Dynamic finite-size effect in the classical spin van der Waals model

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A phenomenological dynamic scaling method is proposed and utilized to interpret finite-size effects in the time-dependent correlation function for the classical spin van der Waals model. We claim that the equations of motion for the classical spin van der Waals model are well defined only within a certain characteristic time. The order of magnitude for the size-dependent maximum time interval of the time-dependent correlation function, analytically obtained in the large N limit via the method of Laplace by Dekeyser and Lee, is found to be related to our characteristic time. As a by-product of this work, time scaling variables are found with which we can completely collapse the time-dependent correlation functions of the classical spin van der Waals model into one curve.

I. INTRODUCTION

Dynamical properties of continuous-spin models are much more intricate and richer in content than static properties. The spin van der Waals model[1–10] attracted some attention, because it has the merit of having exact solutions for the time evolution and the time-dependent correlation function. However, the equations of motion depend explicitly on the number of spins, N, and thereby the time-dependent correlation function also inherits the explicit dependence on N. What this implies is that the time dependence and the size dependence of this model are not independent of each other, and one has to be extremely careful in interpreting the analytic results. Dekeyser and Lee[1] properly treated this aspect of the problem for the time-dependent total spin correlation function away from criticality by introducing the maximum time interval without detailed analysis.

In this work, we would like to show specifically how one can phenomenologically understand the order of magnitude of the maximum time interval, where the analytical result for the spin van der Waals model obtained in the large N limit via the method of Laplace is valid. For the sake of convenience in handling the equations of motion numerically, we have chosen the classical limit of the spin van der Waals model (CSVW).

As we shall see below, it is precisely this piece of information on the maximum time interval which will enable us to determine the time scaling variable. By making use of this time scaling variable within the scaling region, we can completely collapse the time-dependent correlation functions for different N's into one curve.

II. DESCRIPTION OF THE DYNAMIC SCALING METHOD

For a given classical spin Hamiltonian H, the equation of motion for a single spin vector $\mathbf{s}_i$ at site i is given by

$$\dot{\mathbf{s}}_i = -\mathbf{s}_i \times \frac{\delta H}{\delta \mathbf{s}_i}. \tag{1}$$

The equation of motion for the total spin vector $\mathbf{S} = \Sigma_{i=1}\mathbf{s}_i$ is obtained by summing the above equation over i, i.e.,

$$\dot{\mathbf{S}} = -\sum_{i=1} \mathbf{s}_i \times \frac{\delta H}{\delta \mathbf{s}_i}. \tag{2}$$

Before solving Eqs. (1) and (2), we note that both sides of the equations should have the same order of magnitude independent of the number of spins, N. To get an estimate of the order of magnitude we choose to examine the order of magnitude for the ensemble average of absolute value of each side of the equation. We adopt this convention because each variable randomly fluctuates among possible spin configurations. This is conventional in studying finite-size systems.[11] We can determine the order of magnitude on each side from the results of statics through either analytic calculations for some of the quantities or Monte Carlo (MC) simulations.[11] Since the single spin components $\langle |s_i^\alpha| \rangle$ ($\alpha = x,y,z$) are independent of N, the order of magnitude of $\langle |s_i^\alpha| \rangle$ can be set to be $O(1)$. Suppose that the orders of magnitude of the total spin components $\langle |S_x| \rangle$, $\langle |S_y| \rangle$, and $\langle |S_z| \rangle$ are given by $O(N^a)$, $O(N^b)$, and $O(N^c)$, respectively. Here a, b, and c are numbers to be determined. The reason why the total spin components have dependence on N is due to the collective effect.

Next, let us consider the time derivatives. For example, $\langle |\dot{s}_i^\alpha| \rangle$ has the dimensions of $[\langle |s_i^\alpha| \rangle]/[\text{time}]$. Hence we propose to denote its order of magnitude as $O(1/\mathscr{T})$. Here $\mathscr{T}$ has the dimensions of time, and represents the change in the N dependence arising from taking the derivative with respect to the time. Accordingly, $\langle |\dot{S}_x| \rangle \sim O(N^a/\mathscr{T})$, $\langle |\dot{S}_y| \rangle \sim O(N^b/\mathscr{T})$, and $\langle |\dot{S}_z| \rangle \sim O(N^c/\mathscr{T})$. We express the N dependence of $\mathscr{T}$ in the form $\mathscr{T} \sim O(N^e)$. This number e is to be determined by comparing both sides of Eqs. (1) and (2), and can be related to static critical exponents. Even away from the critical point, we can still have nonzero e depending upon the particular spin model under consideration.

