i v_{\mathbf{p}}^j D_{ij}(0,\mathbf{k})\, \delta(\xi_{\mathbf{p}} - \xi_{\mathbf{p}+\mathbf{k}}). \tag{3.10}$$

To obtain the conductivity, the momentum dependent transport time $\tau_{\rm tr}^{-1}(\mathbf{p})$ has to be averaged over the momentum $\mathbf{p}$ with the Boltzmann distribution. The sole effect of the averaging is to replace momentum by its thermal value, $p_i^2 \sim m_B^i T$, $v_i^2 \sim T/m_B^i$, so that $p_\parallel^2 \ll p_\perp^2$, $v_\parallel^2 \gg v_\perp^2$. Due to the above anisotropy $k_z^2 \gg k_\parallel^2$ holds and under this condition Eq. (3.9) simplifies to

$$|\mathbf{k}|^2 v_{\mathbf{p}}^i v_{\mathbf{p}}^j D_{ij}(0,\mathbf{k}) \approx \frac{v_\parallel^2}{\chi_\parallel k_\parallel^2 + \chi_\perp k_z^2} \left(k_z^2 + \frac{\chi_\parallel}{\chi_\perp} k_\parallel^2 \right) \tag{3.11}$$

where we kept only the term proportional to $v_\parallel^2$, neglecting terms proportional to $v_\perp^2$. The next step is the integration over the angle between $\mathbf{v}_\parallel$ and $\mathbf{k}_\parallel$, which gives $(v_\parallel k_\parallel)^{-1}$. The last integrations over $k_\parallel$ and k_z and the thermal averaging over $\mathbf{p}$ yield $\tau_{\rm tr}^{-1} = \tau_\parallel^{-1} + \tau_\perp^{-1}$, where $\tau_\parallel^{-1}$ and $\tau_\perp^{-1}$ are given by:

$$\tau_\parallel^{-1} \sim \frac{T\sqrt{m_B^\perp T}}{\chi_\parallel m_B^\parallel}, \tag{3.12}$$

$$\tau_\perp^{-1} \sim \frac{T\sqrt{m_B^\perp T}}{\chi_\perp m_B^\perp}, \tag{3.13}$$

$$\rho_{ab} \propto \rho_c \propto T^{3/2}. \tag{3.14}$$

The essential difference of the above result from the two dimensional one is an extra factor $\sqrt{T}$, which originates from the 3D density of states. We acknowledge that the $T^{3/2}$ dependence of the resistivity in the context of gauge theory was mentioned in Ref. 9.

There is a simple way to understand Eqs. (3.12) and (3.13): $\tau_\perp$ and $\tau_\parallel$ contain the static chirality fluctuations $\langle B_x^2(\mathbf{k})\rangle \sim \langle B_y^2(\mathbf{k})\rangle \approx T/\chi_\perp$ and $\langle B_z^2(\mathbf{k})\rangle \approx T/\chi_\parallel$, the projected areas onto the xy and yz planes of the contour composed of the path of a boson in unit time, $S_{yz} \sim (m_B^\parallel m_B^\perp)^{-1/2}$ and $S_{xy} \sim (m_B^\parallel)^{-1}$, and the density of states in the parallel and transverse directions, $(m_B^\parallel T)^{1/2}$ and $(m_B^\perp T)^{1/2}$. Considering the products of three factors, $\tau_\perp^{-1}$ and $\tau_\parallel^{-1}$ can be attained, respectively.

As discussed in the Introduction, the anisotropic gauge theory (2.3) is assumed to be valid at temperatures below the dimensional crossover temperature. However, the 3D theory can give an upper limit for the crossover temperature $T_d^\perp$. The interlayer relaxation rate $\tau_\perp^{-1}$ increases with the anisotropy. When it reaches the interlayer hopping amplitude $t_\perp$, the kinetic equation and, as a consequence, Eqs. (3.12) and (3.13), are no longer valid. Accordingly, the out-of-plane conductivity reverses its temperature behavior. It is likely that at $T = T_d^\perp$ the out-of-plane wavelength $1/p_\perp \sim (m_B^\perp T)^{-1}$ reaches the interlayer distance d. Then the condition $\tau_\perp^{-1} \sim t_\perp$ gives a temperature scale of the crossover, $T_{\rm d}^\perp = t_\perp m_B^\perp d \chi_\perp$. If the value of $m_B^\perp$ can be identified with $(t_\perp d^2)^{-1}$, $T_d^\perp \sim \chi_\perp / d$.