We assert that the equations of motion for a system in the thermodynamic limit are well defined only within this characteristic time $\mathscr{T}$. This strongly suggests that we should choose $t/\mathscr{T}$ or equivalently tN^{-e} as the finite-size time scaling variable τ. This assertion and the above proposition will be justified below by examining the time-dependent correlation functions of the CSVW model. Consequently, the time-dependent correlation functions for systems with different N will collapse to form a universal curve within the scaling region.

By a previously given analysis based on the order of magnitude, we arrive at the finite-size scaling form for the time-dependent autocorrelation function, namely,

$$\frac{\langle s_i^\alpha(t)s_i^\alpha(0)\rangle}{\langle(s_i^\alpha)^2\rangle}=F(\tau) \tag{3}$$

for the single spin components, and

$$\frac{\langle S_\alpha(t)S_\alpha(0)\rangle}{\langle S_\alpha^2\rangle}=G(\tau) \tag{4}$$

for the total spin components. Here $F(\tau)$ and $G(\tau)$ are universal dynamic scaling functions. As we shall see below, it is only within the scaling region that the bulk behavior of the system manifests itself. As the time increases beyond the scaling region, corrections to scaling will be more and more pronounced

III. APPLICATION OF THE METHOD

The CSVW is one of a few models for which many static and dynamic quantities in the thermodynamic limit can be obtained exactly, and which have a phase transition. The CSVW model is an infinite-range version of the anisotropic Heisenberg model, and is a microscopic realization of the mean-field theory.

The CSVW model is defined by the Hamiltonian

$$H=-\frac{J}{4N}\sum_{i\neq j}(s_i^x s_j^x+s_i^y s_j^y)-\frac{J_z}{4N}\sum_{i\neq j}s_i^z s_j^z, \tag{5}$$

where s_i^α denotes the α component of a classical Heisenberg spin at site i; J and J_z are the positive coupling constants. Note that the sums run over all pairs i,j of sites in the system. Below we shall distinguish three different limits of the model, namely, the Ising limit if $J=0$ but $J_z\neq 0$, the XY limit if $J\neq 0$ but $J_z=0$, and the Heisenberg limit if $J=J_z\neq 0$. The critical temperature is obtained from $\beta_c J_z=6.0$ in the Ising limit, and from $\beta_c J=6.0$ in the XY and Heisenberg limits, where $\beta_c=1/k_BT_c$.[12]

In terms of total spin components S_α, we can recast Eq. (5) in the form

$$H=-\frac{J}{4N}(S_x^2+S_y^2)-\frac{J_z}{4N}S_z^2, \tag{6}$$

where we have ignored terms which vanish in the thermodynamic limit. For the CSVW model, from Eq. (1), we obtain

$$\dot{s}_i^x=-\frac{J}{2N}S_y s_i^z+\frac{J_z}{2N}S_z s_i^y, \tag{7}$$

$$\dot{s}_i^y=\frac{J}{2N}S_x s_i^z-\frac{J_z}{2N}S_z s_i^x, \tag{8}$$

$$\dot{s}_i^z=-\frac{J}{2N}S_x s_i^y+\frac{J}{2N}S_y s_i^x \tag{9}$$

for the single spin components. Among the total spin components, S_z is a constant of motion and we obtain from Eq. (2)

$$\dot{S}_x=-\frac{J-J_z}{2N}S_zS_y, \tag{10}$$

$$\dot{S}_y=\frac{J-J_z}{2N}S_zS_x. \tag{11}$$

Now let us consider the time-dependent correlations of the model in the previously mentioned limits. All the static quantities given in the following are obtained using the factorization property of static correlation functions as well as analytic calculations for some quantities, and Monte Carlo simulations are used to confirm some of the factorization properties. We have also made use of the finite-size scaling property of the model near the critical point. The static finite-size scaling[13] in the spin van der Waals model was first found by Kittel and Shore,[14] and later developed as a theory by Botet, Jullien, and Pfeuty.[15] Since we are concerned only with the orders of magnitude, we make use of $\langle|A|\rangle\approx\langle A^2\rangle^{1/2}$, where A is either a variable or a composite of variables.

A. Total spin correlations

First of all, we study the time-dependent total spin correlations. Since all the total spin components are constants of motion in the Heisenberg limit, we consider the Ising and XY limits only. Furthermore, S_z is a constant of motion, and thus only the time evolutions in S_x and S_y are nontrivial.

1. Ising limit

Let us consider the Ising limit in three separate temperature regimes, namely, high temperature ($T\gg T_c$), critical ($T\approx T_c$), and low temperature ($T\ll T_c$) regimes. At $T\gg T_c$, since $\langle S_x^2\rangle=\langle S_y^2\rangle=N/3$ and $\langle S_z^2\rangle=2N/(6-\beta J_z)$, we get $\langle|S_x|\rangle\approx\langle S_x^2\rangle^{1/2}\sim O(N^{1/2})$, $\langle|S_y|\rangle\approx\langle S_y^2\rangle^{1/2}\sim O(N^{1/2})$, and $\langle|S_z|\rangle\approx\langle S_z^2\rangle^{1/2}\sim O(N^{1/2})$. Consequently, we have $\langle|\dot{S}_x|\rangle=\langle|\dot{S}_y|\rangle\sim O(N^{1/2}/\mathscr{T})$, $\langle|S_zS_y|\rangle\approx\langle|S_z|\rangle\langle|S_y|\rangle\sim O(N)$, and $\langle|S_zS_x|\rangle\approx\langle|S_z|\rangle\langle|S_x|\rangle\sim O(N)$.[2] Then comparing both sides of Eqs. (10) and (11) yields the maximum time interval $\mathscr{T}\sim O(N^{1/2})$ where the analytic result, obtained in the large N limit, should be valid. Apart from $\hbar$ and the coupling constant, this result is in good agreement with $t_{\max}$ given by Dekeyser and Lee.[1] It follows from the analysis given in the previous section that we can set the time scaling variable as $\tau=tN^{-1/2}$.

At $T\approx T_c$, since $\langle S_x^2\rangle=\langle S_y^2\rangle=N/3$, we have $\langle|S_x|\rangle=\langle|S_y|\rangle\sim O(N^{1/2})$. Also since $\langle S_z^2\rangle\approx N^2\sigma^2$, where $\sigma\sim O(N^{-1/4})$ denotes the long-range order,[15] we get $\langle|S_z|\rangle\sim O(N^{3/4})$. Hence, we obtain $\langle|\dot{S}_x|\rangle=\langle|\dot{S}_y|\rangle\sim O(N^{1/2}/\mathscr{T})$, $\langle|S_zS_y|\rangle\approx\langle|S_z|\rangle\langle|S_y|\rangle\sim O(N^{5/4})$, and $\langle|S_zS_x|\rangle\approx\langle|S_z|\rangle\langle|S_x|\rangle\sim O(N^{5/4})$. Therefore the equations of motion lead to $\mathscr{T}\sim O(N^{1/4})$, and $\tau=tN^{-1/4}$.

TABLE I. Time scaling variables for the time-dependent total spin correlation functions in the Ising and XY limits of the CSVW model.

	Time scaling variable τ		
	$T \gg T_c$	$T \approx T_c$	$T \ll T_c$
Ising	$tN^{-1/2}$	$tN^{-1/4}$	t
XY	$tN^{-1/2}$	$tN^{-1/2}$	$tN^{-1/2}$

At $T \ll T_c$, $\langle S_x^2 \rangle = \langle S_y^2 \rangle = 2N/\beta J_z$, and thus $\langle |S_x| \rangle = \langle |S_y| \rangle \sim O(N^{1/2})$. Since $\langle S_z^2 \rangle \approx N^2 \sigma^2$, where $\sigma \sim O(1)$, we have $\langle |S_z| \rangle \sim O(N)$.[2] Thereby, we get $\langle |\dot{S}_x| \rangle = \langle |\dot{S}_y| \rangle \sim O(N^{1/2}/\mathcal{T})$, and $\langle |S_z S_y| \rangle = \langle |S_z S_x| \rangle \sim O(N^{3/2})$. Hence $\mathcal{T} \sim O(1)$, and $\tau = t$.