On the contrary, $\tau_\parallel^{-1}$ varies smoothly through dimensional crossover and Eq. (3.12) is still valid for in-plane transport. The only difference is that the integration over p_z has to be cut off by the inverse interlayer distance, $1/d$. Therefore, at $T > (2 m_B^\perp d^2)^{-1}$,

$$\tau_\parallel^{-1} \sim \frac{T}{\chi_\parallel m_B^\parallel} \frac{1}{d}. \tag{3.15}$$

This is the well-known T-linear in-plane resistivity in the two-dimensional limit. Note that in this limit $\chi_\parallel d = \chi^{(2{\rm D})}$.

The dimensional crossovers of the in-plane and out-of-plane transport stem from different mechanisms and may occur at different temperatures. Nevertheless, if one assumes that all phenomenological parameters of the out-of-plane part of the model are of the same order $(m_B^\perp d^2)^{-1} \sim \chi_\perp / d \sim t_\perp$, the estimate of the crossover temperature is

$$T_d(x) \sim t_\perp. \tag{3.16}$$

Let us discuss the experimental implications of above results (3.14) and (3.16). According to the experimental data in overdoped $La_{2-x}Sr_xCuO_4$ ρ_{ab} is proportional to T^α, where $1 < \alpha < 2$.[2–5] Notably in Ref. 2 α was found to be very close to 3/2 for $La_{2-x}Sr_xCuO_4$ for $x \geq 0.25$ and this agrees with Eq. (3.14).

In Ref. 2 it was indicated that the $T^{3/2}$ dependence of ρ_{ab} may evolve into linear T dependence as the temperature is lowered, which is contrary to the scenario of this paper. However, a closer examination shows that the available data are insufficient to support the viewpoint.[25] Although the dimensional crossover is not very clearly defined in the data of Ref. 2, T_d $(x = 0.35)$ can be estimated to be around 800 K. Measurements of the c-axis polarized optical spectrum over the doping range $0.1 < x < 0.3$ (Ref. 5) are consistent with the resistivity data. An estimate of $t_\perp$ may be taken from the optical conductiviy data. A Drude-like fitting gave $\tau_\perp^{-1} \sim \max(\omega^\alpha, T^\alpha)$ with $1 < \alpha < 2$. In Ref. 5 the ratio of in-plane and out-of-plane plasma frequencies was also measured. At doping $x = 0.15$ and $x = 0.3$, $\omega_{p,\parallel}/\omega_{p,\perp} \approx 30$ and 10, respectively. These data enable one to get an estimate of $t_\perp$ using the formula:[23,26]

$$\frac{\omega_{p,\perp}}{\omega_{p,\parallel}} = \sqrt{2} \left(\frac{d}{a} \right) \frac{t_\perp}{E_a'},$$

where $a = 3.79$ Å is the lattice spacing in the layer and $d = 13.21$ Å is the intercell distance of $La_{2-x}Sr_xCuO_4$. $E_a' a = \hbar v_F$ and v_F is the in-plane Fermi velocity.[26] According to, Ref. 27 band structure calculations yield $v_F = 3.1 \times 10^7$ cm/s at $x = 0.15$ and $x = 0.20$. Combining all of the above formulas and data we obtain a somewhat low value, $t_\perp \sim 200$ K for $x = 0.3$–0.35. In the overdoped case we may rely on the band theory. The values of hopping amplitudes quoted in Ref. 18 are $t_\parallel \approx 0.5$ eV ~ 6000 K and $t_\perp \approx 0.05$ eV ~ 600 K. The band theory value of $t_\perp$ is not very different from the value of $t_\perp$ obtained above from optical

data.[5] This can be expected, since the interaction in the overdoped case is not as strong as in the case of optimal doping.