2. *XY* limit

At $T \gg T_c$, $\langle S_x^2 \rangle = \langle S_y^2 \rangle = 2N/(6 - \beta J)$, and thus $\langle |S_x| \rangle = \langle |S_y| \rangle \sim O(N^{1/2})$. Similarly, $\langle S_z^2 \rangle = N/3$ implies that $\langle |S_z| \rangle \sim O(N^{1/2})$. Using the above, we find that $\langle |\dot{S}_x| \rangle = \langle |\dot{S}_y| \rangle \sim O(N^{1/2}/\mathcal{T})$, $\langle |S_z S_y| \rangle \approx \langle |S_z| \rangle \langle |S_y| \rangle \sim O(N)$, and $\langle |S_z S_x| \rangle \approx \langle |S_z| \rangle \langle |S_x| \rangle \sim O(N)$. Then $\mathcal{T} \sim O(N^{1/2})$, and $\tau = tN^{-1/2}$.

At $T \approx T_c$, $\langle S_x^2 \rangle = \langle S_y^2 \rangle \approx N^2 \sigma^2$ where $\sigma \sim O(N^{-1/4})$ implies that $\langle |S_x| \rangle = \langle |S_y| \rangle \sim O(N^{3/4})$. Similarly, $\langle S_z^2 \rangle = N/3$ leads to $\langle |S_z| \rangle \sim O(N^{1/2})$. Therefore we have $\langle |\dot{S}_x| \rangle = \langle |\dot{S}_y| \rangle \sim O(N^{3/4}/\mathcal{T})$, $\langle |S_z S_y| \rangle \approx \langle |S_z| \rangle \langle |S_y| \rangle \sim O(N^{5/4})$, and $\langle |S_z S_x| \rangle \approx \langle |S_z| \rangle \langle |S_x| \rangle \sim O(N^{5/4})$. Accordingly, we get $\mathcal{T} \sim O(N^{1/2})$, and $\tau = tN^{-1/2}$.

At $T \ll T_c$, from $\langle S_x^2 \rangle = \langle S_y^2 \rangle \approx N^2 \sigma^2$ where $\sigma \sim O(1)$, we have $\langle |S_x| \rangle = \langle |S_y| \rangle \sim O(N)$. Similarly, from $\langle S_z^2 \rangle = 2N/\beta J$, we get $\langle |S_z| \rangle \sim O(N^{1/2})$.[2] Thereby $\langle |\dot{S}_x| \rangle = \langle |\dot{S}_y| \rangle \sim O(N/\mathcal{T})$, $\langle |S_z S_y| \rangle \approx \langle |S_z| \rangle \langle |S_y| \rangle \sim O(N^{3/2})$, and $\langle |S_z S_x| \rangle \approx \langle |S_z| \rangle \langle |S_x| \rangle \sim O(N^{3/2})$. Then $\mathcal{T} \sim O(N^{1/2})$, and $\tau = tN^{-1/2}$.

Table I summarizes the results of our method applied to the total spin case.

B. Single spin correlations

We now proceed to the problem of the time-dependent single spin correlations, where no finite-size dependence is known at all. The factorization property of the static correlation function still holds due to extremely widely separated peak positions of the respective probability distributions for the variables S_α and s_i^α. This is reasonable in that S_α grows with N while $\langle (s_i^\alpha)^2 \rangle \sim O(1)$, for $(s_i^x)^2 + (s_i^y)^2 + (s_i^z)^2 = 1$. It can also be shown by MC simulations.

1. *Ising limit*

In the Ising limit, s_i^z is a constant of motion. Hence we consider only the x and y components of the equations of the motion for a single spin, i.e., Eqs. (7) and (8) with $J = 0$.

At $T \gg T_c$, $\langle |S_z s_i^y| \rangle \approx \langle |S_z| \rangle \langle |s_i^y| \rangle \sim O(N^{1/2})$ and $\langle |S_z s_i^x| \rangle \approx \langle |S_z| \rangle \langle |s_i^x| \rangle \sim O(N^{1/2})$. Then comparison of both sides of the equations of motion, with $\langle |\dot{s}_i^\alpha| \rangle \sim O(1/\mathcal{T})$, yields the characteristic time $\mathcal{T} \sim O(N^{1/2})$. Again the order of magnitude of the maximum time interval, where analytic results are valid, should be comparable to this quantity. The time scaling variable is given by $\tau = tN^{-1/2}$.

At $T \approx T_c$, $\langle |S_z s_i^y| \rangle \approx \langle |S_z| \rangle \langle |s_i^y| \rangle \sim O(N^{3/4})$ and $\langle |S_z s_i^x| \rangle \approx \langle |S_z| \rangle \langle |s_i^x| \rangle \sim O(N^{3/4})$. Then we get in a similar manner $\mathcal{T} \sim O(N^{1/4})$, and $\tau = tN^{-1/4}$.

At $T \ll T_c$, $\langle |S_z s_i^y| \rangle \approx \langle |S_z| \rangle \langle |s_i^y| \rangle \sim O(N)$ and $\langle |S_z s_i^x| \rangle \approx \langle |S_z| \rangle \langle |s_i^x| \rangle \sim O(N)$. Thus the maximum time interval is of the order $\mathcal{T} \sim O(1)$ and the time scaling variable is given by $\tau = t$.

2. *XY* limit

In the XY limit, s_i^z is not a constant of motion, and thus we have to consider the equations of motion for all three spin components, i.e., Eqs. (7) through (9) with $J_z = 0$.

At $T \gg T_c$, $\langle |S_x| \rangle = \langle |S_y| \rangle \sim O(N^{1/2})$. Then using the factorization property along with $\langle |\dot{s}_i^\alpha| \rangle \sim O(1/\mathcal{T})$, we obtain from the equations of motion that $\langle |S_y s_i^z| \rangle \approx \langle |S_y| \rangle \langle |s_i^z| \rangle \sim O(N^{1/2})$, $\langle |S_x s_i^z| \rangle \approx \langle |S_x| \rangle \langle |s_i^z| \rangle \sim O(N^{1/2})$, and $\langle |S_y s_i^x| \rangle \approx \langle |S_y| \rangle \langle |s_i^x| \rangle \sim O(N^{1/2})$, $\langle |S_x s_i^y| \rangle \approx \langle |S_x| \rangle \langle |s_i^y| \rangle \sim O(N^{1/2})$. Then similar reasoning gives $\mathcal{T} \sim O(N^{1/2})$, and $\tau = tN^{-1/2}$.

At $T \approx T_c$, $\langle |S_x| \rangle = \langle |S_y| \rangle \sim O(N^{3/4})$. Following the same reasoning gives $\mathcal{T} \sim O(N^{1/4})$ and $\tau = tN^{-1/4}$.

At $T \ll T_c$, $\langle |S_x| \rangle = \langle |S_y| \rangle \sim O(N)$. Thereby we again obtain $\mathcal{T} \sim O(1)$ and $\tau = t$.

3. *Heisenberg limit*

Now consider the Heisenberg limit. Here again we have to consider the equations of motion for all three spin components, i.e., Eqs. (7) through (9) with $J_z = J$.

At $T \gg T_c$, $\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle = 2N/(6 - \beta J)$ and $\langle |S_x| \rangle = \langle |S_y| \rangle = \langle |S_z| \rangle \sim O(N^{1/2})$. Then using the factorization property we obtain the orders of magnitude for all the terms in the equations of motion. Hence we obtain $\mathcal{T} \sim O(N^{1/2})$, and thus the time scaling variable is $\tau = tN^{-1/2}$.