Near the optimal doping,[23] $t_\perp \sim 2.4\ \mathrm{meV} = 28\ \mathrm{K}$ at $x = 0.16$, which is also obtained from the optical measurements. The superconducting transition temperature at $x = 0.16$ is $T_c(x = 0.16) = 34\ \mathrm{K}$. Thus near optimal doping the dimensional crossover can possibly be screened by the superconducting transition since $T_d(x = 0.16) \sim t_\perp < T_c(x = 0.16)$.

The estimates of $T_d(x)$ indicate that there is room for the three dimensional regime in overdoped $La_{2-x}Sr_xCuO_4$ ($x > 0.25$).

IV. INCOHERENT TRANSPORT IN TWO DIMENSIONAL REGIME

In this section we consider the out-of-plane transport in the two dimensinal high-temperature regime $T > T_d(x)$, where the interlayer hopping amplitude $t_\perp$ is the smallest rate: the time of hopping $t_\perp^{-1}$ is (i) longer than the in-plane relaxation time and (more importantly) (ii) longer than the characteristic time of all kinds of magnetic fluctuations. This case corresponds to the optimally doped $La_{2-x}Sr_xCuO_4$.

Under the condition (i) the hopping term (2.2) can be treated as a perturbation and under the condition (ii) the approximation which allows us to write the hopping term (2.2) in the form of (2.3) is no longer valid.

It is instructive to compare the c-axis conductivity of a Fermi liquid with that of (2D) gauge theories. In a Fermi liquid electrons in a layer are quasiparticles with some relaxation time and their retarded Green function has a pole in the lower half plane. Then, provided that there is no interlayer scattering, Eq. (2.5) yields

$$\sigma_c^{(0)} \sim e^2 t_\perp^2 m_F \tau. \tag{4.1}$$

Therefore ρ_c and ρ_{ab} have the same temperature dependence.

The situation is different if the electron is not a quasiparticle. Once the interlayer hopping is treated as a perturbation, the electron always decays to true quasiparticles during the interlayer tunneling, so the quantum states of the electron in different layers are incoherent. As a result of this incoherence, the out-of-plane transport is blocked and may be relaxed by thermal processes, which is similar to a semiconducting behavior.

The above case is true of a doped Mott insulator: at sufficiently high temperature electrons decay very fast (on the time scale of $1/J$ or $1/t_\parallel$) into spinons and holons and do not constitute a stable excitation. At this temperature range the gauge interaction is perturbative and the electron Green function is simply a product of fermion and boson Green functions (recall $c_\alpha = f_\alpha b^\dagger$):

$$G_e(x,y) = -\langle f(x) f^\dagger(y) b(y) b^\dagger(x)\rangle \sim G_F(x,y) G_B(y,x). \tag{4.2}$$

Therefore, the propagating character of the fermion Green function $\langle f(x) f^\dagger(y)\rangle \sim e^{i[|\mathbf{x}-\mathbf{y}| - v_F(t_x - t_y)]}$ is blocked by the localized boson Green function $\langle b(y) b^\dagger(x)\rangle \sim T_0/T \exp(-|\mathbf{x}-\mathbf{y}|^2 m_B T)$.

The simplest way to evaluate the integral (2.5) in gauge theory is to rewrite it in the form of Fermi and Bose density-density correlation functions π_F and π_B using the decomposition Eq. (4.2):

$$\pi_F(i\omega, \mathbf{q}) = \sum_{\mathbf{x}} \int_0^\beta d\tau e^{i\mathbf{q}\cdot\mathbf{x} + i\omega\tau} G_F^{(t)}(\mathbf{x},\tau) G_F^{(b)}(-\mathbf{x},-\tau),$$

$$\pi_B(i\omega, \mathbf{q}) = \sum_{\mathbf{x}} \int_0^\beta d\tau e^{i\mathbf{q}\cdot\mathbf{x} + i\omega\tau} G_B^{(t)}(-\mathbf{x},-\tau) G_B^{(b)}(\mathbf{x},\tau).$$