At $T \approx T_c$, from $\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle \approx N^2 \sigma^2$ where $\sigma \sim O(N^{-1/4})$, $\langle |S_x| \rangle = \langle |S_y| \rangle = \langle |S_z| \rangle \sim O(N^{3/4})$. Therefore we obtain $\mathcal{T} \sim O(N^{1/4})$, and the time scaling variable is $\tau = tN^{-1/4}$.

At $T \ll T_c$, from $\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle \approx N^2 \sigma^2$ where $\sigma \sim O(1)$, $\langle |S_x| \rangle = \langle |S_y| \rangle = \langle |S_z| \rangle \sim O(N)$. Then we get thereby $\mathcal{T} \sim O(1)$ and $\tau = t$.

Table II summarizes the results of our method applied to the single spin case.

TABLE II. Time scaling variables for the time-dependent single spin correlation functions in the Ising, XY, and Heisenberg limits of the CSVW model.

	Time scaling variable τ		
	$T \gg T_c$	$T \approx T_c$	$T \ll T_c$
Ising	$tN^{-1/2}$	$tN^{-1/4}$	t
XY	$tN^{-1/2}$	$tN^{-1/4}$	t
Heisenberg	$tN^{-1/2}$	$tN^{-1/4}$	t

IV. SPIN DYNAMICS SIMULATION RESULTS

We now want to determine the order of magnitude for the maximum time interval and justify the time scaling variable

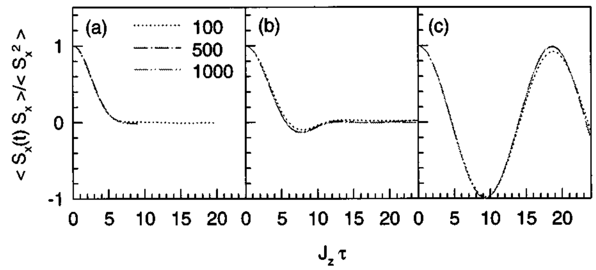

FIG. 1. $\langle S_x(t)S_x(0)\rangle/\langle S_x^2(0)\rangle$ vs corresponding time scaling variables τ at (a) $\beta J_z=3.0$, (b) $\beta J_z=6.0$, and (c) $\beta J_z=9.0$, repectively, in the Ising limit. The numbers of spins are 100, 500, and 1000.

of our method. It is to be noted that the time-dependent total spin correlation function in the critical region is not known and the time-dependent single spin correlation function obtained by Dekeyser and Lee is not given in a form amenable to finite-size scaling analysis. Therefore we shall appeal to the spin dynamics simulation[17] (SDS) of the time-dependent correlation functions for the CSVW model.

Usual SDS's have two types of statistical errors, namely, the static one coming from MC simulation and the dynamic one from integrating out the equations of motion. We shall use the exact time evolution obtained by Dekeyser and Lee[3] (or equivalently the one obtained in Ref. 4 with a slight modification of the coupling constants) to reduce statistical errors, instead of doing numerical integrations. What we have done in the SDS is to obtain ensemble averages of the time-dependent correlations through MC simulation by utilizing the heat-bath algorithm.[16] Hence the errors we commit are of the static type only, and they do not accumulate dynamically. Therefore our results should be quite reliable even for a long time interval without burdening the computer heavily.

In our simulations, we have discarded the first 4000 Monte Carlo steps per spin (MCS) for the equilibration of the system. Ensemble averaging has been carried out by collecting three sets of 100 000 to 200 000 configurations, where successive configurations are separated by 10 to 20 MCS to assure independence between the configurations. The error in our calculation is estimated to be less than 2%.

Figures 1 and 2 show the time-dependent total spin correlation functions $\langle S_x(t)S_x(0)\rangle/\langle S_x^2(0)\rangle$ vs time scaling variables, which were given in Table I, in the Ising and XY

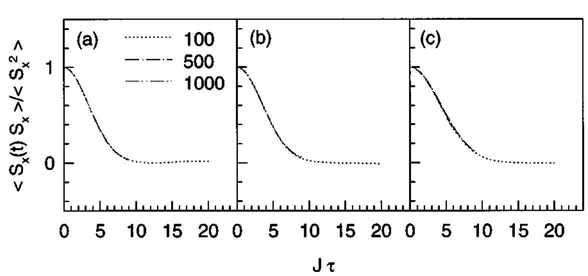

FIG. 2. $\langle S_x(t)S_x(0)\rangle/\langle S_x^2(0)\rangle$ vs corresponding time scaling variables τ at (a) $\beta J=3.0$, (b) $\beta J=6.0$, and (c) $\beta J=9.0$, repectively, in the XY limit. The numbers of spins are 100, 500, and 1000.