The superscripts of the Green functions denote the two layers involved in the hopping process (top and bottom). In terms of π_F and π_B (2.5) takes the form

$$\sigma_c^{(0)} = 2e^2 t_\perp^2 \sum_{\mathbf{q}} \int \frac{d\omega}{2\pi} \times \left(-\frac{\partial n_B(\omega)}{\partial \omega} \right) \mathrm{Im}\pi_F^R(\omega, \mathbf{q})\ \mathrm{Im}\pi_B^R(\omega, \mathbf{q}). \tag{4.3}$$

At small frequency and momentum and at $T > T_0$ the imaginary parts of the polarization operators are

$$\mathrm{Im}\pi_F^R(\omega, \mathbf{q}) = -m_F a^2 \frac{\omega}{v_F |\mathbf{q}|},$$

$$\mathrm{Im}\pi_B^R(\omega, \mathbf{q}) = -\frac{T_0}{T} m_B a^2 \frac{\omega}{v_B |\mathbf{q}|},$$

where a is the lattice constant in a layer and $v_B = (k_B T/m_B)^{1/2}$ is the thermal boson velocity. The T_0/T factor of $\mathrm{Im}\pi_B^R(\omega, \mathbf{q})$ comes from the Bose factor $n_B(\epsilon_{\mathbf{q}} = 0) \approx \exp(\mu_B) = T_0/T$. The momentum integration in Eq. (4.3) is logarithmic and is cut by T/v_F at the lower limit (the condition for the existence of imaginary parts of polarization functions). Due to the exponential decay of the Bose factor $[\partial n_B(\omega)/\partial\omega \propto e^{-|\omega|/T}]$ the frequency integral is convergent at the ultraviolet limit. The main contribution to the frequency integral comes from the infrared region $|\omega| \leqslant T$, in which $-\partial n_B(\omega)/\partial\omega \approx T/\omega^2$. The T/ω^2 singularity is cancelled by ω^2 coming from the numerator of $\mathrm{Im}\pi_F^R\,\mathrm{Im}\pi_B^R$. Thus the frequency integral gives T^2. Using $T_0 \sim x/m_B$ and rearranging other factors, we obtain

$$\sigma_c^{(0)} = \text{const.} \times e^2 t_\perp^2 x m_F^2 \sqrt{m_B T}. \tag{4.4}$$

The c-axis conductivity has been estimated in Ref. 20 in a similar framework. The result of Ref. 20 is, however, different from (4.4). In terms of resistivity (4.4) reads

$$\rho_c \propto \frac{1}{\sqrt{T}}, \tag{4.5}$$

which agrees with the ''semiconducting'' behavior of ρ_c observed experimentally in optimally doped cuprates.

The dimensional crossover to the anisotropic three dimensional regime is complex. In particular, the interlayer hopping process (2.2) evolves into anisotropic gauge theory (2.3) through dimensional crossover and the elucidation of the crossover requires much more sophisticated analysis. Let us just note that the two models (2.1), (2.2), and (2.3) are

essentially different, so that an estimate of the crossover temperature from the high-temperature side may not necessarily coincide with the estimate from the low-temperature side. In any case it is very likely that the crossover temperature in optimally doped cuprates falls below the superconducting transition temperature.

A comment is necessary at this point. In optimal $La_{2-x}Sr_xCuO_4$, near $T=300$ K the out-of-plane resistivity stops decreasing and starts to grow with temperature. This up-turn is attributed to the structural transformation from the high-temperature tetragonal phase to the low-temperature orthorhombic phase.[3] Above this upturn temperature the c-axis conductivity is still much lower than the Mott minimal metallic conductivity ($\approx 10^2$ s/cm) and cannot be considered to be metallic.

V. INELASTIC SCATTERING BY GAUGE FIELDS

Two dimensional gauge theory gives rise to linear temperature dependence of the in-plane resistivity of optimally doped $La_{2-x}Sr_xCuO_4$ at low temperatures where scattering is *elastic*. At sufficiently high temperatures inelastic processes change the linear-T behavior into $T^{3/2}$. This can be understood as follows. From the propagator of the gauge field $D^R(\omega,\mathbf{k})=(\chi k^2-i\gamma\omega/k)^{-1}$ it follows that the energy transfer ω scales like $\omega_{\mathbf{k}}=\chi\gamma^{-1}k^3$. At finite temperature the boson energy is typically of order T. Thus the typical momentum transfer in the scattering of bosons by the gauge field is $(m_BT)^{1/2}$. As a result the typical energy transfer in scattering would be $\omega^*\sim\chi/\gamma(m_BT)^{3/2}$. This is larger than the thermal energy of scattered bosons, i.e., T at $T>T_{\rm in}\sim(\gamma/\chi)^2 1/m_B^3$ and at this temperature the ω dependence of the propagators has to be taken into account. This inelasticity softens the infrared singularity of scattering and thus leads to the less singular temperature dependence of resistivity.

In the three dimensional case the scattering by $D_\perp(\mathbf{k},\omega)$ is almost always elastic (see Sec. III), while the scattering by $D_\parallel(\mathbf{k},\omega)$ can be inelastic at high temperature. It turns out that $T_{\rm in}^{\rm 3D}$ is very close to $T_{\rm in}^{\rm 2D}$. For the three dimensional inelastic regime to be observed, the condition $T_{\rm in}^{\rm 3D}\ll T_d(x)$ should be satisfied. (See the discussion below on the experimental estimate of $T_{\rm in}^{\rm 2D}$ and $T_{\rm in}^{\rm 3D}$.) In the three dimensional inelastic regime we would have $\rho_{ab}\propto T^{3/2}(m_B^\perp T)^{1/2}\propto T^2$, so the anomalous exponent 3/2 cannot be explained. Instead we will discuss the two dimensional case in detail.

In the 2D case Eq. (3.8) reads

$$\tau_{\rm tr}^{-1}(\mathbf{p})\sim\pi\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{|\mathbf{p}\times\hat{\mathbf{k}}|^2}{m_B^2}\int_0^\infty\frac{d\omega}{\pi}{\rm Im}\,D^R(\omega,\mathbf{k})\times[1+n_B(\omega)]\delta(\xi_{\mathbf{p}}-\xi_{\mathbf{p}+\mathbf{k}}-\omega)\frac{|\mathbf{k}|^2}{|\mathbf{p}|^2}. \quad (5.1)$$

After angular integration it becomes

$$\tau_{\rm tr}^{-1}\approx\frac{1}{v_{\mathbf{p}}m_B^2\gamma}\int_0^{|\mathbf{p}|}k^3dk\int d\omega\frac{\omega}{(\omega_{\mathbf{k}}^*)^2+\omega^2}[1+n_B(\omega/T)]. \quad (5.2)$$

At low temperature, $T<T_{\rm in}$, $\omega\sim\omega_{\mathbf{k}}^*\ll T$ and thus $n_B(\omega/T)\sim T/\omega\gg 1$. Then the frequency integral is finite and it gives $(\omega_{\mathbf{k}}^*)^{-1}$. The remaining momentum integration gives a T-linear transport time $\tau_{\rm tr}^{-1}\sim T/(\chi m_B)$.[8,10] Note that the transport time is independent of Landau damping parameter γ, which is not the case in the inelastic regime.