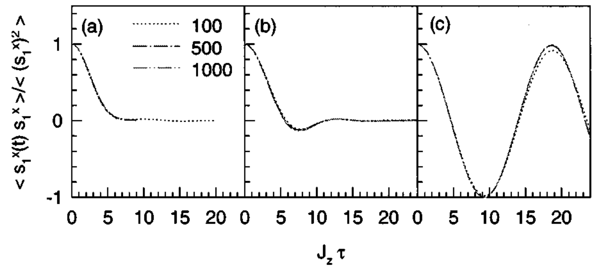

FIG. 3. $\langle s_1^x(t)s_1^x(0)\rangle$ vs corresponding time scaling variables τ at (a) $\beta J_z=3.0$, (b) $\beta J_z=6.0$, and (c) $\beta J_z=9.0$, repectively, in the Ising limit. The numbers of spins are 100, 500, and 1000.

limits in different temperature regimes. The numbers of spins are 100, 500, and 1000. $\langle S_y(t)S_y(0)\rangle/\langle S_y^2(0)\rangle$ also shows an identical behavior due to the symmetry of the model. From the figure, an almost perfect collapse of curves can be seen at least up to $\tau\lesssim3.0$ in the Ising limit and $\tau\lesssim5.0$ in the XY limit, for all temperature regimes. Thus the data are consistent with our assertion made earlier.

Figures 3 through 5 show the time-dependent single spin autocorrelation function $\langle s_1^x(t)s_1^x(0)\rangle/\langle[s_1^x(0)]^2\rangle$ vs time scaling variables, in the Ising, XY, and Heisenberg limits at different temperature regimes. The numbers of spins are 100, 500, and 1000. Although our result is given only for site 1, it is also valid for all other sites. $\langle s_1^y(t)s_1^y(0)\rangle$ shows an identical behavior due to the symmetry of the model. We witness here almost perfect collapse of curves at least up to $\tau\lesssim3.0$ in the Ising limit and $\tau\lesssim2.0$ in the XY and Heisenberg limits, for all temperature regimes, which is consistent with our assertion that the finite-size scaling would be maintained for $\tau\sim O(1)$.

V. DISCUSSION

In this work, we studied the finite-size effect in the time-dependent correlations for the CSVW model with the aim to interpret the maximum time interval given by Dekeyser and Lee. In order to do so, we introduced a phenomenological dynamic scaling method, and determined the order of magnitude for the maximum time interval. It is only within this interval that the large N limit analytic calculation for CSVW is valid. At the same time, we found that there is a time scaling variable τ for the time-dependent correlation function. We adopted SDS to test our method. The SDS results reveal perfect collapse of the curves into a universal scaling

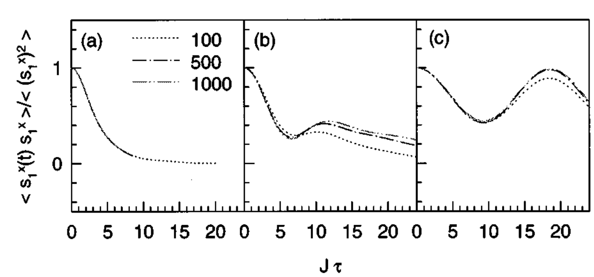

FIG. 4. $\langle s_1^x(t)s_1^x(0)\rangle$ vs corresponding time scaling variables τ at (a) $\beta J=3.0$, (b) $\beta J=6.0$, and (c) $\beta J=9.0$, repectively, in the XY limit. The numbers of spins are 100, 500, and 1000.

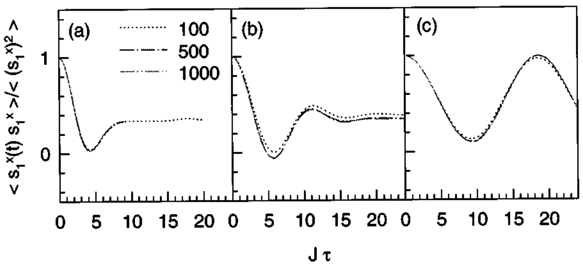

FIG. 5. $\langle s_1^x(t)s_1^x(0)\rangle$ vs corresponding time scaling variables τ at (a) $\beta J=3.0$, (b) $\beta J=6.0$, and (c) $\beta J=9.0$, repectively, in the Heisenberg limit. The numbers of spins are 100, 500, and 1000.

curve within the given time range, showing that our method works quite well. This implies that there is a dynamic scaling phenomenon in the CSVW model, i.e., the characteristic time diverges in the thermodynamic limit. Contrary to statics, the characteristic time can diverge even away from the critical region. Why is it happening? Unlike the nearest neighbor coupled spin models, all the spins are coupled to each other in the CSVW model. Hence correlation in time can be maintained in any physical regime. This induces various dynamic scaling behavior in the CSVW model. Another interesting subject is to explore the relation between our method and the dynamic scaling theory of Halperin and Hohenberg.[18] Presently we do not see any clue for this subject.

Finally, we discuss the reason why our phenomenological method works. Let us from the outset assume Eqs. (3) and (4) as dynamic scaling hypotheses. In order to get the exponent e in the time scaling variable, we make use of the identity $\langle \dot{B}(t)A(0)\rangle = -\langle B(t)\dot{A}(0)\rangle$. Thereby we obtain

$$\frac{\langle(\dot{s}_i^\alpha)^2\rangle}{\langle(s_i^\alpha)^2\rangle} = -\frac{\langle \ddot{s}_i^\alpha s_i^\alpha\rangle}{\langle(s_i^\alpha)^2\rangle} = N^{-2e}\ddot{F}(0) \tag{12}$$

for the single spin components, and

$$\frac{\langle(\dot{S}_\alpha)^2\rangle}{\langle S_\alpha^2\rangle} = -\frac{\langle \ddot{S}_\alpha S_\alpha\rangle}{\langle S_\alpha^2\rangle} = N^{-2e'}\ddot{G}(0) \tag{13}$$

for the total spin components. From the above equations, noting $\langle |A|\rangle \approx \langle A^2\rangle^{1/2}$,

$$\frac{\langle|\dot{s}_i^\alpha|\rangle}{\langle|s_i^\alpha|\rangle} \approx \left(\frac{\langle(\dot{s}_i^\alpha)^2\rangle}{\langle(s_i^\alpha)^2\rangle}\right)^{1/2} = N^{-e}[\ddot{F}(0)]^{1/2}, \tag{14}$$

and

$$\frac{\langle|\dot{S}_\alpha|\rangle}{\langle|S_\alpha|\rangle} \approx \left(\frac{\langle(\dot{S}_\alpha)^2\rangle}{\langle S_\alpha^2\rangle}\right)^{1/2} = N^{-e'}[\ddot{G}(0)]^{1/2}. \tag{15}$$

We can recast Eq. (14) as

$$\langle|\dot{s}_i^\alpha|\rangle = N^{-e}[\ddot{F}(0)]^{1/2}\langle|s_i^\alpha|\rangle, \tag{16}$$

and Eq. (15) as

$$\langle|\dot{S}_\alpha|\rangle = N^{-e'}[\ddot{G}(0)]^{1/2}\langle|S_\alpha|\rangle. \tag{17}$$

Taking a closer look at Eqs. (16) and (17) and comparing with Eqs. (7) through (11) reveal that the orders of magnitude are equivalent if we relate the inner factor like $(J/N)S_\beta$ or $(J_z/N)S_\beta$ $(\beta\neq\alpha)$ with N^{-e} or $N^{-e'}$, depending on the limits. Therefore, it is equivalent to comparing the magnitude of both sides of the equations of motion, and retrieving the N dependence from them. This places our phenomenological dynamic scaling theory on a firm basis.

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