At high temperature, $T>T_{\rm in}$, $\omega\sim\omega_{\mathbf{k}}^*\gg T$ and thus $n_B(\omega/T)\ll 1$. Now the frequency integral is of the order of $\log\Lambda$, where Λ is some high-frequency cutoff. The momentum integral gives p^4. Combining all factors and replacing the boson momentum by its thermal value $(m_BT)^{1/2}$ we obtain in the inelastic limit

$$\tau_{\rm tr}^{-1}\approx\frac{p^3}{m_B\gamma}\propto\frac{T^{3/2}m_B^{1/2}}{\gamma}. \quad (5.3)$$

The value of $T_{\rm in}$ is very sensitive to m_B and can hardly be estimated from the available experimental data. The slope of T-linear resistivity at optimal doping ($\approx 1.0\mu\Omega$ cm/K) gives $\chi^{\rm 2D}$ to be around 500 K. The resistivity data of overdoped $La_{2-x}Sr_xCuO_4$ suggest that $\chi_\parallel^{\rm 3D}$ is of the same order as $\chi^{\rm 2D}$. The one loop value of the damping γ is on the order of 1.[9]

The estimates of m_B which enter into $T_{\rm in}$ vary appreciably depending on the kinds of experiments. Optical conductivity measurements[28] provide the value of m_B at high energy: $m_B\approx 2m_e$, which is almost independent of doping. In particular, $m_B\approx(2-3)m_e$ is almost independent of the probe energy scale *in the overdoped range*. From another side the magnetic susceptibility data provide the value of m_B at low energy: $m_B\approx 15\,m_e$ near optimal doping.[22,29] These estimates of m_B makes the estimate of $T_{\rm in}$ range from 500 K (the susceptibility data) to 10^5 K (the optical data). If one accepts the lower estimate of $T_{\rm in}$ one may exploit the inelastic mechanism in order to explain the $T^{3/2}$ behavior.

In fact the optical estimate, which is close to the band theory value, seems more realistic. This means that the inelastic regime is likely irrelevant for the overdoped cuprates.

VI. CONCLUSION

Anisotropic 3D gauge theory of the normal states of doped Mott insulators is proposed in order to explain the anomalous transport phenomena observed in overdoped cuprates. We argue that $La_{2-x}Sr_xCuO_4$ interpolates between a two dimensional layered *strange* metal and a three dimensional anisotropic anomalous metal for the doping range $x\sim 0.15-0.35$. $La_{2-x}Sr_xCuO_4$ does *not* evolve into an ordinary Fermi liquid in the overdoped range. The anisotropic transport is characterized by a temperature scale $T_d(x)$ of dimensional crossover. At low temperature, $T<T_d(x)$, the charge transport is coherent and three dimensional, and 3D anisotropic gauge theory can describe the charge transport. On the contrary, at high temperature, $T>T_d(x)$, the charge transport is incoherent and two dimensional, and 2D gauge theory with an interlayer hopping term describes charge transport. The crossover of the out-of-plane transport is peculiar: due to strong interaction, electrons do not constitute an elementary excitation and decay into other particles during interlayer tunneling. As a result, the character of the out-of-plane transport may change from coherent to incoherent

and that of the out-of-plane resistivity changes from metallic to semiconductorlike.

There also is another temperature scale $T_{\rm in}$ which separates elastic scattering from inelastic scattering as temperature increases. The detailed temperature dependence of the resistivities is summarized in the tables below.

If $T_d(x) \leqslant T_{\rm in}$ for some doping range, the following behavior of the resistivities is possible:

$T \leqslant T_d(x) \leqslant T_{\rm in}$	$T_d(x) \leqslant T \leqslant T_{\rm in}$	$T_d(x) \leqslant T_{\rm in} \leqslant T$
$\rho_{ab} \propto \rho_c \propto T^{3/2}$	$\rho_{ab} \propto T, \quad \rho_c \propto T^{-1/2}$	$\rho_{ab} \propto T^{3/2}$

If $T_d(x) \geqslant T_{\rm in}$,

$T \leqslant T_{\rm in} \leqslant T_d(x)$	$T_{\rm in} \leqslant T \leqslant T_d(x)$	$T_{\rm in} \leqslant T_d(x) \leqslant T$
$\rho_{ab} \propto \rho_c \propto T^{3/2}$	$\rho_{ab} \propto \rho_c \propto T^2$	$\rho_{ab} \propto T^{3/2}$

This theory may provide a unified approach in understanding the variety of temperature behaviors of the in-plane and out-of-plane resistivity of copper oxides in wide ranges of doping and temperature. The results qualitatively agree with the available experimental data.

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